LUND UNIVERSITY

Department of Astronomy and Theoretical Physics
Theoretical High Energy Physics
Johan Bijnens, Professor

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Prof. Zdeněk Němeček
Dean
Faculty of Mathematics and Physics
Charles University Prague

Re: Report on doctoral thesis Martin Zdrahál

Dear Sir,

The thesis "Construction of pseudoscalar meson amplitudes in chiral perturbation theory using a dispersive approach" is situated in theoretical low-energy particle physics. The main goal is to go beyond standard perturbative chiral perturbation theory and use dispersion relations to systematically include higher order corrections coming from two-body rescattering effects.

The general method used was originally proposed by Jan Stern and collaborators and it is based on the fact that up to NNNLO effects the amplitude for processes with four meson external legs can be written in a simple way with functions depending on only one kinematical parameter. These one-parameter functions then satisfy a number of dispersive constraints and integral relations. This form of the amplitude is also capable of describing the two-body rescattering effects to higher order than NNLO. The first part of the thesis clarifies the assumptions underlying this form of the amplitudes and proofs exactly when it holds. It also extends the theorem to the case of unequal mass parameters. It had earlier only been used for the equal mass case. This is described in Chapters 2-4 and has been published in paper I and II. I especially liked the fact that in the unequal mass case they managed via (3.50) to reduce at NNLO the problem to one numerical integral only.

The second part of the thesis illustrates the techniques for pi-pi scattering to NNLO and extends the known results to the isospin breaking case with inclusion of a different mass for the charged and neutral pion. This is a new result but its main purpose is to chose a good description of the needed partial waves for the later parts.

The third part is the main theoretical new result of this thesis. It extends the method above for meson-meson scattering to the case of a decay meson to three mesons. This requires carefully taking into account the new type of singularities that can appear here using the analysis of Appendix J. The method here is worked out to NNLO in full generality. This has been partially published in paper VI and will be fully published in paper X. This part is entirely new and should be useful in analyzing the experiments as well as tying together the other approaches to the same processes, standard NNLO chiral perturbation theory, the nonrelativistic effective

Postal address Sölvegatan 14 A, SE-223 62 LUND, Sweden *Office address* Sölvegatan 14 A, Lund
Phone direct +46-46-222 04 47, *switch board* +46-46-222 00 00 *Fax* +46-46-222 96 86 *E-mail* bijnens@thep.lu.se *Web* http://www.thep.lu.se/~bijnens

approach of Gasser, Kubis and collaborators and the pure numerical dispersive approach of Colangelo and collaborators. Since the present method is more analytical it will allow for a better understanding of the origins of large effects.

The fourth part contains the application to $\eta \to 3\pi$ and uses the results to obtain a determination of the quark mass ratio $R = (m_s - \hat{m})/(m_d - m_u)$. Here the thesis describes in detail how their method is used together with the NNLO calculation and the KLOE measurement of the Dalitz plot to obtain an improved determination of R. This work is clearly new and a very welcome to the existing literature on the subject.

The large number of appendices describe a number of formulas that are too long to fit in the main text.

The thesis clearly shows that the author has now reached the level where he can perform creative scientific work.

Yours sincerely,

Johan Bijnens
Professor Theoretical High Energy
Physics