In this work we study various aspects of the behaviour of free test particles in Einstein's general relativity and analyze specific physical properties of the background spacetimes. In the first part we investigate geodesic motions in the four-dimensional constant curvature spacetimes, i.e., Minkowski and (anti-)de Sitter universe, with an expanding impulsive gravitational wave. We derive the simple refraction formulae for particles crossing the impulse and describe the effect of nonvanishig cosmological constant. In the second part of this work we present a general method useful for geometrical and physical interpretation of arbitrary spacetimes in any dimension. It is based on the systematic analysis of the relative motion of free test particles. The equation of geodesic deviation is rewritten with respect to the natural orthonormal frame. We discuss the contributions given by a specific algebraic structure of the curvature tensor and the matter content of the universe. This formalism is subsequently used for investigation of the large class of nontwisting spacetimes. In particular, we analyse the motions in the nonexpanding Kundt and expanding Robinson--Trautman family of solutions.