universität wien

Prof. Dr. Gerhard Ecker
Boltzmanngasse 5
A-1090 Wien
Austria

T +43 (1) 4277-515 09
F +43 (1) 4277-95 15
eMail: gerhard.ecker@univie.ac.at

Faculty of Physics

Particle Physics

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Faculty of Mathematics and Physics
Charles University Prague
Ke Karlovu 3
12116 Praha 2
Czech Republic

Evaluation of the thesis of Martin Zdráhal

Construction of pseudoscalar meson amplitudes in chiral perturbation theory using a dispersive approach

The thesis submitted by Martin Zdráhal is an impressive piece of work, both in size and in quality of the research. From my evaluation detailed below it is clear that the author has demonstrated his ability for creative scientific work. Many of the results obtained in this thesis are new and they provide the basis for further scientific investigations in the area of low-energy particle physics.

In the following report, I will go through the separate parts of the thesis, commenting both on the results and on the presentation and suggesting some minor improvements here and there.

In Part I the basis is laid out for the applications in later chapters. Since Quantum Chromodynamics (QCD), the gauge theory of strong interactions, cannot be treated in a straightforward perturbative manner at low energies, effective field theory methods have to be employed. The standard approach is an effective Lagrangian field theory in terms of the dominant degrees of freedom at low energies, the pseudoscalar mesons. The method is called Chiral Perturbation Theory (CHPT) and it has both advantages and weaknesses. In addition to a systematic expansion in the momenta of pseudoscalar mesons and in the quark masses, it incorporates automatically all the symmetries of QCD, especially the spontaneously and explicitly broken chiral symmetry of the theory. Its main drawback is the possible lack of convergence of successive orders in the chiral expansion for certain observables, especially in the case of three light flavours. Moreover, the justification of treating quark masses at the same level as momenta squared in the expansion has often been questioned, again mainly in the three-flavour case. These possible weaknesses of CHPT are the main motivation for the so-called reconstruction theorem presented in detail in the thesis, a successive iterative construction of a dispersion theoretic framework for pseudoscalar amplitudes. Based on previous work of Stern et al. for pion-pion and pion-kaon scattering, the reconstruction theorem is more general than CHPT but encompasses the results up to O(p^6) in the chiral counting, thereby including up to two loops in the Lagrangian framework.

The derivation of the reconstruction theorem in the general case of quasi-elastic scattering is presented in full detail and leaves no questions open. As a small criticism, I would have preferred to see the same conventions used for the factor N in the partial-wave expansion of the amplitudes instead of switching between N=1 and N=2, probably for "historical" reasons. The relation of the approach to CHPT is worked out and a first illustrative application to meson-meson scattering in the isospin limit is discussed.

In Part II the method is applied in detail to pion-pion scattering. The relation between two standard parametrizations, the scattering-length and the subthreshold parametrization, is established. Although the procedure can easily account for different up- and down-quark masses, a complete analysis of isospin violation would have to include electromagnetic corrections, for which the Lagrangian approach is much more adequate. However, there are no general objections against supplementing the strong isospin violation in the dispersive approach by electromagnetic corrections in the standard approach. In CHPT, a full analysis of pion-pion scattering to next-to-next-to-leading order (NNLO) is only available in the limit of isospin violation and it would certainly be useful to undertake a complete study of isospin violation in this channel along the lines described.

Part III contains the derivation of dispersive decay amplitudes for $K \to 3\pi$ and $\eta \to 3\pi$. The full NNLO amplitudes are given for $K \to 3\pi$ in the isospin limit and for $\eta \to 3\pi$ to first order in isospin breaking. The partial results including strong isospin violation would have to be supplemented again by electromagnetic effects. Since the latter are available in the literature in the CHPT framework, a promising project for the future would be a complete analysis of isospin violation for $K \to 3\pi$, especially because many new experimental results have appeared since the CHPT analysis of Bijnens et al.

The main phenomenological analysis of this thesis is presented in Part IV where $\eta \to 3\pi$ decays are investigated. At least to the order discussed, electromagnetic corrections can be ignored in this case. This allows both for an interesting comparison with the NNLO CHPT results and for an independent determination of the isospin-breaking ratio R related to the light quark masses. I found especially the comparison with CHPT very useful where the author demonstrates that the seeming discrepancy between the chiral prediction and experimental results may well be due to inadequate estimates of the so-called low-energy constants (LECs) at NNLO. Moreover, the evaluation of light quark masses m_u, m_d from $\eta \to 3\pi$ and recent lattice studies is an important new and independent determination of these fundamental QCD parameters.

In this context, again some small suggestions for improvement. On p. 156 it is claimed that two-loop amplitudes in CHPT depend on subsets of 102 LECs at NNLO. In fact, physical amplitudes depend in general on subsets of exactly 90 such LECs in the three-flavour case. Not only for personal reasons, I recommend that the original reference for the NNLO chiral Lagrangian be quoted where these LECs are defined. In addition, reference to the resonance saturation on the same page should include the work at O(p^6). I am sure that Martin Zdráhal is well aware of both papers in this connection. In addition, I have difficulties with footnote 4 on p. 158. Since the CHPT amplitude is a special case of the general dispersive amplitude, I would have expected that the analysis shows that there occur at most (instead of at least, as written in the footnote) six independent combinations of LECs in the CHPT amplitude but maybe I am missing the point. Finally, on p. 184 it is written that the value of α obtained from the analysis is smaller than the most precise experimental values as expected. Why was that to be expected?

The thesis contains a large number of carefully prepared tables and figures and in addition 12 appendices where many relevant (mainly mathematical) details are collected.

In summary, the thesis of Martin Zdráhal presents an interesting approach for studying certain processes of pseudoscalar mesons at low energies. It contains a lot of new material presented in a clear and transparent way. The method should give rise to many phenomenological applications, either independently of or in combination with chiral perturbation theory. The thesis documents in a convincing way the ability of Martin Zdráhal for creative scientific work. In conclusion, I therefore recommend that the thesis be accepted by the Faculty of Mathematics and Physics of the Charles University.

Dr. Gerhard Ecker