In this thesis we study sparse classes of graphs and their properties usable for design of algorithms and data structures. Our specific focus is on the concepts of bounded expansion and tree-depth, developed in recent years mainly by J. Nešetřil and P. Ossona de Mendez. We first give a brief introduction to the theory as whole and survey tools and results from related areas of parametrised complexity and algorithmic model theory.

The main part of the thesis, application of the theory, presents two new dynamic data structures. The first is for keeping a tree-depth decomposition of a graph, the second counts appearances of fixed subgraphs in a given graph. The time and space complexity of operations of both structures is guaranteed to be low when used for sparse graphs.