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Evaluation of the Doctoral Thesis of Martin Tancer: Topological and Geometrical Combinatorics

The interaction between topology and combinatorics has a long history dating back to the pioneering work of Poincaré. Indeed, much of the earlier work in algebraic topology was combinatorial in nature, e.g. topological invariants such as Betti numbers were initially defined in terms of triangulations. Nowadays, Topological Combinatorics is a major area of research with a variety of applications ranging from extremal set theory to discrete geometry and to computational topology. Martin Tancer's thesis is a very substantial contribution to topological combinatorics. Let me briefly describe some of the highlights of his work.

A simplicial complex X is d-representable ($X \in \mathcal{K}^d$) if X is the nerve of a family of convex sets in $\mathbb{R}^d$. Let $\mathcal{C}^d$ denote the family of d-collapsible complexes. A fundamental theorem of Wegner asserts that $\mathcal{K}^d \subset \mathcal{C}^d$. The interest in the classes $\mathcal{K}^d$ and $\mathcal{C}^d$ stems (in part) from the fact that while numerous Helly type results in discrete geometry can be naturally formulated in terms of the combinatorics of d-representable

complexes, in many cases the proofs only use the weaker condition of d-collapsibility. Martin Tancer's thesis contains an in depth study of $\mathcal{K}^d$ and $\mathcal{C}^d$, and settles a number of outstanding problems concerning these classes. In particular he obtains the following results:

1. For any d there exists a $q_0(d)$ such that if $q \geq q_0(d)$ then the complex of a projective plane of order q is not d-representable. This answers in the negative a question of Alon, Kalai, Matoušek and the reviewer. The proof is short and ingenious.
2. Deciding d-collapsibility is NP-complete for $d \geq 4$. This is a hard result that depends among other things on certain high dimensional generalizations of Zeeman's dunce hat.
3. For $d \geq 2$, there exists a a topological good cover in $\mathbb{R}^d$ whose nerve is not d-collapsible. This refutes an old conjecture of Wegner. The main step here is a clever construction of a good cover in $\mathbb{R}^2$ consisting of 19 sets with a non-2-collapsible nerve.

Another major chapter in the thesis deals with algorithmic aspects of the embedding problem. In joint work with Matoušek and Wagner, the author proves that deciding embeddability of a k-dimensional complex in $\mathbb{R}^d$ is NP-hard for $d \geq 4$ and $d \geq k \geq \frac{2d-2}{3}$. This remarkable result is closely related to the optimality of the Haefliger-Weber theorem, and has appeared in the prestigious Journal of the European Math. Society.

The thesis is very well written. The formal proofs are accompanied by intuitive explanations and illuminating figures. Most of the results of the thesis have already appeared in leading journals. The author demonstrates an impressive command of sophisticated mathematical techniques in areas ranging from algebraic topology to computational complexity and to probabilistic combinatorics.

Summary: This is an excellent doctoral thesis with numerous first class contributions in the area of Topological Combinatorics. I strongly

recommend awarding Martin Tancer the doctoral degree.

Sincerely,

Roy Meshulam