

Title: Analysis of Krylov subspace methods

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Abstract: After the derivation of the Conjugate Gradient method (CG) and the short review of its relationship with other fields of mathematics, this thesis is focused on its convergence behaviour both in exact and finite precision arithmetic. Fundamental difference between the CG and the Chebyshev semi-iterative method is described in detail. Then we investigate the use of the widespread linear convergence bound based on Chebyshev polynomials. Through the example of the composite polynomial convergence bounds it is showed that the effects of rounding errors must be included in any consideration concerning the CG rate of convergence relevant to practical computations. Furthermore, the close correspondence between the trajectories of the CG approximations generated in finite precision and exact arithmetic is studied. The thesis is concluded with the discussion concerning the sensitivity of the closely related Gauss-Christoffel quadrature. The last two topics may motivate our further research.

Keywords: Conjugate Gradient Method, Chebyshev semi-iterative method, finite precision computations, delay of convergence, composite polynomial convergence bounds, sensitivity of the Gauss-Christoffel quadrature