# UNIVERZITA KARLOVA V PRAZE

## FAKULTA SOCIÁLNÍCH VĚD

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# Shadow Price of Air Pollution Emissions in the Czech energy sector – Estimation from Distance Function

*Diplomová práce*


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## Abstrakt

V této diplomové práci aplikujeme parametrickou input distance funkci zahrnující jak žádoucí, tak nežádoucí výstupy, čímž dosáhneme ucelenější reprezentace produkční technologie. Na základě Shephardovi (1970) teorie duality odvodíme z odhadnuté input distance funkce stínové ceny nežádoucích výstupů v českém energetickém sektoru pro období 2002 – 2007. Mediány našich odhadnutých stínových cen jsou 8374, 1198, 2805, 6051 a 8549 € za tunu PM, $SO_2$, $NO_x$, CO a VOC. Učiníme rozklad odhadnutých stínových cen a testujeme hypotézy, že mezní náklady na zamezení klesají v čase; že mezní náklady na zamezení rostou s klesající úrovní vypouštěných emisí; a že mezní náklady na zamezení rostou s klesající emisní mírou.



**Klíčová slova:** stínové ceny, vzdálenostní funkce, nežádoucí výstupy, mezní náklady na zamezení

**JEL klasifikace:** C61, D24, Q53



## Abstract

This thesis employs a parametric input distance function that incorporates both desirable and undesirable outputs to provide a more complete representation of the production technology. Based on the Shephard (1970) theory of duality, we derive the shadow prices of undesirable outputs in the Czech energy sector on the data over the period 2002 – 2007. The medians of our shadow prices estimates are 8374, 1198, 2805, 6051 and 8549 € per ton of PM, $SO_2$, $NO_x$, CO and VOC, respectively. We decompose shadow prices estimates and test the hypotheses that the marginal abatement cost decline over time; that marginal abatement cost rice with the declining emission level; and that marginal abatement cost rice with declining emission rate.



**Key Words:** shadow prices, distance function, undesirable outputs, marginal abatement cost

**JEL classification:** C61, D24, Q53

## Prohlášení

1. Prohlašuji, že jsem předkládanou práci zpracoval/a samostatně a použil/a jen uvedené prameny a literaturu.

2. Souhlasím s tím, aby práce byla zpřístupněna pro studijní a výzkumné účely.

V Praze dne 28. 7. 2011 Lukáš Rečka

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# Master Thesis Proposal

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**Proposed Topic:**

Shadow Price of Air Pollution Emissions in Czech energy sector – Estimation from *Distance Function*

**Topic Characteristics:**

The main aim of the thesis is to estimate the shadow prices of classical air pollution emissions in Czech energy sector. "The shadow price can be interpreted as the opportunity cost of reducing an additional unit of undesirable output (*emissions of pollutants*) in terms of forgone desirable output, which is equivalent to the marginal cost of pollution abatement to the producer." Bauman et al. (2008), p. 519. To the shadow price estimation I will use optimization of a parameterized Distance Function from which we get the estimated shadow price according to Fare et al. (1993) and Bauman et al. (2008). I will also analyze the factors that affect the marginal abatement costs using econometric regression of annual marginal abatement costs on other output from the distance function optimization (Rate of technological change, Emission rate).

The estimation of emission shadow prices can be very useful by setting of the environmental regulation. The Czech Republic has specific structure of energy sector (big share of brown coal and district heating) and therefore it would be appropriate to have own estimate of the emission shadow prices.

To the analysis I will use data of produced emissions (REZZO database), produced heat and electricity and their prices (ERU/firm reports), fuel inputs (REZZO database/firm reports), labor and capital inputs and revenues (firm reports/CreditInfo) from the period 1993 - 2008. I will cooperate with the Charles University Environment Centre, which will provide a part of the data.

**Hypotheses:**

1. Hypothesis #1: Marginal abatement costs decline over time.
2. Hypothesis #2: Marginal abatement costs rice with the rate of technological change.
3. Hypothesis #3: Marginal abatement costs rice with declining emission rate.

**Methodology:**

I will describe the theoretical concept of Input/Output Distance Function and also the difference between Input and Output Distance function. Then I will describe how to estimate shadow price from Distance Function. Furthermore I will make a literature review of shadow price estimation form Distance Function and other concepts.
I will use a optimized parameterized Input or Output Distance Function to estimate shadow prices of classical pollutants ($SO_2$, $NO_x$, PM, $CO_2$). I will come from Bauman et al. (2008), but in addition I will encompass not only productivity innovations but also end-of-pipe innovations into my analysis. The desirable outputs are electricity and heat in this case. Then I will make econometric regression of estimated annual Marginal Abatement Costs on Rate of technological change and Emission rate to get their influence on Marginal Abatement Costs.

**Outline:**

1. Introduction
2. Literature review
    2.1. Input/Output Distance Function
    2.2. Shadow price estimation form Distance Function
    2.3. Empirical evidence
3. Distance function optimisation
    3.1. Model specification
    3.2. Data description
    3.3. Empirical results
4. Decomposition of Marginal Abatement Costs
5. Results summary and interpretation
6. Conclusions

**Core Bibliography:**


Bauman, Y., Lee, M., Seeley, K. (2008) Does Technological Innovation Really Reduce Marginal Abatement Costs? Some Theory, Algebraic Evidence, and Policy Implications. *Environ Resource Econ* **40**:507–527.

Fare R., S. Grosskopf, C.A.K. Lovell and S. Yaisawarng (1993): "Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach," *Review of Economics and Statistics* **75**. pp. 375-80.

Fare, R., and D. Primont (1995): Multi-Output Production and Duality: Theory and Applications. Kluwer Academic Publishers, Netherlands.

Fare, R (1988): Fundamentals of Production Theory, Springer-Verlag, Berlin.

Pittman, R.W. (1983): "Multilateral Productivity Comparisons with Undesirable outputs", Economic Journal. 93, pp. 883-891.

Shepard (1970): "Theory of Cost and Production Functions", Princeton University Press


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# 1. Introduction

Firms produce desirable outputs by using set of inputs and as by-products the firms can also produce undesirable outputs such as emissions of pollutants. The main aim of this thesis is to estimate the shadow prices of classical air pollutant in the Czech energy sector. Actually, we want to estimate the marginal abatement cost of emission of classical pollutants. However as Bauman, Lee, & Seely (2008, p. 519) write: "*The shadow price can be interpreted as the opportunity cost of reducing an additional unit of undesirable output (emissions of pollutants) in terms of forgone desirable output, which is equivalent to the marginal cost of pollution abatement to the producer.*" Therefore we focus on estimation of shadow prices under which we also understand the marginal abatement cost. For the estimation, we employ the input distance function in quadratic form. We will also try to decompose the marginal abatement costs (MAC) of emissions and analyze the factors that might affect them, such as emission level or emission concentrations. We will test the hypotheses that the marginal abatement cost decline over time in our relative short period of six years; that marginal abatement cost rice with the declining emission level; and that marginal abatement cost rice with declining emission rate.

There are more approaches, how to estimate the shadow prices of emissions. Lee (2005) mentions two approaches to estimating marginal abatement cost of pollutants – *the cost function approach* and *the distance function approach* based on theory of duality (Shephard 1970). Until the eighties of 20$^{th}$ century, the shadow prices of emissions were estimated via the neoclassical cost function from the abatement cost invested to the emission reduction. This approach tends to be bias. Lee (2005, pp. 104-105) notes that "*use of a neoclassical* *cost functions might lead to under-estimation of marginal abatement costs*." Pitman (1983, p. 887) says to this problematic: "*…the most difficult and challenging task is likely to be the assigning of shadow prices to undesirable outputs. Even when there are explicit engineering or econometric estimates available for a particular industry or area, these are likely to be subject to a wide range of error. Where exactly appropriate estimates are not available - as is especially likely for a sample of individual plants - then, of course, an additional error is imposed.*" There is also another critique of the estimation of emission shadow prices from abatement expenditures. As write Hailu & Veeman (2000), the estimation of emission shadow prices from abatement expenditures is very complicated "*because it is increasingly difficult to distinguish between 'productive' and pollution abatement expenditures on capital or other inputs*". (Hailu & Veeman, 2000, p. 252) Based on this, we focus on the second approach based on firm efficiency estimation.

The original studies discussing the effects of undesirable outputs production have focused on the proper measurement of performance of firms producing the undesirable outputs. Pittman (1983) shows how to adjust the productivity indexes. He derives the shadow prices from survey data on abatement expenditures by producers and u ses the data to the construction of an enhanced index of productivity factors. Färe R. , Grosskopf, Lovell, & Pasurka (1989) apply the linear programming approach, which allows that the technology can reflect the scarcity of freely disposable, undesirable outcomes which is regulated. By this, they don't have to estimate the prices of the undesirable outputs explicitly.

The studies following Pitman (1983) and Färe et al. (1989) findings focus already on other approach and aim. Since the nineties, the studies estimate the emission shadow prices rather based on the duality theories. Such shadow prices estimates consider not only the partial information about cost, but also the whole firm's behavior and the technology characterization. The shadow prices are estimated together with the estimation of producing technology and efficiency rate, which are specific for each firm taken into account. The main idea of this method is the estimation of distance function and thereafter the incorporation of Shephard (1970) duality theories. The *output distance function* defines any technology and it is dual to the more familiar revenue function.[1] From incorporating of duality theories into the *output distance function*, we obtain the revenue deflated shadow prices of all outputs. As Färe , Grosskopf, Lowell, & Yaisawarng (1993, p. 374) write: *„Throught the assumption that the observed price of one desirable output equals its shadow price, we may calculte shadow revenue and hence also absolute (undeflated) shadow prices of all other outputs. The absolute shadow prices of the undesirable outputs reflect the opportunity cost, in the terms of forgone revenue, of an incremental decrease in the ability to freely dispose of them.“* This coresponds to the statement in Bauman et al. (2008, p. 519). Generally, the shadow price ratio of any two outputs reflects the relative opporunity cost of those outputs. According to Färe et al. (1993), this means that this ration is equivalent to a marginal rate of transformation. This approach is aplicable to any technology. Färe et al. (1993) inlustrate that this method can be used also in cases if firms face regulation of undesirable outputs and some outputs are non-marketable.

The rest of this thesis is organized as follows. Section 2 provides the theoretical background of different approaches to MAC estimation and broadly discusses different types of the distance function. Section 3 provides overview of empirical results of emission shadow prices estimations via distance function and other approaches. Section 4 describes specification of

[1] Färe, Grosskopf & Nelson (1990) ilustrate that the shadow price of inputs can be derived by moddeling technology if an input distance function which is dual to the cost function.

the used model and also our data. There are also presented our empirical results of shadow price estimation. Section 5 goes over the MAC decomposition. Section 6 provides the results summary and concludes.

# 2. Theory

## 2.1. Methods

As mentioned above, there are several methods to estimate the shadow price of emissions (or MAC). From the methodological point of view, we can divide these methods into two basic groups: bottom-up and top-down approaches. To the bottom-up approaches, we rank engineering studies and economic approaches such as cost functions, distance functions and partial equilibrium models. Between top-down approaches, we count CGE models and econometric models. The description of top-down approaches is beyond the scope of this thesis. In general, CGE and econometric models are top-down models which usually focused on the economy as a whole including interactions between economic sectors but with lack of technological detail. A good description of CGE and econometric models applied for the Czech Republic, you can find in Ščasný et al. (2009). We will now briefly describe the main characteristics and differences of bottom up approaches.

Engineering studies are further divided into retrospective and prospective. The retrospective engineering studies are based on collection of data on observed or reported expenditures on environmental protection. A good example of such survey is Pollution Abatement Costs and Expenditure (PACE) collected and published by Bureau of the Census. These estimates represent the out-of-pocket expenses on environmental protection. Schmalensee (1994) criticizes these surveys based estimates, because they ignore many indirect costs and tend to double-count expenditures that are not part of the final demand.

The main characteristic of the prospective engineering studies is to consider the channels through which pollution can be reduced. Here, we types of possible methods. First; interview among experts focused mainly on costs of possible abatement technologies. According to Gollop & Roberts (1985) they don't take into account all possiblilities to emission reduction such as fuel substitutions. As Gollop & Roberts (1985) write, this often leads to a situation where "*the resulting estimates of the cost of regulation neither adequately reflect the range of control options available to polluters nor take into account how polluters actually have responded to enviromment controls*". (Gollop & Roberts, 1985, p. 81) Therefore the engineering studies tend to overestimate the MAC. The modern engineering studies already include also some fuel substitutions but they are very data demanding. The engineering studies are more appropriate for case studies of a single or very limited number of units, where it is possible to cover all required details. Second; engineering optimization models. These models include usually detailed technological description of the modeled units and a set of available technologies to pollution reduction. They cover the fuel switching very well. The final demand is an exogenous parameter and the pollution is reduced either acording some given constrain or based on some market measure such as environmental tax or price of emission permits. The weakness of these optimization models is that they are very data demanding , especially on fuel prices.

Cost function and distance function are also bottom up approaches but they use observed (ex post) data. The neoclassical cost function approach is based on cost minimization. Therefore we need not only input quantities but also input prices. This is the biggest weakness of this approach. The inconsistency and lack of relevant data are source of difficulties. Lee (2005) argues that firms often fail to minimize their production costs under various regulations. This is a reason why a neoclassical cost function could underestimate the MAC.

The main advantage of the distance function approach over the cost function is the fact that we don't need the information on input prices and regulatory constrains. The shadow prices can be derived through the estimated output distance function by using only the actual data of inputs, desirable goods produced, pollutants emitted and price of at least one output. According to Lee (2005), there are also no assumptions about cost minimizations. The distance function approach is described in detail in the following subsection.

Partial equilibrium (PE) models stand for a complex and perhaps the most challenging method for emission shadow price estimation between the bottom-up approaches. The models usually use two ways how to find the equilibrium point. We either set the emission reduction target and the model finds the optimal combination of measures to reach the given target, or we can set charges on the emission and the model finds the optimal level of emission. PE models are usually focused on detailed analysis of one sector including technological details. More detailed description is beyond the scope of this thesis. A detail description of PE models, you can find in in Rečka (2009).

## 2.2.Distance Function

The distance function represents the relative distance from some observed input-output combination to the production possibility frontier of technology. We can distinguish three types of distance function which are used in the literature in order to estimate the shadow prices of undesirable outputs: The *Shephard output distance function*, the *input distance function* and a generalization of the *Shephard output distance function* – the *directional output distance function*[2]. The first two are most widely used in the literature, but recently also the *directional output distance function* has begun to be used as a tool for efficiency estimation in studies focused on shadow price estimation of undesirable outputs.

### 2.2.1. Production technology

First, we define the production technology which is common for all types of distance function. We will follow Vardanyan & Noh (2006) and Färe, Grosskopf, & Margaristis (2008) and define these functions and their properties.[3]

We denote the input quantities by

$$x = (x_1, \dots, x_N) \in \Re_+^N, \tag{1}$$

good output quantities by

$$y = (y_1, \dots, y_M) \in \Re_+^M, \tag{2}$$

and bad output quantities by

$$b = (b_1, \dots, b_B) \in \Re_+^B. \tag{3}$$

[2] Färe, Grosskopf, & Margaristis (2008) show that *Shepard input* and *output functions* are special cases of *directional distance function*. The *directional distance function* allows set direction vectors both for inputs and outputs. In our theses, we focus on the *directional output distance function* because this form is used in the literature for emission shadow prices estimation. The *directional distance function* allows set direction vectors for outputs, but inputs are held constant.

[3] For our purposes, we modify the notation slightly and we will distinguish the good output (*y*) and bad output (*b*).

According to Färe, Grosskopf, & Margaristis (2008), we assume that the quantites of inputs and outputs are fully divisible real numbers.

The basic characterization of the polluting production technology is the technology set $T$ of all feasible input-output combination:

$$T = [(x, y, b) : x \text{ can produce } (y, b)\ ]. \quad \textbf{(4)}$$

We can rewrite this equivalenty via the output possibilities set, given by

$$P(x) = [(y, b) :\ (x, y, b) \in T]. \quad \textbf{(5)}$$

We assume that the output set is compact for each input vector $x$, because as Färe R. , Grosskopf, Noh, & Weber (2005, p. 474) write: "*An unbounded output set is not physically possible if traditional inputs are given*." Following properties of production technology are common for all types of distance function.

The production technology satisfies the following assumption:

1. Null-Jointness: if $(y, b) \in P(x)$ and $b = 0$, then $y = 0$.
2. Free disposibility of inputs: if $\acute{x} \geq x$ then $P(\acute{x}) \supseteq P(x)$.
3. Weak disposibility of an output vector: $(y, b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta b) \in P(x)$.
4. Free disposability of good outputs: $(y, b) \in P(x)$ and $(y^0, b) \leq (y, b)$ imply $(y^0, b) \in P(x)$.

The null-jointness says that if no bad outputs are produced, it is not possible to produce any good outputs, or in other words, it is not posible to produce any good products without producing bad products at the same time.

Free (strong) disposibility of inputs gives us that if inputs are increased (or not decreased), the output set will not diminish. Färe R. , Grosskopf, Noh, & Weber (2005, p. 472) interpret it so that "*this property implies that inputs are not congesting output*". Furthermore, the weak disposibility of outputs says that any proportional reduction of good and bad output together (at the same rate) is feasible. In other words, for a given inputs $x$, decrease of bad outputs is always possible, if good outputs are shrunk in the same proportion. This is consistent with regulations which require abatement or clean up of pollutants. Finaly, the free disposibility of good outputs means that if some vector of good and bad outputs is feasible, then any vector containing less of good outputs is also feasible. We can interpret it also as that "*we can always 'freely' dispose of some of the good output without any cost*". (Färe R. , Grosskopf, Noh, & Weber, 2005, p. 473)

### 2.2.2. Shephard output and directional output distance function

In this section, we will focus on the *Shephard output distance function* and the *directional output distance function*. We use the traditional definition of the *Shephard output distance function*:

$$D(x, y, b) = inf[\theta : (y, b)/\theta \in P(x)] \qquad \textbf{(6)}$$

and the *directional output distance function:*

$$\vec{D}(x, y, b; g_y, g_b) = sup\left[\boldsymbol{\rho} : (y + \boldsymbol{\rho} g_y, b + \boldsymbol{\rho} g_b) \in P(x)\right], \qquad \textbf{(7)}$$

where $(g_y \in \Re_+^M, g_b \in \Re_+^B)$ are the direction vectors, called the mapping rule. As we can see later in section 3.1, the mapping rule is very crucial for the marginal abatement cost estimation. The solution $\boldsymbol{\rho}^*$, gives the maximum expansion and contraction of desirable and undesirable outputs. The mapping rule $g = (g_y, g_b)$, specifies in which direction an output vector $(y, b) \in P(x)$, is scaled so as to reach the boundary of the output set at $(y + \boldsymbol{\rho}^* g_y, b + \boldsymbol{\rho}^* g_b) \in P(x)$, where $\boldsymbol{\rho}^* = \vec{D}\big(x, y, b; g_y, g_b\big)$. Most authors use mapping rule at the value $g = (1, -1)$, one reason is simplification, but Färe et al. (2005) have further two agrument for this choice. 1) This choice is consistent with the environmental regulation, since the pollutants are reduced, and 2) aggregation – „*Since we have many generating units in our data set, the aggregate industry efficiency is just sum over the individual unit's efficiencies*“ (Färe et al., 2005, p.476). Lee, Park, & Kim (2002) calculate the mappig rule $g = (g_y, g_b)$ by utilizing the annual abatement shedules of pollutants and the production plans of good output as proxy variables for $g_y$ and $g_b$, respectively.

According to Vardanyan & Noh (2006), we can define the *Shephard output distance function* as a measure based on maximal possible proportional expansion of all outputs onto the frontier of production possibilities $P(x)$. In other words, the *Shephard output distance function* maximizes the outputs, while the vector of inputs is held constant. The *Shephard distance function* maximizes the proximity to the production possibility frontier at the value 1.

In contrast to the *Shephard output distance function*, the *directional output distance function* allows for a simultaneous expansion of good outputs and contraction of bad outputs. The *directional output distance function* takes the value of zero for technically efficient output vectors on the boundary of $P(x)$ and positive values indicate the technical inefficient output vectors below the frontier.

Table 1 shows the basic properties of these two types of distance function.[4]

**Table 1 Selected properties of the Shephard and the directional output distance functions**

| | Shephard distance function | Directional distance function |
|---|---|---|
| Representation | $0 < D(x, y, b) \leq 1$ | $\vec{D}(x, y, b; g_y, g_b) \geq 0$ |
| Monotonicity | $\partial D(x, y, b)/\partial x \leq 0,$ | $\partial\vec{D}(x, y, b; g_y, g_b)/\partial x \geq 0,$ |
| | $\partial D(x, y, b)/\partial y \geq 0,$ | $\partial\vec{D}(x, y, b; g_y, g_b)/\partial y \leq 0,$ |
| | $\partial D(x, y, b)/\partial b \leq 0$ | $\partial\vec{D}(x, y, b; g_y, g_b)/\partial b \geq 0$ |
| Output homogeneity of degree +1 | $D(x, \lambda y, \lambda b) = \lambda D(x, y, b),\ \lambda > 0$ | - |
| Translation | - | $\vec{D}(x, y + \rho g_y, b + \rho g_b; g_y, g_b) =$ $\vec{D}(x, y, b; g_y, g_b) - \rho, \quad \rho \in \Re$ |

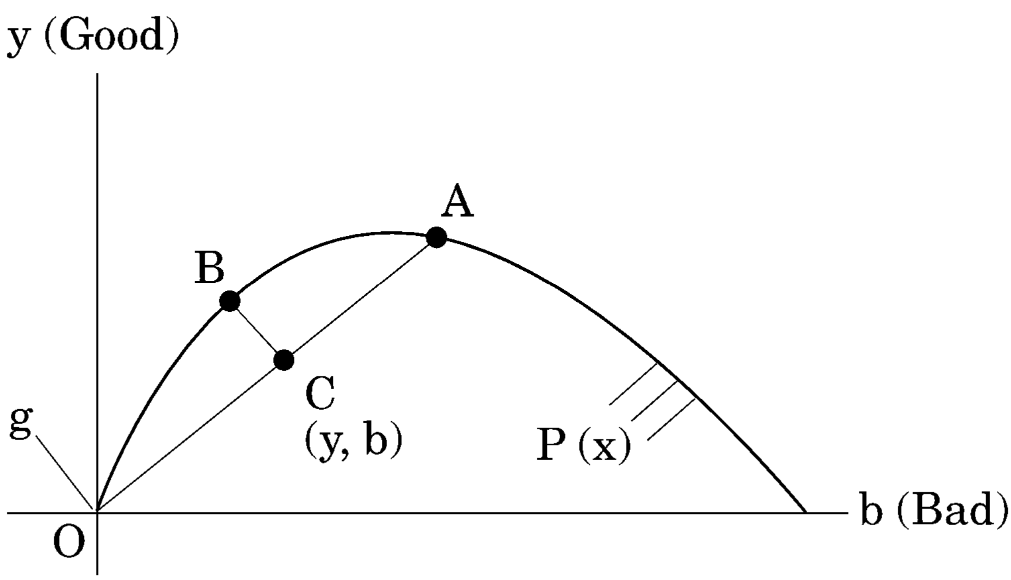
Source: Vardanyan & Noh (2006, p. 179)

Figure 1 illustrates the *directionnal output distance function* and compares it with the *Shephard's output distance function*. Following Chung, Färe, & Grosskopf (1997, p.232), the production technology satisfies the assumption defined on the previsous page. The output set is denoted by $P(x)$, good output by $y$ and bad by $b$. The outputs $(y, b)$ are weakly disposible and $y$ by itself is strongly disposable; the good ouput $y$ is null-joint with $b$, since if $b = 0$, then the $y$ with $(y, b) \in P(x)$ must be equal to zero. *Shephard's output distance function* applied to the ouput vector $(y, b)$ places it on the boundary of $P(x)$ at point $A$, and yields a value of $OC/OA$ (i.e. If goods and bads were both increased by a factor $OC/OA$, the

[4] Vardanyan & Noh (2006) shortly discuss also *hyperbolic* distance function – a variation of Shephard distance function which is defined as $D_H(x, y, b) = inf[\theta : (y/\theta, \theta b) \in P(x)]$. For a detailed characteristics of this variation of Shephard distance function, see Vardanyan & Noh (2006) or Färe R. , Grosskopf, Lovell, & Pasurka (1989).

firm would produce efficiently on its production possibilty frontier.). In contrast, the directional distance function starts at point $C$ and scales in the direction of increased goods and decreased bads and projects $C$ on the boundary at point $B$. In Figure 1 this equals to the ratio $BC/Og$. Thus if the firm moved from $C$ to $B$, it would produce efficiently on its production possibilty frontier.

**Figure 1 Distance Function**




Source: Chung, Färe, & Grosskopf (1997, p. 231)

For better understanding, we follow Chung, Färe, & Grosskopf (1997, p.232) and relate these two distance function to each other. Assume the mapping rule $g = (g_y, g_b)$, we get

$$\vec{D}(x, y, b; g_y, g_b) = \sup\left[\rho : \ D\left(x, (y, b) + \rho(g_y, g_b)\right) \leq 1\right] \quad \textbf{(8)}$$

$$= \sup[\rho : (1 + \rho)D(x, y, b) \leq 1]$$

$$= \sup\left[\rho : \rho \leq \frac{1}{D(x,y,b)} - 1\right]$$

$$= 1/D(x, y, b) - 1$$

We can rewrite this relation also as

$$D(x, y, b) = 1/(1 + \vec{D}(x, y, b; g_y, g_b)). \quad \textbf{(9)}$$

### 2.2.3. Input distance function

The *input distance function* measures the maximum amount by which the input vector can be deflated, given the output vector. We define it according to Hailu & Veeman (2000) as follows:

$$ID(x, y, b) = sup[\lambda : (x/\lambda, y, b) \ \in P(x), \lambda \in \Re_+] \ , \quad \textbf{(10)}$$

In Table 2 you find the basic properties of the *input distance function*.

**Table 2 Input distance function's properties**

| | |
|---|---|
| Representation | $ID(x, y, b) \geq 1$ |
| Monotonicity | $\partial ID(x, y, b)/\partial x \geq 0$, |
| | $\partial ID(x, y, b)/\partial y \leq 0$, |
| | $\partial ID(x, y, b)/\partial b \geq 0$ |
| Input homogeneity of degree +1 | $ID(\lambda x, y, b) = \lambda ID(x, y, b), \ \lambda > 0$ |

The technically efficient production is achieved if the *input distance function* has a value of one. In other words, if the value of the function is bigger than one, the firm uses more inputs than it is optional to the given outputs. From the definition of the *input distance function*, the degree of technical efficiency is defined as

**(11)**

$$TE = \frac{1}{ID(x, y, b)}.$$

Thus, $(1 - TE)$ measures the proportion by which costs could be reduced by improving technical efficiency to optimum, without reducing output.

### 2.2.4. Input, Output and Directional Output Distance Function - pro and con

The input distance function is based on maximal possible proportional reduction of all inputs, while the vector of outputs is held constant and the output distance function maximizes the outputs in proportinal way, while the vertor of inputs is held constant. This is the main conceptual difference between these two types of distance functions. As Hailu & Veeman (2000) write, the input and output distance functions can be related throught the returns to scale parameter. They are equal if the technology has constant returns to scale. Generally, we can find diferent interpretation of these two approaches: The output- based distance function focuses on output expansion, while the input-based distance function concentrates on costs savings.

Kumbhakar, Orea, Rodríguez-Álvarez, & Tsionas (2007) deal in general with the question if we should estimate an input or an output distance function. According to their study, the input distance function is appropriate in the case of cost minimization where output is exogenous and inputs are endogenous. The output distance function is more appropriate in the opposite case.

In case of undesirable outputs, Hailu & Veeman (2000) argue in favour of the *input distance function*. They claim that the *output distance function* is no more reasonable measure if we take into account the undesirable outputs, "*because whether a proportional expansion in outputs (now including undesirable outputs) is socially beneficial depends upon whether the benefits from the expansion in desirable outputs (goods) will more than offset the damage caused by the simultaneous or accompanying expansion in undesirable outputs (bads). The input-based measure of productivity change, on the other hand, continues to serve as a meaningful measure of productivity growth because a proportional savings in inputs or costs, with desirable and undesirable outputs held constant, is an unambiguous indicator of change in social benefits.*" (Hailu & Veeman, 2000, p. 254)

The *directional output distance function* solves the weakness of *output distance function*, because it does not require the proporcinal increase of all outputs. The *directional output distance function* allows different vectors setting for good and bad outputs - i.e. positive vector of good outputs and negative vector of bad outputs. This means that the good outputs can raise while bad outputs decrease. Thus the *directional output distance function* reflects the environmental regulation and the Hailu & Veeman (2000)'s critque is no more actuall for this type of distance function. From this point of view, both *input distance function* and *directional output distance function* can be used to the estimation of emission shadow prices. But if we take into account also the Kumbhakar, Orea, Rodríguez-Álvarez, & Tsionas (2007) study, the *input distance function* is appropriate to estimate emission shadow prices in the energy sector. The reason is that the energy demand is rather exogenous than endogenous, it is very inelastic at least in short-term. Furthermore, the energy demand is partly depended on the weather – exogenous factor. At the same time, the energy sector has to satisfy the demand. Therefore, we can consider the output in energy sector as exogenous.

### 2.2.5. The shadow-pricing model

Due to the duality relationship between cost and revenue function, we can derive the output shadow prices from both *output distance* and *input distance function*. When we derive the output shadow prices from *output distance function*[5], we employ the duality between the *output distance function* and the revenue function and we derive them under the assumption of revenue maximization. We folow Marklund (2003, pp. 11,12) and denote input prices by

$$w = (w_1, \dots, w_N) \in \Re_+^N, \tag{12}$$

good output prices by

$$p = (p_1, \dots, p_M) \in \Re_+^M \tag{13}$$

and bad output prices by

$$r = (r_1, \dots, r_B) \in \Re_+^B. \tag{14}$$

The revenue function is defined as

$$R(x,p,r) = max[py - rb : \ (y,b) \in P(x)]. \tag{15}$$

This can be rewritten as

$$R(x,p,r) = max\big[py - rb : \ \vec{D}\big(x,y,b;g_y,g_b\big) \geq 0\big]. \tag{16}$$

[5] We have shown, that the directional output distance function is generalization of Shephard output distance function, therefore now we use the general formulas connected to the directional output distance function. The derivation of shadow prices from Shephard output distance function is on the same principle.

The directional output distance function can be explress as

$$\vec{D}(x, y, b; g_y, g_b) = \min\left[(R(x,p,r) - (py - rb))/(pg_y + rg_b)\right] \tag{17}$$

To get the explicit shadow-pricing model, we apply the envelope theorem to (17) and get

$$\nabla_{\mathrm{y}}\vec{D}(x, y, b; g_y, g_b) = -\frac{p}{(pg_y + rg_b)} \leq 0, \qquad (pg_y + rg_b) > 0 \tag{18}$$

$$\nabla_{\mathrm{b}}\vec{D}(x, y, b; g_y, g_b) = -\frac{r}{(pg_y + rg_b)} \leq 0, \qquad (pg_y + rg_b) > 0. \tag{19}$$

The absolute shadow prices of the desirable and undesirable outputs we can derive from (18) and (19). Unfortunately, we don't know the value of $(pg_y + rg_b)$ because the shadow prices have not been calculated yet. Now we exploit the assumption that at least one of the good outputs $(y_m)$ is sold on perfectly competitive market. This allows us to take the observed price $(p_m)$ of such good output to be its absolute shadow price. In this case, the absolute shadow prices of all bad outputs can be calculated as

$$r_b = \left( \frac{\partial \vec{D}(x,y,b;g_y,g_b)}{\partial b_b} \Bigg/ \frac{\partial \vec{D}(x,y,b;g_y,g_b)}{\partial y_m} \right) * p_m, \qquad b = 1, \ldots, B \tag{20}$$

where the negative of the expression within brackets is the marginal rate of transformation between the b$^{th}$ bad output and the m$^{th}$ good output, $MRT_{bm}$. The shadow price $r_b$ is equal to the revenue loss, from decreased sales of $y_m$, which has to be undertaken if the bad output $b_b$ is decreased marginally.

Hailu & Veeman (2000) derive the output shadow prices from the *input distance function* under the assumption of cost minimizing but they end with similar formula as in (20). The model is defined as follows:

The cost function is the solution to the minimization problem

$$C(y, b, w) = Min_x \, [w * x : ID(x, y, b) \geq 1, x \in \Re_+^N], \tag{21}$$

where $w \in \Re_+^N$ is the input price vector. Equation (21) is the duality between the cost and the *input distace function* due to Shephard (1970). We again apply the envelope theorem on the first order condition and the optimization problem in (21) yields output shadow price formulas:

$$\nabla_{\mathrm{y}} C(y, b, w) = -C(y, b, w) * \nabla_{\mathrm{y}} \mathrm{ID}(x, y, b) \tag{22}$$

$$\nabla_{\mathrm{b}} C(y, b, w) = -C(y, b, w) * \nabla_{\mathrm{b}} \, \mathrm{ID}(x, y, b) \tag{23}$$

The equations (22) and (23) are obtained from the first order condition for the solutions to (21) and from the fact that the Lagrangian multiplier ($\Lambda(y, b, w)$) is eaqual to the value of the optimized cost function in this case.

"*If we do not have input prices and cannot accurately estimate the cost of production, we can use the foolowing formula derived from* (22) and (23) *to calculate the ratio of the shadow price of output $b$* to that of output $y$" (Hailu & Veeman, 2000, p. 260) :

$$\frac{r_b}{p_m} = \left. \frac{\partial \mathrm{ID}(x, y, b)}{\partial b_b} \middle/ \frac{\partial \mathrm{ID}(x, y, b)}{\partial y_m} \right. \tag{24}$$

We again assume that at least one of the good outputs ($y_m$) is sold on perfectly competitive market. This allows us to take the observed price ($p_m$) of such good output to be its absolute shadow price. So we get:

$$r_b = \left( \frac{\partial \mathrm{ID}(x,y,b)}{\partial b_b} \Big/ \frac{\partial \mathrm{ID}(x,y,b)}{\partial y_m} \right) * p_m, \qquad b = 1, \dots, B \tag{25}$$

The shadow prices for undesirable ouputs are non-positive, as the *input distance function* is non-deacresing undesirable ouputs.

# 3. Lit. review

In this section we provide literature review of empirical studies estimated emission shadow prices. We chronologically illustrate the development of undesirable outputs shadow prices estimation based on *distance function* and also the difference in results based on different specification of the *distance function* in the following subsection.

For a comparison with other methods, we choose a few examples of MAC estimates by other approaches than the shadow price estimation using the *distance function*. Below in

Table 3, you can see the MACs for the Czech Republic from GAINS[6] and GEM-E3[7] models. In Table 4, Bluffstone (1999) analyses abatent costs of air pollutants in Lithuania in years 1993-

[6] Model GAINS developed by IIASA is a technological based macro model. The model gives the MACs for individual abatement technologies and fuel types. For more details about GAINS model and its application for the Czech Republic, see Bízek (2009).

[7] Pye et al. (2008) provide Cost-Benefit Analysis of a revised National Emission Ceilings Directive (NECD). They use CGE model GEM-E3 to assess the macroeconomic impacts of the policy proposals. The authors report

94. The abatement costs are estimated from 366 observation based on profit maximization. Bluffstone finds that marginal abatemetn costs rice with decreasing level of emission. Bluffstone uses OLS and 2SLS method to the estimation and each method leads to different values. 2SLS estimates are signicantly greater than OLS estimates in most cases.

**Table 3 MACs from GAINS and GEM-E3 models**

| €2005 | SO2 | NOx | VOC | PM2.5 | PM |
|---|---|---|---|---|---|
| GAINS | 430 - 4000 | 100 - 10000 | - | 100 - 10000[8] | - |
| GEM-E3/CE | 545 | 1081 | 0 | - | 3253 |
| GEM-E3/S-CE | 785 | 1520 | 0 | - | 7764 |

Source: Ščasný et. al (2008) & Pye et al. (2008)

Table 4 shows the increasing trend of MAC with decreasing emission level and also the differences between the OLS and 2SLS estimates.

emission MACs in year 2020 for scenarios CE and S-CE. ("Baseline scenario assumes all current legislation, including meeting the 2010-national ceilings of the current NECD (at least by 2020), Euro 5/6, and the proposal for a revised directive on industrial emissions and the EURO VI-proposal. It also assumes the full impact of the Climate and Energy Package assuming that the non-ETS targets are met in each Member State and that there is full trade of renewables and JI/CDM is enabled as so that carbon prices do not exceed €30/t CO2.

**Cost-effective measures (CE)** sets national ceilings for 2020 for all five pollutants (incl. PM2.5) in a least-cost way so that all objectives of the TSAP are met in 2020. **S-CE** excludes the Climate and Energy Package from the CE scenario.") Pye et al. (2008, p.2)

[8] The most of MACs of PM2.5 are in the range from 100€ to 2000€ per ton of PM2.5, but for coal some MACs vary also around 10,000€ per ton of PM2.5. This could be caused by the fact that most of Czech coal power plants have already efficient scrubbers and further reduction of particular matters is complicated and very costly.

**Table 4 Marginal abatement costs - Bluffstone (1999)**

| €2005 | 0% emission reduction | 10% emission reduction | | 25% emission reduction | | 40% emission reduction | |
|---|---|---|---|---|---|---|---|
| | | OLS | 2SLS | OLS | 2SLS | OLS | 2SLS |
| SO2 | 13.82 | 27.42 | 45.64 | 40.10 | 86.42 | 57.73 | 127.56 |
| NOx | 25.55 | 63.61 | 63.96 | 115.13 | 126.52 | 165.25 | 179.07 |
| CO | 0.87 | 1.12 | 1.08 | 1.45 | 1.36 | 1.74 | 1.66 |
| PM | 14.88 | 51.16 | 19.40 | - | 26.94 | - | 35.29 |

Source: Bluffstone ,1999, p. 18 (converted from $1995 to €2005)

## 3.1. Distance function

First study estimating shadow prices based on *distance function* using nonparametric linear programming was Färe, Grosskopf, & Nelson (1990). Färe, Grosskopf, Lowell, & Yaisawarng (1993) applied the concept of *output distance function* for shadow price estimation of undesirable output for the first time. They estimated the shadow prices of biochemical oxygen demand (BOD), total suspended solids (TSS), particulates (PM) and sulphur oxides (SOx) for 30 paper and pulp mills in the states Michigan and Wisconsin, USA in 1976. They expressed the shadow prices in negative terms as foregone revenue by reduction of desirable output production. Their average estimates were -1043, 0, -25270 and - 3696 US dollars per one ton of BOD, TSS, PM and SOx, respectively.

Coggins & Swinton (1996) estimate $SO_2$ shadow price on the sample of 14 coal-burning electric power plants in the state of Wisconsin, USA between years 1990 and 1992. They use a parametric *output distance function* in translog form. The sample weighted average $SO_2$ shadow price is estimated on $292.7. But the average shadow price varies across the power plants from $6.2 to $897.[9] They also provide a comparison of shadow prices by boiler type.

[9] 1992 dollars

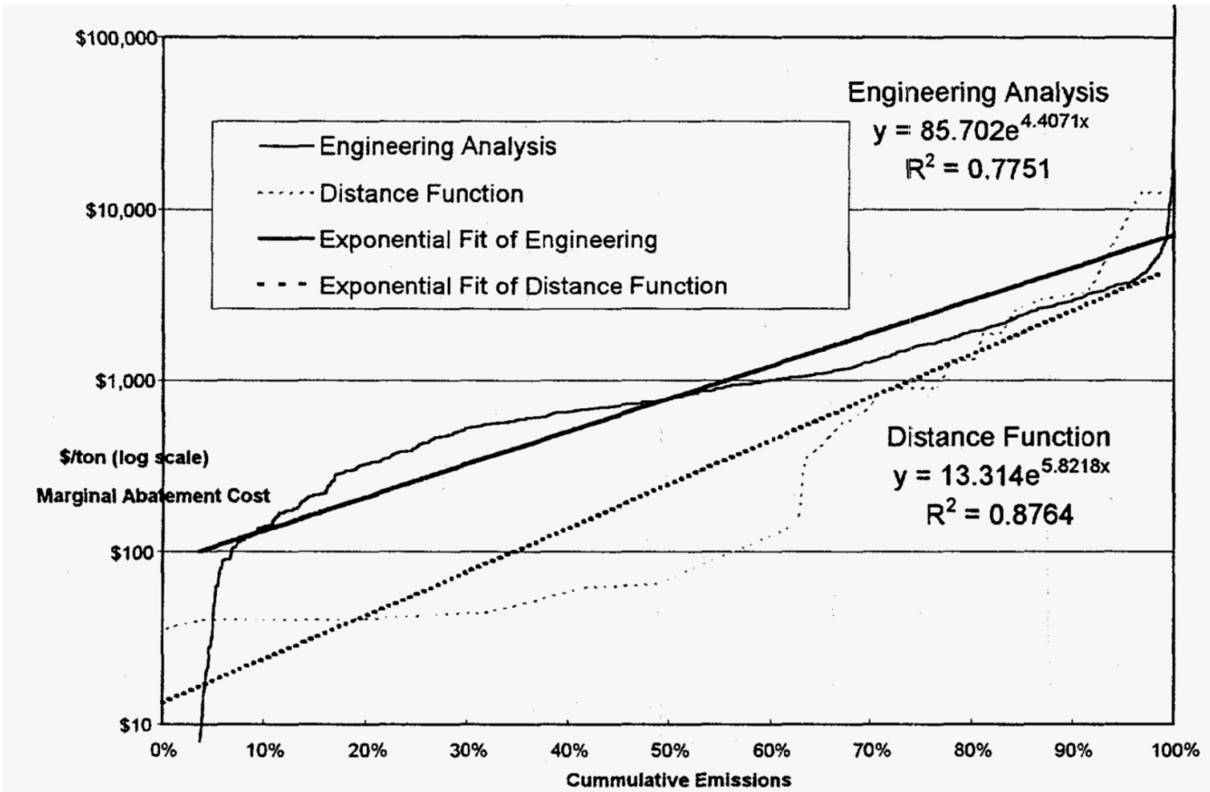
The power plants with dry bottom boiler have mean shadow prices of one ton of $SO_2$ equal to \$326.73 and the plants with cyclone to €175.48 per ton of $SO_2$.

Boyd, Molburg & Prince (1996) analyze the MAC of $SO_2$ in the US energy industry, namely on 62 power plants. They apply *sub-vector output distance function* defined by (Turner 1994) as $D_{SV}(x,y,b) = inf[\theta : (y/\theta, b) \in P(x)]$ and also the *directional output distance function*. From the *sub-vector output distance function* Boyd, Molburg, & Prince (1996) get the mean shadow price of \$1120[10] per ton with the range from \$74 to over \$12000 per ton and median \$876 per ton of $SO_2$ . Based on the *directional output distance function* with mapping rule $\mathrm{g} = (1, -1)$ they come to estimates of shadow price at the value of \$355 for average weighted by total plant emissions, \$1703 and \$787 for mean and median, respectively. Boyd, Molburg & Prince (1996) illustrate the distribution of the MAC over the magnitudes of the plants emissions. Figure 2 plots the MAC estimated by the *directional output distance function* and the MAC coming from engineering cost analysis by Molburg (1996) against the incremental emission reduction for each reduction option at each plant. Figure 2 is normalized to 100 %. Boyd, Molburg & Prince (1996) explaind the differencies in the emission reduction in the area below \$1000 due to smaller data set of their study in comparizon to the engenering analysis and due to that "*low marginal costs in the distance function analysis are attributed to the entire plant emissions*" (Boyd, Molburg, & Prince, 1996, p. 7). The authors get reasonable results only for half of the observations, the other half has negative MAC in terms that there are benefits from increase of produced emissions. This is inconsistent with the economic theory, but Boyd, Molburg & Prince (1996) priovide

[10] This is not a weighted average which could be comparable with the results from other studies. Furthermore this average ignores the outliers on both ends of the estimates range. Therefore, the median is more comparable with the median based on the directional output distance function.

some empirical reasons for such findings, i.e. *„Inconsistency in the regulations between the plants compared in the data set; Inefficinecy or lack of optimization in fuel cost and fuel choice due to the economic regulation of power generation“* (Boyd, Molburg, & Prince, 1996, p. 8). They notes that in order to prevent such "shaky" results, the plants in the study sample should be under a common regulatory structure.

**Figure 2 Comparison of Marginal Abatement Costs based on Directional distance Function and Engineering Analysis**




Source: Boyd, Molburg, & Prince (1996, p. 8)

Swinton (1998) extends the work of Cogginns & Swinton (1993) by including coal-burning power plants from Illinois and Minnesota. In comparison with the sample in Coggins & Swinton (1996), the sample of power plants from Illinois and Minnesota includes power plants which have installed flue gas desulfurization units ("scrubbers") in the followed years.

This allows including explicitly also the abatement capital which is by traditional assumption *non-productive*. Swinton (1998) claims that “it may by desirable to consider abatement efforts before comparing productivity measures” (Swinton, 1998, p. 69), because the *non-productive* abatement capital might be socially beneficial. Therefore, Swinton (1998) separates the capital into productive and abatement capital as two inputs. Swinton (1998) finds the average shadow price of marginally reducing emissions for a plant with a scrubber at the value of $2572 - more than ten times the overall average for plant in the whole sample. However, the plant with a scrubber can emits less than one-tenth that the other plants emit.

Kwon & Yuh (1999) provide estimation of the marginal abatement costs of airborne pollutants in Korea’s power generation sector in the periond of 1990 – 1995 . They use the *translog output distance function* to estimate MACs of SO*x*, NO*x* , TSP and $CO_2$. The authors estimate the mean MACs on 425.5, 201, 210.8 and 5.2 € per ton of SO*x*, NO*x*, TSP and $CO_2$, respectively.

Hailu & Veeman (2000) use *input distance function* approach to shadow price estimation of BOD and TSS on Canadian paper and pulp industry aggregate time series data for the period from 1959 to 1994. They estimate BOD and TSS shadow prices and “the average BOD shadow price increases from $34[11] for the 1970s to $147 per metric ton for the 1980s and to $436per metric ton for the period from 1990 to 1994” (Hailu & Veeman, 2000, p. 270). The average BOD shadow price for the whole 1959 – 1994 period is $ 123. They find also increasing TSS shadow price from $ 100 for 1960s up to $ 663 per metric ton for the first half of 1990s.

[11] 1986 dollars

Swinton (2002) estimates the shadow prices of $SO_2$ for seven power plants operating in Florida during years 1990 – 1998 in order to analyze the potentiol for saving throught the Phase I of new stablished allowance market from 1995 to 1998. He uses the *output distance function* in transcendental logarithmic form for his estimation. His weighted averages of the shadow prices of $SO_2$ vary in time and go from the \$116.95 per ton of $SO_2$ in 1994 up to \$196.69[12] per ton of $SO_2$ in 1996.

Lee, Park, & Kim (2002) use *nonparametric direction distance function* approach and they take into account also the inefficiency in the production process. For these purposes, Lee, Park, & Kim (2002) define also an efficiency rule as $\sigma_g = \sigma_g(\lambda), \sigma_b = \sigma_b(\lambda)$ , whre $\sigma_g$ and $\sigma_b$ are called inefficiency factors and $\lambda$ is a parametr relating $\sigma_g$ to $\sigma_b$. The efficiency rule maps a point $(y, b) \in P(x)$ to corresponding $(y^*, b^*)$ on the boundary $P(x)$ in a way that $\sigma_g(\lambda)y = y^*, \sigma_b(\lambda)b = b^*$. Furthermore they define also an efficiency path (EP) and iso-efficiency path (IEP) as $EP(y^*, b^*) = [(y, b) \in P(x): \sigma_g(\lambda)y = y^*, \sigma_b(\lambda)b = b^*$ and$IEP(y_0, b_0) = [(y, b) \in P(x): D\big(x, \sigma_g^0 y, \sigma_b^0 b\big) = 1]$, respectively. Lee, Park, & Kim (2002) follow Kumbhakar (1996) and estimate the *direction distance function* with the elements $\sigma_g y, \sigma_b b$. They provide also instructions how to caluculate the direction vector in mapping rule. In general, they calculate the directional vector $g = (b, y)$ *"by utilizing the annual abatement schedules of pollutants and the production plans of good output as proxy variables for $b$ and $y$, respectively*." (Lee, Park, & Kim, 2002, p. 371) They estimate the shadow price of SOx, nitrogen oxides (NOx) and TSP in the Korea's electric power industry during 1990-1995, on a sample including 43 power plants. They find that the average shadow prices "are approximately 10% lower than those calculated under the assumption of

[12] 1996 dollars

full efficiency". (Lee, Park, & Kim, 2002, p. 365) They calculate the shadow prices for coal-burned and oil burned power plants separately but present only the average shadow prices for the whole sample. Lee, Park, & Kim (2002) estimate the shadow prices of SOx, NOx and TSP on $3,170, $17,393 and $51,09 per ton of pollutant, respectively.

Marklund (2003) uses *directional output function* to compute shadow prices of oxygen-demanding substances and suspended solids in 12 geographically scattered Swedish pulp and paper plants over the period 1983-1990. He estimates the average shadow prices of oxygen-demanding substances on 5068 SEK per ton with standard deviation 2909.6 and the average shadow price of suspended solids on 793.8 SEK per ton with standard deviation 749.6[13]. Furthermore, Maklund (2003) tests the hypothesis whether there is a positive correlation between population density and pulp plant's MACs. He claims that "population density had a different influence on the two types of emissions. In the case of oxygen-demanding substances, the density contributed negatively to the plants' MACs, indicating that plants located in counties with higher density were targets of laxer environmental regulation. On the other hand, in the case of suspended solids, the density contributed positively to the plants, MACs. That is, plants located in counties with higher population density were more stringently regulated." (Marklund, 2003, pp. 24-25) Furthermore, Marklund (2003) finds that the relative size of the pulp and paper industry in the region is negatively correlated with the MAC of suspended solids for plants located in that region. According to Marklund (2003), this shows that these plants might be targets of less stern environmental regulation.

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[13] After conversion from 1990 SEK to EUR 2005 constant prices, we get €760.2 with s.d. €436.44 and €119.07 with s.d. €112.44, respectively.

Färe et al. (2005) use *quadratic directional output distance function* to measure the technical efficiency of 209 US electric utilities. They estimate the shadow price of $SO_2$ in years 1993 and 1997 and also the output elasticity of substitution between electricity and $SO_2$ in these years. They choose these two years to estimate the effects of *Phase I of the acid rain program* implemented in 1995. They use two approaches to etimate the directional distance function. They use the *loss minimization deterministic procedure* developed by Aigner & Chu (1968) and they also estimate *the directional distance function as a stochastic frontier*. Using a *stochastic frontier* approach they find shadow price of $76 per ton of $SO_2$ in 1993 and $142 per ton of $SO_2$ in 1997. According the authors, these values are close to average prices of market trades. The estimates of shadow prices comming from *deterministic procedure* are around $1100 in 1993 and $1973 in 1997. The authors interpret these results in relation to the previous ones as follows: „These estimates indicate that utilities could reap larger gains in economic efficiency through permit purchase.“ (Färe R. , Grosskopf, Noh, & Weber, 2005, p. 471)

Atkinson & Dorfman (2005) focus on *input distancce function* estimation using the Limited Information Bayesian System Estimator[14]. They use data about 43 US power plants for the yeatrs 1980, 1985, 1990 and 1995. They observe more than 20% reduction of $SO_2$ over the 1980-1995 period. Based on the Bayesian approach, they find the posterior median[15] of MAC at the value of $464.68 per ton of $SO_2$ for the year 1995[16]. Atkinson & Horfman (2005) apply also Generalized Method of Moments (non-Bayesian) in order to estimate the *input distancce function* and to derive the MAC of $SO_2$, but their findings are much less accurate in comparizion to the results from the Bayesian approach and in 1995 they get even negative MAC of $SO_2$.

[14] Description of the Limited Information Bayesian System Estimator is behind the scope of this thesis, for more detail see Atkinson & Horfman (2005)

[15] Posterior median is defined as the median value of a particular parameter or function of parameters from all the draws, for more detail see Atkinson & Horfman (2005)

[16] We mention only the results from 1995, because they are the most relevant form the comparison with other studies.

Lee (2005) estimates the *Shephard input distance function* on data from 38 coal-fired US power plants operating between 1977 and 1986. In contrast to many other studies, Lee estimates the shadow prices of sulphur and ash not in form of forgone output but in form of forgone capital. He gets the overall weighted average estimates 0.076 and 0.058 dollars per pound[17] for sulphur and ash, respectively. Lee (2005) finds substantially variation of the estimates between four geographic regions (*Great Lakes, Midwest, South, and Northeast*) but also within state and even some differences between units within plants. Lee (2005) estimates also the indirect Morishima elasticities of substitution of capital for sulfur and comes to the conclusion that this substitution is relatively high.

Vardanyan & Noh (2006) examine the results of diferent output distance function and mapping rules. The use a panel of observation from the US electricity power industry for 1997-1999. They argue against the assumption that all the parametric methodologies for shadow price estimation of good and bad outputs provide a similar and an adequate approximation to the true production technology. Their study shows that the *estimates are extremely sensitive to the changes in the parameterization methodology, especially to the variation in the mapping regime* (Vardanyan & Noh, 2006, p. 189). Table 5 shows the market price of $SO_2$ allowances and shadow price estimates in dependence on various parametrization methodologies.

[17] constant 1976 US dollars used; after conversion to $ per ton, we get 167.55 and 127.75 (1 pound is equal to 0.454 kilogram.)

**Table 5 $SO_2$ Shadow price estimates and the market price of allowances; various parameterization methodologies**

| | 1997 | | 1998 | | 1999 | |
|---|---|---|---|---|---|---|
| Shephard distance function | 0 | 0 | 0 | 0 | 193.89 | (-55.45) |
| Hyperbolic distance function | 567.09* | (-91.1) | 232.04* | (-15.34) | 1237.78* | (-138.35) |
| Directional distance function $g_b$=-1, $g_y$=2 | 1048.86* | (-69.09) | 539.19* | (-26.12) | 877.22* | (-49.33) |
| $g_b$=-1, $g_y$=3.96 | 398.47* | (-25.02) | 154.05* | (-5.42) | 383.75* | (-17.84) |
| $g_b$=-1, $g_y$=3.97 | 398.38* | (-22.95) | 126.11* | (-3.65) | 383.36* | (-16.41) |
| $g_b$=-1, $g_y$=5 | 216.69* | (-11.03) | 73.27* | (-4.55) | 319.78* | (-13.24) |
| $g_b$=-1, $g_y$=9 | 90.4611 | (-3.89) | 12.82* | (-0.94) | 305.04* | (-12.6) |
| $g_b$=-1, $g_y$=15 | 30.74* | (-1.77) | 0 | 0 | 288.13* | (-12.29) |
| $g_b$=-1, $g_y$=29.4 | 29.68* | (-1.72) | 0 | 0 | 175.87* | (-6.72) |
| Allowance market price | 93.1302 | | 140.85 | | 181.19 | |

*Market price of allowances is the average of the indices published by Cantor Fitzgerald Environmental Brokerage (1997 and 1998) ,The emissions Exchange Corp. (1997–1999), and Fieldston Publications (1997–1999). Bootstrapped standard errors are in parentheses. *Significant at 1% in a two-tailed test of difference from the average market price of allowances.*

Source: Vardanyan & Noh, 2006, p. 188 (converted from 1999$ to 2005€)

Murty, Surender, & Kishore (2007) apply the *directional output distance function* estimation for five coal fired power plants in Adhra Pradesh State of India during the years 1996-97 to 2003-2004[18]*,* namely its parametrical specification as a quadratic functional form. Using this approach, they estimate the combined environmental and technical efficiceny, shadow price of Suspended Particulate Matter (SPM), $SO_2$ and $NO_X$. Furthermore they estimate also the elasticity of substitution between electricity and pollutants. They estimate the distance function as a stochastic frontier and use the directional vector $(g_y, g_b) = (1, -1)$. The mean shadow prices of SPM, $SO_2$ and $NO_X$ are estimated on 106.16, 41.84 and 149.43 US$, respectively. The authors stress a significant variation of the estimated marginal abatement cost of pollutants by year and plant unit. Murty, Surender, & Dhavala (2007) conlude that the "*variation in the shadow price of bad output among firms could be attributed to different levels of compliance to environmental regulation."* (Murty, Surender, & Kishore, 2007, p. 46) Based on correlation analysis between the marginal abatement cost and pollution intensity and electricity generated for each polutant Murty, Surender, & Kishore (2007) find that MAC "*increases with a decrease in pollution concentration and decreases with an increase in firm*

[18] Murty, Surender, & Dhavala (2006) use 480 monthly panel observations.

*capacity.*" (Murty, Surender, & Kishore, 2007, p. 46) This indicates increasing marginal abatement cost in coal-fired power plants.

Bauman et al. (2008) estimate $SO_2$ shadow price in the Korean electric power industry between 1970 and 1998. The average shadow price is \$ 184 per ton of $SO_2$. This is approximately about 40 % lower than the three-year average $SO_2$ shadow price estimation by Coggins & Swinton (1996). Here we note that the same the three-year average $SO_2$ shadow price is \$ 171.3 – even lower. Bauman et al. analyzes also the factors affecting the annual marginal cost of production $SO_2$ abatement – rate of technical change and emission rate. He finds out that "technological innovation increased marginal abatement cost of process $SO_2$ during the 1970-1998 period." (Bauman et al., 2008, p. 522)

Park & Lim (2009) estimates the MAC of $CO_2$ using the *output distance function* in translog functional form. Their data set includes 20 Korean fossil-fuel power plants in time horizon from 2001 to 2004. They find the weighted average MAC of $CO_2$ at the value of €14.04 per ton of $CO_2$. According to type of main fuel, the weighted averages of MAC are €13.04, €12.45 and €11.40 per ton of $CO_2$ for coal-fired, oil and natural gas-fired power plants, respectively.

Maradan & Vassiliev (2005) test whether the MAC of $CO_2$ curve shifts upwards wiht the incresig income, what is one of assumptions connected to the Environmetal Kuznets Curve. They anylzze the evolution of the opportunity cost of $CO_2$ abatement with income of economies. They estimate the Shadow prices of $CO_2$ via the *directional distance function*. They apply the mapping rule $g = (0, g_b)$, where $g_b$is equal to the mean value of bad output in the sample. Maradan & Vassiliev (2005) work with macro-economic cross-section data from 76 developed and developing countries[19] in 1985.*"These include 30 low- and lower-middle income countries and 46 upper-middle and high-income countries. Either gross domestic product (GDP) or consumption has been alternatively considered as proxies for the desirable output. The undesirable output is carbon dioxide ($CO_2$). Each country is assumed to employ four inputs that are labour force, capital, arable land and energy."*[20] (Maradan & Vassiliev, 2005, p. 10) The authors find that *„Shadow prices of $CO_2$ diminish as income per capita grows. … Hence, the developed economies would have to undergo a smaller loss of consumption or GDP if the last unit of the $CO_2$ pollution had to be eliminated."* (Maradan & Vassiliev, 2005, p. 12) Table 6 showes the results of Maradan & Vassiliev (2005) grouped country income, the shadow prices are in milions US$ per Kilo-ton of $CO_2$.

**Table 6 CO2 Shadow prices estimates by income group (US$)**

| Desirable output: GDP | Average shadow price | Highest shadow price | Lowest shadow price |
|---|---|---|---|
| Low-income countries | 5.22 (4.33) | 10.83 | 0.10 |
| Lower-middle income countries | 3.83 (2.38) | 8. 03 | 0.29 |
| Higher-middle income countries | 3.27 (1.96) | 7.27 | 0.01 |
| High-income countries | 1.16 (0.77) | 2.55 | 0.13 |
| Desirable output: Consumption | | | |
| Low-income countries | 4.74 (3.63) | 11.65 | 0.22 |
| Lower-middle income countries | 2.51 (2.04) | 6.68 | 0.17 |
| Higher-middle income countries | 1.09 (0.78 ) | 3.52 | 0.01 |
| High-income countries | 0.60 (0.46) | 1.29 | 0.07 |

Note: standard deviations are in parenthesis.

Source: Maradan & Vassiliev (2005, p. 13)

[19] The sample does not include the Czech Republic.

[20] GDP and consumption are expressed in purchasing power parity US dollars and refer to the year 1985, data on $CO_2$ come from World Development Indicators (WDI) database and according to this, $CO_2$ emissions are counted from the pollution from burning of fossil fuel and cement manufactoring. For more detail about the data, see Maradan & Vassiliev (2005).

Salnykov & Zelenyuk (2006) investigate the shadow prices of $CO_2$, $SO_2$ and NOx with focus on post-comunist countires. They use the *directional output distance function* in translog form for these purposes. The autors work with a data sample from 1995 included 96 countries, which they separate into threee groups: Countries in Transition (CITs), North and South. Inputs are labor, arable land energy consumption and capital stock in each coutry, desirable output is GDP and undesirable outputs are the emissions of pollutants. For example, the shadow prices estimated for the Czech Republic are $106.52, $5,079 and $53,523 per ton of pollutant for $CO_2$, $SO_2$ and NOx, respectively. In the internatnational comparison, Salnykov & Zelenyuk (2006) come to a conclusion that if "*any agreement similar to Kyoto protocol should be in force, under assumption of unchanging technology CITs will be major pollution permit sellers*." Based on the fact that the authors work with the data from year 1995, we don't find this assumption as much reasonable.

Summarizing, the estimates of shadow prices differ not only in dependence on country, year or utility but also based on used methodology. Vardanyan & Noh (2006) clearly show that the role of selectected type of distance function and its properties (i.e. the mapping rule) is crucial for the estimates. In the last 20 years, the autrhors have used all above described types of distance function - *output, input* and *directional output* (ODF, IDF, DDF). Especially in energy sector, the ODF is used very often. In the last years, the authors use the DDF more and more often. The DDF allows us to reflect the real development of good and bad outpus – it don't require the proportional change of boht good and bad outputs in the same direction. The above cited works differ to some degree also in type and quality of the data. For exsample, Kwon & Yuh (1999) use the plant capacities in kW as substitution for capital input.

Table 7 provide a overview of shadow prices estimates and methods described above, in Table 18 in the Appendix A you can find a extended version of this table.

**Table 7 Emission shadow price estimates overview**

| | | SCHADOW PRICES (€2005/t) | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Study** | **Method** | **CO2** | **SOx** | **NOx** | **PM** | **BOD (COD)** | **TSS[21]** | **vector g=(y,b)** |
| Färe et al. (1993) | ODF | - | 9956.7 | - | 68074.9 | 2809.7 | 0 | y>0, b>0 |
| Coggins & Swinton (1996) | ODF | - | 357.1 | - | - | - | - | y>0, b>0 |
| Boyd et al. (1996) | ODF | - | 475.7 | - | - | - | - | y>0, b>0 |
| Swinton (1998) | ODF | - | 254.1 | - | - | - | - | y>0, b>0 |
| Kwon & Yuh (1999) | ODF | 5.2 | 425.5 | 201 | 21210.8 | - | - | y>0, b>0 |
| Hailu-Veeman (2000) | IDF | - | - | - | - | 199.3 | 463 | y=0, b=0 |
| Swinton (2002) | ODF | - | 176.7 | - | - | - | - | y>0, b>0 |
| Lee, Park & Kim (2002) | DDF | - | 3790.5 | 21219.5 | 62333.5 | - | - | y<0, b<0 |
| Marklund (2003) | DDF | - | - | - | - | (760.2) | 119 | y=1, b=-1 |
| Färe et al. (2005) | DDF | - | 106.82 | - | - | - | - | y=1, b=-1 |
| Atkinson & Dorfman (2005) | IDF | - | 501.8544 (only 1995) | - | - | - | - | y=0, b=0 |
| Lee (2005) | IDF | - | 451.4 | - | 344.1 (ash) | - | - | y=0, b=0 |
| Vardanyan & Noh (2006) | | | | **see Table 5** | | | | |
| Murty et al. (2007) | DDF | - | 51.0 | 182.3 | 129.52 (SPM) | - | - | y=1, b=-1 |
| Bauman et al.(2008) | ODF | - | 224.5 | - | - | - | - | y>0, b>0 |
| Park & Lim (2009) | ODF | 14.04 | - | - | - | - | - | - |
| Maradan & Vassiliev (2005) | DDF | 2.13-9.6 | - | - | - | - | - | y=0, b>0 |
| Salnykov & Zelenyuk (2006) | DDF | 115.0 | 5485.3 | 57805 | (results for CZE) | | | - |

[21] BOD, COD and TSS are water pollutants. BOD is a subset of COD.

# 4. Distance function optimization

## 4.1.The empirical model

According to Färe et al. (2005), the distance function can be estimated in a several ways. One of these methods is to use a data envelopment analysis (DEA) model, "*where the output possibility set is constructed as a piecewise linear combination of all observed outputs and inputs"* (Färe et al., 2005, p. 476). Using the DEA, we are able to estimate the distance function the performance but not the shadow prices. Since our main aim is to estimate the shadow price, we require a parametric and differentiable specification distance function. Following Färe et al. (2005) and Vardanyan & Noh (2006), we look for a function satisfying the translation property and that could provide a second-order approximation to a true, but unknown function. The quadratic input distance function satisfies such condition:

$$\begin{aligned}
&ID_{kt}(x_{kt}, y_{kt}, b_{kt}) \\
&= \alpha_0 + \sum_{n=1}^{N} \alpha_n x_{nkt} + \sum_{m=1}^{M} \beta_m y_{mkt} + \sum_{b=1}^{B} \gamma_b b_{bkt} + \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \alpha_{nn'} x_{nkt} x_{n'kt} \\
&+ \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \beta_{mm'} y_{mkt} y_{m'kt} + \frac{1}{2}\sum_{b=1}^{B}\sum_{b'=1}^{B} \gamma_{bb'} b_{bkt} b_{b'kt} \\
&+ \sum_{n=1}^{N}\sum_{m=1}^{M} \delta_{nm} x_{nkt} y_{mkt} + \sum_{n=1}^{N}\sum_{b=1}^{B} \eta_{nb} x_{nkt} b_{bkt} + \sum_{m=1}^{M}\sum_{b=1}^{B} \mu_{mb} y_{mkt} b_{bkt}
\end{aligned} \tag{26}$$

We estimate the parameters of this function by minimizing the total distance between individual observations in the sample and the estimate of the optimal input set frontier solving the following linear programming problem:

$$Min\sum_{k=1}^{K}\sum_{t=1}^{T}[ID_{kt}(x_{kt},y_{kt},b_{kt})-1]$$

$s.t.$

(i) $ID_{kt}(x_{kt},y_{kt},b_{kt})\geq 1;\quad k=1,\ldots,K\ \ t=1,\ldots,T,$

(ii) $ID_{kt}(x_{kt},y_{kt},0)<1;\quad k=1,\ldots,K\ \ t=1,\ldots,T,$

(iii) $\frac{ID_{kt}(x_{kt},y_{kt})}{\partial y_m}\leq 0;\quad k=1,\ldots,K\ \ t=1,\ldots,T\ \ m=1,\ldots,M,$

(iv) $\frac{ID_{kt}(x_{kt},y_{kt},b_{kt})}{\partial b_b}\geq 0;\quad k=1,\ldots,K\ \ t=1,\ldots,T\ \ b=1,\ldots,B,$

(v) $\frac{ID_{kt}(x_{kt},y_{kt},b_{kt})}{\partial x_n}\geq 0;\quad k=1,\ldots,K\ \ t=1,\ldots,T\ \ n=1,\ldots,N,$

(vi) $\sum_{n=1}^{N}\alpha_n=1,$

$\sum_{n=1}^{N}\alpha_{nn'}=0;\ \ n'=1,\ldots,N,$

$\sum_{=1}^{N}\delta_{nm}=0;\quad m=1,\ldots,M,$

$\sum_{=1}^{N}\eta_{nb}=0;\quad b=1,\ldots,B,$

(vii) $\alpha_{nn'}=\alpha_{n'n};\ \ n\neq n',\quad \beta_{mm'}=\beta_{m'm};\quad m\neq m',\quad \gamma_{bb'}=\gamma_{b'b};\ \ b\neq b'.$

Where $k$ and $t$ are indexes of producer and year, respectively; $k = 1 \dots K, t = 1, \dots, T$, $K$ is number of producers and $T$ is number of years. $N, M$ and $B$ are numbers of inputs, good and bad outputs, respectively. The restrictions in (27) are implemented in a way that satisfies all of the input distance function properties in Table 12. The representation property is imposed by the inequality in (i). The null-jointness property is ensured by the restriction in (ii) and implies that the desirable output cannot be produced without producing the undesirable outputs. The monotonicity conditions are imposed by the restrictions (iii) - (v), respectively. Free disposability of good outputs is satisfied by (iii). Free disposability of inputs is imposed by the restriction in (v). The translation property is imposed by the restrictions in (vi). Finally, the symmetry of parameters of the quadratic functional form is ensured by the restriction in (vii).

### 4.2. The data

The directional distance function is estimated using data on the Czech energy industry over the period 2002-2007. Our model has two good outputs (electricity and heat), five bad outputs ($SO_2$, PM, NOx, CO and VOC) and three types of production inputs, including total assets as capital input, number of employees and fuels consumption, i.e. $M = 2$, $B = 5$ and $N = 3$. We have aggregated the fuels consumption from the single types of fuel into one aggregated fuel consumption. However, we still keep the information about the fuel types combusted in each firm. From the original dataset (already cleaned), which has included 88 observations from fifteen firms, we have had to remove the gas heating plants. The reason is that the gas heating plants optimize the profit primary according heat demand and the electricity is only their second product.[22] Since that, it has shown that the estimation of distance function by gas heating plant is problematic and the estimation of emission shadow prices relative to the electricity price is not appropriate. Thus, our model contains nine firms producing electricity and heat with a total of 53 observations (due to one missing observation).

We collected the annual electricity (MWh) and heat (GJ) production of each generating firms in the sample from the Energy Regulatory Office year statistics.[23] The data about total assets and the number of employees are gathered by Creditinfo Czech Republic, s.r.o. - the number of employees was additionally checked in sample firm's annual reports. Fuels consumptions (GJ) and emission data (tons) are gathered by the Czech Hydrometeorological Institute in the REZZO database. The data in our sample come from the REZZO 1 database.[24] Fuel consumptions and emission data are available even on generating unit level but the numbers of employees and information about total assets are available only on firm level data. Therefore we aggregate the fuel consumptions and emission data also on firm level. The aggregation on firm level data has also positive effects because it allows us to include also electricity production from renewable energy sources. This brings another source of

[22] This is common for all heating plants in general, but the gas heating plants are more flexible than coal heating plants and therefore are able to produce electricity only in peak time in much more degree than other heating plants. The price of peak electricity is significantly higher than the average electricity price and therefore the estimate of marginal abatement cost might be bias in this case.

[23] Energy Regulatory Office (ERO) provides statistics about yearly electricity and heat production on its web pages http://www.eru.cz/dias-browse_articles.php?parentId=131&deep=off&type and http://www.eru.cz/dias-browse_articles.php?parentId=136&deep=off&type, respectively.

[24] REZZO - Register of Emissions and Air Polluters - is reporting system operated by the Czech Hydrometeorological Institute in accordance with Act No. 86/2002 Coll., Clean Air Act. It has four categories according to the type of the polluter. Category REZZO 1 is register of emissions from extra large and large pollution sources, it contains stationary sources with installed thermal capacity higher than 5 MW.

substitution between fuels, labor and capital within one firm (e.g. ČEZ). The power electricity price is obtained as a weighted average of daily market averages from the OTE's annual reports. In 2002 the electricity started to be traded on marked in the Czech Republic and since this year OTE has been reporting the electricity price.[25] Since the electricity price is created on the market, we assume that it is common for all firms. The capital input (in thousand CZK) and electricity price (CZK/MWh) are both deflated by the OECD consumer price index (2005=100).[26]

The input distance function is sensitive to fuel mix and it is appropriate to estimate the input distance function for a group of firm with approximately the same fuel mix. Therefore we split our dataset into two samples of data according the coal consumption. Sample A includes firms combusting hard coal and other fuels – there are 18 observations from 3 firms. Sample B includes firms, where brown coal is the main fuel and no hard coal is combusted – there are 35 observations from 6 firms. The summary statistics of the samples are compiled in Table 8 and Table 9, respectively. In the Appendix A, you can find more detail descriptive statistics by years.

[25] In accordance with Act No. 458/2000 Coll., the electricity market in the Czech Republic was opened as of January 1, 2002. Before this date, the electricity prices were fully regulated and the price of power electricity was not available because the Czech Statistical Office reports only the final electricity price including the transmission costs. This is the main reason why our time series begins in year 2002 and not earlier.

[26] For the conversion between CZK 2005 and € 2005, the exchange rate 29.78 CZK/€ is used.

**Table 8 Dataset descriptive statistics - Sample A**

| | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| Capital (mil.CZK 2005) | 92200 | 121000 | 8267 | 296000 |
| Labor | 3095 | 2690 | 752 | 7677 |
| Fuels (TJ) | 142000 | 168000 | 8199 | 407000 |
| Electricity (TWh) | 21000 | 28900 | 149 | 65400 |
| Heat (TJ) | 11600 | 5556 | 348 | 21600 |
| PM (t) | 1078 | 1323 | 17 | 3010 |
| so2 (t) | 25233 | 26061 | 897 | 65621 |
| nox (t) | 23835 | 27976 | 800 | 66075 |
| co (t) | 1615 | 1748 | 56 | 4577 |
| voc (t) | 1498 | 1942 | 29 | 4585 |

**Table 9 Dataset descriptive statistics - Sample B**

| | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|
| Capital (mil.CZK 2005) | 2369 | 2550 | 247 | 8677 |
| Labor | 259 | 126 | 84 | 444 |
| Fuels (TJ) | 7823 | 9523 | 703 | 28000 |
| Electricity (TWh) | 483 | 719 | 12 | 2074 |
| Heat (TJ) | 2536 | 2123 | 307 | 6090 |
| PM (t) | 59.5 | 69.7 | 1.4 | 279.1 |
| so2 (t) | 2521.2 | 2515.8 | 340.7 | 10110.6 |
| nox (t) | 1058.8 | 1262.3 | 64.5 | 4616.4 |
| co (t) | 163.1 | 214.4 | 9.0 | 828.0 |
| voc (t) | 83.1 | 98.0 | 0.5 | 319.7 |

The firms included in our dataset produce from 84 % to 87 % of net electricity production in the Czech Republic in year 2002 and 2007, respectively.

Table 10 provides a summary of annual desirable and undesirable outputs from the whole dataset and also shows the annual price of electricity.

**Table 10 Summary statistics of outputs**

| Year | Electricity (TWh) | Heat (PJ) | PM (tons) | $SO_2$ (tons) | NOx (tons) | CO (tons) | VOC (tons) | Electricity price (CZK2005/MWh) |
|---|---|---|---|---|---|---|---|---|
| 2002 | 59.1 | 54.7 | 3785 | 91131 | 78849 | 6623 | 4951 | 783 |
| 2003 | 65.7 | 53.8 | 3395 | 88931 | 76932 | 5610 | 5020 | 737 |
| 2004 | 66.4 | 52.9 | 3669 | 84056 | 77027 | 5667 | 5026 | 707 |
| 2005 | 64.9 | 55.8 | 3585 | 90466 | 75332 | 5554 | 4952 | 916 |
| 2006 | 67.4 | 51.8 | 3476 | 93032 | 77088 | 5577 | 4566 | 1079 |
| 2007 | 70.9 | 46.3 | 3580 | 94821 | 80867 | 5752 | 5349 | 990 |
| Total | 394.4 | 315.3 | 21489 | 542436 | 466095 | 34783 | 29864 | |

## 4.3. Empirical results

The input distance function is estimated for each sample of data separately. 80 parameters are needed to be estimated in each sample. The parameter estimation for the input distance function is carried out by minimizing the sum of deviation from unity – as described in (27) – subject to 223 and 410 constrains for sample A and B, respectively. There are 11 linear homogeneity conditions and 14 symmetry restrictions for both samples. There are 18 (35) representation conditions, 180 (350) monotonicity conditions relating to inputs, desirable outputs and undesirable outputs in sample A and (B), respectively[27]. Matlab codes were written and solved to compute the parameter estimates. The estimates of the parameters for sample A are shown in Table 11. The estimates for sample B, you can find together with other detailed results in the Appendix B.

[27] Number of representation and monotonicity conditions depends on the number of observation in the sample.

**Table 11 Parameter estimates for input distance function - sample A**

| Parameter | Estimate | Parameter | Estimate | Parameter | Estimate | Parameter | Estimate |
|---|---|---|---|---|---|---|---|
| α0 | -0.05403 | δLE | 1.21E-09 | ηFVOC | 9.70E-06 | γPMSO2 | 0.009372 |
| αL | 1.004185 | δLH | -7.06E-10 | βEE | 2.52E-08 | γSO2SO2 | 0.005072 |
| αK | 0.005053 | δKE | -1.05E-10 | βEH | -2.71E-09 | γSO2NOx | -0.01011 |
| αF | -0.00924 | δKH | -4.54E-10 | βEH | -2.71E-09 | γSO2CO | -0.00581 |
| βE | 0.027604 | δFE | -1.11E-09 | βHH | -1.56E-09 | γSO2VOC | -0.06892 |
| βH | 0.006934 | δFH | 1.16E-09 | μEPM | -4.34E-06 | γPMNOx | 0.019267 |
| γPM | -2.75409 | ηLPM | -1.85E-06 | μESO2 | 3.37E-08 | γSO2NOx | -1.01E-02 |
| γSO2 | -1.19304 | ηLSO2 | -9.70E-07 | μENOx | -1.83E-05 | γNOxNOx | 0.021023 |
| γNOx | -2.85841 | ηLNOx | -2.05E-06 | μECO | -2.92E-05 | γNOxCO | 0.020313 |
| γCO | 0.3061 | ηLCO | -5.14E-06 | μEVOC | -1.45E-05 | γNOxVOC | 0.055508 |
| γVOC | -1.26581 | ηLVOC | -5.76E-06 | μHPM | -1.86E-07 | γPMCO | -0.42882 |
| αLL | -5.39E-10 | ηKPM | -1.63E-06 | μHSO2 | -5.91E-07 | γSO2CO | -0.00581 |
| αLK | 4.12E-10 | ηKSO2 | 1.51E-07 | μHNOx | -2.36E-06 | γNOxCO | 0.020313 |
| αLF | 1.27E-10 | ηKNOx | 8.28E-07 | μHCO | -3.10E-06 | γCOCO | -0.29343 |
| αLK | 4.12E-10 | ηKCO | 3.97E-07 | μHVOC | -1.10E-06 | γCOVOC | 0.368614 |
| αKK | -1.15E-09 | ηKVOC | -3.94E-06 | γPMPM | -0.01636 | γPMVOC | -0.29207 |
| αKF | 7.38E-10 | ηFPM | 3.48E-06 | γPMSO2 | 0.009372 | γSO2VOC | -0.06892 |
| αLF | 1.27E-10 | ηFSO2 | 8.19E-07 | γPMNOx | 0.019267 | γNOxVOC | 0.055508 |
| αKF | 7.38E-10 | ηFNOx | 1.22E-06 | γPMCO | -0.42882 | γCOVOC | 0.368614 |
| αFF | -8.65E-10 | ηFCO | 4.74E-06 | γPMVOC | -0.29207 | γVOCVOC | -0.74942 |

The estimated value of the input distance function (IDF) is very close to one in most cases, which implies very high technical efficiency. This could be partly caused by the relative small size of the data samples, but on the other hand we can find very similar results also in the literature (e.g. Hailu & Veeman (2000)). The firm averages of the IDF value together with emission rates (ER) for each pollutant are displayed in Table 12.

**Table 12 Firm averages of IDF values and Emission Rates**

| Firm | Value of IDF | ER PM (t/PJ) | ER $SO_2$ (t/PJ) | ER $NO_x$ (t/PJ) | ER CO (t/PJ) | ER VOC (t/PJ) |
|---|---|---|---|---|---|---|
| 1 | 1.058503 | 7.8 | 161.4 | 166.6 | 10.7 | 11.2 |
| 2 | 1.000039 | 7.1 | 264.0 | 131.6 | 21.3 | 10.1 |
| 3 | 1.000041 | 11.6 | 373.3 | 154.0 | 20.4 | 10.1 |
| 4 | 1.007060 | 6.7 | 322.1 | 194.6 | 17.8 | 6.9 |
| 5 | 1.004077 | 5.0 | 130.8 | 97.3 | 8.7 | 3.3 |
| 6 | 1.000040 | 7.2 | 386.9 | 132.5 | 12.4 | 12.1 |
| 7 | 1.000043 | 7.4 | 498.9 | 163.0 | 15.3 | 15.6 |
| 8 | 1.000043 | 5.1 | 656.3 | 124.5 | 49.9 | 13.4 |
| 9 | 1.000043 | 5.1 | 656.3 | 124.5 | 49.9 | 13.4 |

The emission shadow prices are derived from the input distance function as described in (25). The shadow prices are in term of forgone output (electricity) and therefore are in negative terms. For better convenience, we present the result already as marginal abatement cost in positive terms. The overall emission-weighted average (WA) of marginal abatement costs is 5223, 1726, 2450, 4946 and 5921 € per ton of PM, $SO_2$, $NO_x$, CO and VOC with standard deviation 54150, 3274, 6704, 24502 and 25595, respectively. For comparison with other MAC estimations for the Czech Republic we use the median of the MACs, because neither Salnykov & Zelenyuk (2006) nor the estimates from GEM-E3 and GAINS model provide emission-weighted averages of MACs. The medians of our MACs are 8374, 1198, 2805, 6051 and 8549 € per ton of PM, $SO_2$, $NO_x$, CO and VOC, respectively. The weighted averages of MACs of are drifted mainly by the two biggest frims in the data set - 1 = ČEZ, a.s. and 4 = Dalkia Česká republika, a.s.. The estimated MACs vary across firms and also over time as it is shown in the following 3 tables. For sample A, the medians of MACs are 6256, 1491, 2210, 3092 and 10548 per ton of PM, $SO_2$, $NO_x$, CO and VOC, respectively. For sample B, the medians of MACs are 8670, 847, 3293, 7005 and 7398 per ton of PM, $SO_2$, $NO_x$, CO and VOC, respectively.

**Table 13 Summary statistics of MAC (€2005/t)**

| | PM | $SO_2$ | $NO_x$ | CO | VOC |
|---|---|---|---|---|---|
| WA | 5223 | 1726 | 2450 | 4946 | 5921 |
| Mean | 26857 | 2087 | 5312 | 16014 | 18715 |
| Median | 8374 | 1198 | 2805 | 6051 | 8549 |
| S.d. | 54150 | 3274 | 6704 | 24502 | 25595 |
| Min | 36 | 80 | 4 | 10 | 42 |
| Max | 240764 | 20886 | 36903 | 122657 | 116223 |

**Table 14 Annual weighed averages of marginal abatement costs (€2005/t)**

| | PM | $SO_2$ | $NO_x$ | CO | VOC |
|---|---|---|---|---|---|
| 2002 | 4090 | 930 | 3451 | 3792 | 11594 |
| 2003 | 4598 | 680 | 533 | 2799 | 2969 |
| 2004 | 4877 | 952 | 2130 | 3226 | 6441 |
| 2005 | 6709 | 2028 | 2976 | 6849 | 4679 |
| 2006 | 7629 | 2770 | 2590 | 4477 | 4919 |
| 2007 | 3546 | 2844 | 2978 | 8681 | 4959 |

**Table 15 Firm averages of marginal abatement costs (€2005/t)**

| Firm | PM | $SO_2$ | $NO_x$ | CO | VOC |
|---|---|---|---|---|---|
| 1 | 236 | 1134 | 1802 | 2506 | 2860 |
| 2 | 19110 | 8313 | 6442 | 10091 | 33075 |
| 3 | 3621 | 1031 | 7947 | 17516 | 12274 |
| 4 | 7489 | 1329 | 2889 | 3389 | 8678 |
| 5 | 16809 | 2502 | 4560 | 30467 | 62372 |
| 6 | 168074 | 2052 | 17432 | 62163 | 30052 |
| 7 | 9659 | 700 | 2460 | 6430 | 6384 |
| 8 | 8112 | 524 | 1466 | 5550 | 5046 |
| 9 | 4958 | 1015 | 2309 | 4009 | 5490 |

# 5. Decomposition of Marginal Abatement Costs

In order to analyze the factors that might affect the marginal abatement costs of pollution, we will test the hypotheses that the marginal abatement cost decline over time; that marginal abatement costs rice with declining emission level; and that marginal abatement costs rice with declining emission rate. We run following six Fixed-effects models with robust standard errors for all pollutants (28 – 33), where $ER$ is Emission Rate and $EL$ is Emission level[28]. We use the robust standard errors because of heterogeneity of the data. We have only 53 observations in our panel dataset – 9 firms over 6 years. Not all models fit the data properly and not all are significant. There is a problem of multicollinearity between $lnER_{it}$ and $lnEL_{it}$ in Model 6 (The multicollinearity was confirmed also by additional tests by all pollutants.) and therefore this model is rather only illustrative.

Model 1 $$lnMAC_{it} = \alpha + \beta lnER_{it} + \delta lnyear_{it} + \mu_i + v_{it} \tag{28}$$

Model 2 $$lnMAC_{it} = \alpha + \beta lnER_{it} + \mu_i + v_{it} \tag{29}$$

Model 3 $$lnMAC_{it} = \alpha + \gamma lnEL_{it} + \delta lnyear_{it} + \mu_i + v_{it} \tag{30}$$

Model 4 $$lnMAC_{it} = \alpha + \gamma lnEL_{it} + \mu_i + v_{it} \tag{31}$$

Model 5 $$lnMAC_{it} = \alpha + lnyear_{it} + \mu_i + v_{it} \tag{32}$$

Model 6 $$lnMAC_{it} = \alpha + \beta lnER_{it} + \gamma lnEL_{it} + \delta lnyear_{it} + \mu_i + v_{it} \tag{33}$$

[28] Emission Rate is defined as ton of pollutants per input (ton/PJ). Emission level means the absolute amount of emission produced by the firm.

Table 16 provides results all the models for all pollutants. The asterisk in the right marks which of the pair of Model 1 and 2 or the pair of Model 3 and 4 is better based on the Log-likelihood, Schwarz, Akaike and Hannan-Quinn criteria.

Generally, if we don't take into account the biased Model 6 (where are a few exceptions), all coefficients by $lnER_{it}$ are negative for all pollutants with exception of VOC, which is in accordance with the theory that MACs rice with declining emission rate. Unfortunately, the results are significant only for $NO_x$ and CO. By VOC, the $\beta$ and $\gamma$ are negative in model with time term (Models 1and 3) and positive in models without the time term (Models 2 and 4), but in all cases the coefficient are very close to zero and insignificant. The coefficients by $lnEL_{it}$ are negative for all other pollutants with exception of $SO_2$. This also confirms the hypothesis that MACs rice with the declining emission level. But again, the results are significant only for $NO_x$ and CO. The hypothesis that MACs decline over time, has the least support in the data. In the Model 5, the coefficients by time are positive by all pollutants with exception of VOC, they are significant for $SO_2$ and significant at 10% significance level. In other models, where time is included, the time coefficients are also positive with exception of Model 3 by PM and VOC models. The positive time coefficients (although only insignificant) could confirm the finding in Bauman et al. (2008). We have the data from time period 2002-2007. In this period, most of the innovations in energy sector were production process innovation, because the main end-of-pipe innovations were made already in the 90ties of 20$^{th}$ century. Bauman et al. (2008) shows that some production process innovations increase marginal abatement costs – on our data, it would lead to positive time coefficients.

These econometric models are not ideal. There is a problem of non-normality of residuals in all models and tests for differing group intercepts confirm the heterogeneity of the data which lead to different intercepts among the firms. We focus now only on models with the best fit and significant results by the detailed interpretation. We can see relatively high positive time coefficient in $SO_2$ Model 5 at 5% significance level, which indicates, that $SO_2$ MACs rice rapidly with time in our short period. The time coefficients in both CO and models have the same interpretation. Based on the $NO_x$ Models 1 – 4, we cannot reject the hypotheses that MACs rice with declining emission rate and with declining emission level at 1% significance level. We cannot reject the hypotheses that MACs rice with declining emission rate and with declining emission at 5% significance level also by CO Models 1 – 4.

**Table 16 Results with significant results**

| | Model | α | | β | | γ | | δ | | |
|---|---|---|---|---|---|---|---|---|---|---|
| PM | 1 | -84.8906 | (2110.35) | -0.92889 | (0.601855) | - | | 12.5185 | (277.557) | |
| | 2 | 10.299 | (1.15257) | -0.93409 | (0.608989) | - | | - | | * |
| | 3 | 63.2481 | (2162.19) | - | | -0.83351 | (0.608987) | -6.74556 | (284.354) | |
| | 4 | 11.945 | (2.53854) | - | | -0.82967 | (0.616949) | - | | * |
| | 5 | -681.512 | (2198.89) | - | | - | | 90.7579 | (289.208) | |
| | 6 | -552.967 | (2183.2) | -2.59013 | (2.00375) | 1.71802 | (1.86321) | 73.566 | (286.938) | |
| SO2 | 1 | -2896.08 | (1122.75) | -0.73469 | (0.711176) | - | | 382.39 | (147.356) | |
| | 2 | 10.1031 | (5.43116) | -0.53024 | (0.942093) | - | | - | | * |
| | 3 | -2825.71 | (1285.43) | - | | 0.27623 | (0.532863) | 372.287 | (168.955) | * |
| | 4 | 5.90042 | (6.82548) | - | | 0.14346 | (0.854566) | - | | |
| | 5 | -2794.5 | (1271.92) | - | | - | | 368.473 | (167.29) | |
| | 6 | -3801.08 | (610.13) | -4.19515 | (1.75964) | 3.77598 | (1.34287) | 500.077 | (80.1973) | |
| NOx | 1 | -1186.64 | (1111.11) | -3.27965 | (1.17058) | - | | 159.255 | (145.861) | |
| | 2 | 25.2681 | (6.11488) | -3.49655 | (1.23328) | - | | - | | |
| | 3 | -240.35 | (1267.34) | - | | -3.53826 | (1.05396) | 35.9967 | (165.895) | |
| | 4 | 33.9638 | (8.10376) | - | | -3.62553 | (1.12842) | - | | * |
| | 5 | -1914.5 | (1206.37) | -1.10489 | (1.43257) | - | | 252.847 | (158.667) | |
| | 6 | -387.96 | (1383.3) | -2.70804 | (1.62095) | - | | 55.3475 | (181.076) | |
| CO | 1 | -3585.5 | (965.251) | -2.15888 | (0.909945) | - | | 473.54 | (127.016) | * |
| | 2 | 14.7397 | (3.13291) | -2.10811 | (1.08498) | - | | - | | |
| | 3 | -3039.05 | (1099.22) | - | | -2.18128 | (0.903957) | 402.314 | (144.494) | |
| | 4 | 20.3564 | (5.51889) | - | | -2.29057 | (1.0801) | - | | |
| | 5 | -3436.76 | (1429.61) | - | | | | 453.157 | (188.029) | |
| | 6 | -3335.72 | (1004.01) | -1.21499 | (0.906613) | -1.01324 | (1.18335) | 441.011 | (131.964) | |
| VOC | 1 | 3803.31 | (2544.14) | -0.05324 | (0.105309) | - | | -499.06 | (334.595) | * |
| | 2 | 8.654 | (0.149037) | 0.060848 | (0.0720876) | - | | - | | |
| | 3 | 3846.31 | (2589.66) | - | | -0.07836 | (0.145008) | -504.687 | (340.542) | * |
| | 4 | 8.5182 | (0.439375) | - | | 0.060985 | (0.10243) | - | | |
| | 5 | 3753.89 | (2475.46) | - | | - | | -492.575 | (325.584) | |
| | 6 | 4175.6 | (2843.63) | 1.48807 | (2.79587) | -1.52865 | (2.79589) | -547.582 | (373.515) | |

(Standard errors in parentheses)

# 6. Conclusions

We have provided an overview of methodologies to MACs estimation and described types of distance function and discussed their advantages and disadvantages. Based on the discussed arguments, the *direction output distance function* and the *input distance function* are appropriate for shadow price estimation of undesirable outputs and in particular, the *input distance function* is the best choice for shadow price estimation of undesirable outputs in the energy sector. This is based on the Kumbhakar, Orea, Rodríguez-Álvarez, & Tsionas (2007) findings that the *input distance function* is appropriate in those cases of cost minimization where output is exogenous and inputs are endogenous. We have shown that this is the case of energy sector. Hailu & Veeman (2000) argue further in favour of the *input distance function*, because by the reductnion of inputs the undesirable outputs don't increase and the society has pure benefits from the costs minimization and from the emission reduction. Thus, we have chosen the *input distance function* for our emission shadow prices estimation. The *input distance function* measures the maximum amount by which the input vector can be deflated, while the output vector is held constant. Its optimal value is one and if the *input distance function* has value greater than one, the firm uses more inputs than in optimum to produce the same output. The shadow prices are derived from the estimated distance function using the Shephard (1970) duality between revenue and cost function. The big advantage of this approach is that we don't need to know the input prices. For estimation of shadow prices of all outputs it is sufficient to know only the price of one freely traded output.

In the literature review, we have shown which types of distance function are used for pollutants shadow prices estimation in the literature. The authors use different type of distance function and also different functional forms. Also the data levels go from unit level to country level. The choice of the distance function type has been shown as crucial. Most of the shadow prices estimates are made for $SO_x$. The estimates vary across the studies from €0 in some Vardanyan & Noh (2006) distance function specifications to €9957 per ton of $SO_x$ in Färe et al. (1993). Our median estimate of $SO_2$ shadow prices is €1198 per ton of $SO_2$.

We have applied the input distance function in quadratic form on firm level data over the period 2002-2007. We have found that the distance function is sensitive also to structure of fuelmix. Most studies apply the distance function either on homogeneous firm level data (e.g. coal power plants) or on aggregated data (sectoral or country level). We have relative heterogeneous firm level data and therefore we have split our dataset into two samples according to fuelmix structure. The overall medians of our MACs are 8374, 1198, 2805, 6051 and 8549 € per ton of PM, $SO_2$, $NO_x$, CO and VOC, respectively. Our estimates are lower than the estimates for the Czech Republic at values of 5485 and 57805 € per ton of $SO_x$ and $NO_x$ in Salnykov & Zelenyuk (2006), respectively, but are higher than the estimates from the GEM-E3 model – 7764, 785, 1520 and 0 € per ton of PM, $SO_2$, $NO_x$ and VOC in scenario S-CE. Our estimates are also within the range from GAINS model.

In order to analyze the factors that might affect the marginal abatement costs of emission, we have tested the hypotheses that the marginal abatement cost decline over time; that marginal abatement costs rice with declining emission level; and that marginal abatement costs rice with declining emission rate. The second two hypotheses we cannot reject at least by $NO_x$ and CO. By other pollutants the results also support the hypotheses but are no significant. The emission level and emission rate are correlated, therefore we cannot say if the MACs really decline also due to increasing the level of emission produced by the firm or if the MACs decline only due to increasing emission rates. The first hypothesis that the MACs decline over time, we cannot confirm, because it has no support in the data. On the contrary, most results (although only insignificant) indicate that the MACs rice over time. We have short time series to make some conclusions about time trend of MACs, but the increasing MACs in time would be in accordance with Bauman et al. (2008) findings that production process innovations can increase marginal abatement costs.

There are two ways for further research. Either to employ the input distance function on aggregated – sectoral level data. This should allow working with longer time series, because the GDP could be a proxy for the desirable output and the problem with availability of market electricity price only since 2002 will fall away. Or the second – and more challenging – way is to acquire the unit level data about employees and capital and employ the input distance function these data. This would bring another view on the marginal abatement costs according the plant size and combusted fuel.

# References


Atkinson, S. E., & Dorfman, J. H. (2005). Bayesian measuremenm of productivity and efficency in the presence of undesirable outputs: credition electric utilities for reducing air pollution. *Journal of Econometrics* (126), pp. 445-468.

Bauman, Y., Lee, M., & Seely, K. (2008). Does Technological Innovation Really Reduce Marginal Abatement Costs? Some Theory, Algebraic Evidence, and Policy Implications. *Environmental and Resource Economics , 40*, pp. 507-527.

Bízek, J. (2009). *Integrated Approach to Climate Change Mitigation and Air Quality Protection.* Bachelor Thesis. Charles University Prague, Faculty of Social Science, Institute of Economic Studies.

Bluffstone, R. (1999). Are the cost of pollution abatement lower in Central and Eastrn Europe? *Environment and Development Economics , 4*, pp. 449-470.

Boyd, G., Molburg, J., & Prince, R. (1996). Alternative Methods of Marginal Abatement cost Estimation: Non-parametric Distance Functions. *Proceedings of the USAEE/IAEE 17th Conference 86-95* .

Coggins, J., & Swinton, J. (1996). The price of pollution: a dual approach to valuing SO2 allowance. *Journal of Environmental Economics and Management , 30*, pp. 58-72.

Färe, R., Grosskopf, S., & Margaristis, D. (2008). Effiency and Productivity: Malmquist and More. In H. O. Fried, C. A. Lovell, & S. S. Schnidt, *The Measurent of Productive Efficiency and Productive Growth.* New York: Oxford University Press, Inc.

Färe, R., Grosskopf, S., & Nelson, J. (1990). On Price Efficiency. *International Economic Review , 31*, pp. 709-720.

Färe, R., Grosskopf, S., Lovell, K., & Pasurka, C. (1989). Multilateral productivity comparison when some products are undesirable: a non-parametric approach. *The Review of Economics and Statistics , 71*, pp. 90-98.

Färe, R., Grosskopf, S., Lowell, C., & Yaisawarng, S. (1993). Derivation of Shadow Prices for Undesirable Outputs: A Distance Function Approach. *Review of Economocs and Statistics , 75*, pp. 374-380.

Färe, R., Grosskopf, S., Noh, D.-W., & Weber, W. (2005). Caharacteristics of a polluting technology: therory and practice. *Journal of Econometrics* , pp. 469-492.

Gollop, F. M., & Roberts, M. J. (1985, February). Cost-Minimizing Regulation of Sulfur Emissions: Regional Gains in Electric Power. *The Review of Economics and Statistics , 67* (1), pp. 81-90.

Hailu, A., & Veeman, T. S. (2000). ) Environmentally sensitive productivity analysis of the Canadian pulp and paper industry, 1959–1994: An input distance function approach. *Journal of Environmental Economics and Management , 40*, pp. 251-274.

Chung, Y., Färe, R., & Grosskopf, S. (1997). Productivity and Undesirable Outputs: A Directional Distance Function Approach. *Journal of Environmental Management , 51*, pp. 229-240.

K.U.Leuven. (2008). *The GEM-E3 Model*. Retrieved from http://www.gem-e3.net/

Kumbhakar, S. C., Orea, L., Rodríguez-Álvarez, A., & Tsionas, E. G. (2007). Do we estimate an input or an output distance function? An application of the mixture approach to European railways. *J Prod Anal , 27*, pp. 87-100.

Kwon, O. S., & Yuh, W.-C. (1999). Estimation of the marginal abatemetn costs of airborne pollutants in Korea's power generation sector. *Energy Economics , 21*, pp. 547-560.

Lee, J.-D., Park, J.-B., & Kim, T.-Y. (2002). Estimation of the shadow prices of pollutants with production/environment inefficiency taken into account: a nonparametric directionnal distance function approch. *Journal of Environmental Management , 64*, pp. 365-375.

Lee, M. (2005). The shadow price of substituable sulfur in the US electric power plant: A distance function approach. *Journal of Environmental Management , 77*, pp. 104-110.

Maradan, D., & Vassiliev, A. (2005). Marginal Costs of Carbon Dioxide Abatement: Empirical Evidence from Cross-Country Analysis. *Swiss Journal of Economics and Statistics , III* (141), pp. 377-410.

Marklund, P.-O. (2003, December 16). Analyzing Interplant Marginal Abatement Cost Differences: A Directional Output Distance Funstion Approach. *Scandinavian working Papers in Economics* .

Murty, M. N., Surender, K., & Kishore, K. D. (2007). Measuring Environmental Efficiency of Industry: A case of Study of Thermal Power Generation in India. *Environmental and Resource Economics , 38*, pp. 31-50.

Park, H., & Lim, J. (2009). Valuation of marginal CO2 abatement options for electric power plants. *Energy policy , 37*, pp. 1834-1841.

Pye, S., Holland, M., Van Regemorter, D., Wagner, A., & Watkiss, P. (2008). *Analysis of the Costs and Benefits of Proposed Revisions to the National Emission Ceilings Directive - NEC CBA Report3. National Emission Ceilings for 2020 based on the 2008 Climate & Energy Package.* AEA Energy & Environment.

Rečka, L. (2009). *Optimisation of the Czech energy system.* Bachelor thesis. Charles University Prague, Faculty of Social Science, Institute of Economic Studies.

Salnykov, L. M., & Zelenyuk, V. (2006). Parametric estimation of environmental efficiencies and shadow prices of environmental pollutants: cross-country approach. *EERC working paper* .

Schmalensee, R. (1994). The costs of environmental protection. In M. Kotowski, *Balancing Economic Growth and Environmental Goals.* Washington, DC: American Council for Capital Formation, Center for Policy Research.

Swinton, J. R. (1998). At What Cost do We Redeuce Pollution? Shadow Prices of SO2 Emission. *The Energy Jurnal* , pp. 63-83.

Swinton, J. R. (2002). The Potential for Cost Savings in the Sulfure Dioxide Allovance Market: Empiriral Evidence from Florida. *Land Economics , 78* (3), pp. 390-404.

Ščasný, M., Pavel, J., & Rečka, L. (2008, September). Analýza nákladů na snížení emisí znečišťujících látek vypouštěných do ovzduší stacionárními zdroji. Praha.

Ščasný, M., Píša, V., Pollitt, H., & CHewpreecha, U. (2009). Analyzing Macroeconomic Effects of Environmental Taxation in the Czech Republic with the Econometric E3ME Model. *Czech Journal of Economics and Finance , 59* (5), pp. 460-491.

Vardanyan, M., & Noh, D.-W. (2006). Approximation pollution abatement cost via alternative specifiacation of a multi-output technology:A case of the US electric utility industry. *Journal of Environmental Management , 80*, pp. 177-190.

# Appendix A

**Table 17 Descriptive statistics by years**

| Time | | Capital (mil.CZK2005) | Labor | Fuels (TJ) | Electricity(MWh) | Heat (TJ) | PM (t) | $SO_2$ (t) | $NO_x$ (t) | CO (t) | VOC (t) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2002 | mean | 26800 | 1267 | 52600 | 6566082 | 6077 | 420.5 | 10125.6 | 8761.0 | 735.9 | 550.1 |
| | s.d. | 68200 | 2431 | 121000 | 17800000 | 5867 | 977.7 | 20085.7 | 20314.6 | 1471.7 | 1347.5 |
| | min | 251 | 91 | 826 | 13423 | 370 | 5.8 | 449.2 | 130.7 | 16.1 | 0.8 |
| | max | 209000 | 7677 | 373000 | 54100000 | 16900 | 3010.5 | 62746.5 | 62480.9 | 4576.9 | 4131.1 |
| 2003 | mean | 29100 | 1160 | 49500 | 7304557 | 5979 | 377.2 | 9881.2 | 8548.0 | 623.3 | 557.8 |
| | s.d. | 74800 | 2139 | 114000 | 20100000 | 5778 | 912.6 | 18771.5 | 20147.6 | 1210.3 | 1367.6 |
| | min | 252 | 90 | 855 | 13159 | 368 | 6.1 | 535.4 | 100.6 | 12.8 | 13.5 |
| | max | 228000 | 6780 | 350000 | 60900000 | 17200 | 2801.0 | 58745.2 | 61881.4 | 3762.1 | 4195.4 |
| 2004 | mean | 29300 | 1198 | 51600 | 7381327 | 5881 | 407.6 | 9339.5 | 8558.6 | 629.7 | 600.1 |
| | s.d. | 76100 | 2102 | 120000 | 20300000 | 5784 | 964.1 | 18192.2 | 19991.3 | 1241.1 | 1361.2 |
| | min | 247 | 88 | 821 | 12700 | 354 | 4.3 | 387.6 | 133.8 | 9.0 | 13.0 |
| | max | 232000 | 6629 | 370000 | 61600000 | 17600 | 2968.0 | 56916.6 | 61468.3 | 3869.4 | 4211.4 |
| 2005 | mean | 34500 | 1231 | 52500 | 7208199 | 6200 | 398.3 | 10051.8 | 8370.2 | 617.1 | 550.2 |
| | s.d. | 90000 | 2118 | 119000 | 19600000 | 6832 | 919.9 | 18284.9 | 19303.8 | 1154.3 | 1330.0 |
| | min | 249 | 87 | 803 | 12894 | 349 | 3.1 | 387.6 | 99.1 | 11.9 | 0.9 |
| | max | 274000 | 6618 | 366000 | 59500000 | 21600 | 2835.4 | 56718.7 | 59265.4 | 3585.3 | 4078.7 |
| 2006 | mean | 36800 | 1199 | 52600 | 7489195 | 5751 | 386.2 | 10336.9 | 8565.3 | 619.6 | 507.4 |
| | s.d. | 97100 | 2054 | 119000 | 20500000 | 6301 | 892.4 | 19312.7 | 19824.1 | 1241.7 | 1212.5 |
| | min | 257 | 85 | 759 | 12448 | 336 | 1.4 | 340.7 | 64.5 | 10.4 | 0.6 |
| | max | 296000 | 6404 | 368000 | 62000000 | 20300 | 2749.6 | 59723.4 | 60833.4 | 3842.2 | 3722.1 |
| 2007 | mean | 41600 | 1288 | 63100 | 8860476 | 5792 | 447.6 | 11852.6 | 10108.3 | 719.0 | 668.6 |
| | s.d. | 103000 | 2077 | 140000 | 22900000 | 6193 | 1025.4 | 22393.1 | 22797.9 | 1419.2 | 1587.4 |
| | min | 259 | 84 | 703 | 12056 | 307 | 3.1 | 378.7 | 72.8 | 14.8 | 0.5 |
| | max | 296000 | 6146 | 407000 | 65400000 | 19400 | 2973.3 | 65620.6 | 66075.4 | 4159.9 | 4585.2 |
| Total | mean | 32900 | 1223 | 53500 | 7442039 | 5950 | 405.5 | 10234.6 | 8794.2 | 656.3 | 570.5 |
| | s.d. | 81300 | 2053 | 116000 | 19200000 | 5836 | 901.6 | 18549.6 | 19378.0 | 1229.4 | 1300.8 |
| | min | 247 | 84 | 703 | 12056 | 307 | 1.4 | 340.7 | 64.5 | 9.0 | 0.5 |
| | max | 296000 | 7677 | 407000 | 6.54E+07 | 21600 | 3010.5 | 65620.6 | 66075.4 | 4576.9 | 4585.2 |

**Table 18 Emission shadow price estimates overview - extended**

| | | | DATA | | | | | | SCHADOW PRICES (€2005/t) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Study | Method | Function | Country | Data type | # obs. | (firms x Y) | Years | Sector | CO2 | SOx | NOx | PM | BOD (COD) | TSS | g=(y,b) |
| Färe et al. (1993) | ODF | translog | US[29] | firm | 30 | 30x1 | 1976 | pulp | - | 9956.7 | - | 68074.9 | 2809.7 | 0 | y>0, b>0 |
| Coggins & Swinton (1996) | ODF | translog | US[a] | firm | 42 | 14x3 | 1990-92 | power | - | 357.1 | - | - | - | - | y>0, b>0 |
| Boyd et al. (1996) | ODF | | US | firm | 29 | 29x1 | 1989 | power | - | 475.7 | - | - | - | - | y>0, b>0 |
| Swinton (1998) | ODF | | US[b] | firm | 123 | 41x3 | 1990-92 | power | - | 254.1 | - | - | - | - | y>0, b>0 |
| Kwon & Yuh (1999) | ODF | translog | Kor | firm | 57 | 10x6 | 1990-95 | power | 5.2 | 425.5 | 201.0 | 21210.8 | - | - | y>0, b>0 |
| Hailu-Veeman (2000) | IDF | translog | Can | sector | 36 | 1x36 | 1959-94 | pulp | - | - | - | - | 199.3 | 463.3 | y=0, b=0 |
| Swinton (2002) | ODF | transcendental logarithmic | US[c] | firm | 63 | 7x9 | 1990-98 | power | - | 176.7 | - | - | - | - | y>0, b>0 |
| Lee et al. (2002) | DDF | nonparametric | Kor | firm | 258 | 43x6 | 1990-95 | power | - | 3790.5 | 21219.5 | 62333.5 | - | - | y<0, b<0 |
| Marklund (2003) | DDF | quadratic | Swe | firm | 86 | 12x8 | 1983-90 | pulp | - | - | - | - | (760.2) | 119.1 | y=1, b=-1 |
| Färe et al. (2005) | DDF | quadratic / stochastic | US | firm | 418 | 209x2 | 1993&97 | power | - | 1505.8/ 106.8 | - | - | - | - | y=1, b=-1 |
| Atkinson & Dorfman (2005) | IDF | Bayesian approach | US | firm | | 43x4 | 1980, 85, 90, 95 | power | - | 501.85[30] | - | - | - | - | y=0, b=0 |
| Lee (2005) | IDF | | US | firm | 380 | 38x10 | 1977-86 | power | - | 451.4 | - | 344.1 (ash) | - | - | y=0, b=0 |
| Vardanyan & Noh (2006) | ODF, DDF, hyperbolic | | US | firm | 627 | 209x3 | 1997-99 | power | | | **see Table 2** | | | | |
| Murty et al. (2007) | DDF | quadratic | Ind | firm | 480[31] | 5x8 | 1997-2004 | power | - | 51.0 | 182.3 | 129.52 (SPM) | - | - | y=1, b=-1 |
| Bauman et al. (2008) | ODF | translog | Kor | sector | 29 | 1x29 | 1970-98 | power | - | 224.5 | - | - | - | - | y>0, b>0 |
| Park & Lim (2009) | ODF | translog | Kor | firm | 80 | 20x4 | 2001-04 | power | 14 | - | - | - | - | - | - |
| Maradan & Vassiliev (2005) | DDF | | World | cntry | 76 | 76x1 | 1985 | econ. | 2.1 - 9.6 | - | - | - | - | - | y=0, b>0 |
| Salnykov & Zelenyuk (2006) | DDF | translog | PCC[d] | cntry | 96 | 96x1 | 1995 | econ. | 115 | 5485.3 | 57804.8 | (results for CZE) | | | - |

[29] Michigan & Wisconsin; [a] Wisconsin; [b] Wisconsin, Illinois and Minnesota; [c] Florida; [d] Post Communist countries.
[30] Only for year 1995
[31] Monthly data

# Appendix B

**Table 19 Parameter estimates for input distance function - Sample B**

| Parameter | Estimate | Parameter | Estimate | Parameter | Estimate | Parameter | Estimate |
|---|---|---|---|---|---|---|---|
| α0 | -124.299 | δLE | -6.9E-10 | ηFVOC | 1.1E-06 | γPMSO2 | -0.00162 |
| αL | 0.999636 | δLH | -7.5E-12 | βEE | 1.06E-08 | γSO2SO2 | 5.99E-05 |
| αK | 0.000325 | δKE | 1.12E-09 | βEH | 1.22E-09 | γSO2NOx | -0.00095 |
| αF | 3.94E-05 | δKH | 1.08E-10 | βEH | 1.22E-09 | γSO2CO | 8.88E-05 |
| βE | -0.00201 | δFE | -4.2E-10 | βHH | 1.07E-10 | γSO2VOC | 0.001112 |
| βH | -0.00023 | δFH | -1E-10 | μEPM | 3.71E-06 | γPMNOx | 0.002884 |
| γPM | 0.59676 | ηLPM | 1.91E-06 | μESO2 | -4.7E-07 | γSO2NOx | -0.00095 |
| γSO2 | 0.021757 | ηLSO2 | -1.7E-07 | μENOx | -2.2E-06 | γNOxNOx | 0.002475 |
| γNOx | 0.134003 | ηLNOx | 3.79E-08 | μECO | -6.6E-06 | γNOxCO | -0.00041 |
| γCO | 0.201734 | ηLCO | -1.1E-06 | μEVOC | -3.9E-06 | γNOxVOC | -0.01225 |
| γVOC | 0.263357 | ηLVOC | -2.8E-07 | μHPM | 1.45E-06 | γPMCO | -0.01338 |
| αLL | -3.3E-10 | ηKPM | -2.2E-06 | μHSO2 | -1.4E-07 | γSO2CO | 8.88E-05 |
| αLK | 2.77E-10 | ηKSO2 | 6.41E-09 | μHNOx | 1.35E-07 | γNOxCO | -0.00041 |
| αLF | 5.46E-11 | ηKNOx | -3.2E-07 | μHCO | -2.3E-07 | γCOCO | -0.00544 |
| αLK | 2.77E-10 | ηKCO | 5.32E-07 | μHVOC | -4.1E-07 | γCOVOC | -0.00688 |
| αKK | -3.1E-10 | ηKVOC | -8.2E-07 | γPMPM | -0.08316 | γVOCPM | 0.071053 |
| αKF | 3E-11 | ηFPM | 2.62E-07 | γPMSO2 | -0.00162 | γSO2VOC | 0.001112 |
| αLF | 5.46E-11 | ηFSO2 | 1.64E-07 | γPMNOx | 0.002884 | γNOxVOC | -0.01225 |
| αKF | 3E-11 | ηFNOx | 2.79E-07 | γPMCO | -0.01338 | γCOVOC | -0.00688 |
| αFF | -8.5E-11 | ηFCO | 5.83E-07 | γPMVOC | 0.071053 | γVOCVOC | 0.025149 |

**Table 20 IDF estimates and marginal abatement costs - Sample A**

| Firm | Year | Value of IDF | PM MAC (€2005/t) | SO2 MAC (€2005/t) | NOx MAC (€2005/t) | CO MAC (€2005/t) | VOC MAC (€2005/t) | Technical efficiency |
|---|---|---|---|---|---|---|---|---|
| 1 | 2002 | 1.258476 | 86 | 757 | 3256 | 3115 | 10177 | 0.794612 |
| 1 | 2003 | 1.034359 | 165 | 226 | 162 | 404 | 126 | 0.966782 |
| 1 | 2004 | 1.001058 | 628 | 572 | 773 | 1108 | 5214 | 0.998943 |
| 1 | 2005 | 1.024928 | 410 | 1382 | 2343 | 4313 | 42 | 0.975679 |
| 1 | 2006 | 1.02994 | 68 | 2291 | 1642 | 123 | 1471 | 0.97093 |
| 1 | 2007 | 1.002257 | 60 | 1578 | 2634 | 5972 | 131 | 0.997749 |
| 4 | 2002 | 1.00892 | 4662 | 236 | 3284 | 71 | 20902 | 0.991159 |
| 4 | 2003 | 1.004156 | 9326 | 425 | 2077 | 3068 | 11692 | 0.995861 |
| 4 | 2004 | 1.009946 | 9889 | 1725 | 8765 | 11734 | 15243 | 0.990152 |
| 4 | 2005 | 1.00761 | 3888 | 1403 | 1262 | 326 | 2572 | 0.992448 |
| 4 | 2006 | 1.00569 | 8374 | 1981 | 1291 | 3017 | 251 | 0.994343 |
| 4 | 2007 | 1.006036 | 8792 | 2206 | 656 | 2120 | 1406 | 0.994 |
| 5 | 2002 | 1.004029 | 24639 | 2521 | 6015 | 35949 | 87923 | 0.995987 |
| 5 | 2003 | 1.003993 | 15437 | 1202 | 2361 | 37573 | 80869 | 0.996023 |
| 5 | 2004 | 1.004342 | 7851 | 1134 | 1999 | 10 | 32098 | 0.995676 |
| 5 | 2005 | 1.004077 | 695 | 2565 | 1801 | 11037 | 46202 | 0.995939 |
| 5 | 2006 | 1.004017 | 25357 | 5520 | 4603 | 19245 | 10919 | 0.995999 |
| 5 | 2007 | 1.004005 | 26876 | 2070 | 10582 | 78985 | 116223 | 0.996011 |

**Table 21 IDF estimates and marginal abatement costs - Sample B**

| Firm | Year | Value of IDF | PM MAC (€2005/t) | SO2 MAC (€2005/t) | NOx MAC (€2005/t) | CO MAC (€2005/t) | VOC MAC (€2005/t) | Technical efficiency |
|---|---|---|---|---|---|---|---|---|
| 2 | 2002 | 1.000037 | 706 | 2151 | 4035 | 124 | 3810 | 0.999963 |
| 2 | 2003 | 1.00004 | 927 | 3470 | 123 | 4538 | 2432 | 0.99996 |
| 2 | 2004 | 1.000041 | 13240 | 3310 | 4743 | 2156 | 7398 | 0.999959 |
| 2 | 2005 | 1.000049 | 35193 | 9356 | 5580 | 8934 | 49947 | 0.999951 |
| 2 | 2006 | 1.000039 | 37263 | 10706 | 11945 | 12103 | 59053 | 0.999961 |
| 2 | 2007 | 1.000036 | 27329 | 20886 | 12226 | 32690 | 75810 | 0.999964 |
| 3 | 2002 | 1.00004 | 6599 | 1758 | 7459 | 15694 | 14869 | 0.99996 |
| 3 | 2003 | 1.000041 | 8466 | 831 | 9065 | 15958 | 12729 | 0.999959 |
| 3 | 2004 | 1.000039 | 5074 | 687 | 7128 | 13465 | 11959 | 0.999961 |
| 3 | 2005 | 1.00004 | 404 | 1511 | 9459 | 21453 | 21105 | 0.99996 |
| 3 | 2006 | 1.000039 | 180 | 1323 | 8797 | 23883 | 12809 | 0.999961 |
| 3 | 2007 | 1.000041 | 1001 | 80 | 5774 | 14646 | 175 | 0.999959 |
| 6 | 2002 | 1.00004 | 216055 | 2345 | 7796 | 48543 | 38998 | 0.99996 |
| 6 | 2003 | 1.000041 | 148548 | 642 | 3732 | 26593 | 40043 | 0.999959 |
| 6 | 2004 | 1.00004 | 137588 | 85 | 16577 | 34233 | 18846 | 0.99996 |
| 6 | 2005 | 1.00004 | 182983 | 2266 | 36903 | 122657 | 62021 | 0.99996 |
| 6 | 2006 | 1.000039 | 240764 | 2142 | 30797 | 107515 | 2476 | 0.999961 |
| 6 | 2007 | 1.000039 | 82506 | 4835 | 8785 | 33435 | 17929 | 0.999961 |
| 7 | 2002 | 1.000041 | 8296 | 440 | 2326 | 5292 | 4024 | 0.999959 |
| 7 | 2003 | 1.00004 | 7626 | 633 | 1519 | 5450 | 6151 | 0.99996 |
| 7 | 2004 | 1.000039 | 8932 | 572 | 2391 | 5516 | 4843 | 0.999961 |
| 7 | 2005 | 1.000109 | 10843 | 755 | 2985 | 7005 | 6347 | 0.999891 |
| 7 | 2006 | 1.000041 | 11859 | 1060 | 3293 | 8385 | 11145 | 0.999959 |
| 7 | 2007 | 1.000043 | 10400 | 741 | 2246 | 6932 | 5793 | 0.999957 |
| 8 | 2002 | 1.000042 | 6147 | 246 | 2627 | 6051 | 3254 | 0.999958 |
| 8 | 2003 | 1.00004 | 12633 | 523 | 467 | 4844 | 8549 | 0.99996 |
| 8 | 2004 | 1.00004 | 17511 | 371 | 4 | 2780 | 4959 | 0.99996 |
| 8 | 2005 | 1.000044 | 457 | 518 | 939 | 4624 | 2879 | 0.999956 |
| 8 | 2006 | 1.000081 | 7527 | 847 | 2664 | 8451 | 5660 | 0.999919 |
| 8 | 2007 | 1.000038 | 4398 | 641 | 2096 | 6550 | 4972 | 0.999962 |
| 9 | 2002 | 1.00004 | 36 | 1081 | 2805 | 2986 | 10346 | 0.99996 |
| 9 | 2003 | 1.00004 | 4880 | 1198 | 1553 | 4249 | 9463 | 0.99996 |
| 9 | 2004 | 1.000041 | 8670 | 797 | 2531 | 4008 | 1996 | 0.999959 |
| 9 | 2005 | 1.00004 | 343 | 702 | 2813 | 3233 | 5583 | 0.99996 |
| 9 | 2006 | 1.00004 | 10861 | 1295 | 1843 | 5570 | 63 | 0.99996 |

# Appendix C

The Matlab code consists of 3 m.files.[32] One with the input output function, one with all constrains and restrictions, and the last one that runs the optimization and computes the shadow prices. We present here the code for the sample A. The code for sample B is on the same principle only longer due to higher number of observations.

```
function f = HC_IDF_oneF(par,x,y,b)

A0=par(1);
A=par(2:4);
B=par(5:6);
C=par(7:11);
AA=par(12:20);
AB=par(21:26);
AC=par(27:41);
BB=par(42:45);
BC=par(46:55);
CC=par(56:80);



f=A0+x(1,:)*A+y(1,:)*B+b(1,:)*C+1/2*kron(x(1,:),x(1,:))*AA+1/2*kron(y(1,:),y(1,:))*
BB+1/2*kron(b(1,:),b(1,:))*CC+kron(x(1,:),y(1,:))*AB+kron(x(1,:),b(1,:))*AC+kron(y(
1,:),b(1,:))*BC;


n=18;

for i=2:n
    f = f + A0+
x(i,:)*A+y(i,:)*B+b(i,:)*C+1/2*kron(x(i,:),x(i,:))*AA+1/2*kron(y(i,:),y(i,:))*BB+1/
2*kron(b(i,:),b(i,:))*CC+kron(x(i,:),y(i,:))*AB+kron(x(i,:),b(i,:))*AC+kron(y(i,:),
b(i,:))*BC;

end;
```

```
function [c,ceq]=HC_IDF_oneF_Const18(par,x,y,b)


A0=par(1);
A=par(2:4);
B=par(5:6);
C=par(7:11);
AA=par(12:20);
AB=par(21:26);
AC=par(27:41);
BB=par(42:45);
BC=par(46:55);
CC=par(56:80);

r1=sum(A)-1;

r21=sum(AA(1:3));
r22=sum(AA(4:6));
```

[32] The code is too complicated in some parts and can be simplified.

```
r23=sum(AA(7:9));

r31=sum(AB(1:2:6));
r32=sum(AB(2:2:6));

r41=sum(AC(1:5:15));
r42=sum(AC(2:5:15));
r43=sum(AC(3:5:15));
r44=sum(AC(4:5:15));
r45=sum(AC(5:5:15));


r5=sum(TA);


p1=AA(2)-AA(4);
p2=AA(3)-AA(7);

p10=AA(6)-AA(8);

m1=BB(2)-BB(3);

g1=CC(2)-CC(6);
g2=CC(3)-CC(11);
g3=CC(4)-CC(16);
g4=CC(5)-CC(21);
g5=CC(8)-CC(12);
g6=CC(9)-CC(17);
g7=CC(10)-CC(22);
g8=CC(14)-CC(18);
g9=CC(15)-CC(23);
g10=CC(20)-CC(24);




c1=1-
(A0+x(1,:)*A+y(1,:)*B+b(1,:)*C+1/2*kron(x(1,:),x(1,:))*AA+1/2*kron(y(1,:),y(1,:))*B
B+1/2*kron(b(1,:),b(1,:))*CC+kron(x(1,:),y(1,:))*AB+kron(x(1,:),b(1,:))*AC+kron(y(1
,:),b(1,:))*BC);

c2=1-
(A0+x(2,:)*A+y(2,:)*B+b(2,:)*C+1/2*kron(x(2,:),x(2,:))*AA+1/2*kron(y(2,:),y(2,:))*B
B+1/2*kron(b(2,:),b(2,:))*CC+kron(x(2,:),y(2,:))*AB+kron(x(2,:),b(2,:))*AC+kron(y(2
,:),b(2,:))*BC);

c3=1-
(A0+x(3,:)*A+y(3,:)*B+b(3,:)*C+1/2*kron(x(3,:),x(3,:))*AA+1/2*kron(y(3,:),y(3,:))*B
B+1/2*kron(b(3,:),b(3,:))*CC+kron(x(3,:),y(3,:))*AB+kron(x(3,:),b(3,:))*AC+kron(y(3
,:),b(3,:))*BC);

c4=1-
(A0+x(4,:)*A+y(4,:)*B+b(4,:)*C+1/2*kron(x(4,:),x(4,:))*AA+1/2*kron(y(4,:),y(4,:))*B
B+1/2*kron(b(4,:),b(4,:))*CC+kron(x(4,:),y(4,:))*AB+kron(x(4,:),b(4,:))*AC+kron(y(4
,:),b(4,:))*BC);

c5=1-
(A0+x(5,:)*A+y(5,:)*B+b(5,:)*C+1/2*kron(x(5,:),x(5,:))*AA+1/2*kron(y(5,:),y(5,:))*B
B+1/2*kron(b(5,:),b(5,:))*CC+kron(x(5,:),y(5,:))*AB+kron(x(5,:),b(5,:))*AC+kron(y(5
,:),b(5,:))*BC);

c6=1-
(A0+x(6,:)*A+y(6,:)*B+b(6,:)*C+1/2*kron(x(6,:),x(6,:))*AA+1/2*kron(y(6,:),y(6,:))*B
B+1/2*kron(b(6,:),b(6,:))*CC+kron(x(6,:),y(6,:))*AB+kron(x(6,:),b(6,:))*AC+kron(y(6
,:),b(6,:))*BC);

c7=1-
(A0+x(7,:)*A+y(7,:)*B+b(7,:)*C+1/2*kron(x(7,:),x(7,:))*AA+1/2*kron(y(7,:),y(7,:))*B
```

```
B+1/2*kron(b(7,:),b(7,:))*CC+kron(x(7,:),y(7,:))*AB+kron(x(7,:),b(7,:))*AC+kron(y(7
,:),b(7,:))*BC);

c8=1-
(A0+x(8,:)*A+y(8,:)*B+b(8,:)*C+1/2*kron(x(8,:),x(8,:))*AA+1/2*kron(y(8,:),y(8,:))*B
B+1/2*kron(b(8,:),b(8,:))*CC+kron(x(8,:),y(8,:))*AB+kron(x(8,:),b(8,:))*AC+kron(y(8
,:),b(8,:))*BC);

c9=1-
(A0+x(9,:)*A+y(9,:)*B+b(9,:)*C+1/2*kron(x(9,:),x(9,:))*AA+1/2*kron(y(9,:),y(9,:))*B
B+1/2*kron(b(9,:),b(9,:))*CC+kron(x(9,:),y(9,:))*AB+kron(x(9,:),b(9,:))*AC+kron(y(9
,:),b(9,:))*BC);

c10=1-
(A0+x(10,:)*A+y(10,:)*B+b(10,:)*C+1/2*kron(x(10,:),x(10,:))*AA+1/2*kron(y(10,:),y(1
0,:))*BB+1/2*kron(b(10,:),b(10,:))*CC+kron(x(10,:),y(10,:))*AB+kron(x(10,:),b(10,:)
)*AC+kron(y(10,:),b(10,:))*BC);

c11=1-
(A0+x(11,:)*A+y(11,:)*B+b(11,:)*C+1/2*kron(x(11,:),x(11,:))*AA+1/2*kron(y(11,:),y(1
1,:))*BB+1/2*kron(b(11,:),b(11,:))*CC+kron(x(11,:),y(11,:))*AB+kron(x(11,:),b(11,:)
)*AC+kron(y(11,:),b(11,:))*BC);

c12=1-
(A0+x(12,:)*A+y(12,:)*B+b(12,:)*C+1/2*kron(x(12,:),x(12,:))*AA+1/2*kron(y(12,:),y(1
2,:))*BB+1/2*kron(b(12,:),b(12,:))*CC+kron(x(12,:),y(12,:))*AB+kron(x(12,:),b(12,:)
)*AC+kron(y(12,:),b(12,:))*BC);

c13=1-
(A0+x(13,:)*A+y(13,:)*B+b(13,:)*C+1/2*kron(x(13,:),x(13,:))*AA+1/2*kron(y(13,:),y(1
3,:))*BB+1/2*kron(b(13,:),b(13,:))*CC+kron(x(13,:),y(13,:))*AB+kron(x(13,:),b(13,:)
)*AC+kron(y(13,:),b(13,:))*BC);

c14=1-
(A0+x(14,:)*A+y(14,:)*B+b(14,:)*C+1/2*kron(x(14,:),x(14,:))*AA+1/2*kron(y(14,:),y(1
4,:))*BB+1/2*kron(b(14,:),b(14,:))*CC+kron(x(14,:),y(14,:))*AB+kron(x(14,:),b(14,:)
)*AC+kron(y(14,:),b(14,:))*BC);

c15=1-
(A0+x(15,:)*A+y(15,:)*B+b(15,:)*C+1/2*kron(x(15,:),x(15,:))*AA+1/2*kron(y(15,:),y(1
5,:))*BB+1/2*kron(b(15,:),b(15,:))*CC+kron(x(15,:),y(15,:))*AB+kron(x(15,:),b(15,:)
)*AC+kron(y(15,:),b(15,:))*BC);

c16=1-
(A0+x(16,:)*A+y(16,:)*B+b(16,:)*C+1/2*kron(x(16,:),x(16,:))*AA+1/2*kron(y(16,:),y(1
6,:))*BB+1/2*kron(b(16,:),b(16,:))*CC+kron(x(16,:),y(16,:))*AB+kron(x(16,:),b(16,:)
)*AC+kron(y(16,:),b(16,:))*BC);

c17=1-
(A0+x(17,:)*A+y(17,:)*B+b(17,:)*C+1/2*kron(x(17,:),x(17,:))*AA+1/2*kron(y(17,:),y(1
7,:))*BB+1/2*kron(b(17,:),b(17,:))*CC+kron(x(17,:),y(17,:))*AB+kron(x(17,:),b(17,:)
)*AC+kron(y(17,:),b(17,:))*BC);

c18=1-
(A0+x(18,:)*A+y(18,:)*B+b(18,:)*C+1/2*kron(x(18,:),x(18,:))*AA+1/2*kron(y(18,:),y(1
8,:))*BB+1/2*kron(b(18,:),b(18,:))*CC+kron(x(18,:),y(18,:))*AB+kron(x(18,:),b(18,:)
)*AC+kron(y(18,:),b(18,:))*BC);


dy1_1=B(1)+1/2*(y(1,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(1,:)*BC(1:5));
dy1_2=B(1)+1/2*(y(2,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(2,:)*BC(1:5));
dy1_3=B(1)+1/2*(y(3,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(3,:)*BC(1:5));
dy1_4=B(1)+1/2*(y(4,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(4,:)*BC(1:5));
dy1_5=B(1)+1/2*(y(5,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(5,:)*BC(1:5));
dy1_6=B(1)+1/2*(y(6,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(6,:)*BC(1:5));
dy1_7=B(1)+1/2*(y(7,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(7,:)*BC(1:5));
dy1_8=B(1)+1/2*(y(8,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(8,:)*BC(1:5));
dy1_9=B(1)+1/2*(y(9,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(9,:)*BC(1:5));
```

```
dy1_10=B(1)+1/2*(y(10,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(10,:)*BC(1:5));
dy1_11=B(1)+1/2*(y(11,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(11,:)*BC(1:5));
dy1_12=B(1)+1/2*(y(12,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(12,:)*BC(1:5));
dy1_13=B(1)+1/2*(y(13,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(13,:)*BC(1:5));
dy1_14=B(1)+1/2*(y(14,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(14,:)*BC(1:5));
dy1_15=B(1)+1/2*(y(15,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(15,:)*BC(1:5));
dy1_16=B(1)+1/2*(y(16,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(16,:)*BC(1:5));
dy1_17=B(1)+1/2*(y(17,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(17,:)*BC(1:5));
dy1_18=B(1)+1/2*(y(18,:)*BB(1:2))+(x(1,:)*AB(1:2:6))+(b(18,:)*BC(1:5));


dy2_1=B(2)+1/2*(y(1,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(1,:)*BC(6:10));
dy2_2=B(2)+1/2*(y(2,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(2,:)*BC(6:10));
dy2_3=B(2)+1/2*(y(3,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(3,:)*BC(6:10));
dy2_4=B(2)+1/2*(y(4,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(4,:)*BC(6:10));
dy2_5=B(2)+1/2*(y(5,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(5,:)*BC(6:10));
dy2_6=B(2)+1/2*(y(6,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(6,:)*BC(6:10));
dy2_7=B(2)+1/2*(y(7,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(7,:)*BC(6:10));
dy2_8=B(2)+1/2*(y(8,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(8,:)*BC(6:10));
dy2_9=B(2)+1/2*(y(9,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(9,:)*BC(6:10));
dy2_10=B(2)+1/2*(y(10,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(10,:)*BC(6:10));
dy2_11=B(2)+1/2*(y(11,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(11,:)*BC(6:10));
dy2_12=B(2)+1/2*(y(12,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(12,:)*BC(6:10));
dy2_13=B(2)+1/2*(y(13,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(13,:)*BC(6:10));
dy2_14=B(2)+1/2*(y(14,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(14,:)*BC(6:10));
dy2_15=B(2)+1/2*(y(15,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(15,:)*BC(6:10));
dy2_16=B(2)+1/2*(y(16,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(16,:)*BC(6:10));
dy2_17=B(2)+1/2*(y(17,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(17,:)*BC(6:10));
dy2_18=B(2)+1/2*(y(18,:)*BB(3:4))+(x(2,:)*AB(1:2: 6))+(b(18,:)*BC(6:10));


db1_1=-(C(1)+1/2*(b(1,:)*CC(1:5))+(x(1,:)*AC(1:5:15))+(y(1,:)*BC(1:5:10)));
db1_2=-(C(1)+1/2*(b(2,:)*CC(1:5))+(x(2,:)*AC(1:5:15))+(y(2,:)*BC(1:5:10)));
db1_3=-(C(1)+1/2*(b(3,:)*CC(1:5))+(x(3,:)*AC(1:5:15))+(y(3,:)*BC(1:5:10)));
db1_4=-(C(1)+1/2*(b(4,:)*CC(1:5))+(x(4,:)*AC(1:5:15))+(y(4,:)*BC(1:5:10)));
db1_5=-(C(1)+1/2*(b(5,:)*CC(1:5))+(x(5,:)*AC(1:5:15))+(y(5,:)*BC(1:5:10)));
db1_6=-(C(1)+1/2*(b(6,:)*CC(1:5))+(x(6,:)*AC(1:5:15))+(y(6,:)*BC(1:5:10)));
db1_7=-(C(1)+1/2*(b(7,:)*CC(1:5))+(x(7,:)*AC(1:5:15))+(y(7,:)*BC(1:5:10)));
db1_8=-(C(1)+1/2*(b(8,:)*CC(1:5))+(x(8,:)*AC(1:5:15))+(y(8,:)*BC(1:5:10)));
db1_9=-(C(1)+1/2*(b(9,:)*CC(1:5))+(x(9,:)*AC(1:5:15))+(y(9,:)*BC(1:5:10)));
db1_10=-(C(1)+1/2*(b(10,:)*CC(1:5))+(x(10,:)*AC(1:5:15))+(y(10,:)*BC(1:5:10)));
db1_11=-(C(1)+1/2*(b(11,:)*CC(1:5))+(x(11,:)*AC(1:5:15))+(y(11,:)*BC(1:5:10)));
db1_12=-(C(1)+1/2*(b(12,:)*CC(1:5))+(x(12,:)*AC(1:5:15))+(y(12,:)*BC(1:5:10)));
db1_13=-(C(1)+1/2*(b(13,:)*CC(1:5))+(x(13,:)*AC(1:5:15))+(y(13,:)*BC(1:5:10)));
db1_14=-(C(1)+1/2*(b(14,:)*CC(1:5))+(x(14,:)*AC(1:5:15))+(y(14,:)*BC(1:5:10)));
db1_15=-(C(1)+1/2*(b(15,:)*CC(1:5))+(x(15,:)*AC(1:5:15))+(y(15,:)*BC(1:5:10)));
db1_16=-(C(1)+1/2*(b(16,:)*CC(1:5))+(x(16,:)*AC(1:5:15))+(y(16,:)*BC(1:5:10)));
db1_17=-(C(1)+1/2*(b(17,:)*CC(1:5))+(x(17,:)*AC(1:5:15))+(y(17,:)*BC(1:5:10)));
db1_18=-(C(1)+1/2*(b(18,:)*CC(1:5))+(x(18,:)*AC(1:5:15))+(y(18,:)*BC(1:5:10)));


db2_1=-(C(2)+1/2*(b(1,:)*CC(6:10))+(x(1,:)*AC(2:5:15))+(y(1,:)*BC(2:5:10)));
db2_2=-(C(2)+1/2*(b(2,:)*CC(6:10))+(x(2,:)*AC(2:5:15))+(y(2,:)*BC(2:5:10)));
db2_3=-(C(2)+1/2*(b(3,:)*CC(6:10))+(x(3,:)*AC(2:5:15))+(y(3,:)*BC(2:5:10)));
db2_4=-(C(2)+1/2*(b(4,:)*CC(6:10))+(x(4,:)*AC(2:5:15))+(y(4,:)*BC(2:5:10)));
db2_5=-(C(2)+1/2*(b(5,:)*CC(6:10))+(x(5,:)*AC(2:5:15))+(y(5,:)*BC(2:5:10)));
db2_6=-(C(2)+1/2*(b(6,:)*CC(6:10))+(x(6,:)*AC(2:5:15))+(y(6,:)*BC(2:5:10)));
db2_7=-(C(2)+1/2*(b(7,:)*CC(6:10))+(x(7,:)*AC(2:5:15))+(y(7,:)*BC(2:5:10)));
db2_8=-(C(2)+1/2*(b(8,:)*CC(6:10))+(x(8,:)*AC(2:5:15))+(y(8,:)*BC(2:5:10)));
db2_9=-(C(2)+1/2*(b(9,:)*CC(6:10))+(x(9,:)*AC(2:5:15))+(y(9,:)*BC(2:5:10)));
db2_10=-(C(2)+1/2*(b(10,:)*CC(6:10))+(x(10,:)*AC(2:5:15))+(y(10,:)*BC(2:5:10)));
db2_11=-(C(2)+1/2*(b(11,:)*CC(6:10))+(x(11,:)*AC(2:5:15))+(y(11,:)*BC(2:5:10)));
db2_12=-(C(2)+1/2*(b(12,:)*CC(6:10))+(x(12,:)*AC(2:5:15))+(y(12,:)*BC(2:5:10)));
db2_13=-(C(2)+1/2*(b(13,:)*CC(6:10))+(x(13,:)*AC(2:5:15))+(y(13,:)*BC(2:5:10)));
db2_14=-(C(2)+1/2*(b(14,:)*CC(6:10))+(x(14,:)*AC(2:5:15))+(y(14,:)*BC(2:5:10)));
```

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db2_15=-(C(2)+1/2*(b(15,:)*CC(6:10))+(x(15,:)*AC(2:5:15))+(y(15,:)*BC(2:5:10)));
db2_16=-(C(2)+1/2*(b(16,:)*CC(6:10))+(x(16,:)*AC(2:5:15))+(y(16,:)*BC(2:5:10)));
db2_17=-(C(2)+1/2*(b(17,:)*CC(6:10))+(x(17,:)*AC(2:5:15))+(y(17,:)*BC(2:5:10)));
db2_18=-(C(2)+1/2*(b(18,:)*CC(6:10))+(x(18,:)*AC(2:5:15))+(y(18,:)*BC(2:5:10)));


db3_1=-(C(3)+1/2*(b(1,:)*CC(11:15))+(x(1,:)*AC(3:5:15))+(y(1,:)*BC(3:5:10)));
db3_2=-(C(3)+1/2*(b(2,:)*CC(11:15))+(x(2,:)*AC(3:5:15))+(y(2,:)*BC(3:5:10)));
db3_3=-(C(3)+1/2*(b(3,:)*CC(11:15))+(x(3,:)*AC(3:5:15))+(y(3,:)*BC(3:5:10)));
db3_4=-(C(3)+1/2*(b(4,:)*CC(11:15))+(x(4,:)*AC(3:5:15))+(y(4,:)*BC(3:5:10)));
db3_5=-(C(3)+1/2*(b(5,:)*CC(11:15))+(x(5,:)*AC(3:5:15))+(y(5,:)*BC(3:5:10)));
db3_6=-(C(3)+1/2*(b(6,:)*CC(11:15))+(x(6,:)*AC(3:5:15))+(y(6,:)*BC(3:5:10)));
db3_7=-(C(3)+1/2*(b(7,:)*CC(11:15))+(x(7,:)*AC(3:5:15))+(y(7,:)*BC(3:5:10)));
db3_8=-(C(3)+1/2*(b(8,:)*CC(11:15))+(x(8,:)*AC(3:5:15))+(y(8,:)*BC(3:5:10)));
db3_9=-(C(3)+1/2*(b(9,:)*CC(11:15))+(x(9,:)*AC(3:5:15))+(y(9,:)*BC(3:5:10)));
db3_10=-(C(3)+1/2*(b(10,:)*CC(11:15))+(x(10,:)*AC(3:5:15))+(y(10,:)*BC(3:5:10)));
db3_11=-(C(3)+1/2*(b(11,:)*CC(11:15))+(x(11,:)*AC(3:5:15))+(y(11,:)*BC(3:5:10)));
db3_12=-(C(3)+1/2*(b(12,:)*CC(11:15))+(x(12,:)*AC(3:5:15))+(y(12,:)*BC(3:5:10)));
db3_13=-(C(3)+1/2*(b(13,:)*CC(11:15))+(x(13,:)*AC(3:5:15))+(y(13,:)*BC(3:5:10)));
db3_14=-(C(3)+1/2*(b(14,:)*CC(11:15))+(x(14,:)*AC(3:5:15))+(y(14,:)*BC(3:5:10)));
db3_15=-(C(3)+1/2*(b(15,:)*CC(11:15))+(x(15,:)*AC(3:5:15))+(y(15,:)*BC(3:5:10)));
db3_16=-(C(3)+1/2*(b(16,:)*CC(11:15))+(x(16,:)*AC(3:5:15))+(y(16,:)*BC(3:5:10)));
db3_17=-(C(3)+1/2*(b(17,:)*CC(11:15))+(x(17,:)*AC(3:5:15))+(y(17,:)*BC(3:5:10)));
db3_18=-(C(3)+1/2*(b(18,:)*CC(11:15))+(x(18,:)*AC(3:5:15))+(y(18,:)*BC(3:5:10)));


db4_1=-(C(4)+1/2*(b(1,:)*CC(16:20))+(x(1,:)*AC(4:5:15))+(y(1,:)*BC(4:5:10)));
db4_2=-(C(4)+1/2*(b(2,:)*CC(16:20))+(x(2,:)*AC(4:5:15))+(y(2,:)*BC(4:5:10)));
db4_3=-(C(4)+1/2*(b(3,:)*CC(16:20))+(x(3,:)*AC(4:5:15))+(y(3,:)*BC(4:5:10)));
db4_4=-(C(4)+1/2*(b(4,:)*CC(16:20))+(x(4,:)*AC(4:5:15))+(y(4,:)*BC(4:5:10)));
db4_5=-(C(4)+1/2*(b(5,:)*CC(16:20))+(x(5,:)*AC(4:5:15))+(y(5,:)*BC(4:5:10)));
db4_6=-(C(4)+1/2*(b(6,:)*CC(16:20))+(x(6,:)*AC(4:5:15))+(y(6,:)*BC(4:5:10)));
db4_7=-(C(4)+1/2*(b(7,:)*CC(16:20))+(x(7,:)*AC(4:5:15))+(y(7,:)*BC(4:5:10)));
db4_8=-(C(4)+1/2*(b(8,:)*CC(16:20))+(x(8,:)*AC(4:5:15))+(y(8,:)*BC(4:5:10)));
db4_9=-(C(4)+1/2*(b(9,:)*CC(16:20))+(x(9,:)*AC(4:5:15))+(y(9,:)*BC(4:5:10)));
db4_10=-(C(4)+1/2*(b(10,:)*CC(16:20))+(x(10,:)*AC(4:5:15))+(y(10,:)*BC(4:5:10)));
db4_11=-(C(4)+1/2*(b(11,:)*CC(16:20))+(x(11,:)*AC(4:5:15))+(y(11,:)*BC(4:5:10)));
db4_12=-(C(4)+1/2*(b(12,:)*CC(16:20))+(x(12,:)*AC(4:5:15))+(y(12,:)*BC(4:5:10)));
db4_13=-(C(4)+1/2*(b(13,:)*CC(16:20))+(x(13,:)*AC(4:5:15))+(y(13,:)*BC(4:5:10)));
db4_14=-(C(4)+1/2*(b(14,:)*CC(16:20))+(x(14,:)*AC(4:5:15))+(y(14,:)*BC(4:5:10)));
db4_15=-(C(4)+1/2*(b(15,:)*CC(16:20))+(x(15,:)*AC(4:5:15))+(y(15,:)*BC(4:5:10)));
db4_16=-(C(4)+1/2*(b(16,:)*CC(16:20))+(x(16,:)*AC(4:5:15))+(y(16,:)*BC(4:5:10)));
db4_17=-(C(4)+1/2*(b(17,:)*CC(16:20))+(x(17,:)*AC(4:5:15))+(y(17,:)*BC(4:5:10)));
db4_18=-(C(4)+1/2*(b(18,:)*CC(16:20))+(x(18,:)*AC(4:5:15))+(y(18,:)*BC(4:5:10)));


db5_1=-(C(5)+1/2*(b(1,:)*CC(21:25))+(x(1,:)*AC(5:5:15))+(y(1,:)*BC(5:5:10)));
db5_2=-(C(5)+1/2*(b(2,:)*CC(21:25))+(x(2,:)*AC(5:5:15))+(y(2,:)*BC(5:5:10)));
db5_3=-(C(5)+1/2*(b(3,:)*CC(21:25))+(x(3,:)*AC(5:5:15))+(y(3,:)*BC(5:5:10)));
db5_4=-(C(5)+1/2*(b(4,:)*CC(21:25))+(x(4,:)*AC(5:5:15))+(y(4,:)*BC(5:5:10)));
db5_5=-(C(5)+1/2*(b(5,:)*CC(21:25))+(x(5,:)*AC(5:5:15))+(y(5,:)*BC(5:5:10)));
db5_6=-(C(5)+1/2*(b(6,:)*CC(21:25))+(x(6,:)*AC(5:5:15))+(y(6,:)*BC(5:5:10)));
db5_7=-(C(5)+1/2*(b(7,:)*CC(21:25))+(x(7,:)*AC(5:5:15))+(y(7,:)*BC(5:5:10)));
db5_8=-(C(5)+1/2*(b(8,:)*CC(21:25))+(x(8,:)*AC(5:5:15))+(y(8,:)*BC(5:5:10)));
db5_9=-(C(5)+1/2*(b(9,:)*CC(21:25))+(x(9,:)*AC(5:5:15))+(y(9,:)*BC(5:5:10)));
db5_10=-(C(5)+1/2*(b(10,:)*CC(21:25))+(x(10,:)*AC(5:5:15))+(y(10,:)*BC(5:5:10)));
db5_11=-(C(5)+1/2*(b(11,:)*CC(21:25))+(x(11,:)*AC(5:5:15))+(y(11,:)*BC(5:5:10)));
db5_12=-(C(5)+1/2*(b(12,:)*CC(21:25))+(x(12,:)*AC(5:5:15))+(y(12,:)*BC(5:5:10)));
db5_13=-(C(5)+1/2*(b(13,:)*CC(21:25))+(x(13,:)*AC(5:5:15))+(y(13,:)*BC(5:5:10)));
db5_14=-(C(5)+1/2*(b(14,:)*CC(21:25))+(x(14,:)*AC(5:5:15))+(y(14,:)*BC(5:5:10)));
db5_15=-(C(5)+1/2*(b(15,:)*CC(21:25))+(x(15,:)*AC(5:5:15))+(y(15,:)*BC(5:5:10)));
db5_16=-(C(5)+1/2*(b(16,:)*CC(21:25))+(x(16,:)*AC(5:5:15))+(y(16,:)*BC(5:5:10)));
db5_17=-(C(5)+1/2*(b(17,:)*CC(21:25))+(x(17,:)*AC(5:5:15))+(y(17,:)*BC(5:5:10)));
db5_18=-(C(5)+1/2*(b(18,:)*CC(21:25))+(x(18,:)*AC(5:5:15))+(y(18,:)*BC(5:5:10)));
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dx1_1=-(A(1)+1/2*(x(1,:)*AA(1:3:9))+(y(1,:)*AB(1:2))+(b(1,:)*AC(1:5)));
dx1_2=-(A(1)+1/2*(x(2,:)*AA(1:3:9))+(y(2,:)*AB(1:2))+(b(2,:)*AC(1:5)));
dx1_3=-(A(1)+1/2*(x(3,:)*AA(1:3:9))+(y(3,:)*AB(1:2))+(b(3,:)*AC(1:5)));
dx1_4=-(A(1)+1/2*(x(4,:)*AA(1:3:9))+(y(4,:)*AB(1:2))+(b(4,:)*AC(1:5)));
dx1_5=-(A(1)+1/2*(x(5,:)*AA(1:3:9))+(y(5,:)*AB(1:2))+(b(5,:)*AC(1:5)));
dx1_6=-(A(1)+1/2*(x(6,:)*AA(1:3:9))+(y(6,:)*AB(1:2))+(b(6,:)*AC(1:5)));
dx1_7=-(A(1)+1/2*(x(7,:)*AA(1:3:9))+(y(7,:)*AB(1:2))+(b(7,:)*AC(1:5)));
dx1_8=-(A(1)+1/2*(x(8,:)*AA(1:3:9))+(y(8,:)*AB(1:2))+(b(8,:)*AC(1:5)));
dx1_9=-(A(1)+1/2*(x(9,:)*AA(1:3:9))+(y(9,:)*AB(1:2))+(b(9,:)*AC(1:5)));
dx1_10=-(A(1)+1/2*(x(10,:)*AA(1:3:9))+(y(10,:)*AB(1:2))+(b(10,:)*AC(1:5)));
dx1_11=-(A(1)+1/2*(x(11,:)*AA(1:3:9))+(y(11,:)*AB(1:2))+(b(11,:)*AC(1:5)));
dx1_12=-(A(1)+1/2*(x(12,:)*AA(1:3:9))+(y(12,:)*AB(1:2))+(b(12,:)*AC(1:5)));
dx1_13=-(A(1)+1/2*(x(13,:)*AA(1:3:9))+(y(13,:)*AB(1:2))+(b(13,:)*AC(1:5)));
dx1_14=-(A(1)+1/2*(x(14,:)*AA(1:3:9))+(y(14,:)*AB(1:2))+(b(14,:)*AC(1:5)));
dx1_15=-(A(1)+1/2*(x(15,:)*AA(1:3:9))+(y(15,:)*AB(1:2))+(b(15,:)*AC(1:5)));
dx1_16=-(A(1)+1/2*(x(16,:)*AA(1:3:9))+(y(16,:)*AB(1:2))+(b(16,:)*AC(1:5)));
dx1_17=-(A(1)+1/2*(x(17,:)*AA(1:3:9))+(y(17,:)*AB(1:2))+(b(17,:)*AC(1:5)));
dx1_18=-(A(1)+1/2*(x(18,:)*AA(1:3:9))+(y(18,:)*AB(1:2))+(b(18,:)*AC(1:5)));


dx2_1=-(A(2)+1/2*(x(1,:)*AA(2:3:9))+(y(1,:)*AB(3:4))+(b(1,:)*AC(6:10)));
dx2_2=-(A(2)+1/2*(x(2,:)*AA(2:3:9))+(y(2,:)*AB(3:4))+(b(2,:)*AC(6:10)));
dx2_3=-(A(2)+1/2*(x(3,:)*AA(2:3:9))+(y(3,:)*AB(3:4))+(b(3,:)*AC(6:10)));
dx2_4=-(A(2)+1/2*(x(4,:)*AA(2:3:9))+(y(4,:)*AB(3:4))+(b(4,:)*AC(6:10)));
dx2_5=-(A(2)+1/2*(x(5,:)*AA(2:3:9))+(y(5,:)*AB(3:4))+(b(5,:)*AC(6:10)));
dx2_6=-(A(2)+1/2*(x(6,:)*AA(2:3:9))+(y(6,:)*AB(3:4))+(b(6,:)*AC(6:10)));
dx2_7=-(A(2)+1/2*(x(7,:)*AA(2:3:9))+(y(7,:)*AB(3:4))+(b(7,:)*AC(6:10)));
dx2_8=-(A(2)+1/2*(x(8,:)*AA(2:3:9))+(y(8,:)*AB(3:4))+(b(8,:)*AC(6:10)));
dx2_9=-(A(2)+1/2*(x(9,:)*AA(2:3:9))+(y(9,:)*AB(3:4))+(b(9,:)*AC(6:10)));
dx2_10=-(A(2)+1/2*(x(10,:)*AA(2:3:9))+(y(10,:)*AB(3:4))+(b(10,:)*AC(6:10)));
dx2_11=-(A(2)+1/2*(x(11,:)*AA(2:3:9))+(y(11,:)*AB(3:4))+(b(11,:)*AC(6:10)));
dx2_12=-(A(2)+1/2*(x(12,:)*AA(2:3:9))+(y(12,:)*AB(3:4))+(b(12,:)*AC(6:10)));
dx2_13=-(A(2)+1/2*(x(13,:)*AA(2:3:9))+(y(13,:)*AB(3:4))+(b(13,:)*AC(6:10)));
dx2_14=-(A(2)+1/2*(x(14,:)*AA(2:3:9))+(y(14,:)*AB(3:4))+(b(14,:)*AC(6:10)));
dx2_15=-(A(2)+1/2*(x(15,:)*AA(2:3:9))+(y(15,:)*AB(3:4))+(b(15,:)*AC(6:10)));
dx2_16=-(A(2)+1/2*(x(16,:)*AA(2:3:9))+(y(16,:)*AB(3:4))+(b(16,:)*AC(6:10)));
dx2_17=-(A(2)+1/2*(x(17,:)*AA(2:3:9))+(y(17,:)*AB(3:4))+(b(17,:)*AC(6:10)));
dx2_18=-(A(2)+1/2*(x(18,:)*AA(2:3:9))+(y(18,:)*AB(3:4))+(b(18,:)*AC(6:10)));


dx3_1=-(A(3)+1/2*(x(1,:)*AA(3:3:9))+(y(1,:)*AB(5:6))+(b(1,:)*AC(11:15)));
dx3_2=-(A(3)+1/2*(x(2,:)*AA(3:3:9))+(y(2,:)*AB(5:6))+(b(2,:)*AC(11:15)));
dx3_3=-(A(3)+1/2*(x(3,:)*AA(3:3:9))+(y(3,:)*AB(5:6))+(b(3,:)*AC(11:15)));
dx3_4=-(A(3)+1/2*(x(4,:)*AA(3:3:9))+(y(4,:)*AB(5:6))+(b(4,:)*AC(11:15)));
dx3_5=-(A(3)+1/2*(x(5,:)*AA(3:3:9))+(y(5,:)*AB(5:6))+(b(5,:)*AC(11:15)));
dx3_6=-(A(3)+1/2*(x(6,:)*AA(3:3:9))+(y(6,:)*AB(5:6))+(b(6,:)*AC(11:15)));
dx3_7=-(A(3)+1/2*(x(7,:)*AA(3:3:9))+(y(7,:)*AB(5:6))+(b(7,:)*AC(11:15)));
dx3_8=-(A(3)+1/2*(x(8,:)*AA(3:3:9))+(y(8,:)*AB(5:6))+(b(8,:)*AC(11:15)));
dx3_9=-(A(3)+1/2*(x(9,:)*AA(3:3:9))+(y(9,:)*AB(5:6))+(b(9,:)*AC(11:15)));
dx3_10=-(A(3)+1/2*(x(10,:)*AA(3:3:9))+(y(10,:)*AB(5:6))+(b(10,:)*AC(11:15)));
dx3_11=-(A(3)+1/2*(x(11,:)*AA(3:3:9))+(y(11,:)*AB(5:6))+(b(11,:)*AC(11:15)));
dx3_12=-(A(3)+1/2*(x(12,:)*AA(3:3:9))+(y(12,:)*AB(5:6))+(b(12,:)*AC(11:15)));
dx3_13=-(A(3)+1/2*(x(13,:)*AA(3:3:9))+(y(13,:)*AB(5:6))+(b(13,:)*AC(11:15)));
dx3_14=-(A(3)+1/2*(x(14,:)*AA(3:3:9))+(y(14,:)*AB(5:6))+(b(14,:)*AC(11:15)));
dx3_15=-(A(3)+1/2*(x(15,:)*AA(3:3:9))+(y(15,:)*AB(5:6))+(b(15,:)*AC(11:15)));
dx3_16=-(A(3)+1/2*(x(16,:)*AA(3:3:9))+(y(16,:)*AB(5:6))+(b(16,:)*AC(11:15)));
dx3_17=-(A(3)+1/2*(x(17,:)*AA(3:3:9))+(y(17,:)*AB(5:6))+(b(17,:)*AC(11:15)));
dx3_18=-(A(3)+1/2*(x(18,:)*AA(3:3:9))+(y(18,:)*AB(5:6))+(b(18,:)*AC(11:15)));


c=[c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12 c13 c14 c15 c16 c17 c18;
```

```
   dy1_1 dy1_2 dy1_3 dy1_4 dy1_5 dy1_6 dy1_7 dy1_8 dy1_9 dy1_10 dy1_11 dy1_12
dy1_13 dy1_14 dy1_15 dy1_16 dy1_17 dy1_18;
   dy2_1 dy2_2 dy2_3 dy2_4 dy2_5 dy2_6 dy2_7 dy2_8 dy2_9 dy2_10 dy2_11 dy2_12
dy2_13 dy2_14 dy2_15 dy2_16 dy2_17 dy2_18;
   db1_1 db1_2 db1_3 db1_4 db1_5 db1_6 db1_7 db1_8 db1_9 db1_10 db1_11 db1_12
db1_13 db1_14 db1_15 db1_16 db1_17 db1_18;
   db2_1 db2_2 db2_3 db2_4 db2_5 db2_6 db2_7 db2_8 db2_9 db2_10 db2_11 db2_12
db2_13 db2_14 db2_15 db2_16 db2_17 db2_18;
   db3_1 db3_2 db3_3 db3_4 db3_5 db3_6 db3_7 db3_8 db3_9 db3_10 db3_11 db3_12
db3_13 db3_14 db3_15 db3_16 db3_17 db3_18;
   db4_1 db4_2 db4_3 db4_4 db4_5 db4_6 db4_7 db4_8 db4_9 db4_10 db4_11 db4_12
db4_13 db4_14 db4_15 db4_16 db4_17 db4_18;
   db5_1 db5_2 db5_3 db5_4 db5_5 db5_6 db5_7 db5_8 db5_9 db5_10 db5_11 db5_12
db5_13 db5_14 db5_15 db5_16 db5_17 db5_18;
   dx1_1 dx1_2 dx1_3 dx1_4 dx1_5 dx1_6 dx1_7 dx1_8 dx1_9 dx1_10 dx1_11 dx1_12
dx1_13 dx1_14 dx1_15 dx1_16 dx1_17 dx1_18;
   dx2_1 dx2_2 dx2_3 dx2_4 dx2_5 dx2_6 dx2_7 dx2_8 dx2_9 dx2_10 dx2_11 dx2_12
dx2_13 dx2_14 dx2_15 dx2_16 dx2_17 dx2_18;
   dx3_1 dx3_2 dx3_3 dx3_4 dx3_5 dx3_6 dx3_7 dx3_8 dx3_9 dx3_10 dx3_11 dx3_12
dx3_13 dx3_14 dx3_15 dx3_16 dx3_17 dx3_18];


ceq=[r1 r21 r22 r23 r31 r32 r41 r42 r43 r44 r45 r5 p1 p2 p10 m1 g1 g2 g3 g4 g5 g6
g7 g8 g9 g10];
```

---

```
Input= [Labor Capital Fuels];
Good=[Electricity Heat];
Bad=[PM so2 nox co voc];


x=Input;
y=Good;
b=Bad;


par=ones(80,1);
par0=zeros(80,1);




option=optimset('Algorithm','interior-point','TolX',1e-100,'MaxFunEvals',30000);

[par,fval]=fmincon(@(par)HC_IDF_oneF(par,x,y,b,t),par0,[],[],[],[],[],[],@(par)BC_b
ez11_oneF_Const24(par,x,y,b,t),option);

A0=par(1);
A=par(2:4);
B=par(5:6);
C=par(7:11);
AA=par(12:20);
AB=par(21:26);
AC=par(27:41);
BB=par(42:45);
BC=par(46:55);
CC=par(56:80);






    n=18;
    df=ones(n,1);
for i=1:n
    df(i) = A0+
x(i,:)*A+y(i,:)*B+b(i,:)*C+1/2*kron(x(i,:),x(i,:))*AA+1/2*kron(y(i,:),y(i,:))*BB+1/
```

```
2*kron(b(i,:),b(i,:))*CC+kron(x(i,:),y(i,:))*AB+kron(x(i,:),b(i,:))*AC+kron(y(i,:),
b(i,:))*BC;
end

    dy1=ones(n,1);
for i=1:n
    dy1(i)= B(1)+1/2*sum(y(i,:)*BB(1:2))+sum(x(i,:)*AB(1:2:6))+sum(b(i,:)*BC(1:5));
end

    db1=ones(n,1);
for i=1:n

db1(i)=(C(1)+1/2*sum(b(i,:)*CC(1:5))+sum(x(i,:)*AC(1:5:15))+sum(y(i,:)*BC(1:5:10)))
;
end
    db2=ones(n,1);
for i=1:n

db2(i)=(C(2)+1/2*sum(b(i,:)*CC(6:10))+sum(x(i,:)*AC(2:5:15))+sum(y(i,:)*BC(2:5:10))
);
end
    db3=ones(n,1);
for i=1:n

db3(i)=(C(3)+1/2*sum(b(i,:)*CC(11:15))+sum(x(i,:)*AC(3:5:15))+sum(y(i,:)*BC(3:5:10)
));
end
    db4=ones(n,1);
for i=1:n

db4(i)=(C(4)+1/2*sum(b(i,:)*CC(16:20))+sum(x(i,:)*AC(4:5:15))+sum(y(i,:)*BC(4:5:10)
));
end
    db5=ones(n,1);
for i=1:n

db5(i)=(C(5)+1/2*sum(b(i,:)*CC(21:25))+sum(x(i,:)*AC(5:5:15))+sum(y(i,:)*BC(5:5:10)
));
end


P_PM=ones(n,1);
for i=1:n
    P_PM(i)=P_el(i)*db1(i)/dy1(i);
end
P_SO2=ones(n,1);
for i=1:n
    P_SO2(i)=P_el(i)*db2(i)/dy1(i);
end
    P_NOx=ones(n,1);
for i=1:n
        P_NOx(i)=P_el(i)*db3(i)/dy1(i);
end
    P_CO=ones(n,1);
for i=1:n
    P_CO(i)=P_el(i)*db4(i)/dy1(i);
end
    P_VOC=ones(n,1);
for i=1:n
    P_VOC(i)=P_el(i)*db5(i)/dy1(i);
end

MAC=[P_PM P_SO2 P_NOx P_CO P_VOC];


R= [df MAC];
```