Title: Krylov subspace methods: Theory, applications and interconnections
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Abstract: After recalling of properties of Chebyshev polynomials and of stationary iterative methods, this thesis is focused on the description of Conjugate Gradient Method (CG), the Krylov method of the choice for symmetric positive definite matrices. Fundamental difference between stationary iterative methods and Krylov subspace methods is emphasized. CG is derived using the minimization of the quadratic functional and the relationship with several other fields of mathematics (Lanczos method, orthogonal polynomials, quadratic rules, moment problem) is pointed out. Effects of finite precision arithmetic are emphasized. In compliance with the theoretical part, the numerical experiments examine a bound derived assuming exact arithmetic which is often presented in literature. It is shown that this bound inevitably fails in practical computations. The thesis is concluded with description of two open problems which can motivate further research.

Keywords: Krylov subspace methods, convergence behaviour, numerical stability, spectral information, convergence rate bounds