Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Seller Strategies for Virtual Auctions Using Real Currencies

Author: **Bc. Waheed Brown**

Supervisor: **PhDr. Martin Gregor Ph.D.**

Academic Year: **2012/2013**

Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

The author grants to Charles University permission to reproduce and to distribute copies of this thesis document in whole or in part.

Prague, May 15, 2013

Signature

Acknowledgments

The author is grateful especially to the staff at DiabloHub.com who are referenced by their online handles; Anuiran, kojasou, and Artishir. A special thank you also to my thesis supervisor who sanity checked the theory and lent invaluable guidance, wisdom, and experience. The usual caveat applies.

And of course, credit is due to Blizzard Entertainment, Inc., for being ambitious enough to create a real money auction house for virtual goods, within a video game.

Abstract

This thesis focuses on finding Bayesian equilibria for sellers in virtual auctions using real currencies. Existing literature for real-world auctions is examined from the perspective of economic theory, game theory, and pricing strategies. Next, computer science theory is reviewed to identify applications of real-world auction models in video games. Finally, the video game Diablo 3, the first to have a real currency auction house for virtual goods, will be examined as a case study. This thesis contributes to the known literature by analyzing the Diablo 3 real currency auction house and identifying seller strategies to be applied in future virtual auction houses and economies using real currencies.

JEL Classification	C73, D44, D58
Keywords	repeated games, auctions, general equilibrium

Author's e-mail	`waheedbrown@hotmail.com`
Supervisor's e-mail	`gregor@fsv.cuni.cz`

Abstrakt

Tato práce se zameruje na hledání Bayesovské rovnováhy pro prodejce ve virtuálních aukcích, které používají skutecné peníze. Na úvod se prozkoumá existující literatura o aukcích z hlediska ekonomické teorie, teorie her tvorby a cenové strategie. Dále budou analyzovány príspevky z informatiky, které povedou k zjištení zpusobu aplikace aukcních modelu ve videohrách. Hlavním príspevkem bude analýza video hry Diablo 3, která jako první má aukcní dum pro virtuální zboží založený na reálných penezích. Tato práce cílí být príspevkem k aukcní literature v tom smyslu, že analyzuje strategie prodávajících v prípade, kdy se kombinuje virtuální ekonomika a reálné peníze. Identifikace strategií lze použít pri konstrukci budoucích virtuálních aukcních domu a pri analýze virtuální ekonomiky realizované pomocí skutecných penez.

Klasifikace JEL	C73, D44, D58
klícová slova	opakované hry, aukce, všeobecné rovnováhy

E-mail autora	`waheedbrown@hotmail.com`
E-mail vedoucího práce	`gregor@fsv.cuni.cz`

Contents

List of Tables

List of Figures

Acronyms

CDF Cumulative Distribution Function

D3RMT Diablo 3 Real Money Auction House Tracker

GRETL Gnu Regression, Econometrics and Time-series Library

IPVP Independent Private-Values Paradigm

LHS Left-Hand-Side

NE (Bayesian) Nash Equilibrium

ORP Optimum Reserve Price

OSP Optimum Selling Price

PD Prisoner's Dilemma

PDF Probability Density Function

RHS Right-Hand-Side

Master Thesis Proposal

Author	Bc. Waheed Brown
Supervisor	PhDr. Martin Gregor Ph.D.
Proposed topic	Seller Strategies for Virtual Auctions Using Real Currencies

Topic characteristics Video games have long been an indulgence for the purposes of entertainment and socializing. However, since the monetization of online gaming, publishers have implemented economic models in video games in an effort to increase revenue. The most cutting-edge revenue model in video games is the real currency auction house in Diablo 3, a fantasy game where online players embark on demon-slaying quests to amass goods and treasure. Economic ingenuity comes in the form of in-game (virtual) goods being available for auction in real-world currencies such as the US dollar and the euro. Almost exactly mimicking conventional online auction houses such as eBay, Diablo 3's virtual goods, real currency auction house is a living socioeconomic experiment on seller-buyer strategies for goods that only function in a virtual world.

Hypotheses An equilibrium price function can be constructed, which results in a stable likelihood function, predicting the probability of a successful sale that maximizes profitability. Explaining the terminology: an equilibrium price function determines the value of an auctioned good after historical winning bids converge to a relatively constant price, in practice this means a reduction in price variance to one that is less logarithmic but more linear (flat) over time; a likelihood function is a probabilistic term in an equation; finally, a successful sale should maximize profitability for the seller only, as this paper focuses on an optimum selling price, not an optimum bidding price.

Methodology The research methodology in this thesis consists of spending time with the D3RMT auction house index. Observation of the data directly leads to the idea of an arbitrage price and a suspicion that the optimum selling price equality may not yield intelligent results.

Outline

1. Introduction
2. Background
3. Hypothesis
4. Review of Existing Research
5. Methodology
6. Theoretical Analysis
7. Empirical Analysis
8. Applications
9. Conclusion

Core Bibliography

1. Bagchi, A. et al., 2001. "Seller-focused algorithms for online auctioning."
2. Bajari, P. & Hortacsu, A., 2003. "The Winner's Curse, Reserve Prices, and Endogenous Entry: Empirical Insights from eBay Auctions." *RAND Journal of Economics*, **34(2)**, pp.329–55.
3. Gibbons, R., 1992. "Game Theory for Applied Economists." Princeton University Press.
4. Hendricks, K. & Porter, R.H., 2007. "Chapter 32 An Empirical Perspective on Auctions." *In M. Armstrong and R. Porter, ed. Handbook of Industrial Organization.* Elsevier, pp. 2073–2143.
5. Mas-Colell, A., Whinston, M.D. & Green, J.R., 1995. "Microeconomic Theory." Oxford University Press, Inc.
6. Paarsch, H.J. & Hong, H., 2006. "An Introduction to the Structural Econometrics of Auction Data." The MIT Press.
7. Segal I., 2003. "Optimal Pricing Mechanisms with Unknown Demand." *The American Economic Review*, **93(3)**, pp.509–529.

Author Supervisor

Chapter 1

Introduction

To begin with, this thesis reviews the existing literature for real-world auctions. Economic game theory concepts are explored to the point of application to virtual economies. The literature then introduces economic concepts that are specific to video games with computer science literature forming as a bridge between the economic theory and video game practice. The later sections of this thesis evaluate Diablo 3's auction house and construct rational seller strategies. Evaluations of Bayesian equilibria, fairness, and models for future games conclude the thesis.

From an economics perspective, virtual economies using real currencies serve as compelling microcosms of real-world economic problems. Especially in light of the 2007-2009 crisis, simulations of economies using real players as control variables is a relevant pursuit indeed. Specifically the identification of cheating and deviation strategies will have direct relevance to not only the construction of future video games but also to applications in the real world, such as bond auctions (Section 9.1).

Video games have long been an indulgence for the purposes of entertainment and socializing. However, since the monetization of online gaming, publishers have implemented economic models in video games in an effort to increase revenue. The most cutting-edge revenue model in video games is the real currency auction house in Diablo 3, a fantasy game where online players embark on demon-slaying quests to amass goods and treasure. Economic ingenuity comes in the form of in-game (virtual) goods being auctionable for real-world currencies such as the US dollar and the euro. Almost exactly mimicking conventional online auction houses such as Ebay, Diablo 3's virtual goods, real currency auction house is a living socioeconomic experiment on seller-buyer strategies for goods that only function in a virtual world.

This thesis attempts to identify rational seller strategies to maximize profits and achieve Bayesian equlilibria. Bayesian equilibria are crucial for virtual economies as they are the closest approximations to fairness for player interactions. Specifically, over several repeated auctions in the Diablo 3, real currency auction house, the values of goods tend to normalize. Seeing each individual auction as a game in a series of repeated games, bidder behavior reaches an equilibrium where they do not deviate too much from the previous winning bid price. The idea of auctions as a repeated game between bidders is further explored in Section 2.4. Following that, auctions as a signaling game between a bidder and a seller are evaluated in Section 2.5. Additionally, auctions as a single stage Bayesian game are presented in Section 2.6 as a race to the bottom.

After a discussion on game theory the hypothesis is reviewed. In support of the hypothesis statement are definitions of the terminology as well as a time series plot, illustrating the main point of the thesis. The next section is a review of existing research and literature. Following a loose chronology that matches their appearances in the thesis, the cited texts are discussed and validated in their contribution to this paper. Following this is a section on the methodology used in determining the optimum selling price, which leads into the theoretical analysis section. A section on the empirical analysis is divided into data collection, OLS regression, correlation of currencies, and actually using the optimum selling price in practice. The bulk of the experimental investigation is conducted using MATLAB. Trial runs are designed to observe numerical convergence to the optimum selling price. From these experiments, it is determined that a correction factor is needed that incorporates the logarithmic decay of prices over time. Prior to the final section on the paper's conclusions there is a section on applications of the optimum selling price. These last two sections serve as a discussion of the real world limitations of the optimum selling price equality given in Equation 6.15.

Chapter 2

Background

2.1 Glossary

There are hundreds of items that can be traded in the Diablo 3 real currency auction house. To simplify an academic analysis of the auctions a convenient, high level glossary is provided in the below two tables. For the duration of the thesis, focus will be put on the Blacksmith Plan good called Sage's Plight, abreviated as "sp" when used to name GRETL variables.

Table 2.1: Main Categories of Diablo 3 Auction House Goods

Item:	An object such as a sword, armor, helmet, etc. that can be immediately used without the necessity of any crafting being applied to it.
Crafting:	One of the three main categories of goods that can be bought/sold in the auction house; the other categories are Items and Gems.
Gems:	One of the three main categories of goods that can be bought/sold in the auction house; the other categories are Items and Crafting. Gems are goods that can be added to an item to increase its potency. For example, a sword can have a gem added to it to give it greater efficacy in battle.

Source: https://eu.battle.net/support/en/games/diablo3/auction-house

Table 2.2: Crafting Sub-Categories

Crafting Materials:	Raw materials required for crafting (fabricating) items.
Blacksmith Plans:	Plans that a blacksmith character in the game can use to craft a finished, usable item. Some examples are plans to make a sword or armor.
Jeweler Designs:	A design scheme that a jeweler character in the game can use to incorporate a gem into an item.
Pages of Training:	In-game craftspeople (such as blacksmiths and jewelers) can have their skills improved if they acquire pages of training. These improvements in craftsmanship allow craftspeople to fabricate more advanced and powerful items.

Source: https://eu.battle.net/support/en/games/diablo3/auction-house

2.2 Auction Size and Context

The Diablo 3 auction house is a sub-game within the Diablo 3 action/adventure computer game. When players defeat monsters in the game they are rewarded with goods such as items, types of crafting goods, and gems. These goods can then be traded for real currency in the auction house sub-game. As of April 4, 2013, the number of active Diablo 3 players was 1818 (Xfire 2013). As of April 5, 2013, the number of people online was 4791 (DiabloProgress.com 2013). There is much contention over the accuracy of these numbers as the publisher of Diablo 3 does not officially release them (Battle.net 2013c). After reading through the Battle.net forums however, it seems that the players themselves estimate the number of participants to be over one million (Battle.net 2013c). For the purposes of this paper, the Xfire number of 1818 players will be assumed the most accurate. Xfire actually registers its users and tracks their participation in Diablo 3, effectively making the Xfire estimate a lower bound for the number of auction house participants.

2.3 Auction Rules

The Diablo 3 auction house is a second-price auction where the winner pays the second highest bid price; that is, the bid placed right before his or her winning one. In the online bidding lexicon, this is also known as a proxy bidding system. Also important is the facility for maximum and incremental bids. In proxy bidding systems such as Ebay and the Diablo 3 auction house, players can set a maximum bid, and then an incremental amount for increasing that bid as other players compete for the good.

Diablo 3 has a real currency auction house for each of the following currencies: GBP, EUR, RUB, USD, MXN, BRL, ARS, CLP, and AUD (Battle.net 2013b). This thesis focuses on the USD real currency auction house only. To begin with goods can be searched for in either the gold (in-game currency) or real money auction house, for each world currency. Search criteria are based on the various characteristics of goods and their price ranges. When a good appears for sale in the auction houses, the bidders can see the following information: the current highest bid; the minimum bid, which is the seller's reserve price; and the time remaining until bidding closes. Note that the bidders cannot see any of: the seller's actual valuation of the good, which is the secret reserve price; the identity of the seller; or the identity of the current highest bidder.

Bidder actions are limited in the real currency auction house in that they must have a fully registered and currently active PayPal account. Proxy bidding (also called incremental or automatic bidding) occurs in 5% increments of the bidder's maximum bid price (Battle.net 2013a). The absolute minimum value for increments is 10 gold in the gold auction house and $0.10 USD (or equivalent) in the real currency auction house. There is no defined maximum for a bid increment amount.

Seller actions are limited in the following ways: no secret reserve price is allowed; the minimum bid price can be disabled so that bidders can bid as low as the like, down to the defined minimum of $1.25 USD; sellers can define a direct purchase price for their item, which allows the bidder to buy the good directly, superseding the auction; listing an item for sale costs $1 USD and there is a clearing fee of 15% to transfer earnings to a PayPal account.

More detailed rules specific to good types are available on Battle.net (2013b). Most relevant to this thesis are the rules for minimum and maximum bids. This restriction has a significant influence on seller strategies as it controls negative auction phenomena such as a race to the bottom (Grandy 1989) and the winner's curse (Bajari & Hortacsu 2003). Specifically, the minimum listing price for a good in any geographical region (including the EU) is $1.25 USD. The maximum price that a seller can list a good is $250 USD. Interestingly, average prices on D3RMT were observed to be near the floor of this range, and almost never above $140 USD.

As an example, consider an auction for a good with only two bidders. The first bidder will set a maximum bid, r, and an incremental amount, i, that is small relative to the amount of r. Bidder two will then place a bid that is greater than r, which automatically causes the first bidder to place a new bid of $r + i$. As a result, bidder two will then increase his or her bid by some incremental amount. This process iterates automatically in the computerized auction until either some final maximum bid amount is reached by either player, or the auction time expires. The winning bidder will then pay the price right before the final increment was made on the bidding price; hence the penultimate bid will be the actual sale price.

2.4 Vickrey Auctions as a Repeated Game (bidders vs. bidders)

The game theory concepts explored in this section were referenced from Gibbons (1992, pg. 88). And as a quick introduction, a Vickrey auction is a sealed bid auction in which the price paid by the winner is the second-highest bid (Vickrey 1961, pg. 8). To illustrate the idea of a repeated game it is often easiest to start with the classic Prisoner's Dilemma (PD).

Figure 2.1: Prisoner's Dilemma

		P_2	
		b_1	b_2
P_1	a_1	1,1	5,0
	a_2	0,5	4,4

Source: Modification of Albert W. Tucker's 1950 discription.

In Figure 2.1, player one (P_1) can choose a strategy of a_1 or a_2, likewise player two (P_2) can choose a strategy of b_1 or b_2. If the players cooperate then they both get a payout of four, if they both defect then they each get a payout of one. Conclusively, if the players distrust each other then they will avoid a zero payout. Hence the Nash equilibrium (NE) for this game is (a_1, b_1), the shaded cell in Figure 2.1.

The Prisoner's Dilemma can be restated in terms of the Diablo 3 auction house (Figure 2.2). A limited definition of the Diablo 3 auction house can be given as follows: let an auction for a good be a game, G, of two periods, T = 2, defined as G(2); let the number of bidders be limited to two players, P_1 and P_2; P_1 has a maximum bid predetermined at v_0, the maximum valuation for the good (Section 6.3); P_2 has a maximum bid predetermined at v_1, which is a lower bid than v_0, specifically, $v_0 \geq v_1$; let the smallest possible bid be v_N, where N is the total number of valuations for a period, t; let v_{N-1} be the next highest bid, such that $v_0 \geq v_1 \geq \ldots \geq v_{N-1} \geq v_N$.

Figure 2.2: Period t = 1 of the Diablo 3 Auction House Game

		P_2	
		v_1	v_{N-1}
P_1	v_0	2,1	1,0
	v_N	0,1	0,1

Source: author's design.

Figure 2.3: Period t = 2 of the Diablo 3 Auction House Game

		P_2	
		v_1	v_{N-1}
P_1	v_0	4,2	3,1
	v_N	2,2	2,2

Source: author's design.

Next the payouts for the Diablo 3 two period game, G(2), can be defined as follows: let payouts be denoted as "payout 1, payout 2", where "payout 1" is the payout for player one and "payout 2" is the payout for player two; payouts are written within the cells of a game table; a payout of 0 means a loss for that player for that period, a payout of 1 means the player continues to the next period, and a payout of 2 means that player has won the game; more intuitively, a payout of 0 means the player has dropped out of the auction, a payout of 1 means the player is continuing to the next round of the auction (unless it is the final round, period t=2), and a payout of 2 means that player has won the auctioned good. Assuming that rational players always choose the Nash equilibrium of (v_0, v_1) using the process outlined in Gibbons (1992, pg. 12), the payouts in period t=2 will always be the period t=1 payouts plus the NE values of 2 for player one and 1 for player two (Figure 2.3).

The final step is to define the players' strategies. Assuming that players are rational, in period t=1 player one will choose a strategy of v_0 and player two a strategy of v_1; thus the Nash equilibrium of (v_0, v_1). In period t=2, the maximum valuation, v_0, is increased to facilitate the idea of higher bids being needed to win. Remember that player one by definition always has a higher limit for his or her possible maximum bid, hence player two can only ever bid a maximum valuation of v_1 per period. With period t=2 being the final period, and with both rational players again picking the Nash equilibrium of (v_0, v_1), the player able to make the higher bid wins. Conclusively, the player who defines a higher value for their maximum possible bid will always win the auction.

This two period concept of a maximum ceiling bid is directly scalable to the idea of the Diablo 3, real currency auction house. In an auction made up of multiple periods (a finite time limit determined by the seller) any number of players can bid on a good. Often players outbid each other in preselected increments, up until the closing time of the auction. Since this computerized incremental bidding happens at the temporal speed of digitized computers (not at human speed) setting a maximum ceiling bid that is higher than all other bidders is a competitive strategy for winning a good. The real skill is in determining whether this maximum ceiling bid is an overvaluation of the good itself. This is why a strategy of v_1 for player two still gives a payout (Figure 2.3); if v_0 is an overvaluation from the perspective of player two, then it is still a rational strategy to lose; hence the winner's curse (Bajari & Hortacsu 2003, pg. 5). Intuitively, a payout for v_1 means that player two benefits from saving his or her money by losing the auction.

What makes this whole process a Bayesian game, with Bayesian Nash equilibria is the fact that bidders do not know each other's maximum ceiling bids. This incomplete information game becomes a signaling game once the seller is involved. The idea of a Vickery auction as a signaling game is presented in Section 2.5.

2.5 Vickrey Auctions as a Signaling Game (sellers vs. bidders)

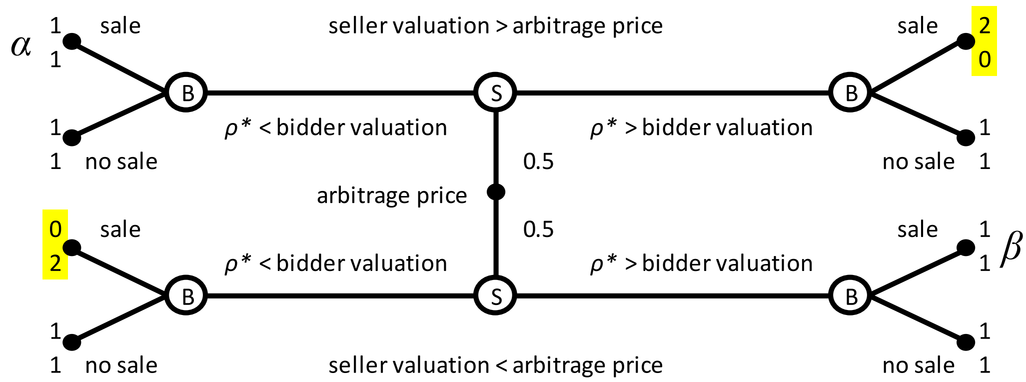
In summary, bidders signal the value of a good based on the values of historical winning bids. The seller then uses this as a signal to determine the starting bid (optimum selling price, Section 6.6 of this paper) of the good. Finally the winning bidder reads the seller's signal and picks a maximum ceiling bid that will win the auction. This is why it is key for the bidders to determine a good's valuation through finding the Bayesian Nash equilibrium in a repeated game (Section 2.4). Sellers can use the valuation of the winning bid, v_0, as the starting signal for determining the optimum selling price.

The Diablo 3 signaling game G can be defined as follows, using the methods described in Gibbons (1992, pg. 183). First a clarification about the first signal sent by the bidder, the one determined by the historical winning bids, selected by the repeated game Bayesian Nash equilibrium. In reality, this bidder signal can be ignored by the seller, because of the existence of an arbitrage price (Section 6.5). Restated briefly, players can sell and purchase goods in an alternative Diablo 3 auction house that used gold, the in-game currency. Gold can be bought or sold at a daily exchange rate. Rational players can arbitrage goods between the real currency and in-game currency auction houses. This means that a rational seller would base their optimum selling price on the arbitrage price, not on the historical winning bids in the real currency auction house. For this reason, it can be assumed that the seller now sends the first message in the signaling game (Figure 2.4).

Briefly explained: the arbitrage price is determined by the gold (in-game currency) auction house; it is assumed that the arbitrage price has an equal probability of being above or below the seller's valuation of the good; the optimum scenario for the seller is a sale above both the arbitrage price and the optimum selling price (the shaded payouts on the top right of Figure 2.4); the optimum scenario for the bidder is a sale below both the arbitrage price and the bidder valuation (the shaded payouts on the lower left). The top number in a pair of payouts is the seller's payout, with the bottom being the bidder's.

Going into more detail, all other payout nodes are balanced with a payout of 1 for both the seller and the bidder. The reason for this is that first of all, a failure to sell is of no consequence to anybody: the seller can turn to the gold auction house and "break even" at the arbitrage price while the bidder

Figure 2.4: Diablo 3 as a Seller-Buyer Signaling Game



Source: author's design.

gets to save his or her money. The two edge cases of sales at nodes α and β in Figure 2.4 are ambiguous because the final sale price could feasibly be between the arbitrage price and the players' valuations. Of course with the shaded nodes, if a player has a payout of 0 it means that he or she would be more rational to buy or sell the good at the arbitrage price in the gold (in-game currency) auction house. Conclusively, rational players would not bother with the real currency auction house at all; it would be more rational to simply buy or sell the good at the arbitrage price and avoid the risk and effort of "game playing".

But evidently, not all players in the Diablo 3 real currency auction house are rational. This is clearly visible in the divergence of historical sale prices between both the real currency and gold auction houses. By using the daily exchange rate for USD/gold, it is quite clear that arbitrage opportunities exist. Either players are being willfully ignorant or are simply playing the real currency auction house "games" for entertainment. There is a third possibility however; irrational players may not be aware of the historical pricing information. Conversely, proof that rational players do exist is evident in the form of the Diablo 3 auction house indices on (D3RMT 2012).

2.6 Vickrey Auctions as a Race to the Bottom (sellers vs. sellers)

In the case of two sellers competing for the best selling price, each player wants to choose a lower selling price than the other. Defining some game rules: the game is limited to one period only, with just two players (sellers); player one (P_1) always goes first and player two (P_2) always goes last; player two is always willing to value a good lower than player one; neither player knows the other's valuations but player two can see player one's selling price in the auction house (since player one posts his or her sale first); equilibrium can be found using the process outlined in Gibbons (1992, pg. 12). This game is illustrated in Figure 2.5, with the shaded cell being the Nash equilibrium.

Figure 2.5: Race to the Bottom

		P_2: v_1	P_2: v_N
P_1	v_0	0,2	0,1
	v_{N-1}	1,0	0,1

Source: author's design.

This game is called a race to the bottom (Grandy 1989) as the Bayesian Nash equilibrium of (v_{N-1}, v_N) gives payouts when choosing the lowest valuations of a good; hence sellers are competing to put the cheapest prices on their goods in order to beat each other's prices. In the end, the seller who valuates a good the cheapest will win the sale. Interestingly, the race to the bottom game is an extremely close approximation of seller behavior in the Diablo 3 auction house. When observing a time series of winning bids for a good, there is a near logarithmic decay in the sale price over time. Part of this is due to the increased rationality of bidders over time but Figure 2.5 also contributes to the phenomenon; rational sellers will play the race to the bottom game if they want to make sales.

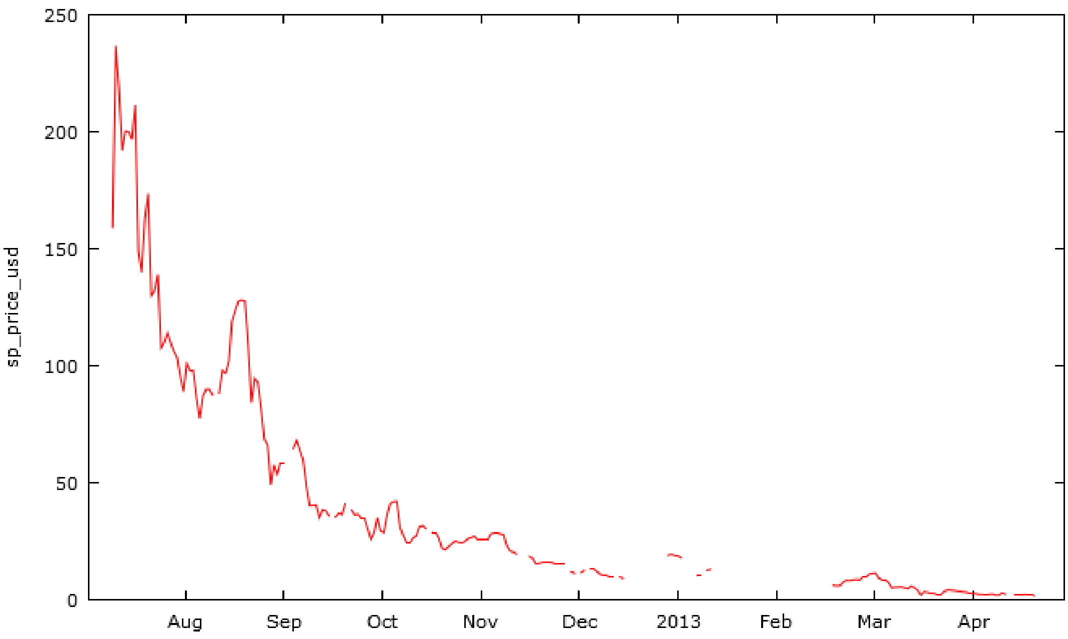
Chapter 3

Hypothesis

An equilibrium price function can be constructed, which results in a stable likelihood function, predicting the probability of a successful sale that maximizes profitability. Explaining the terminology: an equilibrium price function determines the value of an auctioned good after historical winning bids converge to a relatively constant price, in practice this means a reduction in price variance to one that is less logarithmic but more linear (flat) over time, this is illustrated in Figure 3.1; a likelihood function is a probabilistic term in an equation, it will be shown in Section 6.6 that two probabilistic expressions are used, both the cumulative distribution function and the probability density function; finally, a successful sale should maximize profitability for the seller only as this paper focuses on an optimum selling price, not an optimum bidding price.

A secondary hypothesis is that the optimum selling price may be better calculated through computational brute force. In addition to pursuing a closed-form mathematical expression for the optimum selling price, experiments are conducted to observe the convergnce of the distribution of prices. This secondary hypothesis is a unique contribution by this paper, supplementing the referenced literature on optimum pricing. In pursuit of this secondary thesis, an entire section on experiments is devoted to the analysis of computational methods.

Figure 3.1: Time Series Plot of sp_price_usd



Source: author's calculation *GRETL*

Chapter 4

Review of Existing Research

The primary focus of the literature was to build a foundation of economic theory for this thesis. To meet this purpose, the first paper to be reviewed was Segal (2003). This paper, provides an archetypal model for seller pricing strategies in an auction house with unknown buyer distributions. After a discussion on identifying monopoly profits for a standard profit maximization problem, Segal (2003, pg. 510) introduces unknown buyer distributions and auctions with high numbers of buyers to identify profit convergence for monopoly profits.

For the econometrical analysis of the auction house data, a reference was used from one of the seminal authors on econometrics, Harry J Paarsch. Mr. Paarsch's work at the University of British Columbia is considered a paradigm for empirical studies on auctions (Paarsch 1992, pg. 191). This reference presents mathematical models for bidder behavior, equilibrium bidding, probability density functions of bids and "out of" methods for out of sample predictions. Paarsch's most applicable conclusions on the optimal reserve price (optimum selling price) were found in another publication a few years later (Paarsch 1997, pg. 339). One deficiency of Paarsch (1992; 1997) is that there are not many computational examples for use with software. Rather than attempting to build algorithms based on the raw theory one of Mr. Paarsch's later works is consulted (Paarsch & Hong 2006, pp. 66, 313, 314).

Out of the Paarsch readings comes the idea of a second-price auction, also known as a Vickrey auction (Paarsch & Hong 2006, pg. 30). The idea of a second-price auction was first described in Vickrey (1961, pg. 8); hence the adoption of the term, Vickrey auction. As the moniker suggests, a second-price auction is one where the winner pays the amount of the second-highest bid, and not that of the winning bid. Vickrey (1961, pg. 8) argues that the second-price construction of an auction increases an auction's efficiency without giving bias in favor of the sellers. Unfortunately, a Vickrey auction is still prone to the negative bidder strategy of the winner's curse and the negative seller strategy of a race to the bottom.

In order to challenge the mathematical model for the optimal reserve price equation proposed by Paarsch & Hong (2006, pg. 66) preference relations for auction players were researched. The main reference for preference relations was Mas-Colell *et al.* (1995, pp. 50, 57) for an evaluation of utility maximization problems and expenditure minimization problems. To establish the feasibility of optimal reserve price equation, Milgrom & Weber (1982, pg. 1107) was consulted, revealing the same equation as Paarsch & Hong (2006, pg. 5) for the equilibrium bid price. This is significant because the equilibrium bid price can be used to derive the optimal reserve price (optimum selling price), thus corroborating with the conclusions of Paarsch & Hong (2006, pg. 66).

The main game theory reference for this thesis is the work of Gibbons (1992). The tabular notation of game stages used in this thesis is based on this book. Additionally, tabular methods for findng equilbrium strategies and the design of signalling game diagrams follow the book's guidelines. It is actually beneficial to read the subchapters of Gibbons (1992, pp. 12, 88, 183) referenced in the Background section of this paper.

After the emergence and success of Ebay, Paarsch & Hong (2006, pg. 301) amalgamated the earlier work on empirical analysis with the computational power of MATLAB. MATLAB is a mathematical software package that allows for data visualization and analysis and the design of computer algorithms based on mathematical models. With a rich selection of MATLAB code samples, Paarsch & Hong (2006, pg. 301) is an invaluable toolbox of empirical analysis and computational methods. As a result, the MATLAB tools were referenced for this thesis but modified for use with the Diablo 3 auction house data, rather than with the data from Ebay.

One concept common in most online auctions that has debatable impact in the Diablo 3 auction house is that of the "winner's curse" (Bajari & Hortacsu 2003, pg. 5). Most closely related to the behavior of Ebay auctions, the winner's curse explains that to have the winning bid for a good a bidder must pay over the good's actual book value. Additionally, if there are many bidders for a good, the amount by which the winning bid must exceed the book value increases. The root cause for the winner's curse is information asymmetry about the good being auctioned. In the case of the Diablo 3 auction house, near perfect information about a good is available in the form of an average price index (D3RMT 2012). Since the Diablo 3 auction house has considerably fewer goods than Ebay, coupled with the fact that virtual goods are all of uniform quality, information asymmetry is virtually eliminated when players track the goods on the D3RMT index. Yet curiously, bidders will still pay above the book value (average daily price on D3RMT) in the Diablo 3 auction house. The winner's curse is not explored in great detail in either the theoretical or empirical analyses parts of this thesis, but it is still an important concept with respect to player behavior. In the conclusion of this thesis, explanations are given as to why the winner's curse persists in a virtual goods action with almost perfect information.

One important note on the winner's curse with Ebay is that this concept was identified as a problem nearly ten years ago in Bajari & Hortacsu (2003, pg. 5). Ebay has since taken measures to encourage final bids to be closer to the book value of goods; or stated plainly, increased the amount of information available to bidders. Features such as seller rankings, the establishment of webpage storefronts, better search technology, and maximum selling prices have all helped to reduce information asymmetry. Although not formally reviewed in this thesis, it would be informative to track how close final bid prices are to the "buy it now" or maximum selling prices. The "buy it now" price augments the auction by providing an alternative equilibrium point for the Bayesian game, or a fixed convergence point for bidding prices.

As common in auctions as the winner's curse is the concept of a race to the bottom, reviewed in Grandy (1989). Where the winner's curse focuses on the consequences of being a winning bidder, the race to the bottom focuses on a similar scenario for the seller. In order to be a winning seller in a competitive auction house (where sellers compete directly in selling the same goods) the equilibrium strategy is to out price the competition. Particularly in a repeated game with a set minimum price, sellers compete to assign the most affordable price that is both above his or her individual, minimum valuation but below the minimum valuation of competitors. Consequently, over a successive series of game stages (multiple consecutive auctions for the same good) the good's sale price converges to the minimum allowable price. In the case of Diablo 3, this phenomenon hurts the entire auction house as a whole as the good's sale price often falls so far below its true valuation that both bidders and sellers alike eventually lose interest.

After reviewing the case of a real good auction house, the next piece of literature, Fields *et al.* (2011), lays the groundwork for economic models in video games. Specifically, the concept of virtual goods is evaluated from both an economic and video game perspective. Next, Fields *et al.* (2011) explores currency models in video games. Interestingly, the cornerstone for successful video game currency models (as of 2012) is the Alan Greenspan interpretation of neo-Keynesian macroeconomics. This concept is further indulged with discussions on single and dual currency models, soft and hard currencies, and closed and open economies. Finally, a section on "honest gameplay" (Fields *et al.* 2011) segues quite nicely into the game theory idea of Bayesian equilibria, where a currency model equilibrium is the pareto efficiency of prices and quantities among players.

Finally, Bagchi *et al.* (2001, pg. 2) evaluates seller strategies in auctions from a computer science perspective. Although beyond the domain of economics, the game theory elements of this paper provide insights into profit maximization. Bagchi *et al.* (2001, pg. 2) then modifies the standard model for online algorithms to enhance the competitiveness (fairness, pareto efficiency) of a variety of auctioning scenarios. The concept of fairness is defined as all players being equally rational in that they all know the same set of information (or have the same lack of knowledge about each other's valuations). Pareto efficiency is defined as neither player benefiting from a deviation from their strategies, once an equilibrium state has been achieved.

Extending the scope of the thesis to some basic applications of the optimum selling price, bond auctions are reviewed in Hendricks & Porter (2007, ch. 3) and Bartha & Quinn (2011). Coincidentally, municipal bond auctions in the United States also allow the idea of a reserve price to have uniform characteristics in both first-price and second-price (Vickrey) auctions. Furthermore, state bonds from various trouble economies in the Eurozone can be used to gauge investor confidence. What makes bonds a special auction case is that consumer temperaments (valuations) and the risk for contagion seem to depend more on bond spreads than they do on the bond principle.

Although not a part of the academic or journalistic body of research on auctions, the referenced Diablo 3 websites provide invaluable data and details on the real currency auction house for virtual goods. The publisher of Diablo 3, Blizzard Entertainment, Inc., keeps all public documentation of the real currency auction house on their branded gaming website, Battle.net. Battle.net not only contains the auction house rules and definitions but also many forums on characteristics of the game. These forums contain everything from updates on errata to clarifications on player misconceptions. The second Diablo 3 website is the D3RMT auction house index. D3RMT.com (recently rebranded as DiabloHub.com) is a fan built collection of four indices, one for each of the auction houses: European gold (gold being the in-game currency), European EUR (€), North American gold, and North American USD ($). This is directly analogous to the idea of the NASDAQ indices for technology stocks, except that in the case of the D3RMT indices the instruments being listed and tracked are the final sale prices of the goods available for auction.

Supplementary video game websites were referenced in order to obtain usage statistics on Diablo 3. DiabloProgress.com displays the number of Diablo 3 players who are currently online; although not the number who are actively participating in the auction house, nor any usable historical data. A second website, Xfire.com (read cross-fire-dot-com) also gives a figure for the ambiguously defined “number of players”. For the sake of this thesis, the Xfire.com numbers are taken as the lower bound of daily players and DiabloProgress.com as the upper bound. It is vital to note, however, that discussions in the Battle.net forums, and especially reports to the media by Blizzard staff associated with the game, put the player count at over one million per day (Schramm 2013). Again, these reports give ambiguous statistics on the number of daily users in the real currency auction house. Adding to the confusion is the fact that interest in the game is decreasing over time (Mu-hyun 2012); hence an accurate representation of the number of auction house participants is difficult to confirm. Economists will be proud to learn that the majority of Diablo 3 players favored gaming the auction house over playing the actual game of killing monsters (Schramm 2013).

Chapter 5

Methodology

The research methodology in this thesis consisted of spending time with the D3RMT auction house index. Observation of the data directly led to the idea of an arbitrage price and a suspicion that the optimum selling price equality would not yield intelligent results. The overall process for the methodology was as follows: give the Diablo 3 auction house a real world context, in terms of the number of participants; define the Diablo 3 auction house terminology and rules; define equilibrium for a Diablo 3 auction house good as the result of a series of three types of games, repeated games, signaling games, and a race to the bottom; focus only on the Diablo 3, North American USD, real money auction house; use the Diablo 3, North American Gold, in-game currency auction house for determining a regression model and the arbitrage price; restrict the hypothesis to question only whether a feasible optimum selling price equation can be found; direct the review of existing research to support only Bayesian equilibria in second-price auctions, supporting literature is consulted to justify any concepts used during the course of the thesis; the theoretical analysis focuses on the construction and utility of an optimum selling price equation, supporting theory is also presented; the empirical analysis includes both a statistical analysis and a numerical determination of the optimum selling price; to further investigate the theory, a computational analyis of the optimum selling price is performed through a series of MATLAB experiemnts; prior to the conclusion, some applications of the optimum selling price are reviewed so as to give the final discussion some perspective; and the thesis concludes that the hypothesis is incorrect because the winning bid prices for items decay logarithmically over time, thus making the results of the optimum selling price calculation seem far too high for practical use.

Chapter 6

Theoretical Analysis

6.1 Vickrey Auction

The type of auction used in Diablo 3 and Ebay is called a second price or Vickrey Auction. An explanation of Vickrey auctions is given quite concisely in Paarsch & Hong (2006, pg. 30). Described briefly, a Vickrey auction is a sealed bid auction in which the price paid by the winner is the second-highest bid (Vickrey 1961, pg.8). For this reason, a Vickrey auction is also known as a second-price auction. A further augmentation is a second-price, sealed bid auction which means that bidders submit their bids in envelopes. In general this means that bidders do not know each other's bids. For Diablo 3 and Ebay (and most second-price, sealed-bid internet auctions) this translates into bids being anonymous in the online auction house. The exception, in the case of Ebay, is that the seller knows the bidders' identities.

To get a clearer understanding of how a second-price auction works, it helps to consider a basic example. The participant who placed the highest bid wins the auction, but he or she pays the price tendered by the nearest opponent (Paarsch & Hong 2006, pg. 30). Another important theoretical concept with Vickrey auctions is that the winning bid can be treated as the opportunity cost of the good being auctioned. This is key to viewing Vickrey auctions from an economics perspective as opportunity costs can then be subject to applications of microeconomic theory.

For a detailed explanation of the structure of information presented to bidders, please see the Independent Private-Values Paradigm in the next section.

6.2 Independent Private-Values Paradigm

The Independent Private-Values Paradigm (IPVP) is an effective starting point for evaluating the structure of information presented to bidders. This theoretical model for information structure will serve as the basis for discussions further on in this thesis. The IPVP concept and its explanation was referenced from Paarsch & Hong (2006, pg. 25). First, the survivor function is presented as a proportion of the population having demand for a good when the price is v.

$$S_V(v) = [1 - Pr(V \leq v)] \tag{6.1}$$

The survivor function has a random variable V representing the value of a good, v is the price of the good at which a certain number of potential bidders N will want to bid on that good, and $Pr()$ represents the probability of an event occurring.

With the survivor function defined, it is possible to construct an expected demand curve for bidders (Figure 6.1). Along the horizontal axis is the value $NS_V(p)$ and along the vertical access is the value p, where p is some price for a good being auctioned. So for potential bidders N, each one is assumed to demand at most one unit of the good. This theoretical assumption allows the aggregate demand to be a maximum value of N when p is at a theoretical minimum of zero.

At this point it is useful to introduce a cumulative distribution function (CDF) of bids. This CDF will be for the entire pool of potential bidders N, with $v_{(N:N)}$ being the lowest bid, $v_{(2:N)}$ being the second highest bid, and $v_{(1:N)}$ being the highest bid.

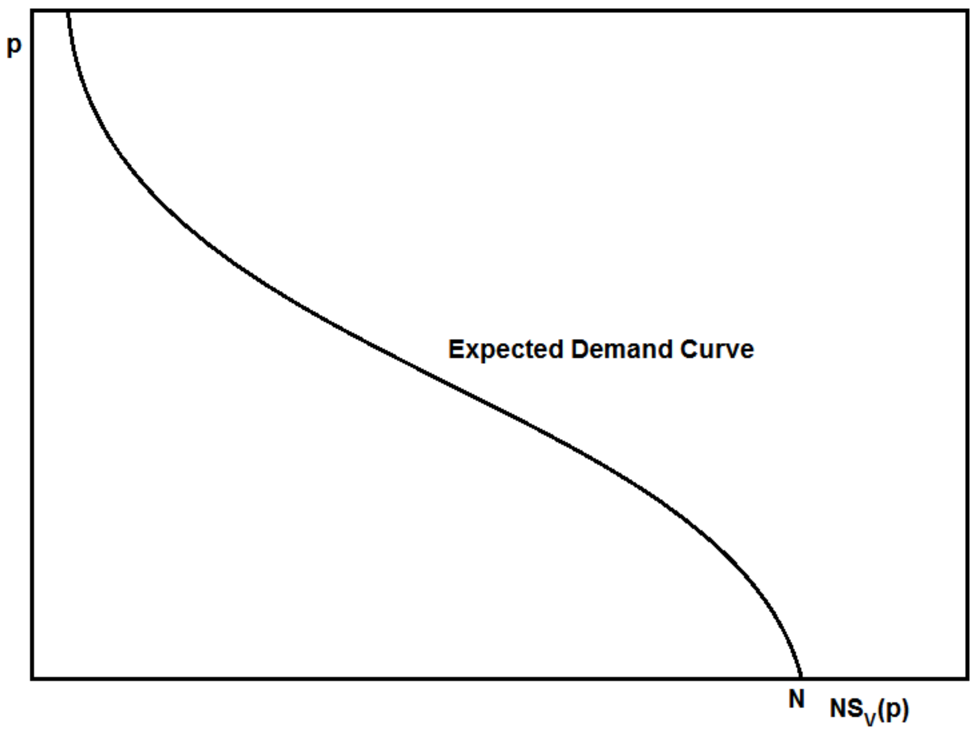
$$F_V(v) = Pr(V \leq v) \tag{6.2}$$

$$v_{(1:N)} \geq v_{(2:N)} \geq \ldots \geq v_{(N:N)} \tag{6.3}$$

The CDF makes it clear where the second-price, $v_{(2:N)}$, sits in the distribution of all bids for the pool of N bidders.

Although the focus of this paper is on a Bayesian equilibrium pricing functions, Bayesian equilibrium bid function can now be defined. Most of the heavy lifting in defining the bid function was performed by Paarsch & Hong (2006, pg. 5), but a basic understanding of the survivor function, expected demand

Figure 6.1: Expected Demand Curve



Source: Paarsch & Hong (2006).

curve, and cumulative distribution function allows an early understanding of the bid function.

$$\sigma(V) = v - \frac{\int_0^v F_V(u)^{N-1}\,\mathrm{d}u}{F_V(v)^{N-1}} \tag{6.4}$$

In the bid function (Equation 6.4), u represents an arbitrary bid from the CDF while v represents the maximum bid. This allows other bidders to determine what the minimum winning bid would be. The theory concludes that when the survivor function equals the bid function, then a minimum winning bid has been found. The minimum winning bid can be interpreted as $V = s^{-1}(s)$. Savvy bidders can thus reverse-engineer the bidding function too determine the value of a rational bid.

$$S_V(v) = \sigma(V) \tag{6.5}$$

$$V = \sigma^{-1}(s) \tag{6.6}$$

Looking at the expected demand curve in Figure 6.1, it becomes clear that at the maximum, or winning price $p = v_{(1:N)}$ there will only be one survivor.

$$NS_V(v) \rightarrow (1)S_V(v_1) = S_V(v_1) = [1 - \Pr(V \leq v_1)] = [1 - 0] = 1 \tag{6.7}$$

As a result, the expected demand function is for a pool of only one bidder. This causes the survivor function to yield only one survivor. Hence the expected demand and survivor functions give the same value: only one person demanding the good at the winning price $p = v_{(1:N)}$ (or simply v_1, in reduced notiation) means there can only be one survivor. And thus, the survivor is the winner.

6.3 Defining the Set of Valuations and Prices for an Auctioned Good

For simplicity, the number of valuations for a good is the same as the number of possible prices, N. Also note that the elements in the sets are ordered from highest to lowest; where v_0 is the highest valuation for an auctioned good and v_N is the lowest. Please consult the source reference, Paarsch & Hong (2006, pg. 2), for further reading on valuation sets.

$$v_{(0:N)} = v_{(1:N)} = v_{(2:N)} = \ldots = v_{(N:N)} \tag{6.8}$$

$$v_0 = v_1 = v_2 = \ldots = v_N \tag{6.9}$$

Let X be the set of valuations for a good, defined as:

$$X = \{v \mid v \in \mathbb{R}, v_0 \geq v_1 \geq v_2 \geq \ldots \geq v_N\} \tag{6.10}$$

where N is the total number of valuations.

$$\rho_{(0:N)} = \rho_{(1:N)} = \rho_{(2:N)} = \ldots = \rho_{(N:N)} \tag{6.11}$$

$$\rho_0 = \rho_1 = \rho_2 = \ldots = \rho_N \tag{6.12}$$

Let Y be the set of prices for a good, defined as:

$$Y = \{\rho \mid \rho \in \mathbb{R}, \rho_0 \geq \rho_1 \geq \rho_2 \geq \ldots \geq \rho_N\} \tag{6.13}$$

where N is the total number of valuations.

6.4 Determining an Initial Valuation

As stated in Paarsch & Hong (2006, pg. 2), the determination of valuations (and auction preference relations in general) is often shrouded in ambiguity. In this section an attempt is made to justify the choice of the expected value of the good's price as its minimum valuation.

First some notes on terminology: the terms "minimum valuation" and "initial valuation" mean the same thing. The initial valuation is the starting point on the "thermometer" of valuations (Paarsch 1992, pg. 338) from the perspective of the bidder. Conversely, the minimum valuation is the lowest price at which the seller values the good. Above the minimum valuation, the seller believes that he or she is making an advantageous profit, above the actual value of the good. The maximum valuation is the highest possible price that the pool of bidders is willing to pay for the good. For ease of reference, definitions for the set of valuations and prices for an auctioned good can be found in Section 6.3.

Next the notation needs to be clearly explained so that more attention can be paid to the determination of seller preferences in the case of the Diablo 3 real currency auction house. In the below Equation 6.14: ρ is an arbitrary price from the complete distribution of possible prices at which a bidder is willing to bid; $\rho_{(*:N)}$ or simply ρ^* is the optimum selling price at which the seller should initially post the good in the auction house; $\rho_{(1:N)}$ or simply ρ_1 is the winning or highest bid price, where N is the number of past winning bids; $\rho_{(2:N)}$ or simply ρ_2 is the second highest price, which is the second-price in the second-price auction model, which is the price that the winning bidder actually pays; $\rho_{(0:N)}$ or simply ρ_0 is the minimum selling price, equivalent to v_0, the initial valuation. The cumulative distribution function (CDF) for the complete set of valuations for a good is written as $F_V(\rho^*)$ and the corresponding probability density function is written as $f_V(\rho^*)$.

$$\rho^* = v_0 + \frac{1 - F_V(\rho^*)}{f_V(\rho^*)} \tag{6.14}$$

There is still one key concept yet to be explained: how does the complete set of valuations $\{v_i\}_{i=1}^N$ relate to the complete set of prices $\{\rho_i\}_{i=1}^N$? Some assumptions are necessary at this point. The first key assumption is that the number of valuations is equal to the number of prices, N. The second assumption is that the current valuation is always the arbitrage price, the good's value in gold (in-game currency) converted to American dollars (USD) on the date that the bid is placed, (Section 6.5). If there is no gold/USD exchange rate for a date, than the most recent exchange rate is used. The third assumption is that the complete set of optimum selling prices is calculable, using Equation 6.14. The last assumption is that the minimum selling price, ρ_0 is always equal to the arbitrage price of the good. A rational seller would not sell a good for less than this price.

6.5 The Arbitrage Price

For ease of understanding, the seller valuation of a good is assumed to be the arbitrage price of the good in the gold (in-game currency) auction house. The principles of a rational seller are as follows: (i) she can buy gold at an exchange rate of gold/USD; (ii) she can then buy the desired good in the gold auction house; (iii) she can then sell this same good in the USD auction house; (iv) therefore the value of the good purchased in gold and then converted to USD (at the USD/gold exchange rate) is the arbitrage price; (v) a rational seller would thus not want to auction the good in the USD auction house for less than the arbitrage price. In auction terms: the price paid for the purchase of the good in the most recent gold auction is assumed to be the seller's valuation of the good, v_0 (which is also restricted to equal ρ_0).

6.6 The Optimum Selling Price

As described in Paarsch & Hong (2006, pg. 66) the optimum selling price for a Vickrey auction is the optimum reserve price (ORP). The ORP equation is given as:

$$\rho^* = v_0 + \frac{1 - F_V(\rho^*)}{f_V(\rho^*)} \tag{6.15}$$

The variable v_0 represents the seller's valuation of the auctioned good. In the case of Diablo 3, v_0 will be the arbitrage price (Section 6.5) at the date of the most recently completed auction. $F_V(\rho^*)$ is the cumulative distribution function (CDF) of the entire set of possible seller valuations. Similarly, $f_V(\rho^*)$ is the probability density function (PDF) of the entire set of possible seller valuations. A more complete explanation of the set of valuations can be found in Section 6.3. The most crucial aspect of the optimum selling price equation is that the right-hand-side does not automatically evaluate to the left-hand-side; only when the equality holds true does ρ^* actually represent the optimum selling price.

A sound argument can be made that the structure of the optimum selling price equality in Equation 6.15 may not be the most rational, from the perspective of the seller. However, employing probability distributions approximates player preferences without a quest through behavioral economics. An exhaustive employment of preference relations from microeconomic theory can effectively abstract the behavior of a rational player. The desired path to deriving an equation using preference relations would be through the Utility Maximization Problem (Mas-Colell *et al.* 1995, pg. 50) and the Expenditure Minimization Problem (Mas-Colell *et al.* 1995, pg. 57). A more succient description using a profit maximazation approach can be found in Segal (2003, pg. 513). However there is another academic source that supports the idea of a cumulative distribution function divided by a probability density function as providing the equilibrium bid price: Milgrom & Weber (1982, pg. 1107). Paarsch & Hong (2006, pg. 66) extend this idea to create the optimal reserve price (optimum selling price) equality. It proved difficult to find other academic sources that gave feasible alternatives for the second-price auction equilibrium bid price or optimal reserve price.

6.7 The Optimum Selling Price as an Indefinite Solution

As introduced in Section 6.6, the optimum selling price is an equality that must be solved for a specific dollar value, ρ^*. For convenience the optimum selling price is restated:

$$\rho^* = v_0 + \frac{1 - F_V(\rho^*)}{f_V(\rho^*)} \tag{6.16}$$

One inhibiting factor in the usage of the optimum selling price equality is that it is an indefinite expression, with no apparent closed-form solution (although one is found in the next section). The starting point for illustrating this limitation is Equation 6.17.

$$\rho^* = v_0 + \sigma e^{\frac{(\rho^* - \mu)^2}{2\sigma^2}} \sqrt{2\pi}(1 - \frac{1}{2} erfc(\frac{\mu - \rho^*}{\sigma\sqrt{2}})) \tag{6.17}$$

Focusing on the cumulative distribution function (CDF) specially gives Equation 6.18.

$$F_V(\rho^*) = \frac{1}{2} erfc(\frac{\mu - \rho^*}{\sigma\sqrt{2}}) \tag{6.18}$$

Proceeding further, the error function is isolated so that the infeasible portion of the optimum selling price equality can be identified (Equation 6.20).

$$erfc(\frac{\mu - \rho^*}{\sigma\sqrt{2}}) = 1 - erf(\frac{\mu - \rho^*}{\sigma\sqrt{2}}) \tag{6.19}$$

$$erf(\frac{\mu - \rho^*}{\sigma\sqrt{2}}) = \frac{2}{\sqrt{\pi}} \int_0^{\frac{\mu - \rho^*}{\sigma\sqrt{2}}} e^{-t^2}\, \mathrm{d}t \tag{6.20}$$

6.8 The Optimum Selling Price as a Closed-Form Expression

The optimum selling price given in Equation 6.15 can be reduced to a closed-form expression, thus allowing the feasible calculation of ρ^*. The first step is to express the optimum selling price in terms of only the PDF function, $f_V(\rho^*)$, using the property that the PDF is the derivative of the CDF (Equation 6.21).

$$\rho^* = v_0 + \frac{1 - \int f_V(\rho^*)\,\mathrm{d}v}{f_V(\rho^*)} \tag{6.21}$$

$$\rho^* - v_0 = \frac{1 - \int f_V(\rho^*)\,\mathrm{d}v}{f_V(\rho^*)} \tag{6.22}$$

$$(\rho^* - v_0) f_V(\rho^*) = 1 - \int f_V(\rho^*)\,\mathrm{d}v \tag{6.23}$$

After some algebraic manipulation, the derivative of the whole expression can be taken (Equation 6.24). This step is necessary because taking the derivative removes the integral expression of the CDF, which is itself an indefinite integral (shown in Section 6.7).

$$(\rho^* - v_0) f'_V(\rho^*) = -f_V(\rho^*) \tag{6.24}$$

$$\rho^* = v_0 - \frac{f_V(\rho^*)}{f'_V(\rho^*)} \tag{6.25}$$

Algebraic manipulation yields a new expression for the optimum selling price, which no longer has a CDF term (Equation 6.25). At this point the algebraic expressions for $f_V(\rho^*)$ and $f'_V(\rho^*)$ can be substituted into the equality. What results is a closed-form expression (Equation 6.28) that can now be algebraically reduced into something manageable.

$$f_V(\rho^*) = \frac{e^{-\frac{(\rho^*-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \tag{6.26}$$

$$f'_V(\rho^*) = \frac{(\mu - \rho^*) e^{-\frac{(\rho^*-\mu)^2}{2\sigma^2}}}{\sigma^3\sqrt{2\pi}} \tag{6.27}$$

$$\rho^* = v_0 - \frac{\sigma^3\sqrt{2\pi} e^{-\frac{(\rho^*-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}(\mu - \rho^*) e^{-\frac{(\rho^*-\mu)^2}{2\sigma^2}}} \tag{6.28}$$

More algebra gets the inequality into the form of a second-degree quadratic equation. This allows the use of the quadratic formula to determine a definite value for ρ^*. Although ρ^* has two roots in this case, the feasible root will be a positive real number; and based on the auction house rules (Section 2.3), ρ^* must also be above \$1.25.

$$\rho^* = v_0 - \frac{\sigma^2}{(\mu - \rho^*)} \tag{6.29}$$

$$\rho^* = \frac{(v_0 + \mu) \pm \sqrt{(v_0 + \mu)^2 - 4(v_0\mu - \sigma^2)}}{2} \tag{6.30}$$

Thus the positive real value of the optimum selling price depends only on the seller's valuation, the mean, and the variance of the historical winning bid prices of the good (Equation 6.30).

Chapter 7

Empirical Analysis

7.1 Data Collection

The most challenging part of the research methodology was the first phase of data collection. Since the topic of study was virtual auctions using real currencies, there was a very limited selection of potential examples. The largest virtual auction using real currencies as of May 2012 was within a computer game named Diablo 3, produced by Blizzard Entertainment in the United States. This was in fact the beginning of the difficulties in data acquisition.

When contact, Blizzard Entertainment informed me that they do not release any data for research purposes. The justification for this was that they receive too many academic data requests. To overcome this obstruction, a third party website that provides analytics on the Diablo 3 auction house was used as a data source (D3RMT 2012). This website aggregates daily price data to provide an average sale price for commodities in the auction house. To ensure sufficient randomization and avoid selection bias, a variety of ten goods were chosen in the US dollar auction market place.

The Diablo 3 auction house is administered geographically. For example, participants using the Internet from a North American location are automatically restricted to the North American auction house. This is also the case for the EU. To further randomize the data, participation in the auctions was not undertaken, only observation of the prices set by other sellers.

7.2 OLS Regression

Note that for the empirical model section the Blacksmith Plan good "Sage's Plight" was used. The characteristics of the historical prices of this good are similar to those of other top selling goods in the auction house.

Time series plots of both USD prices and gold (in-game currency) prices were evaluated to determine a basic regression model. The results of an OLS regression on this model are in Figure 7.1.

$$sp\_price\_usd = \alpha + \beta(sp\_price\_gold) + u \qquad (7.1)$$

The OLS regression indicated that both the constant, α, and the coefficient for the explanatory variable, β, were very significant, with p-values of 1.12×10^{-6} and 7.67×10^{-134} respectively. A strong value of 0.95 was also calculated for the R^2 of the model. The complete results of the OLS regression are in Figure 7.1, with a scatter plot in Figure 7.2.

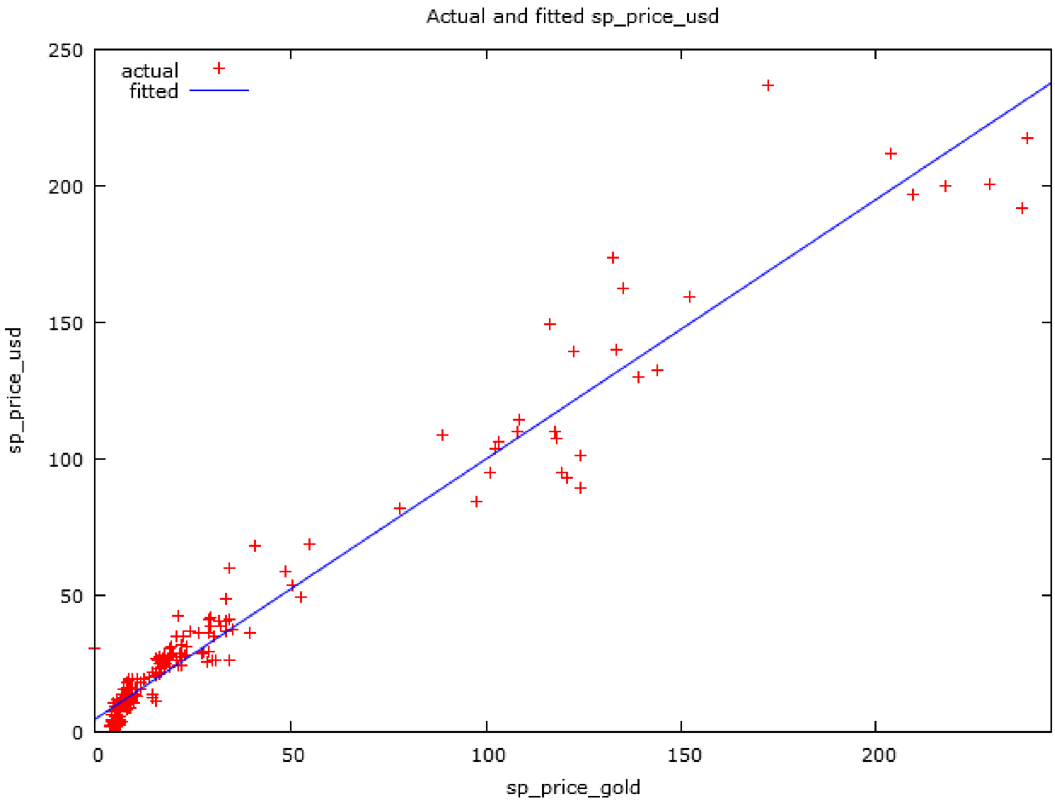
Figure 7.1: OLS Regression Results

```
Model 1: OLS, using observations 2012/07/10-2013/04/23 (T = 201)
Missing or incomplete observations dropped: 87
Dependent variable: sp_price_usd

                 coefficient   std. error   t-ratio    p-value
  --------------------------------------------------------------
  const           4.44011      0.883541      5.025    1.12e-06  ***
  sp_price_gold   0.952836     0.0150454    63.33     7.67e-134 ***

Mean dependent var   35.19617   S.D. dependent var   48.00966
Sum squared resid    21791.30   S.E. of regression   10.46442
R-squared            0.952729   Adjusted R-squared   0.952491
F(1, 199)            4010.758   P-value(F)           7.7e-134
Log-likelihood      -756.1457   Akaike criterion     1516.291
Schwarz criterion    1522.898   Hannan-Quinn         1518.965
```

Source: author's computation (GRETL).

Observation of the OLS regression results and scatter plot suggests that the model design is sound. Some comments should be made as to why. Since all auction goods are distributed with an inherent gold price, gold is effectively the reserve currency in the game. There is absolutely no necessity on the parts of players to ever convert gold into USD (or any other real world currency). As a result, any change in the gold/USD exchange rate will have a near immediate impact on the sale prices of goods in both the gold and USD auction houses. In the next section, this suspected correlation is statistically tested.

Figure 7.2: OLS Regression Scatter Plot

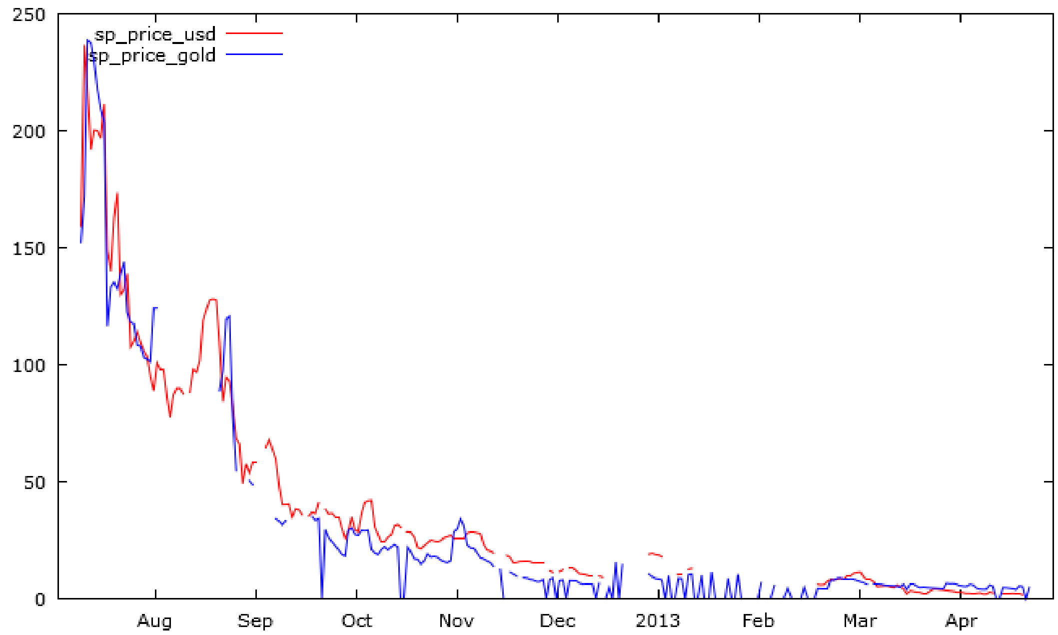


Source: author's computation (GRETL).

7.3 Correlation of USD Prices with Gold Prices

Visually inspecting the time series gives anecdotal evidence of a high correlation between the USD price and the gold (in-game currency) price in the auction house. Specifically looking at the Blacksmith Plan good “Sage’s Plight”, with its gold value converted to USD at the daily gold/USD exchange rates, the time series overlap quite closely (Figure 7.3).

Figure 7.3: Time Series Plots of USD Price with Gold Price of Sage’s Plight



Source: author’s computation (GRETL).

The distribution of prices is highest for values below \$16 (Figure 7.4). However, this distribution is not uniform across time, which distorts the calculation of the optimum selling price. The prices decrease almost logarithmically over time, rather than having a linear increase or decrease, or ideally a random walk nature. A serious deficiency of the optimum selling price equality is that its accuracy diminishes drastically the further the historical prices deviate from a random walk model.

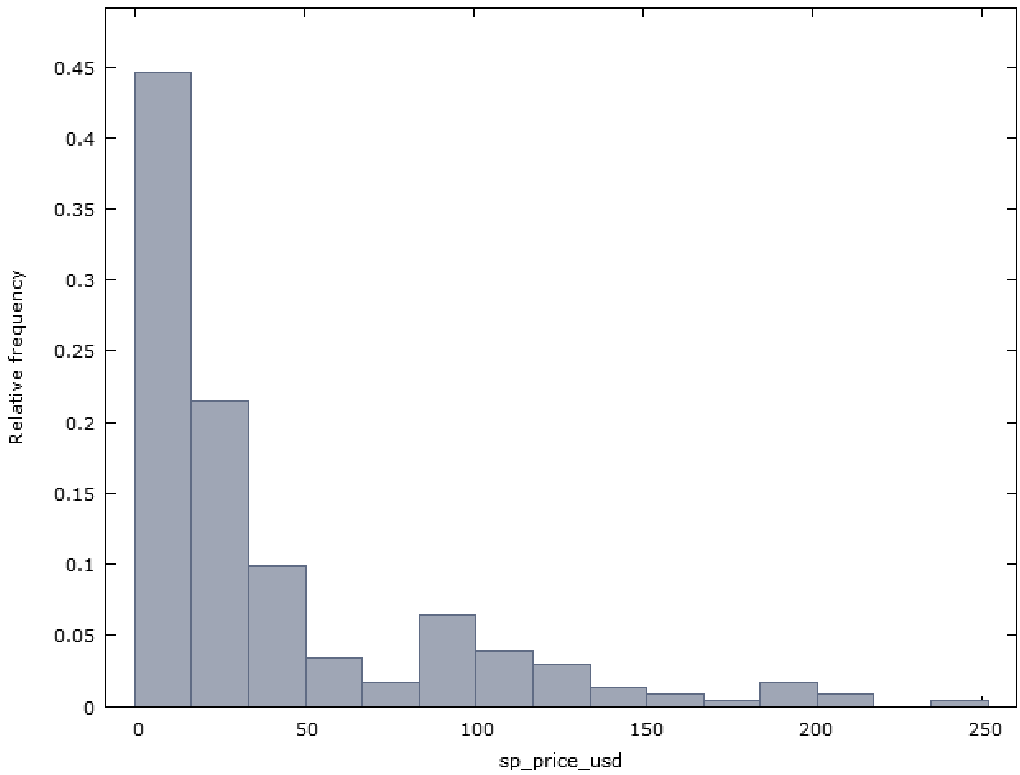
A test for correlation between the USD and gold prices gave a correlation coefficient of 0.98. This suggests that the correlation between prices in the gold auction house and the real currency auction house is significant.

Figure 7.4: Frequency Distribution of USD Prices for Sage's Plight

```
Frequency distribution for sp_price_usd, obs 1-288
number of bins = 15, mean = 40.107, sd = 48.2906

       interval          midpt   frequency    rel.     cum.

           < 16.793     8.3964       104     44.64%   44.64% ****************
    16.793 - 33.586     25.189        50     21.46%   66.09% *******
    33.586 - 50.379     41.982        23      9.87%   75.97% ***
    50.379 - 67.171     58.775         8      3.43%   79.40% *
    67.171 - 83.964     75.568         4      1.72%   81.12%
    83.964 - 100.76     92.361        15      6.44%   87.55% **
    100.76 - 117.55     109.15         9      3.86%   91.42% *
    117.55 - 134.34     125.95         7      3.00%   94.42% *
    134.34 - 151.14     142.74         3      1.29%   95.71%
    151.14 - 167.93     159.53         2      0.86%   96.57%
    167.93 - 184.72     176.33         1      0.43%   97.00%
    184.72 - 201.51     193.12         4      1.72%   98.71%
    201.51 - 218.31     209.91         2      0.86%   99.57%
    218.31 - 235.10     226.70         0      0.00%   99.57%
          >= 235.10     243.50         1      0.43%  100.00%

Missing observations = 55 (19.10%)
```

Source: author's computation (GRETL).

Figure 7.5: Frequency Distribution of USD Prices for Sage's Plight



Source: author's computation (GRETL)

Figure 7.6: Correlation of USD Price with Gold Price

```
corr(sp_price_usd, sp_price_gold) = 0.97607831
Under the null hypothesis of no correlation:
 t(199) = 63.3305, with two-tailed p-value 0.0000
```

Source: author's computation (GRETL).

7.4 Calculating the Optimum Selling Price

Restating Equation 6.15, derived in the theoretical section of this paper, the optimum selling price can be numerically computed:

$$\rho^* = v_0 + \frac{1 - F_V(\rho^*)}{f_V(\rho^*)} \tag{7.2}$$

Before presenting the empirical determinations of the various optimum selling prices, a note must be made on approximating the seller valuations, v_0, for each good. Taking Paarsch & Hong (2006, pg. 2) as the authoritative text on auction good valuations, it was found that economists often "guess" as to the seller valuation of goods. In fact, Paarsch & Hong (2006, pg. 65) rationalize that a normalized probability distribution of valuations is often a sufficient approximation.

In the case of the Diablo 3 real currency auction house, there is no established precedent for how sellers valuate a virtual good. Specifically, in order to estimate an exact value, v_0, one would need to develop an informed set of preference relations for the seller. Defining an important restriction, the seller valuation of a good is assumed to be the arbitrage price; which is the good's value in gold (in-game currency) converted to American dollars (USD) on the date that the bid was placed. This rationale may seem over simplified, but with seller valuation procedures being a complete unknown, this simplification acts to reduce the number of variables in the equation.

For the crafting plan "Sage's Plight", the optimum selling price for March 25, 2013 calculated as follows. The values of the variables for the Blacksmith Plan "Sage's Plight" are: \$4.62 USD for v_0, where v_0 is the arbitrage price, calculated as the North American Gold index price for March 24, 2013, converted to USD at the Gold/USD exchange rate for the same date; \$45.02 for the mean, determined as the average of all daily average prices from July 10, 2012 to March 24, 2013 in the North American USD index; and \$49.29 for the standard deviation, calculated using the same historical data as the mean. The result is a ρ^* of \$78.098, with the most recent winning bid being only \$2.45.

$$\rho^* = \frac{(v_0 + \mu) \pm \sqrt{(v_0 + \mu)^2 - 4(v_0\mu - \sigma^2)}}{2} \tag{7.3}$$

$$\rho^* = \frac{(4.62 + 45.02) \pm \sqrt{(4.62 + 45.02)^2 - 4(4.62 \times 45.02 - 49.29^2)}}{2} \tag{7.4}$$

The outrageous discrepancy between the calculated optimum selling price and the most recent winning bid is intuitive, once understood. Looking at the time series plot of the Blacksmith Plan good "Sage's Plight" (Figure 3.1) the historical prices are clearly shown to be decaying over time, almost logarithmically. This is due to not only players becoming more rational over time, but also due to the number of non-rational players decreasing. Past the first three months of a video game's launch, fewer new (non-rational) players will purchase the game.

Chapter 8

Experiments

8.1 Optimum Pricing Algorithm

In order to conduct experiments with the optimum selling price equation, an algorithm was derived using the indefinite solution. Rather than reducing the optimum selling price equality to the state of Equation 6.30, the original form in Equation 6.15 will be used. By running MATLAB experiments to actually compute the optimum selling price, one can directly observe the convergence of ρ to ρ^*.

The notation for the below algorithm is based on that used in Maurer & Ralston (2004, pg.174). This is in an arbitrary language called pseudo code, which is used for illustrative purposes. This code is then translated into an executable ".m" MATLAB file. The MATLAB source code is available in Figure A.1 in the Appendix).

The algorithm is explained as follows, from start to finish. In line 1 the algorithm begins. In line 2 the algorithm makes its first guess at the optimum selling price, the minimum price from the complete distribution of past winning bids. Line 3 is the beginning of the iterative section, so that all of lines 4 through 11 are part of a single iteration. Also in line 3 is the condition for remaining within the iteration: while the guess price is smaller-than-or-equal-to the maximum price (from the complete distribution of past winning bids) the current iteration may continue; alternatively once the guess price is greater than the maximum price then the algorithm jumps out of its iterative section (to line 12). Line 4 contains the right-hand-side (RHS) of Equation 6.15, the optimum reserve price, and assigns it to a value, ρ. Note that this is not the optimum reserve price, ρ^*, as it is not yet known whether the full equality

Algorithm 1 Optimum Pricing Algorithm

start algorithm
$price_{guess} \leftarrow price_{min}$
while $price_{guess} \leq price_{max}$ **do**
  $\rho \leftarrow v_0 + \frac{1-CDF(price_{guess})}{PDF(price_{guess})}$
  if $price_{guess} = \rho$ **then**
    $\rho^* \leftarrow price_{guess}$
    end algorithm
  else
    $price_{guess} \leftarrow price_{guess} + \$0.10$ USD
    next iteration
  end if
end while
end algorithm

Source: author's design (MATLAB).

in Equation 6.15 is satisfied: so far it is only known that the RHS of Equation 6.15 evaluates to ρ. Line 5 tests if the equality in Equation 6.15 is satisfied; if it is then the guess price is indeed the optimum selling price, which means its value can be assigned to ρ^*, in line 6; and consequently the algorithm is exited in line 7, having served its purpose. The resulting ρ^* using this algorithmic method is \$51.63.

Figure 8.1: Program Output optimum_reserve_price.m

```
Arbitrage Price = 4.62
rho_star = 51.63
rho_star_star = 3.9441
```

Source: author's computation (MATLAB).

But what if ρ^* is not found in the first iteration, or many iterations? Line 8 handles this situation by stating that "if the equality in Equation 6.15 is not satisfied then do the following . . . "; this quotation is neatly encompassed in the pseudo code term **else**. Line 9 is the next guess made by the algorithm, simply the current (failed) guess incremented by ten American cents. Line 10 then instructs the algorithm to jump back up to line 3 and perform the whole optimum selling price calculation and test for equality one more time, with the guess price now being ten cents higher.

8.2 Optimum Pricing Algorithm Exceptions

In the worst case scenario of an optimum selling price ρ^* never being found, then the iterative section in Equation 6.2 terminates when the guess price is greater than the maximum price, in line 3. When this test fails, the algorithm jumps to line 12 and then continues to line 13, where it ends, with no ρ^* ever being found. However, by properties of the PDF and CDF functions, this outcome is infeasible (but not impossible). At some point along the PDF and CDF curves, there will be a convergence of the RHS of Equation 6.1 to the value of v_0, the arbitrage price (Section 6.5). As the algorithm iterates from the minimum price to the maximum price, the value of ρ is steadily decreasing. By observing this computational behavior (Figure B.2) it is a characteristic of the data population that ρ will decrease from a number smaller than the maximum price, to a number greater than the arbitrage price of v_0.

$$\lim_{price_{guess} \to price_{max}} v_0 + \frac{1 - CDF(price_{guess})}{PDF(price_{guess})} = v_0 \tag{8.1}$$

This assumes that the value of the CDF approaches one, so that the second term on the left-hand-side (LHS) of Equation 8.1 collapses to zero. In reality though, there is always some positive, non-zero value attributed to the second term.

It is possible that ρ^* may never found, in the case where v_0 (the arbitrage price) is smaller than the minimum price. Additionally when the empirical CDF and PDF have too few samples relative to the \$0.10 increments in the guess price, the optimum pricing algorithm may fail. This very occurrence is reproduced computationally in Figure B.2, which is summarized below in Figure 8.2.

Figure 8.2: Program Output - non_convergence.m

```
Arbitrage Price = 1.25
min_price = 2.03
rho_star = 0
rho_star_star = -Inf
```

Source: author's computation (MATLAB).

As is shown, the algorithm failed to equate the LHS to the RHS of Equation 6.15. A complete evaluation of this failure is presented sequentially in the below three figures.

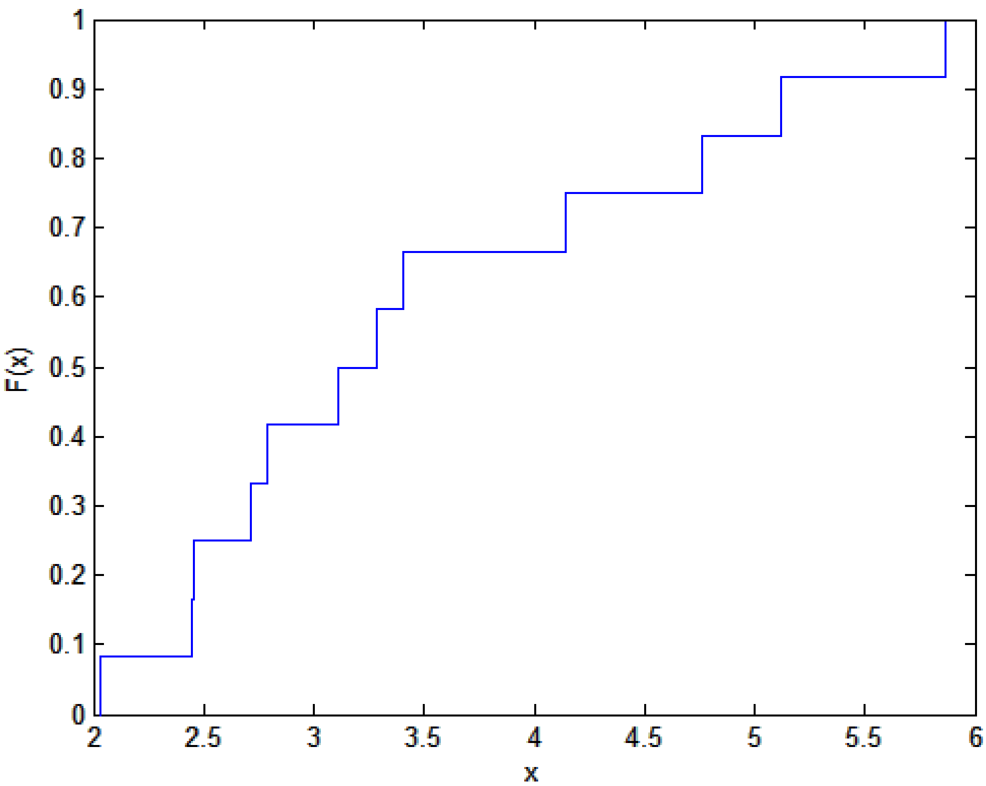
Truncating the dataset to its last twelve rows made the CDF and PDF to coarse. In the below Figure 8.3, the third column is the sale price for the given date, the forth is the mean, the fifth is the standard deviation, and the sixth is the arbitrage price.

Figure 8.3: Input Data - dataset.dat (truncated to last 12 rows)

247	13-Mar-13	4.76	47.37	49.63	5.43
248	14-Mar-13	5.87	47.16	49.59	5.27
249	15-Mar-13	5.12	46.94	49.56	4.98
250	16-Mar-13	4.14	46.73	49.53	6.24
251	17-Mar-13	2.03	46.50	49.50	3.88
252	18-Mar-13	3.41	46.29	49.47	6.25
253	19-Mar-13	3.11	46.07	49.44	6.26
254	20-Mar-13	2.79	45.86	49.41	5.36
255	21-Mar-13	2.71	45.65	49.38	4.74
256	22-Mar-13	2.46	45.44	49.35	4.93
257	23-Mar-13	2.45	45.23	49.32	4.77
258	24-Mar-13	3.29	45.02	49.29	4.62

Source: author's computation (D3RMT).

Figure 8.4: CDF of Non-Convergent Dataset



Source: author's computation (MATLAB).

As the guess price was incremented by \$0.10 the value of ρ kept decrementing at too great an interval. The results of this are clearly visible when looking at the output of the non-convergence MATLAB code in Figure B.2.

8.3 Optimum Pricing Algorithm with Logarithmic Correction

As a final correcting step, the log_e (abbreviated as ln) of the resulting ρ^* is used. This is a crude form of correcting for the logarithmic decay of prices. When observing the data on auctioned good prices, there is a correlation between good prices and the length of time from the opening of the auction house (the data sample begins on July 10, 2012). Based on observation, the prices for goods over the past three months are an order of magnitude smaller than the prices in the first three months (assuming a total sample lifetime of at least nine months, from July 2012 through April 2013). The end result of taking the ln of ρ^* is that it results in a usable number for the optimum selling price, ρ^{**}. Otherwise, ρ^* takes the entire distribution of prices, including the order of magnitude higher prices in the first three months of the auction house's inception.

Algorithm 2 Optimum Pricing Algorithm with Logarithmic Correction

start algorithm
$price_{guess} \leftarrow price_{min}$
while $price_{guess} \leq price_{max}$ **do**
  $\rho \leftarrow v_0 + \frac{1-CDF(price_{guess})}{PDF(price_{guess})}$
  if $price_{guess} = \rho$ **then**
    $\rho^* = price_{guess}$
    $\rho^{**} = log_e(\rho^*)$
    end algorithm
  else
    $price_{guess} \leftarrow price_{guess} + \$0.10$ USD
    next iteration
  end if
end while
end algorithm

Source: author's design (MATLAB).

A possible extension to this paper would be to rigorously determine a correction method that takes into account the probability distribution, mean, and variance of a good. One idea would be to see how much the sample population can be truncated (from the auction house's inception to the present) so that the standard deviation of the good's price is minimally centered around zero. However, such a noble quest is beyond the scope of this paper. The $ln(\rho^*) = \rho^{**}$ method yields an acceptable result for illustrating the usage of the optimum pricing algorithm. And mathematically, the counterintuitive value of ρ^* is in fact algorithmically sound as it takes into account the entire probability distribution of prices, over the life of the auction house.

One potential weakness with the ρ^{**} correction is that any ρ^* below a value of one will yield a negative value for ρ^{**}. Coincidentally, there is a set floor on auction prices of $1.25 USD in the Diablo 3 real currency auction house (Battle.net 2012). Hence the ρ^{**} bootstrap is computationally sound and feasible for the desired experiments.

Chapter 9

Applications

9.1 Bond Auctions

This is a special case where the idea of an optimum selling price has real world applications. In the case of California municipal bonds examined in Hendricks & Porter (2007, ch. 3) the reserve price can be used to identify expected revenues for bidders. Using the methods discussed in this thesis, the idea of an optimum reserve price for sellers can be applied, thus helping the bond issuer maximize its auction revenues.

Another application of the optimum selling price to bond auctions is in the use of bond auctions as a litmus test for investor confidence (Bartha & Quinn 2011). In the case of the current Eurozone crisis, auctions for PIIGS bonds serve as a signal for investor confidence in the issuing country. It is not only the bond sale price that matters but also the bond spread, or yield price that serves as an investor confidence signal. One interesting expansion of the optimum selling price idea would be to derive an optimum bond spread, that factors in bidder confidence. This may touch on areas of behavioral finance, but on a rudimentary level investor preferences could be approximated using historical distributions.

9.2 The Diablo 3 Auction House Goods Index

The Diablo 3 Real Money Auction House Price Tracker, or D3RMT, is a collection of indices that list the historical prices of virtual goods for auction (D3RMT 2012). Although the name D3RMT is now defunct (the website was rebranded as DiabloHub in the first quarter of 2013) it is used in this research paper for the sake of consistency. There are many goods in the Diablo 3 videogame that are available for auction, unfortunately not all of them are comprehensively represented in D3RMT. This was not an inhibiting factor towards an academic investigation, as many of the more popular items are listed. Specifically, this thesis focused on the North American Gold (in-game currency) and North American USD (real currency) auction houses, and their corresponding indices on D3RMT.

Although the landing page for D3RMT is misleading, one only needs to click on the link for "Price Tracker" at the top of the page. Once this new page loads, a selection box on the right gives a choice of four indices: North American Gold, North American USD, European Gold, and European EUR. Assuming that the North American USD index is selected, a vast array of goods will then be displayed on the page. Each good listing contains the following information; the most recent sale price, the sale price from three days ago, the sale price from five days ago, and the sale price from seven days ago. Clicking on the image of a good opens a more detailed page. Here a histogram can be modified to show the complete history of sale prices for the good since the auction house first opened on July 10, 2012. Basic functionality for the goods' histograms includes; displaying all data points, displaying daily averages, displaying monthly averages, and limiting or expanding the date range of displayed data points.

Chapter 10

Conclusion

The theoretical analysis of this thesis concludes that a feasible, closed-form expression for calculating the optimum selling price was realizable. In practice, however, the numerical calculation of the optimum selling price gives a value that is significantly higher than the most recent winning bid. This is due to the number of rational players increasing over time in proportion to the number of irrational players. Evidently, the optimum selling price equality cannot compensate for this behavior as it assumes a constant mean throughout the set of repeated auctions for the same good. Perhaps this is also due to a change in player preferences over time, which is why the idea of deriving an optimum selling price expression through preference relations was suggested. Academic reasoning hints that if the utility maximization and expenditure minimization problems could be combined (in a mathematically feasible expression) then the logarithmic decay in the utility of a good could somehow also be incorporated. This speculation would be a tantalizing topic for an extension of the thesis, or a new one altogether. The existing literature that came closest to this approach was Segal (2003, pg. 513).

The results of the experiments were not as initially expected. Using the closed-form solution model for determining an optimum selling price for Blacksmith's Plan "Sage's Plight" gave a value of \$78.098. The algorithmic method gave a slightly better value of \$51.63, and an even more realistic value of \$3.94 when using logarithmic correction. Stressing these results, it appears that the observation of logarithmic decay of prices over time proved useful. Of course the *log* of the closed-form model results can also be taken, giving an optimum selling price of \$4.36. This leads to the conclusion that the method of evaluating the distribution of prices is less important than recognizing the pattern of prices over time. Contrary to the methods proposed in Segal (2003, pg.513) and Paarsch & Hong (2006, pg. 66), recognizing the pattern of the growth or decay of prices gives a significantly better estimate of an optimum selling price.

Independent of the theoretical and empirical analyses of this thesis is the behavior of the players (both bidders and sellers) in the Diablo 3 auction house. What is impressive is that since the auction house's inception in mid-2012, the players themselves have built electronic indices to track the in-game goods (D3RMT 2012). Quite aptly, the programmers of the D3RMT Diablo 3 auction house index transformed the goods into commodities. The price for which a good is auctioned contributes to the commodity's price in the form of a daily average. Without going into the full details of the D3RMT index it was a surprise to find that large arbitrage opportunities still persist.

What seems to affect the price of a commodity is the per-day highest bid that uniformed players are willing to pay for a good. As an example, a commodity could have a daily average of 10€ but a player in the auction house may still be willing to pay 15€ for it. Viewing a timeseries plot of a commodity's daily average price over its lifetime does show a convergence to an efficient price, but players seem to ignore this information when they participate in auctions. However there seems to be a subset of "professional" players who are members of the D3RMT index (and probably other indices) who use advanced features copied from real indices. An example of this is the use of email notifications for when a commodity reaches a certain average daily price threshold.

Commenting on the behavior of players, there is evidence that fewer people overall are playing Diablo 3, thus there are fewer players in the auction house (Mu-hyun 2012). This may explain the apparent long term convergence of commodity prices versus the seemingly uncorrelated values of winning bids. With more players, especially new players who would have less information on the price of goods, there would be greater variation in the value of winning bids. Over time, with the removal of less informed players, there would be a trend towards price convergence. However, in spite of convergence, there would still be sufficient outliers of uninformed players to provide bids that are significantly higher than the average price of a commodity on the D3RMT index.

One conclusion that can be drawn from D3RMT commodity price convergence and the drop in auction participants is that in order for a virtual auction to be efficient there needs to be enough of an incentive to play. This is perhaps the biggest difference between an auction house like Ebay and the one in Diablo 3; players on Ebay have long term incentives to play repeatedly. The most notable structural difference in Ebay is the ability for sellers to run an actual business in the auction house, complete with a webpage storefront and advertising within the auction house itself. Diablo 3 provides no webpage storefronts for sellers and no advertising. Also, Ebay has non-uniform goods. Although Diablo 3 has a variety of virtual goods, their prices all seem to converge to a value below 10€. Additionally, the utility of the goods is limited to a near uniform utility of being either a weapon or an in-game tradable gem. Ebay comparatively has an infinite number of disparate goods with prices ranging from the minute to the extreme, with utilities spanning the entire range of human consumption. This it seems is the key to Ebay's efficiency, where participants tend to bid at values similar to the book value of an auctioned good; Ebay auctions function as a marketplace or a bazar while Diablo 3 auctions appear as a pastime or hobby, or entertainment with money; I.E., gambling.

Bibliography

BAGCHI, A., A. CHAUDHARY, R. GARG, M. T. GOODRICH, & V. KUMAR (2001): "Seller-focused algorithms for online auctioning."

BAJARI, P. & A. HORTACSU (2003): "Economic insights from internet auctions: A survey." *Working Paper 10076*, National Bureau of Economic Research.

BARTHA, E. & E. QUINN (2011): "Bond auctions test contagion fears." *Wall Street Journal* .

BATTLE.NET (2013a): "Battle.net - diablo III - auction house - bidding." https://us.battle.net/support/en/article/diablo-iii-auction-house-functionality#q4.

BATTLE.NET (2013b): "Battle.net - diablo III - auction house - transaction fees." https://eu.battle.net/support/en/article/diablo-iii-auction-house-general-information#q4.

BATTLE.NET (2013c): "Battle.net - diablo III - forum." http://eu.battle.net/d3/en/forum/topic/5967189342.

D3RMT (2012): "Diablo III RMT." http://www.d3rmt.com/.

DIABLOPROGRESS.COM (2013): "DiabloProgress - diablo 3 rankings." http://www.diabloprogress.com/.

FIELDS, T., G. MARQUES, & B. COTTON (2011): *Social Game Design: Monetization Methods and Mechanics*. Elsevier.

GIBBONS, R. (1992): *Game Theory for Applied Economists*. Princeton University Press.

GRANDY, C. (1989): "New jersey corporate chartermongering, 1875-1929." *The Journal of Economic History* **49(3)**: pp. 677–692.

HENDRICKS, K. & R. H. PORTER (2007): "Chapter 32 an empirical perspective on auctions." In M. ARMSTRONG AND R. PORTER (editor), "Handbook of Industrial Organization," volume Volume 3, pp. 2073–2143. Elsevier.

MAS-COLELL, A., M. D. WHINSTON, & J. R. GREEN (1995): *Microeconomic Theory.* Oxford University Press, Inc.

MAURER, S. B. & A. RALSTON (2004): *Discrete Algorithmic Mathematics.* A K Peters, Limited.

MILGROM, P. R. & R. J. WEBER (1982): "A theory of auctions and competitive bidding." *Econometrica* **50(5)**: p. 1089.

MU-HYUN, C. (2012): "Diablo 3 popularity waning fast." *The Korea Times* .

PAARSCH, H. J. (1992): "Deciding between the common and private value paradigms in empirical models of auctions." *Journal of Econometrics* **51(1–2)**: pp. 191–215.

PAARSCH, H. J. (1997): "Deriving an estimate of the optimal reserve price: An application to british columbian timber sales." *Journal of Econometrics* **78(1)**: pp. 333–357.

PAARSCH, H. J. & H. HONG (2006): *An Introduction to the Structural Econometrics of Auction Data.* The MIT Press.

SCHRAMM, M. (2013): "Diablo 3 director jay wilson: Auction houses 'really hurt' game." http://www.joystiq.com/2013/03/28/diablo-3-director-jay-wilson-auction-houses-really-hurt-game/.

SEGAL, I. (2003): "Optimal pricing mechanisms with unknown demand." *The American Economic Review* **93(3)**: pp. 509–529.

VICKREY, W. (1961): "Counterspeculation, auctions, and competitive sealed tenders." *The Journal of Finance* **16(1)**: p. 8–37.

XFIRE (2013): "Xfire - diablo III." http://www.xfire.com/games/d3.

Appendix A

MATLAB Code

Table A.1: Variable Dictionary - MATLAB

cdf_rho:	The normal cumulative distribution function of the set of prices in price_usd, at the guess_price.
dataset:	The complete set of data for a single good, including the historical prices from both the USD and gold (in game currency) auction houses. Note that the gold price is converted to USD at an exchange rate of USD/gold for that same date.
guess_price:	An arbitrary price within the range of prices in price_usd. guess_price is selected according to the rules in the optimum pricing algorithm.
iteration:	An index tracking the current iteration of the iterative part of the optimum pricing algorithm.
max_price:	The maximum price from price_usd.
mean_price:	The statistical mean of the set of prices in price_usd.
min_price:	The minimum price from price_usd.
pdf_rho:	The normal probability density function of the set of prices in price_usd, at the guess_price.
price_usd:	The complete set of all historical prices for a single good in the North American USD auction house.
price_gold_to_usd:	The complete set of all historical prices for a single good in the North American Gold auction house. Note that the gold price is converted to USD at an exchange rate of USD/gold for that same date.
rho:	The resulting value of the right-hand-side of Equation 6.1; not to be confused with ρ^* or the left-hand-side of the equation.
rho_star:	ρ^* in the optimum selling pricing formula, Equation 6.1.
std_dev_price:	The standard deviation of the set of prices in price_usd.
v0:	The arbitrage price, v_0. This is equal to the most recent price of the good in the North American Gold auction house. Note that the gold price is converted to USD at an exchange rate of USD/gold for that same date.

Figure A.1: Source Code - optimum_reserve_price.m

```
% Load the data.
load ('C:/matlab/dataset.dat');
price_usd = dataset(:,3);
price_gold_to_usd = dataset(:,6);
v0 = price_gold_to_usd(numel(price_usd));

% Descriptive Statistics.
min_price = min(price_usd);
max_price = max(price_usd);
mean_price = mean(price_usd);
std_dev_price = std(price_usd);

% Find Optimum Reserve Price: rho_star.
guess_price = min_price;
iteration = 1;
rho_star = 0;
while (guess_price <= max_price)
    % Compute the cdf of the normal distribution with
    % mean of mean_price and
    % standard deviation of std_dev_price at
    % the value of the guess_price,
    % guess_price ranging from min_price to max_price,
    % incrementing by $0.10 (10 cents USD) per iteration.
    cdf_rho = cdf('Normal', guess_price, mean_price, std_dev_price);
    pdf_rho = pdf('Normal', guess_price, mean_price, std_dev_price);
    rho = v0 + ((1 - cdf_rho)/pdf_rho);

    % Output the value of rho for each iteration.
    fprintf(1,'Iteration %d, rho = %g\n', iteration, rho);

    % Stop when left-hand-side equals right-hand-side of rho equation.
    if ((guess_price < (rho + 0.1)) && (guess_price >= (rho - 0.1)))
        rho_star = guess_price;
        break;
    else
        guess_price = guess_price + 0.1;
        iteration = iteration + 1;
        continue;
    end
end

% Final Output.
fprintf(1,'\nArbitrage Price = %g\nrho_star = %g\nrho_star_star = %g\n', v0, rho_star, log(rho_star));
```

Source: author's computation (MATLAB).

Figure A.2: Source Code - non_convergence.m

```
% Load the data.
load ('C:/matlab/dataset.dat');
price_usd = dataset(:,3);
price_gold_to_usd = dataset(:,6);
v0 = 1.25;

% Descriptive Statistics.
min_price = min(price_usd);
max_price = max(price_usd);
mean_price = mean(price_usd);
std_dev_price = std(price_usd);

% Find Optimum Reserve Price: rho_star.
guess_price = min_price;
iteration = 1;
rho_star = 0;
while (guess_price <= max_price)
    % Compute the cdf of the normal distribution with
    % mean of mean_price and
    % standard deviation of std_dev_price at
    % the value of the guess_price,
    % guess_price ranging from min_price to max_price,
    % incrementing by $0.10 (10 cents USD) per iteration.
    cdf_rho = cdf('Normal', guess_price, mean_price, std_dev_price);
    pdf_rho = pdf('Normal', guess_price, mean_price, std_dev_price);
    rho = v0 + ((1 - cdf_rho)/pdf_rho);

    % Output the value of rho for each iteration.
    fprintf(1,'Iteration %d, guess_price = %g, rho = %g\n', iteration, guess_price, rho);

    % Found rho_star when left-hand-side equals right-hand-side.
    if ((guess_price < (rho + 0.1)) && (guess_price >= (rho - 0.1)))
        rho_star = guess_price;
    end

    guess_price = guess_price + 0.1;
    iteration = iteration + 1;
end

% Final Output.
fprintf(1,'\nArbitrage Price = %g\nmin_price = %g\n', v0, min_price);
fprintf(1,'rho_star = %g\nrho_star_star = %g\n', rho_star, log(rho_star));
```

Source: author's computation (MATLAB).

Appendix B

MATLAB Code Output

Figure B.1: Program Output - optimum_reserve_price.m (output truncated for readabilty)

```
>> optimum_reserve_price
Iteration 1, rho = 131.87
Iteration 2, rho = 131.544
Iteration 3, rho = 131.219
Iteration 4, rho = 130.895
Iteration 5, rho = 130.572
Iteration 6, rho = 130.251
Iteration 7, rho = 129.93
Iteration 8, rho = 129.611
Iteration 9, rho = 129.293
Iteration 10, rho = 128.976
Iteration 11, rho = 128.66
Iteration 12, rho = 128.345
Iteration 13, rho = 128.032
Iteration 14, rho = 127.719
Iteration 15, rho = 127.408
Iteration 16, rho = 127.098
Iteration 17, rho = 126.789
Iteration 18, rho = 126.481
Iteration 19, rho = 126.174
Iteration 20, rho = 125.868
...
Iteration 479, rho = 53.0284
Iteration 480, rho = 52.9523
Iteration 481, rho = 52.8764
Iteration 482, rho = 52.8007
Iteration 483, rho = 52.7252
Iteration 484, rho = 52.6499
Iteration 485, rho = 52.5748
Iteration 486, rho = 52.4998
Iteration 487, rho = 52.4251
Iteration 488, rho = 52.3505
Iteration 489, rho = 52.2762
Iteration 490, rho = 52.202
Iteration 491, rho = 52.128
Iteration 492, rho = 52.0542
Iteration 493, rho = 51.9806
Iteration 494, rho = 51.9071
Iteration 495, rho = 51.8339
Iteration 496, rho = 51.7608
Iteration 497, rho = 51.6879

Arbitrage Price = 4.62
rho_star = 51.63
rho_star_star = 3.9441
```

Source: author's computation (GRETL).

Figure B.2: Program Output - non_convergence.m

```
>> non_convergence
Iteration 1, guess_price = 2.03, rho = 6.98675
Iteration 2, guess_price = 2.13, rho = 6.35098
Iteration 3, guess_price = 2.23, rho = 5.80683
Iteration 4, guess_price = 2.33, rho = 5.33903
Iteration 5, guess_price = 2.43, rho = 4.93513
Iteration 6, guess_price = 2.53, rho = 4.58492
Iteration 7, guess_price = 2.63, rho = 4.28003
Iteration 8, guess_price = 2.73, rho = 4.01353
Iteration 9, guess_price = 2.83, rho = 3.77967
Iteration 10, guess_price = 2.93, rho = 3.5737
Iteration 11, guess_price = 3.03, rho = 3.39161
Iteration 12, guess_price = 3.13, rho = 3.23008
Iteration 13, guess_price = 3.23, rho = 3.08628
Iteration 14, guess_price = 3.33, rho = 2.95785
Iteration 15, guess_price = 3.43, rho = 2.84277
Iteration 16, guess_price = 3.53, rho = 2.73934
Iteration 17, guess_price = 3.63, rho = 2.6461
Iteration 18, guess_price = 3.73, rho = 2.5618
Iteration 19, guess_price = 3.83, rho = 2.48536
Iteration 20, guess_price = 3.93, rho = 2.41588
Iteration 21, guess_price = 4.03, rho = 2.35254
Iteration 22, guess_price = 4.13, rho = 2.29466
Iteration 23, guess_price = 4.23, rho = 2.24165
Iteration 24, guess_price = 4.33, rho = 2.19297
Iteration 25, guess_price = 4.43, rho = 2.14818
Iteration 26, guess_price = 4.53, rho = 2.10687
Iteration 27, guess_price = 4.63, rho = 2.0687
Iteration 28, guess_price = 4.73, rho = 2.03335
Iteration 29, guess_price = 4.83, rho = 2.00055
Iteration 30, guess_price = 4.93, rho = 1.97006
Iteration 31, guess_price = 5.03, rho = 1.94166
Iteration 32, guess_price = 5.13, rho = 1.91517
Iteration 33, guess_price = 5.23, rho = 1.89042
Iteration 34, guess_price = 5.33, rho = 1.86725
Iteration 35, guess_price = 5.43, rho = 1.84553
Iteration 36, guess_price = 5.53, rho = 1.82514
Iteration 37, guess_price = 5.63, rho = 1.80597
Iteration 38, guess_price = 5.73, rho = 1.78792
Iteration 39, guess_price = 5.83, rho = 1.7709

Arbitrage Price = 1.25
min_price = 2.03
rho_star = 0
rho_star_star = -Inf
```

Source: author's computation (MATLAB).