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Bakalářská práce

2012 **Kryštof Wolf**

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Electricity futures option pricing

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Abstract

The main focus of this thesis is the futures option pricing in electricity sector. We begin with description of fundamental features of futures options and describe specifics of electricity and markets where it is traded, which influenc option pricing. Further on, we will choose, according to defined criteria, three pricing methods resp. models that will be subject of our interest for the rest of the work – Black model, binomial model and Monte Carlo simulation. These models will be then briefly described and the basic idea of their approach to option pricing will be introduced. In the last, most important part, of this thesis we deal with empirical testing of all the three above mentioned pricing methods using data obtained from European Energy Exchange in Leipzig. In order to find best model for electricity futures option pricing we first calculate option premiums according to single models, these estimates are compared with the market premiums and based on the average percentage difference between these two values the most accurate model is chosen. To the author’s best knowledge, the thesis presents the broadest empirical testing of futures option pricing models in electricity sector.

Keywords

Futures options, electricity markets, Black model, binomial model, Monte Carlo simulation, accuracy testing

Abstrakt

Hlavní zaměření této práce je oceňování futuritních opcí obchodovaných v elektronickém sektoru. Začátek práce je věnován popisu základních vlastností futuritních opcí společně se specifiky trhu s elektřinou, které ovlivňují jejich oceňování. Dále jsou dle předem stanovených kritérií vybrány tři modely pro oceňování opcí, které budou předmětem našeho zájmu po zbytek práce – jsou jimi Blackův model, binomický model a konečně simulace Monte Carlo. Tyto modely jsou potom krátce popsány společně s předtavením základních principů na něchž jsou založeny jejich přístupu k oceňování. V poslední části je testována přesnost těchto modelů za použití dat získaných z European Energy Exchange v Lipsku. Za účelem nalezení nejvhodnějšího modelu pro oceňování futuritních opcí v sektoru elektřiny nejprve spočítáme opčí prémie dle jednotlivých modelů, tyto odhady jsou poté porovnány se skutečnými tržními prémiemi a na základě průměrné procentuální odchylky mezi těmito dvěma veličinami je vybrán nejpřesnější model. Podle autorova nejlepšího vědomí práce prezentuje nejširší empirický test modelů pro oceňování futuritních opcí v sektoru elektřiny.

Klíčová slova

Opce na futures, trhy s elektřinou, Blackův model, binomický model, Monte Carlo simulace, testování přesnosti

Rozsah práce: 96 216 znaků

Declaration

Hereby I declare that I compiled this bachelor thesis independently, using only the listed literature and resources and that this thesis was not used to obtain another academic degree.

I also grant to Charles University permission to produce and to distribute copies of this document in whole or in part.

Prague, May 13, 2012

Prohlášení

Prohlašuji, že jsem předkládanou práci zpracoval samostatně, použil jen uvedené prameny a literaturu a že práce nebyla použita k získání jiného titulu.

Zároveň souhlasím s tím, aby práce byla zpřístupněna pro studijní a výzkumné účely.

V Praze dne 13.05.2012 ....

Kryštof Wolf

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Electricity futures option pricing

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Předběžná náplň práce

Cílem této bakalářské práce je shrnutí stávajících modelů pro oceňování opcí a ověření jejich platnosti na Phelix opcích s podkladovým aktivem ve formě futures kontraktu, jež jsou reálně obchodované na Europian Energy Exchange v Lipsku.
Po obecném úvodu do problému energetických derivátů a jejich specifik, se v druhé, teoretické části bakalářské práce zaměřím na představení teoretických modelů pro oceňování opcí (binomický, Black – Scholesův a jump diffusion model).
Jako další přístup k oceňování opcí bude zmíněna simulační metoda Monte Carlo.
Třetí část pak bude věnována aplikaci konkrétních dat (cen futures a opčních kontraktů) z EEX na výše zmíněné modely a jejich ověření i pro poměrně specifické opce na elektrickou energii. Závěrem práce by mělo být porovnání výstupů ze zkoumaných modelů a výběr nejvhodnějšího z nich pro oceňování energetických opcí.

Předběžná struktura:
1. Úvod do problematiky energetických opcí
2. Modely pro jejich oceňování
3. Aplikace dat, vzájemné porovnání modelů
4. Závěr

Předběžná náplň práce v anglickém jazyce

The aim of this bachelor thesis is a summary of existing option pricing models and its validation on Phelix options written on futures contracts, which are traded on Europian Energy Exchange in Leipzig.
After the general introduction into problem of energy derivatives and its specifics, I will focus on introduction of current option pricing models (binomial, Black – Scholes and jump diffusion model). As another approach to option pricing will be mentioned simulation Monte Carlo.
In the third part I will apply concrete data (prices of futures and option contracts) from EEX to models mentioned above and verify whether the models are valid also for quite specific electricity options. In the end of this work I will compare results from examined models and try to find the best one for energy options pricing.

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List of abbreviations and symbols

Abbreviations

CAPM	Capital asset pricing model
CBOE	Chicago Board Options Exchange
EEX	European Energy Exchange
GDP	Gross domestic product
HDD	Heating degree day
MC	Monte Carlo
OTC	Over-the-counter
Q-Q	Quantile-quantile
SD	Standard deviation
S/K	Skewness/Kurtosis test
S-W	Shapiro-Wilk test

Symbols

δ	Percentage difference between market and model price
Δ	Average difference between market and model price
ε	Random number sampled from a standard normal distribution
$\aleph(x)$	Standard normal distribution function
Π	Value of hedged portfolio
σ	Volatility
S	Spot price of underlying asset (generally)
F	Price of underlying futures contract
K	Expiration price
t	Current time
T	Expiration date
$(T-t)$	Time to maturity
r	Risk-free interest rate
C	Premium of call option
P	Premium of put option
k	Number of periods in binomial model or Monte Carlo simulation
u	Growth coefficient
U	Increase in underlying price in percents
d	Drop coefficient
D	Decrease in underlying price in percents
p	Probability of increase in underlying price
y	Convenience yield

List of tables and figures

Tables

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Introduction

Since the introduction of Black-Scholes model in 1973 there has been a boom with both theoretical and empirical papers (e.g. Bates (1995) or Baksi, Cao and Chen (1997) and many others) handling with option pricing. Unfortunately, to the author's best knowledge, almost all papers are devoted to pricing of options written on stocks, while the other underlying assets (namely futures contracts) are mostly ignored and nearly no sources (especially empirical ones) for these topics are available. This fact is based on two reasons – first is much higher popularity of stock options compared to other types, second is a better availability of the data for stock option pricing models testing[1]. Nevertheless the fact is that options written on different underlying assets than stocks (especially futures and commodities) become more and more popular in last decade.

Next drawback of current works dealing with pricing models testing and comparison is ignoring of simulation approach to option pricing, namely Monte Carlo simulation. This is done despite the fact that this powerful tool is nowadays plentifully used in almost every science discipline.

Second current topic, apart from futures options and simulation methods expansion, in field of option pricing is pricing of energy resp. electricity options. After the liberalization of electricity markets (which is an issue of several last decades, somewhere even an actual one) the need of pricing electricity derivatives became an issue of the day. As no models created specifically for electricity derivatives pricing are available, there is no other possibility than to use already existing ones. Nevertheless not even the suitability of available models for use in electricity sector has been studied in more details yet.

In this thesis we try to combine these two current issues. Our main goal is to find the most convenient pricing approach resp. concrete model for electricity futures option pricing which is able to capture the real price development and its use is not unnecessarily complicated. Moreover we also provide reader with an overview of basic models designated for futures option pricing, introduce Monte Carlo simulation as an

[1] Mainly due to CBOE (Chicago Board Options Exchange)

alternative to option pricing formulas and also examine the influence of non-standard electric energy price development, as it is described in first chapter, on option pricing.

For this purpose we have chosen three pricing models – Black model, binomial model and already mentioned Monte Carlo simulation - one representative for each pricing approach (analytical, numerical and simulation). These models will be tested on options traded on European Energy Exchange (EEX) in Leipzig, Germany.

Our thesis is divided into three parts. In the first one we will handle with option and futures option fundamentals and also mention electricity market specifics influencing option pricing. In the second part we will move to option pricing itself. Apart from factors affecting option price we will introduce the basic theoretical framework for all three chosen models. The last, empirical part is then devoted to testing of single models using prices of futures options which are traded on EEX and to an effort to find the best one for electricity futures option pricing.

1. Electricity futures option fundamentals

In the first chapter of our thesis we are going to deal with basic characteristics of financial instrument we are interested in – futures options on electric energy.

In the first part we will discuss general fundamentals of options and markets where they are traded. In the second one we will namely focus on futures options and derive important relationships which will be useful during pricing models derivation - put-call parity and price bounds, both with respect to features of futures as an underlying asset. The last third part will be devoted to energy resp. electricity markets, especially we will focus on its specifics affecting option pricing.

1.1 Introduction to options

1.1.1 Options

Option is a derivative financial instrument which gives its owner the right but not the obligation to buy or sell an agreed amount of particular commodity at agreed (strike) price on agreed (expiration) date. For this right pays purchaser so called option premium. In the expiration date the option holder decides to exercise (activate) or not to exercise the option based on the parity value (difference between current price of underlying asset and strike price). If this difference is positive the option is called to be in-the-money and will be exercised, if both prices are approximately equal – at-the-money and finally if the difference is negative – out-of-the-money. In the last case the option is not exercised and becomes worthless.

There are several groups according to which we can divide options:

1. According to right connected with the option:
    - **call option** gives the owner right to buy an underlying asset when exercised
    - **put option** is then connected with right to sell
2. According to underlying asset:
    - we can find options written on various assets such as stock, bond, futures contract, interest rate or any type of commodity

3. According to position:
   - in **long position** the option is bought because the price of underlying asset is expected to rise till expiration date
   - contrarily in **short position** the option is sold and price of underlying asset expected to fall

1.1.2 Option markets

Next classification of options mentioned in this work is according to place where they are traded:

Exchange-traded options are the first case. These options are traded at option or futures exchange and have standardized contracts, especially when talking about underlying asset, expiration date or strike price. Options are settled through a clearing house which lowers the risk for both sides. Thanks to a standardized form there usually are quite sophisticated and accurate pricing models available.

Over-the-counter (OTC) options are the second type, which is the opposite of Exchange-traded options. These option contracts come from individual needs of single traders and so we can only hardly find some common features. OTC options are not listed on an exchange and so risk connected with default of one of sides has to be taken into account. Usually at least one of traders is an important and big financial institution (most often an investment bank) which is in most cases able to set the price for which it is ready to buy or sell any option. Among most popular options traded over-the-counter belong those written on foreign exchange and interest rates[2]. Due to non-standardized character it is difficult to find some general pricing model which would capture price development of such options.

[2] For more information about OTC trading, its specifics and possible threats see Duffie, Gârleanu and Pedersen (2005)

1.1.3 Option value

As this thesis is devoted to option pricing we should also mention two different values we distinguish at options – intrinsic and time one:

1. The **intrinsic value**, which is the present value of an option measured as difference between actual price of underlying asset and strike price. Thanks to the fact that intrinsic value cannot be negative we can express it as $max\{0, S_t - K\}$ (for call option) resp. $max\{0, K - S_t\}$ (for put option), where S_t is a spot price of underlying asset in time *t* and *K* strike price.
2. The second, a **time value**, reflects the potential that in time period till expiration the price of underlying asset will rise (resp. fall) and make the option profitable when exercised. Development of time value heavily depends on time left to expiration of the option and volatility of underlying asset price but it always converges to zero as $t \rightarrow expiration\ date$. As time value is a complement to intrinsic value it can be easily calculated as

$$Time\ value = Option\ value - Intrinsic\ value \qquad (1)$$

1.1.4 Option styles

Nowadays there is a huge number of various option styles. In effort to satisfy any need derivative traders have new and new styles arise - some of them only as a combination of already existing ones, some of them completely new. As option styles are not the main goal of this theses we choose only those ones which are most favourite and most frequently used among all power derivative traders. Due to the specific requirements in power sector, the most often traded options there are of European, American, Asian and swing style[3]:

European option: In the case of European option the only one exercise date (which is the same as expiration date) is allowed. On this date the option holder can use his option right to sell or buy particular asset. Thanks to the fact that Phelix options, which we will use for testing models accuracy, are of an European style, we will focus on it in following discussion.

[3]According to Vehviläinen (2001)

American option: This option style allows, contrary to European one, for more than one exercise date. It can be any date till option expiration or, more typically, there is a given period (week or month) during which the option can be exercised. Despite the fact that Phelix futures options are European ones, most of the futures options (which we are interested in) are of an American style.

Asian option: Asian option allows, as the European one, for a single exercise date. The difference between these two types is in the price settlement. While the European option offers discrete, the Asian one an average price settlement. Thanks to this fact Asian options are most frequently used in energy or power sector.

Swing option: Swing option has again only one exercise date but it allows for changes in quantity of appropriate underlying asset which is traded. This “trading freedom” is usually limited by given maximal and minimal bounds for possible trade. We can distinguish two main types of swing options – price-driven and demand-driven. Price-driven options are characterised by the fact that both sides can buy as well as sell. By contrast, in case of demand driven-options one of counterparties only can take or refuse the delivery of underlying asset[4].

1.1.5 Reasons to trade options

In the financial markets there are several types of traders using derivatives, namely options, for different purposes. Arbitrators, who try to take advantage of discrepancy between prices of the same asset in two different markets, hedgers, who protect themselves from the risk associated with price of an underlying asset or speculators, whose only aim is to make money by speculating on future price development. Now we will briefly discuss seven main reasons[5] why to enter option market:

1. **Hedging** – Hedging is the original and still very important motive for option trading. Options are not the only one derivative convenient for hedging but it is definitely the most popular one – especially because of its flexibility and high number of option strategies available, which allows traders to create any possible hedge position they want to.

   As hedging is very often a primary aim of option trading we will now briefly discuss, using concrete examples, three basic option hedging strategies[6]:

[4] For detail explanation and more informations about swing options se Kluge (2006)
[5] From Wolfinger (2008)
[6] For details and more complicated strategies see ASX (2011)

Buying protective puts – Trader buys a protective put in the case he holds stocks which have appreciated and he wants to protect the profit: Trader holds a stock which has increased its value from €20 to €40 and now he wants to protect the profit just in case the stock would decline in next period. In this case he buys a protective 35 put which gives him right to sell the stock for €35, this ensures him at least €15 profit per one stock (of course minus the option premium paid).

Writing covered calls – Trader writes a covered call in case he does not expect price of underlying to rise till option expiration but want to make some extra cash: Let trader hold the same stock as in previous example. If he does not expect its price to rise, he can write a call at €45 which will bring him, in case of no or only a small increase in stock price which means no exercising of the option, extra cash in form of received option premium.

Creating a price collar – Last strategy, price collar, combines two previous ones. In this case option premium received from written call is used to pay premium when buying put. Trader can in this way determine a floor and ceiling, or range, in which he wants to operate. Thanks to the premium trade-off this operation requires only small or even no financing.

2. **Insurance** – Options can also serve as insurance which will ensure sufficient demand or supply (important chiefly for factories in order not to have to stop the production because of insufficient demand or material supply) or certain prices in future time, i.e. work as insurance against increases (for consumers) or decreases (for producers) in prices. Such insurance is usually expensive but with proper trading techniques the costs can be minimized or even eliminated.
3. **Speculation** – Making money in the derivatives market is probably the main goal of today traders. They try to predict future development of underlying asset price and speculate on its growth or decline.
4. **Leverage** – Leverage, which is highly connected with the speculation, is the reason why traders looking for profit trade options instead of other securities. In the financial market there are only a few instruments allowing for profits which can reach hundreds or thousands of percent of initial investment, options are one of them.

5. **No need to be bullish all the time** – When one trades options, there is no need of growing market to make a profit. Thanks to variety of option contracts trader can create such a position that prospers when market stagnates or even moves lower.
6. **Limited risk** – Another fact which attracts all traders is the possible limited risk connected with options trading. In this case, of course, the profits are limited as well but still, compared to other securities, the risk/return[7] ratio is much more interesting in case of options.
7. **Indexing** – Option trading also enables diversification of portfolio. Except for options on single stocks one can trade ones on the major indexes (e.g. S&P, DJIA, etc.) as well.

1.2 Futures options

As we have already mentioned, in this thesis we will be interested in options written on a futures contract. Taking this into consideration we will now move from general option fundamentals to discussion of features that are specific for futures options.

1.2.1 Futures options specifics

Options written on a futures contract are a specific class of options, which requires several adjustments if we want to apply general pricing methods developed originally for stock options. The main reason for this is that if the option is exercised, its holder receives another derivative (futures), instead of the underlying asset itself as in the other options case. Other specifics come from the unique features of futures contract and can be summed up into three following points[8]:

- The futures price is a compounded value of the spot price:

$$F = Se^{r(T-t)} \tag{2}$$

  It says that buying a cash instrument and holding it to maturity yields the same as buying the futures contract. This equality allows us to adjust most of relations which holds for spot options to futures ones.

[7] Discussed in more details in Pilipovic (2007)
[8] As did it Brenner, Courtadon and Subrahmanyam (1985)

- Usual put-call parity does not hold any more[9].
- Value of an European option written on particular asset will have the same price as European futures option written on futures based on the same asset, provided that both options have same maturity date and exercise price[10].

1.2.2 Futures options popularity

Especially in recent time the futures options become more and more popular. This happens at the expense of options on commodities themselves. Why is it so? Why traders prefer to hold a future contract to commodity itself? Option market simply follows the general trend in exchange trading and looks for new instruments that will be able to satisfy market requirements better. Futures contracts are more liquid, easier to trade and bring lower transaction costs than underlying commodity itself. If we consider that, apart from hedging, options are most frequently used for speculative purposes it is much more logical and practical for traders to hold a futures contract than some physical asset. Next advantage for speculators is that exercising of futures options often does not lead to delivery of underlying asset as the futures options can be settled in cash.

1.2.3 Put-call parity

Now we derive an important relationship between price of call and put futures option both with strike price K and time to expiration $(T-t)$. We will follow Hull (2009) and create two different portfolios:

Portfolio A: a European call option plus an amount of cash equal to $Ke^{-r(T-t)}$

Portfolio B: a European put option plus a long futures contract plus an amount of cash equal to $F_0e^{-r(T-t)}$

The basic idea behind the put-call parity is that the cash in portfolio A can be invested at risk-free rate r and grows to K at time T. Value of this portfolio depends on whether the option is exercised or not – if $F_T \leq K$ call option is not exercised and value

[9] More about this in next subsection

[10] For detail explanation see Brenner, Courtadon, Subrahmanyam (1985)

of portfolio is K, on the other hand if $F_T > K$ option is exercised and portfolio is worth F_T. If we put previous findings together we can write value of A as:

$$max\{K, F_T\} \tag{3}$$

We can make a similar consideration in case of portfolio B. Now the cash can be invested at the risk-free rate to grow to F_0 at time T and the conditions for exercising will be reversed to the previous ones. Now we sum up all information about payoffs in time T we know – payoff of put option if it is exercised which is $K - F_T$, zero otherwise, payoff of futures contract which is $(F_T - F_0)$ – and add yield from invested amount of money - F_0, to obtain overall value of portfolio B:

$$F_0 + (F_T - F_0) + \ max\{K - F_T, 0\} = \ max\{K, F_T\} \tag{4}$$

From the fact that both portfolios has the same value at expiration time and from the nature of European option (which cannot be exercised earlier than in expiration date) we simply deduce that also values in the present time have to be equal. As the futures contract from portfolio B is worth zero today we can writte:

$$C + Ke^{-r(T-t)} = P + F_0 e^{-r(T-t)}, \tag{5}$$

where C is price of call option and P price of the put one.

1.2.4 Bounds for futures options

From the put-call parity we can directly derive next important feature which shows to be important during derivation of pricing formulas in single models.

The worst case that can happen is non-exercising the option. Value of the option (put or call) is then zero, i.e. the first bound is:

$$C \geq 0 \; resp. \; P \geq 0 \tag{6}$$

For bound in the case of exercising the option we will use already mentioned put-call parity. From the non-negativity of price of a call option results:

$$P + F_0 e^{-r(T-t)} \geq Ke^{-r(T-t)} \tag{7}$$

$$P \geq Ke^{-r(T-t)} - F_0 e^{-r(T-t)} = e^{-r(T-t)}(K - F_0) \tag{8}$$

For call option we will use the same arguments and obtain:

$$C \geq e^{-r(T-t)}(F_0 - K) \tag{9}$$

If we put both cases together we obtain a general lower bound for put:

$$P \geq max\{0, e^{-r(T-t)}(K - F_0)\} \tag{10}$$

resp. call:

$$C \geq max\{0, e^{-r(T-t)}(F_0 - K)\} \tag{11}$$

The upper bounds proceed directly from definition of option itself. Price of put option cannot ever exceed the discounted strike price:

$$P \leq Ke^{-r(T-t)} \tag{12}$$

and price of call cannot be higher than price of underlying asset, this has to hold in any time period t, so:

$$C \leq F_t \tag{13}$$

1.3 Electricity market and options specifics

1.3.1 Electricity market specifics

Trading with electricity brings many various problems to those ones who try to handle it and predict future development of its price or volumes sold:

Commodity market, in comparison with the financial one, is much harder to model. This is simply caused by tangible character of assets traded which is connected with additional costs of transportation or storage. Energies then are the most complicated group of commodities and to cut a long story short electric energy, especially due to impossibility of storage, is unique and trickiest one among all others.

The impossibility of storage is of course not the only one reason why electricity makes so much trouble to all who try to predict its price development. In our discussion we will try to summarize the most important ones, for this purpose we will follow Pilipovic (2007).

At first we should mention a **complex price behaviour** which is caused by high number of fundamental drivers. Transportation and its price, weather and technological issues belong among the most important ones but we can mention many others, maybe less important, but still significant factors affecting price of power as political decisions, environmental policies or limited amount of fossil fuels and its allocation etc. This fact makes all tries to create relatively simple models that capture the essence of the market impossible.

If we try to predict a future development of price of any commodity a development of both, supply and demand needs to be predicted as exactly as possible. And here comes second problem of energies, namely electricity. Not only price but supply and demand as well are influenced, compare to most of other kinds of commodities, by high and mostly unpredictable factors.

Among supply drivers we should mention two important ones that only energy sector has to face - storage and production:

Storage – Storage is not the issue when talking about natural gas or oil – there it is possible and we can treat these fossil fuels as any other commodity. Storing power is, from obvious reasons, not possible. This fact brings several more or less serious problems we have to face when valuing electricity and its derivatives. Most of them will be discussed in next chapter.

Production - Problem of volumes of future production is strongly connected with impossibility of storage because we cannot simply produce more electricity and have it in reserve[11]. In order to prevent shortage of power causing steeply rising prices or even blackouts, power producers try to develop various models predicting future electricity demand. Despite the fact that these models usually take into account plenty of variables affecting the demand as GDP of country, price development of power and gas (as its closest substitute for heating), HDD (heating-degree days) and many others and are quite successful, they are not able to catch market reactions to unexpected events which can cause massive demand shocks. Another point of view on this problem is the impossibility of production itself. With the exception of natural disasters or political decisions (e.g. nuclear power restriction in Germany) this is mainly caused by the fact that substantial part of reserves of fossil fuels and elements used as a nuclear fuel (which are the most important source for electricity production) is allocated in

[11] With the exception of pumped-storage hydroelectricity which is used only rarely anyway.

politically unstable countries like Russia or Arabian countries in north Africa. Possible stop in supply makes then the production several times costlier or even impossible.

The situation is maybe even more complicated on the demand side. Although there are many demand drivers (as weather, electricity and gas price, etc.), we will focus on those mentioned in Pilipovic (2007) which are unique for energy market – seasonality and convenience yield:

Seasonality - The fact that power consumption differs among single months or seasons is quite obvious. The highest consumption is usually in July and August, when the temperatures used to be highest and people use air condition, and in January and February when electricity is one of energies used for heating. This is a well known fact and energy traders usually prepare for it. Much more "dangerous" are unexpectedly high or low temperatures – extremely cold winter or unexpectedly hot summer can easily cause sharply rising prices due to insufficient supply.

Convenience yield – Generally the convenience yield is a total benefit received from holding of appropriate commodity minus the cost connected with this holding, such a storage cost. The convenience yield can be then positive (if benefits from holding exceed storage cost) or negative (vice versa). As we will see later the convenience yield (resp. impossibility of its expression due to storage problem) is the key difference between pricing of "usual" commodity and electricity.

There are also other, less important, features of energy sector which are unique and make its modelling difficult. Among all we can mention relatively lower liquidity compared to financial market, various levels of regulation or higher need of exotic derivatives. Comparison of energy and money market is summarized in table 1:

Table 1: Comparison of energy and money market

Issue	In Money Markets	In Energy Markets
Maturity of market	Several decades	Relatively new
Fundamental price drives	Few, simple	Many, complex
Impact of economic cycles	High	Low
Frequency of events	Low	High
Impact of storage and delivery; the convenience yield	None	Significant
Correlation between short- and long-term pricing	High	Lower, "split personality"
Seasonality	None	Key to natural gas and electricity
Regulation	Little	Varies from little to very high
Market activity ("liquidity")	High	Lower
Market centralization	Centralized	Decentralized
Complexity of derivative contracts	Majority of contracts are relatively simple	Majority of contracts are relatively complex

Source: **Pilipovic (2007)**

1.3.2 Power vs. commodity option pricing

As we have already mentioned, in commodity option pricing there is an important role of a convenience yield. This measure subtracts from the overall benefits from holding appropriate commodity the costs connected with this holding. It complicates modelling of price development because the future price is now not only an interest-bearing current spot price. This problem was solved by Brennan and Schwartz (1985) who presented the relationship between spot price, S, and future price, F, of contract of maturity T:

$$F(t,T) = S(t)e^{(r-y)(T-t)} \tag{14}$$

where r is the risk-free rate and y the convenience yield.

This relationship allows us to interpret the convenience yield as a dividend paid to the commodity owner. As we accept this interpretation we can use for the commodity option pricing a formula derived by R. Merton which is used for dividend-paying stocks[12].

Unfortunately this does not hold, due to an obvious reason (non-storability), for power options. Eydeland and Geman (1998) had explained the impossibility of usage of above mentioned Merton's formula and summarized the difference between "usual" commodity and power in following three points:

- Convenience yield is defined as difference between return from owning the commodity for delivery and the cost of storage. Because of impossibility of storing power, these quantities cannot be specified.
- The non-storability causes breakdown of above mentioned relationship between spot and future prices on storable commodities or stocks.
- Using the spot price evolution models for pricing power options is not optimal, since hedges involving the underlying asset cannot be implemented, because they require buying and holding power for a certain period of time.

If we summarize above mentioned facts we find out that the convenience yield makes commodity option pricing more complicated. Nevertheless these obstacles were already solved and convenient pricing models were developed. Next problem comes with impossibility of storage of power which makes the use of commodity option pricing models inappropriate.

[12] For details about the model see Merton (1973)

1.3.3 Option usage in electricity sector

Generally most of reasons for trading options in power sector are the same as we have mentioned them in sub-chapter 1.1.5, here we will briefly introduce power option specifics and discuss some of them in more details.

As in the general case, also in power market, the main reason why to enter derivative market is an effort to minimize risk connected with energy trading. Generally we can find five groups of risk in power trading[13] – market (or price) risk, volumes of sales risk, credit/default risk, operational risk and political risk. Of course not all of them can be minimized using options and so will only be interested in market, volumes of sales and operational risk.

Problems and their solutions connected with market risk are quite obvious. Energy traders prefer paying small amount of money in form of premium for call option and having guaranteed prices to unsure situation and possible high amounts paid to power producers in time of excessive demand. The same relationship holds than between power traders and consumers.

Risk connected with volumes of sales is carried by power producers – power plants. Possible stop in production (due to insufficient demand) and following restart of power plant is operation which is too expensive to producers can afford it. The solution is quite simple – they sell options with slightly smaller than expected expiration price which will ensure them future demand.

By operational risk is meant the risk connected with possible forced stop in production (different than insufficient demand), e.g. lack of fossil or nuclear fuel in power plant. This uncertainty can be again simply solved by long call option which ensures sufficient delivery in any future time.

Other reasons such as speculating or taking advantage of financial leverage are the same as already mentioned and so do not require any further discussion.

[13]Division adopted from Energy Information Administration, U.S. Department of Energy (2002)

2. Option pricing methods

In the second chapter of this work we are going to deal with option pricing itself. We will mention all relevant measurable variables which influence price of an option and are taken into account in most pricing models. In the second part we will introduce criteria which led to choice of examined models and briefly present assumptions, theoretical framework and derivation of single chosen models and methods.

2.1 Factors affecting option price

According to most of relevant books and articles (e.g. Hull (2007)) and also pricing models we are going to examine, there are six variables that somehow influence price of an option:

1. The current price of underlying asset (in our case futures contract)
2. The strike price
3. The volatility of underlying asset price
4. The risk-free interest rate
5. The dividends and other payments expected during the life of the option
6. The time to expiration

Each of these variables has different direction and intensity in which influences the price of option. Direction is usually known and we summarize it in following table (will be discussed in more details later):

Table 2: Factors affecting option price

Variable	European call	European put
Price of futures contract	+	-
Strike price	-	+
Volatility	+	+
Risk-free rate	+	-
Dividends expected	-	+
Time to expiration	**?(+)**	**?(+)**

+ **means that increase in variable causes the option price to rise**
- **means that increase in variable causes the option price to fall**
? direction of influence on the option price is unknown or uncertain

Source: Hull (2007)

The intensity of each variable is then generally unknown and depends on many other factors[14]. It can also change with another variable: For example change in underlying price influences option premium much less one year than one week before expiration when almost whole change is reflected in the option premium. The fact that different models give each variable different importance or sometimes even omit some of them can serve us as an evidence of unclear opinion on this problem.

Price of futures contract and strike price

First two variables are connected through definition of option payoff and so can be described together. When the option is exercised, the payoff (for call option) will be the difference between futures and strike price: $F_t - K$. The higher payoff, the higher price of option and so increase in futures price makes price of the option to rise, on the other hand increase in strike price makes the option less valuable.

In the case of put option the payoff is defined reversely (as $K - F_t$) and so the impact of increase in both variables is inverse as well.

Volatility

Next variable, volatility, is closely connected with the previous one – futures price. This variable is included into pricing model as a measure of risk or uncertainty in underlying asset price development. Volatility is usually defined as standard deviation of the return provided by one unit of underlying asset in one year. For a practical purposes there are two ways how to compute the standard deviation (volatility) [15] – first is on the base of historical data, in second case one can use a pricing model and current data:

1. **Implied volatility approach**

   Name of first approach comes from the fact that it expresses volatility of the underlying asset price which is implicit in the market price of the option according to appropriate model. If we assume that prices determined by the model reflect the real price development we can use current prices available at the exchange or option market for expressing the volatility. As we know all other variables (time to expiration, underlying and strike price, interest rate and dividends if we consider them) we can simply substitute for them and express the last unknown - volatility.

[14] Nevertheless there exist works dealing with this topic. For interested readers we can recommend Reynaerts and Vanmaele (2003), Fu and Hu (1995) or Timsina (2007)
[15] According to Kotzé (2005)

Unfortunately in most cases simple expressing is not possible due to complicated formulas used for option pricing, in this case couple of approximation formulas are available[16].

As the biggest advantage of implied volatility we can mention its clear definition and "theoretical purity". The main disadvantage of implied volatility approach we should point out, is the fact that it assumes absolute correctness of the model predictions, which is at least a questionable assumption. Since we are trying to examine models accuracy, this assumption disables us to use this approach.

2. Historical approach

The historical volatility is computed based on historical data. In most cases we have available spot prices of underlying asset for a given period from which we are able to compute standard deviation which is usually taken as a measure of volatility. The historical volatility estimate is thus given by:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(u_i - \bar{u})^2}{n-1}} \tag{15}$$

where

$$u_i = \ln(F_i) - \ln(F_{i-1}) = \ln\left(\frac{F_i}{F_{i-1}}\right)^{17} \tag{16}$$

and $\bar{u}$ is mean defined as

$$\bar{u} = \frac{\sum_{j=1}^{n} u_j}{n} \tag{17}$$

To enable comparison of volatility between periods with various length, it is usually expressed in annual terms. This is done by multiplying standard deviation by factor h which expresses the number of used periods in one year, i.e. if we use daily data $h = 356$ (or more frequently 252 which is an approximate number of trading days), $h = 52$ if weekly data and $h = 12$ if monthly ones are used. The annual volatility is then:

[16] As this is not goal of this work we only state references to literature handling with this topic. See for example Dumas, Fleming and Whaley (1998) or Chance (1996)

[17] Follows from log-normal distribution of underlying prices, for details see Kotzé (2005)

$$\sigma_{annual} = \sigma * \sqrt{h} \tag{18}$$

The biggest problem we have to face when using historical volatility is the ambiguity of measuring quantities. It is not clear which should be the length of time period in which we measure volatility, frequency in which we measure (hour, day, week) and the price we use (opening, closing, average). Generally it can be said that there is no one correct answer – longer time period means more data which should make the calculation more precise, on the other hand there is a higher probability that we include some exceptional occasion which may devaluate our results, the threat of outdated estimate also has to be taken into account, etc. Nevertheless common combination in most of works dealing with volatility estimation is daily returns and the period between one month and one year[18]. Next argument for historical volatility critics is the fact that value obtained by this way can be outdated (depends on whether the volatility changes from period to period or is stable for long time). On the other hand the biggest plus of historical volatility is the simplicity of its estimation and no dependence on pricing model correctness.

Influence of volatility on option price is not straightforward. Usually higher risk means worse situation for owner of appropriate asset, not in the case of options. Holder of call option profits from increases of underlying asset price, on the other side the risk of decreasing price is limited since the maximum the holder can lose is price of the option. To sum it up holders can with higher volatility achieve more than lose, it means that the risk connected with higher volatility is carried by the option issuer, who logically requires higher option premium[19]. A similar argument holds also for put options and so higher volatility means higher option value regardless of option type.

Risk-free rate

As mentioned in Hull (2007) there are two relevant consequences of changes in risk-free rate. Increase in this rate causes the expected return required by investors to rise. Second consequence is the decrease in present value of any future cash flow. Result of impact of both factors together causes call option value to increase, put option value to decrease.

[18] According to Bajerová (2010)

[19] Discussed in more details in Natenberg (1994)

It is also important to point here out that we explore impact of each variable ceteris paribus, i.e. other factors being equal. It is particularly important in case of risk-free rate because its changes usually influence other variables as well – especially underlying asset price.

Dividends (and other payments) expected

Due to the fact that in this work we are handling with futures options and all pricing models used assume no dividends or other payments we will not discuss this topic in details. We can only say that amount of money paid on dividends is negatively correlated with underlying asset price (e.g. price of stock declines after dividend payment) which then influences the option price as discussed in first part of this chapter (i.e. the higher dividends, the lower underlying asset price which means decrease in call resp. increase in put option premium).

Time to expiration

Generally the impact of longer time to expiration on option price is positive, i.e. the more time to expiration, the higher price. This is cause by higher probability of potential change in underlying asset price that would cause higher profit for option holder. This argument again, as in the case of volatility, holds for put as well as call option.

The question marks in table from Hull (2007) are caused by the fact that Hull takes into account possible dividends paid in longer time period which make the effect uncertain:

"Consider two European call options on a stock: one with an expiration date in 1 month, the other with an expiration date in 2 months. Suppose that a very large dividend is expected in 6 weeks. The dividend will cause the stock price to decline, so that the short-life option could be worth more than the long-life option."

As already mentioned in our thesis we will not take dividends into account and so we can adopt the positive influence argument.

2.2 Most appropriate model criteria

Choice of the model is the key issue for all financial derivatives traders. Today not only accuracy of appropriate model plays role, as exchange trading becomes a profession where decisions have to be made in several minutes, also ease of usage is a factor that should be taken into account.

For evaluation of models in this thesis we decided to use three criteria which are mentioned in Pilipovic (2007):

- the ability to capture market reality
- ease of implementation
- ease of maintenance

To find the most accurate model one needs to consider particular characteristics of the market he is interested in and the commodity traded there. Types of options, level of deregulation of market or the fact that electricity cannot be stored. These are only some of factors that have to be considered when choosing the model. According to these criteria we have chosen three models or methods which will be tested on data from European Energy Exchange (EEX) in Leipzig, Germany.

The ease of maintenance and implementation are also important factors of good usage of the model. In reality these two criteria heavily depend on conditions trader has – software, funds for various tools or support of his trading group. In this thesis this will be mostly a subjective opinion of the author. Generally we can say that the more accurate the model is, the more complicated is its use and maintenance. Our goal is to find the best ratio between these two groups of criteria when accuracy of the model will have slightly more importance than remaining two factors.

2.3 Types of valuation methods

According to Gregor (2005) we can distinguish three groups of valuation methods using discrete or continuous approach:

- analytical methods (Black – Scholes model, Black model, Stochastic volatility model, Merton model, Jump diffusion model, etc.)
- numerical methods (binomial and trinomial model)
- simulation methods (Monte-Carlo and Quasi-Monte-Carlo simulation)

The analytical methods (or closed-form solutions) were the first massively used option pricing tools. Namely the Black-Scholes formula (Black and Scholes 1972) meant a huge revolution in option trading. Closed-form solution is generally a solution to differential equation which expresses the change in price of option according to factors affecting this price – mainly time and price of underlying asset. It means that after solving the differential equation all we need is to plug in all relevant variables and obtain the price of option. This procedure is easy, quick and also adequately precise.

Problems of analytical methods come with more complicated options. When used for valuation of exotic options or the ones written on underlying asset with high number of price drivers, the solution of the differential equation becomes very difficult or even impossible. In this case significant simplifications and approximations are necessary which of course lowers the accuracy of the method. For our study we choose the Black model which is an adjusted version of Black-Scholes designated for futures contracts as underlying. The more complicated models, which eliminate some of the restrictive assumption (such as Jump-diffusion or Stochastic volatility model), were excluded because of problematic estimating of additional parameters (e.g. jump component) which very often leads to bad results in papers handling with pricing models testing[20].

Numerical approach (or tree method) is also sometimes called a discrete version of Black-Scholes model to which the tree converges as number of steps goes to infinity. Generally the numerical methods consist in building a tree for all possible prices of underlying asset when upward and downward (or neutral in trinomial model) steps have assigned probability and growth or drop coefficients. The present value of weighted sum from the final step, where weights are corresponding probabilities, is the searched option price. Generally if we compare any numerical model with Black or Black-Scholes it should be less accurate (as it is only its discrete approximation), on the other hand this shortcoming should be, at least partially, compensated by easier implementation and maintenance. Due to the fact that binomial model has easier use and converges to the trinomial one relatively quickly[21] we decided to use it for the empirical part.

Simulation methods became very popular in last few decades when a huge progress in computer technology occurred. Monte Carlo simulation was not originally developed for option pricing but as a universal tool for solution of problems with many degrees of freedom (or dimensions) in physics, mathematics or even biology. With new, more complicated (exotic) options and financial derivatives as a whole, this method found its place also in financial markets. The basic idea of Monte Carlo method is to find a mean value which is a result of random process. We create a computer model with appropriate parameters and sufficient number of simulations is performed. Obtained data can be than treated by using common statistical tools. Difference between Monte Carlo and

[20] For example Filáček (1998) or Chrobok (2010)
[21] See Rubinstein (2000)

Quasi Monte Carlo method is in use of quasi-random[22] (Halton, Faure, Wozniakowski, ...) instead of pseudorandom sequences[23]. From the simulation methods we chose simple Monte Carlo preferring much simpler use to slightly more precise results.

2.4 Option pricing models theory

As mentioned above in this chapter three chosen models or methods will be presented. We introduce assumptions which models are based on, the basic idea of each model and its derivation or a hint providing the most important steps.

2.4.1 Black model

After publication of the most famous option pricing model, Black-Scholes, several improvements adjusting the model for different kinds of underlying or reducing number of restrictive assumptions were presented. One of these is also the Black model which is designed for futures option pricing.

In the literature we can find several ways how the basic differential equation is formed. In this work will be described the original approach of F. Black presented in "*The Pricing of commodity contracts"* (1975).

Assumptions

Due to the fact that financial or option markets are extraordinarily complicated, the author was forced to accept several simplifying assumptions. We state them in the original form:

1) The futures price follows a random walk (or Brownian motion[24]) in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible futures prices at the end of any finite interval is distributed log-normally, with constant variance rate σ^2.
2) All of the parameters of the capital asset pricing model (CAPM), including the expected return on the market, the variance of the return on the market, and the short-term interest rate, are constant through time.
3) Taxes and transaction costs are zero.

[22] For more details see Gregor (2005)
[23] Sequence which seems to be random but is generated by deterministic algorithm
[24] Which is described by Wiener process, for more information see Csôrgô (1979)

Apart from these there are some more assumptions which are not explicitly mentioned in Black's paper but also necessary for deriving the valuation formula. Most of them come from the similarity with original Black-Scholes model:

4) The option is European
5) There are no penalties to short selling
6) Both options and underlying futures contracts are perfectly divisible and traded on efficient markets
7) It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate
8) There are no dividends or other payments connected to underlying asset

When all these conditions hold the price of option will depend only on price of underlying futures contract, time left to expiration and several known constants mentioned in assumptions.

Derivation of Black formula

For derivation itself we need to construct a hedged portfolio consisting of long position in futures contract and short position in option. Its value will not depend on price of underlying futures contract but on time and abovse mentioned constants only.

We denote $C(F,t)$ the value of option as a function of time and price of underlying asset, than number of options needed to hedge one stock is:

$$\frac{\partial C(F,t)}{\partial F} \tag{19}$$

Now we can construct risk-neutral portfolio with value Π:

$$\Pi = C - F\frac{\partial C(F,t)}{\partial F} \tag{20}$$

By differentiating this equation we obtain change in value of the portfolio in a short interval:

$$dC - dF\frac{\partial C(F,t)}{\partial F} \tag{21}$$

Now we use Ito's lemma[25] to decompose the expression $dC = C(F+dF,t+dt) - C(F,t)$:

$$dC = \frac{\partial C(F,t)}{\partial F}dF + \frac{1}{2}\frac{\partial^2 C(F,t)}{\partial F^2}\sigma^2 F^2 dt + \frac{\partial C(F,t)}{\partial t}dt \tag{22}$$

[25] Based on Taylor series, for the general form of Ito's lemma se Pilipovic (2007)

If we plug this expression back into equation (21) and use the fact that the return on hedge portfolio must be at the risk-free rate *r*, we obtain:

$$\frac{\partial C(F,t)}{\partial F}dF + \frac{1}{2}\frac{\partial^2 C(F,t)}{\partial F^2}\sigma^2 F^2 dt + \frac{\partial C(F,t)}{\partial t}dt - \frac{\partial C(F,t)}{\partial F}dF = \left(C - F\frac{\partial C(F,t)}{\partial F}\right) r\Delta t \tag{23}$$

After some computation and rearrangements we obtain differential equation for Black model:

$$\frac{\partial C(F,t)}{\partial t} + \frac{1}{2}\frac{\partial^2 C(F,t)}{\partial F^2}\sigma^2 F^2 - rC = 0 \tag{24}$$

This equation is almost identical with the one from original Black-Scholes model except for the term $rF\frac{\partial C(F,t)}{\partial F}$. This difference is caused by no need of borrowing of money from the bank for our hedge to the option price because the futures contract is an agreement that carries no cost of financing.

If we solve this differential equation given the boundary constraint[26]:

$$C(F,t) = max\{0, F-K\}, \tag{25}$$

we obtain the famous Black formula for European call option:

$$C_{Black}(F,K,t) = Fe^{-r(T-t)}\aleph(d_1) - Ke^{-r(T-t)}\aleph(d_2) = e^{-r(T-t)}[F\aleph(d_1) - K\aleph(d_2)], \tag{26}$$

where:

$$d_1 = \frac{ln\frac{F}{K} + \left(\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \tag{27}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = \frac{ln\frac{F}{K} + \left(-\frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}} \tag{28}$$

[26] The solution itself is beyond the scope of this text as it uses a Fourier transformation and various substitutions. For detail solution of differential equation see for example Norstad (2011)

and $\aleph(x)$ is the normal distribution function, i.e.:

$$\aleph(x) = \int_{-\infty}^{x} \frac{e^{-\frac{y^2}{2}}}{\sqrt{2\pi}} dy \tag{29}$$

Now the model provides us with quite simple formula where all variables are known (time to expiration, strike and underlying asset price) or can be estimated from historical data (volatility).

From the put-call we can easily derive corresponding formula for European put option. It has to hold:

$$C + Ke^{-r(T-t)} = P + Fe^{-r(T-t)} \tag{30}$$

If we substitute for price of call option, C, and express price of put, P, we obtain:

$$\begin{aligned} P_{Black}(F,K,t) &= \\ Fe^{-r(T-t)}\aleph(d_1) - Ke^{-r(T-t)}\aleph(d_2) &+ Ke^{-r(T-t)} - Fe^{-r(T-t)} \\ &= Fe^{-r(T-t)}(\aleph(d_1) - 1) - Ke^{-r(T-t)}(\aleph(d_2) - 1) \end{aligned} \tag{31}$$

As for normal distribution function holds:

$$\aleph(x) = 1 - \aleph(-x), \tag{32}$$

we can write:

$$P_{Black}(F,K,t) = e^{-r(T-t)}(K\aleph(-d_2) - F\aleph(-d_1)) \tag{33}$$

2.4.2 Binomial model

Six years after the boom in option trading caused by publication of Black-Scholes model another approach to option pricing was presented by Cox, Ross and Rubinstein (1979). They avoid the need of solution of any differential equation and use quite simple numerical approach instead. This method consists in building a tree of all possible values of underlying asset prices and corresponding option prices. Each change in price (increase or decrease) has defined coefficient and probability, with which it occurs, based on historical data. Binomial tree was a basis for trinomial tree, which extended the original model by allowing for a “neutral” step.

In this thesis we will present the original model which is adjusted (according to Hull (2007)) for futures option pricing. This adjustment consists in different way of additional parameters estimation as will be presented in empirical part of the thesis.

Assumptions

As mentioned above the binomial model "pays" for its simplicity by quite high number of more or less restrictive assumptions which have to be fulfilled:

- There exist a perfect market for underlying asset, i.e. the agents are rational and cannot influence price, there are no transaction costs, taxes or any other costs connected with trading, market is liquid
- There are no financial flows connected with underlying asset (e.g. dividends, storage costs, ...)
- Risk-free interest rate is constant in time, one can borrow or lend any amount of money for this rate
- Market is effective, arbitrage is not possible
- Price of underlying asset follows a binomial process
- Both underlying asset and options are perfectly divisible

Derivation

When trying to build the binomial tree we should start with a one-period case. There are only two possibilities of underlying futures contract development – either its price increase by D percent (with probability q) or decrease by U percent (with probability (1-q)). Now we define growth (u) and drop (d) coefficients as:

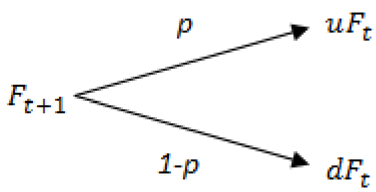
$$u = (1 + \frac{U}{100}) \tag{34}$$

and

$$d = (1 + \frac{D}{100}) \tag{35}$$

From the definition we know that price of underlying contract is, with corresponding probabilities, one of two possible values:

Figure 1: Binomial tree – one period case



Source: Author

Now we should focus on price of European option which is unknown in time *t*. Nevertheless in $t+1$ we know the price for both values of underlying contract, these are:

$$C_{t+1}^{u} = max\{0, uF_t - K\} \tag{36}$$

$$C_{t+1}^{d} = max\{0, dF_t - K\} \tag{37}$$

In order to obtain price in time *t* we use a similar approach as in Black model derivation. We create a hedged portfolio consisting of h-futures contracts and one short position in call option. Yield on this portfolio is riskless in *t+1*. Value of such a portfolio in time *t* is:

$$\Pi_t = hF_t - C_t \tag{38}$$

Value of the portfolio in *t+1* depends on whether the price of futures contract rises:

$$\Pi_{t+1}^{u} = h(F_{t+1}^{u} - F_t) - C_{t+1}^{u} \tag{39}$$

or falls:

$$\Pi_{t+1}^{d} = h\left(F_{t+1}^{d} - F_t\right) - C_{t+1}^{d} \tag{40}$$

Based on the fact that our portfolio is riskless we are able to derive amount of futures contracts (*h*) needed. Riskless portfolio means that its value has to stay unchanged whether the price of underlying asset *F* rises or falls, i.e.:

$$hF_{t+1}^{u} - C_{t+1}^{u} = hF_{t+1}^{d} - C_{t+1}^{d} \tag{41}$$

From this equation we can easily obtain:

$$h = \frac{C_{t+1}^u - C_{t+1}^d}{F_{t+1}^u - F_{t+1}^d} \tag{42}$$

h is a important option characteristic describing sensitivity of price of option on changes of underlying asset price, it is called hedge ratio.

From the no-arbitrage assumption we also know that the initial investment has to be the same as the present value of Π_{t+1}^uresp. Π_{t+1}^d:

$$\Pi_t = \frac{hF_{t+1}^u - C_{t+1}^u}{(1+r)} \tag{43}$$

$$\Pi_t = \frac{hF_{t+1}^d - C_{t+1}^d}{(1+r)} \tag{44}$$

After substituting for Π_t from (38) and h from (42) into any of two equations and expressing C_t we obtain:

$$C_t = \frac{1}{1+r}\left[\frac{(1+r)-d}{u-d}C_{t+1}^u + \frac{u-(1+r)}{u-d}C_{t+1}^d\right] \tag{45}$$

If we set

$$p = \frac{1-d}{u-d} \tag{46}$$

and

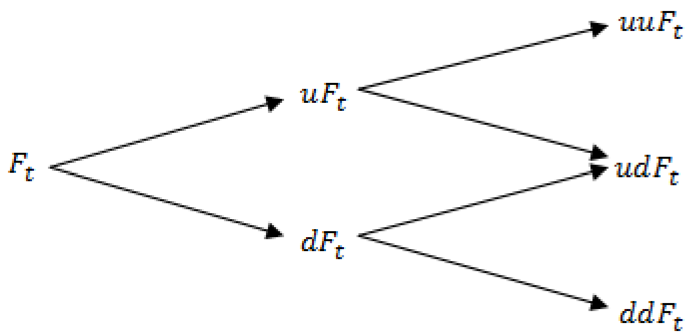
$$1-p = \frac{u-1}{u-d}, \tag{47}$$

We can write the pricing formula for one-period model:

$$C_t = \frac{pC_{t+1}^u + (1-p)C_{t+1}^d}{1+r} \tag{48}$$

Now we extend previous results to two-period model. We simply add second period when price of underlying can rise or fall with the same probabilities as in one period case:

Figure 2: Binomial tree – two period case



Source: Author

Now we express value of the option in time *t+1* in terms of possible prices in *t+2*:

$$C_{t+1}^{u} = \frac{pC_{t+2}^{uu} + (1-p)C_{t+2}^{ud}}{1+r} \tag{49}$$

resp.

$$C_{t+1}^{d} = \frac{pC_{t+2}^{du} + (1-p)C_{t+2}^{dd}}{1+r} \tag{50}$$

The formula for overall price of option which we are interested in remains the same as derived above, i.e.:

$$C_t = \frac{pC_{t+1}^{u} + (1-p)C_{t+1}^{d}}{1+r} \tag{51}$$

So price in time *t* in terms of *t+2* is:

$$C_t = \frac{p^2C_{t+2}^{uu} + 2p(1-p)C_{t+2}^{ud} + (1-p)^2C_{t+2}^{dd}}{(1+r)^2} \tag{52}$$

After substituting for C_{t+2} by known expiration values for call option:

$$C_t = \frac{1}{(1+r)^2}(p^2max\{0, u^2F_t - K\} + 2p(1-p)max\{0, udF_t - K\} + (1-p)^2max\{0, d^2F_t - K\}) \tag{53}$$

Now we can see that probability for each state follows a binomial distribution a so the equation can be simplified by using binomial probability function $\binom{k}{j}p^j(1-p)^{k-j}$, where *k* is a number of periods in the model and j number of rises of price of underlying asset:

$$C_t = \frac{1}{(1+r)^2}\sum_{j=0}^{2}\binom{2}{j}p^j(1-p)^{2-j}max\{0, u^j d^{2-j}F_t - K\} \tag{54}$$

In the same way we can go on and extend our model to any number of periods. All that left us to do to obtain a universal pricing formula for binomial tree with *k* period is to generalise the previous model as follows:

$$C_t = \frac{1}{(1+r)^k}\sum_{j=0}^{k}\binom{k}{j}p^j(1-p)^{k-j}max\{0, u^j d^{k-j}F_t - K\} \tag{55}$$

Although this formula is valid it is in quite impractical form – especially due to *max* function contained. We can see that for all $j = 1,2, \dots, k$ for which:

$$K > u^j d^{k-j}F_t, \tag{56}$$

value of whole addend in the sum is zero and so it can be omitted. We need to find the lowest $a \in N$ that for all $j \geq a$ holds:

$$u^a d^{k-a}F_t > K \tag{57}$$

After taking a logarithm of both sides and expressing *a* we obtain:

$$a > \frac{\ln\left(\frac{K}{d^k F_t}\right)}{\ln\left(\frac{u}{d}\right)} \tag{58}$$

As mentioned above now for all $j \geq a$ holds:

$$max\{0, u^j d^{k-j}F_t - K\} = u^j d^{k-j}F_t - K \tag{59}$$

and we can rewrite the formula into more suitable form:

$$C_t = \frac{1}{(1+r)^k} \sum_{j=a}^{k} \binom{k}{j} p^j (1-p)^{k-j} (u^j d^{k-j} F_t - K), \tag{60}$$

where a satisfies condition (58).

The procedure of obtaining formula for put option is similar to the one we have used in Black model. After substituting for call price in put-call parity and expressing put price we obtain:

$$P_t = \frac{1}{(1+r)^k} \sum_{j=a}^{k} \binom{k}{j} p^j (1-p)^{k-j} (K - u^j d^{k-j} F_t) \tag{61}$$

where a satisfies conditions:

$$a > \frac{\ln\left(\frac{K}{d^k F_t}\right)}{\ln\left(\frac{u}{d}\right)} \tag{62}$$

The fact that this condition is the same as in the case of call option follows from complementarity[27] of call and put option.

2.4.3 Monte Carlo simulation

Despite the fact that Monte Carlo simulation is primarily used for valuation of even more complicated options (such as options where the payoff is dependent on a basket of underlying assets[28]), we decided to include it into our group of methods that will be examined. This is especially due to above mentioned fact that no model is adequately able to price electricity as underlying asset, we think that pricing by simulation should solve this problem.

Even though it can seem that Monte Carlo simulation is, due to need of modern computer technology, an issue of last two or three decades, it is not so. This tool for solving various problems across the whole world of science was used already in 1940s. As a tool for option pricing was then Monte Carlo introduced by Phelim Boyle in 1977. As already mentioned the simulation methods differentiate themselves from other option pricing techniques in the way the potential future underlying prices are

[27] If call is in the money, put is out of money and vice versa
[28] See Bolia and Juneja (2005)

generated. Monte Carlo is based on performing large number (usually thousands) of possible paths of underlying price development. The price of option from the final step is than discounted as in the case of binomial tree and its mean value is calculated.

Assumptions

In the case of Monte Carlo most of assumptions are identical with that mentioned in Black model:

- Price of underlying asset follows a Wiener process and is log-normally distributed with known and constant variance rate
- Risk-free rate is constant over time
- There are no dividends or other payments connected with underlying asset

Pricing process

For more detailed explanation we follow Kaplan (2008) and divide the pricing process into two phases:

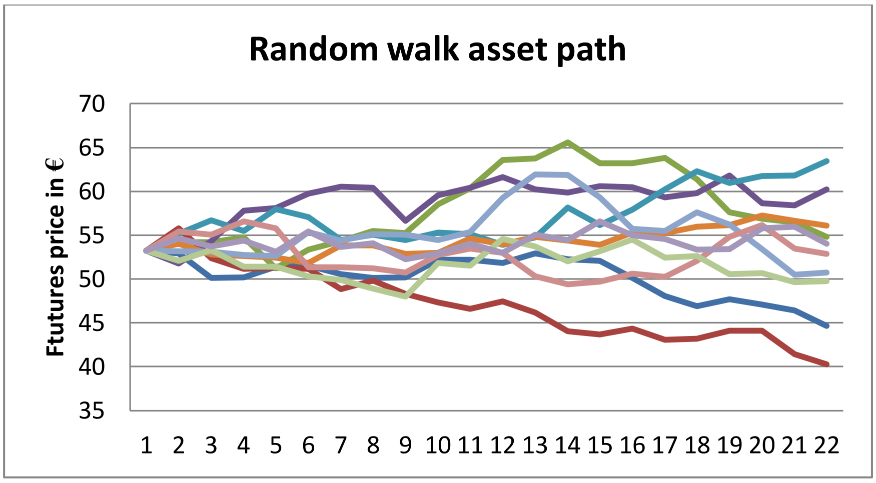
1. **Simulating asset path** – First step in using Monte Carlo method is generating the above mentioned large number of possible paths of underlying price future development. This is done by selecting appropriate stochastic equation according to which the underlying price is expected to behave. As we assume a log-normal distribution of underlying prices, we will simulate them using the Weiner process, i.e. the one-period-later price (F_{t+1}) is simulated as:

$$F_{t+1} = F_t * \exp\left(\left(r - \frac{\sigma^2}{2}\right) + \sigma\varepsilon\right) \quad (63)$$

where F_t is price of futures contract today
r is the expected return (or risk-free rate)
σ is volatility
ε is a random number sampled from a standard normal distribution, i.e. $\varepsilon \in N(0,1)$

Example of ten paths with initial price €53.25, $\sigma = 15\%$ and $r = 1{,}5\%$ (both in annual terms) can be found in following figure:

Figure 3: Random walk asset path



Source: Author

2. **Pricing the option** – Once the asset price paths have been simulated they are used to price the option in similar way as in binomial model. We define payoff function for price of underlying asset after k simulated periods (F_{t+k}) European call:

$$C_{t+k} = max\{0, F_{t+k} - K\} \quad (64)$$

resp. put:

$$P_{t+k} = max\{0, K - F_{t+k}\} \quad (65)$$

After calculating the payoff for each simulation we find the mean value which is the searched price of the option in time $t + k$. All that left to do now is to discount the option price to present time in usual way.

3. Empirical model testing

In the last third chapter of our thesis we move from theory to option pricing itself. In the first part we will describe dataset which will be used for option pricing and estimate all necessary variables. In the second part we will compare prices obtained by application of data to the models with the market ones, evaluate models precision and choose the best model for electricity futures option pricing.

3.1 Data description

As already mentioned, our three chosen models will be tested on prices of Phelix Options which are traded on European Energy Exchange in Leipzig, Germany. Phelix Options are European-style options which lead to opening of the corresponding Phelix Futures position at the respective exercise price upon exercising of a call or put option. Options can be written on the respective next five Phelix Baseload Month Futures, next six Phelix Baseload Quarter Futures and next three or four Phelix Baseload Year Futures. The delivery rate amounts to 1 MW per contract[29].

For our thesis we decided to use Phelix Baseload Year Futures. This choice was made based on several reasons - first we tried to avoid problems with seasonality, which influences all other options (those on month or quartet futures), second Year Futures are the most liquid ones among all others, which provides us with most information from the market and finally third reason, the possibility of various expiry dates (namely four – end of March, June, September and mid of December), which is only possible at Phelix Year Options. The last fact helps to make the dataset more diverse which should make our results more reliable. The examined options are both call and put options on futures with delivery period in 2012.

As the examined period we have chosen the whole year 2011, i.e. from 3.1.2011 to 30.12.2011. This decision was made in order to ensure sufficient number of data and also cover option prices from different parts of the year.

[29] From: http://www.eex.com/en/

3.1.1 Data restriction

Not all observations from above mentioned period will be really used in testing of single models. In order to prevent devaluation of our results we decided to settle three criteria which should filter the “problematic” data:

1. The expiration price differs from the spot one maximally about 10 %, i.e. $\frac{F(t)}{K} \in (0.9\,;1.1)$
2. Time to expiration is at least 30 days
3. Price of the option is at least €0.1

First two criteria[30] are accepted in order to prevent high differences between model and market price caused by border values. In the cases when option is deeply in-the-money or out-of-money or is close to its expiration, the approximation which is used in the pricing models usually fails, which could seriously affect and devaluate our results.

Due to the fact, that the percentage difference is going to be used for model evaluation, reason for the third criterion is quite obvious. If we include smaller prices, a difference between market and model price, which is insignificant for practical purposes, would cause percentage differences even in thousands of percents.

3.1.2 Dataset characteristics

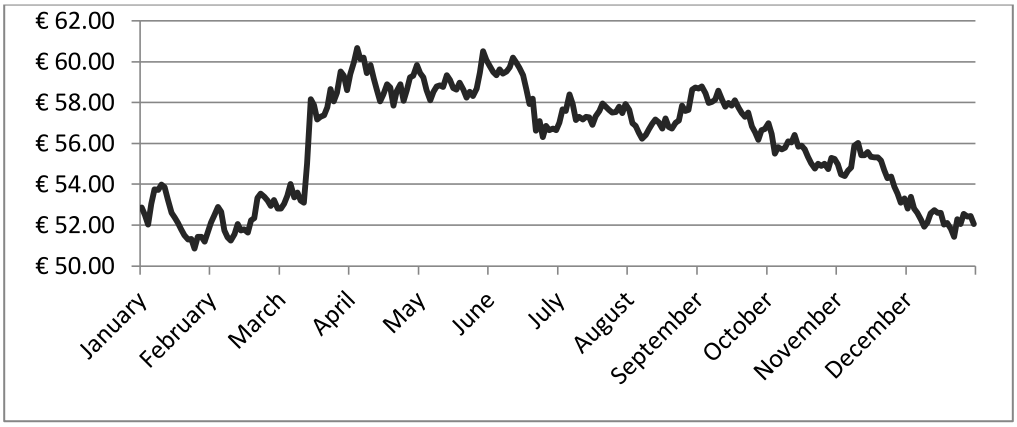
The resulting dataset, after elimination of data violating restrictive criteria, contains 2643 observations, put and call options approximately half-and-half (1328 call and 1315 put options).

Underlying price

In the case of our dataset the underlying price is price of Phelix Baseload Year Futures 2012, i.e. futures of which delivery period is the whole year 2012.

[30] Inspired by Filáček (1998)

Figure 4: Underlying price development



Source: Author

As we can see in graph the price development of Phelix Year Futures in 2011 was quite uninteresting, with the only one exception in March when the price increased about more than 10 % (more than €5) within three days.

Table 3: Underlying price characteristics

	Average price	Maximum	Minimum	Median	SD
Phelix Baseload Year Futures 2012	**56.03**	**60.68**	**50.84**	**56.72**	**2.68**

Source: Author

Strike price

Strike price is the first variable which is restricted by one of our three criteria. Nevertheless the criterion, which eliminates all observations for which $\frac{F(t)}{K} \notin (0.9\,;1.1)$, did not exclude any of them as options with strike price that differ from the underlying one by more than 10 % were not actually traded.

The strike price moves, according to underlying one, between €50 and €65, the most frequent one was €54:

Figure 5: Strike price

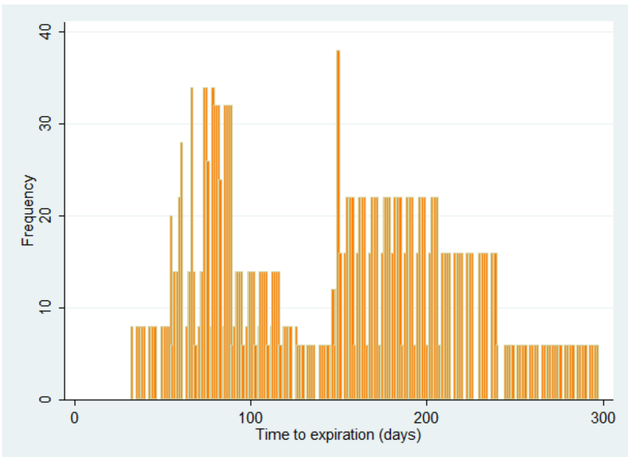
Source: Author

An important fact that cannot be seen from the histogram is prevalence of options with strike price higher than the current underlying one. This fact is understandable as general expectation about power price development is its growth.

Time to expiration

As the examined period is whole year 2011 and expiration dated differ from the end of March to the mid of December 2011, the spectrum of times to expiration is quite diverse. In concrete numbers it ranges from 33 days (due to restrictive criterion) to 297 days. As we can see from histogram attached below most of observations is situated between 70 and 230 days to expiration.

Figure 6: Time to expiration



Source: Author

More detailed information can be also found in following table:

Table 4: Time to expiration

	Average	Maximum	Minimum	Median	SD
Time to expiration (days)	151.86	297	33	159	66.81

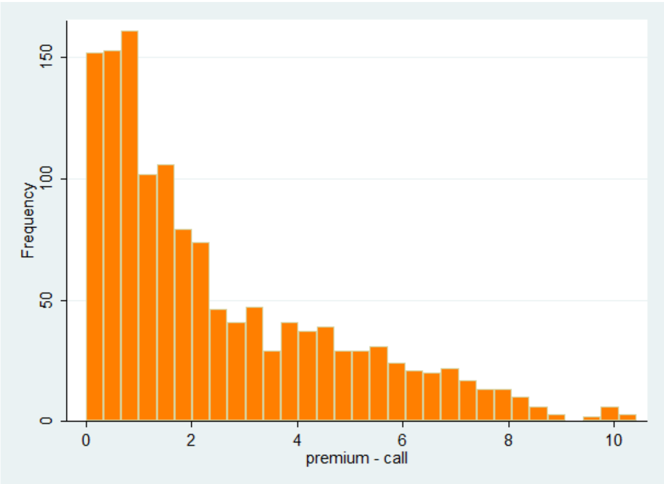
Source: Author

Option premium

Option premium, or also option price, is the most important characteristics of every observation as it is going to be compared with the estimated price and based on this comparison the model evaluation will be done. This is also the only one characteristic which significantly differs among put and call options and so we will distinguish between these two option types when describing it.

The lower bound is from above mentioned reasons created by restrictive criterion and is for both types €0.1, the maximal values are than in case of both types above €10.

Figure 7: Option premium - call

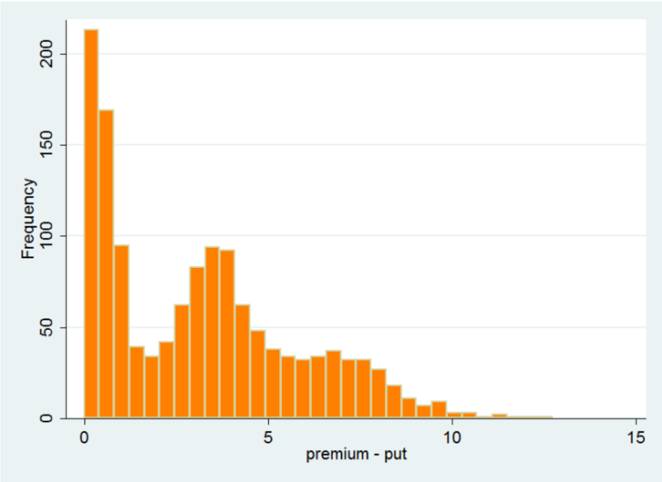


Source: Author

From attached histograms we can see that option premiums for put options are generally higher (which is also confirmed by average value in table 5). This is mainly caused by the fact described in the part about strike price (the most options have strike price higher than current underlying one)[31].

[31] As we already know from 2.1 higher strike means lower premium for call and higher premium for put options.

Figure 8: Option premium - put



Source: Author

Overview of option premium characteristics can be again found in following table:

Table 5: Data set characteristics

	Average	Maximum	Minimum	Median	SD
Option premium - overall	2.84	12.75	0.1	2.18	2.48
Option premium – call (€)	2.65	10.43	0.1	1.83	2.26
Option premium – put (€)	3.29	12.75	0.1	3.13	2.61

Source: Author

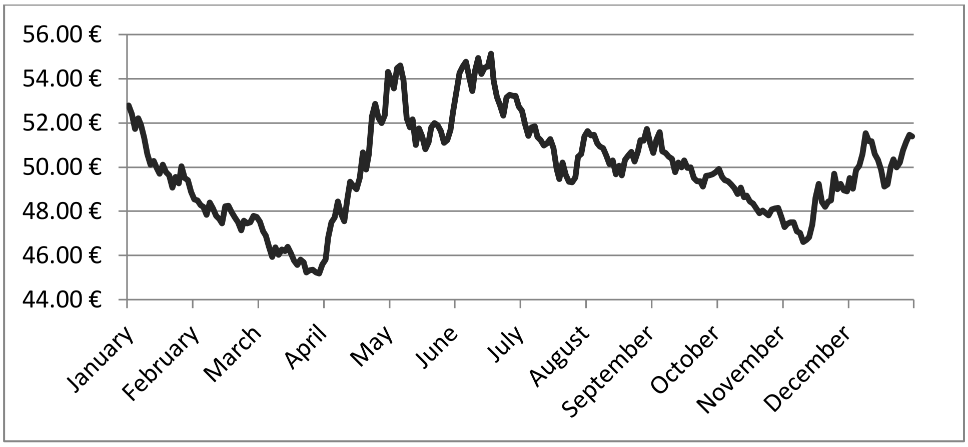
3.1.3 Historical dataset description

Apart from the data that will be used directly for option pricing we will need a set of historical underlying asset prices as well. It will serve us for estimation of historical volatility and will be used for testing of models assumptions as well. For this purpose will be used Phelix Baseload Year Futures 2011 (i.e. futures with year 2011 as delivery period), again from whole year 2010.

In the examined period the price development of Phelix Futures was quite volatile and offered both – periods of constantly declining price (especially first third of the year) but also periods when price rose sharply about almost €10 within one month (in April). Such a behaviour is a huge advantage over other underlying assets (namely stocks), which usually behave according to some trend and in the examined period move in one direction only (rise or fall all the time). Such a development can lead to violation of some of pricing models assumptions (most often log-normal distribution of underlying price).

The overall price development can be seen in following figure:

Figure 9: Underlying futures price development



Source: Author

We also mention the most important characteristics in table 6:

Table 6: Historical dataset characteristics

	Average price	Maximum	Minimum	Median	SD
Phelix Baseload Year Futures 2011	**49.88**	**55.13**	**45.19**	**49.87**	**2.23**

Source: Author

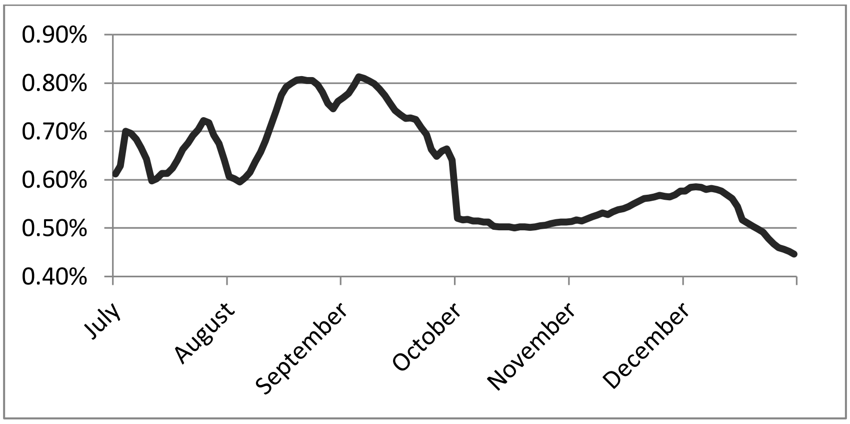
3.2 Risk-free interest rate

According to Mařík (2005) there are three basic possibilities that can be used, for practical purposes, as a proxy for risk-free interest rate – the short-dated government bonds, the inter-bank lending rate or triple AAA rated corporate bonds. In the financial markets usually first two proxies are used, corporate bonds only rarely. As we deal with options traded in Europe, resp. Germany our choice of risk-free interest rate proxies shrinks to three-month German government bonds and the Euro Interbank Offered Rate (EURIBOR). As in the examined period the development of both rates is very similar (i.e. moves between 0.5% and 1%) and not even literature gives a clear recommendation of one of them[32] we decided to use EURIBOR as the risk-free interest rate.

[32] For discussion of advantages and drawbacks of single proxies see Mařík (2005) or Officer and Bishop (2008)

EURIBOR is a daily rate based on the average interest rates at which Eurozone banks offer to lend unsecured funds to other banks in the interbank market. As all used models assume constant value of interest rate we will use an average of daily values in second half of year 2010[33]. In selection of period for average interest rate calculation a compromise between sufficient number of observations and its actuality was made.
In the examined period the interest rates were generally very low, the EURIBOR which we are interested in have not exceed 0,85%, the overall development in the above mentioned period can be seen in following figure:

Figure 10: EURIBOR development



Source: Author

Low interest rates in 2010 were caused mainly by expanding sovereign-debt crisis and by European Central Bank efforts to provide financial markets with additional liquidity to stabilize the situation. One of tools supporting such a policy is just setting of low interest rates. As the situation in South Europe countries (mainly Greece, Spain and Portugal) did not look to improve in near future data obtained from this period showed to be a good source for 2011 interest rate estimation.
The resulting average value of EURIBOR rates from second half of 2010 which will be used in all pricing models is then $\mathbf{0,616}$ %.

3.3 Volatility estimation

As already explained in 2.1 we are going to use historical approach for volatility estimation. We will follow the very same process as it is described in above mentioned chapter.

For estimation of volatility, which we are going to use in pricing models, we use daily returns from Phelix futures prices traded on EEX in the period between

[33] Data obtained from http://www.itistimed.com/

04.01.2010 and 30.12.2010, which results into a dataset containing 251 prices. If we follow the procedure mentioned above, the obtained daily volatility estimation will be:

$$\sigma = 0.0099 \quad (66)$$

In annual terms than:

$$\sigma_{annual} = 0.0099 * \sqrt{251} = \mathbf{15.68}\% \quad (67)$$

3.4 Model assumptions discussion

A famous statement says that "model is just as good as are the assumptions it proceeds from". In this chapter we are going to discuss these assumptions and assess, based on our dataset, the probability that they are fulfilled in reality. For obvious reasons we will focus on those ones that can be empirically tested, the rest of assumptions will be discussed only briefly.

3.4.1 Option and underlying market efficiency

One of the basic assumptions, if not the most important one, which is common for all pricing models introduced in this work is an assumption of efficient markets – both option and underlying one. As the only one possibility of testing option market efficiency is testing of hypothesis of equality between theoretical and market price, we will have to accept the same procedure as the authors of single models[34] and just assume this feature.

In the case of underlying (futures) market a test of weak efficiency is possible. However the market efficiency testing is not the main goal of this work, moreover it is beyond its scope and so we will only assume this feature as well[35].

3.4.2 Black model

First and probably the most important assumption (apart from market efficiency) of Black model is the assumption that price of underlying futures contract follows the Brownian motion which results in log-normal distribution of possible prices.

This assumption will be tested on the same dataset that was used for volatility analysis, i.e. futures prices from year 2010.

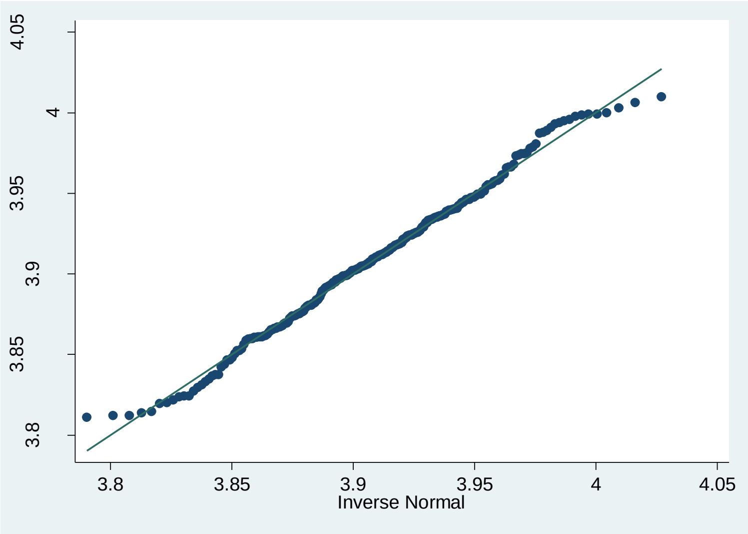
[34] e.g. Rubinstein (1985) or Black (1976)

[35] Readers interested in this topic are refered to Zhang, Sanning and Shaffer (2010) or Maberly (2006)

For testing itself we will use the fact that if variable *x* is log-normally distributed than *log(x)* is distributed normally[36]. In order to do so we create a natural logarithm of examined variable, futures price, and move to analyses.

Usually the first step in finding appropriate distribution of data is a graphic analysis. For this purpose we will use Q-Q plot with normal quantiles on the horizontal axis:

Figure 11: Q-Q plot for normality of data



Source: Author

According to Q-Q plot the normal distribution of our variable seems to be probable. In order to confirm our conjecture two tests of normality were performed. In both cases the null hypothesis, which says that data are normally distributed, is tested against the alternative. Neither the first, Skewness/Kurtosis (S/K) test:

Figure 12: Skewness/Kurtosis test for normality

Skewness/Kurtosis tests for Normality

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	joint adj chi2(2)	joint Prob>chi2
logF	252	0.9959	0.1717	1.88	0.3900

Source: Author

[36] For prove of this statement see e.g. Limpert, Stahel and Abbt (2001)

nor the second one, Shapiro-Wilk (S-W):

Figure 13: Shapiro-Wilk test for normality

Shapiro-Wilk W test for normal data

Variable	Obs	W	V	z	Prob>z
logF	252	0.99015	1.798	1.366	0.08601

Source: Author

gives us enough evidence to reject the null on 5%-level of significance. Due to the construction of both tests we cannot certainly say that our data follows log-normal distribution but it seems very likely.

The rest of Black model assumptions can be divided into two groups:

1. In the first group there are assumptions which are, in case of our dataset, fulfilled. Among these we can classify the requirement of European option style, no payments connected with underlying asset till the option maturity or possibility of borrowing money at risk-free rate (or rate very close to it).
2. The second group contains assumptions which are generally very simplifying and unrealistic but its violation does not cause much concern among option traders. First example of such assumption can be constancy of all CAPM parameters (i.e. volatility, variance of the return and interest rates). All of these usually change from day to day and so obviously violates one of the assumptions, nevertheless an approximation of changing variables by its mean value shows to be more than sufficient. Next three assumptions which are not met, but its violation is generally tolerated, are no penalties for short selling, perfect divisibility of both options and underlying asset and finally zero taxes and transaction costs.

In the case of Black model we did not find any severe violation of assumptions and so our expectation of precise results provided by Black model seems to be reasonable.

3.4.3 Binomial model

As the binomial model is a discrete approximation of the Black-Scholes resp. Black model, most of assumptions which are these models based on are common for both of them. The main difference between two approaches to option pricing, each represented by one of above mentioned models, is in the modelling of underlying asset price development and this is also the only one assumption which we are going to discuss. As the name of model indicates in case of binomial model we assume the underlying price to follow a binomial process. In reality this does not obviously hold, nevertheless the real price development can be by binomial process, with proper parameters as will be estimated in next chapter, successfully approximated. For the rest of assumptions hold the same arguments as mentioned in the case of Black model.

Violation of binomial process assumption is much more serious than violation of all assumptions in the case of Black model together. Nevertheless this is nothing unexpected and we can consider it as price paid by binomial model for its simplicity. With respect to this we can expect slightly worse results than the ones obtained from Black model.

3.4.4 Monte Carlo simulation

As already said in 2.4.3 all assumptions, which MC simulation is based on, are the same as for Black model and so next discussion is not necessary.

Just like in the case of Black model the crucial assumption of log-normal distribution of underlying asset price is fulfilled and so there is no obstacle in Monte Carlo simulation usage.

3.5 Option pricing

In this chapter we will finally move to the option pricing itself, we will describe estimating of additional parameters, state the final form of option pricing formulas and in the attached tables compare characteristics of market and model premiums.

3.5.1 Black model

The pricing process using Black model is less complicated than in the case of other two models. This is especially due to a no need of additional adjustments of any variable and a simple pricing formula that can be used for any option regardless of time to expiration.

As already mentioned both variables - risk-free interest rate and volatility - need no adjustments and so can be used in their annual forms. The time to expiration is then adjusted to two previous variables and is included as a part of the year that is left to exercise date.

The resulting pricing formula is than

$$C_{Black}(F,K,t) = e^{-0.0062(T-t)}[F\aleph(d_1) - K\aleph(d_2)] \quad (68)$$

for call, resp.

$$P_{Black}(F,K,t) = e^{-0.0062(T-t)}(K\aleph(-d_2) - F\aleph(-d_1)) \quad (69)$$

for put option, where

$$d_1 = \frac{ln\frac{F}{K} + \left(\frac{1}{2}0{,}1568^2\right)(T-t)}{0{,}1568\sqrt{T-t}} \quad (70)$$

and

$$d_2 = d_1 - 0{,}1568\sqrt{T-t} \quad (71)$$

Table 7: Market & model premium comparison - Black model

	Average	Maximum	Minimum	Median	SD
Option premium - market	2,84	12.75	0.1	2.18	2.48
Option premium – model	2.91	12.74	0.01	2.21	2.45

Source: Author

On a first glance we can see that characteristics of both dataset are very similar and so we can state, without any proper analysis, that results obtained from Black model will belong among the best ones.

3.5.2 Binomial model

Despite the fact that the binomial model fundamentals and derivation are much more comprehensible and intuitive than Black's one, its use is a little bit more complicated. First obstacle is an adjustment of interest rates and volatility to length of used periods, second one is then a need of additional parameters estimation.

Parameters adjustment

As already mentioned in the case of binomial tree we are going to need to adjust variables, namely volatility and interest rate, to length of periods used. As the times to expiration differ from 33 to 297 days, we decided to use 14-days periods. During the choice of the length of one period a compromise between accuracy (which increases with more periods) and difficulty of use (which increase with number of periods as well) was made. To adjust both variables we need to find number of periods used in one year, which is, after rounding, 26.

An annual level of volatility is 15,68 %, to find a value which corresponds with 14-days period, we use a reversed approach than in the volatility calculation:

$$\sigma = \frac{0.1568}{\sqrt{26}} = \mathbf{3.08}\% \tag{72}$$

Another possibility would be calculating of volatility directly from 14-day data. As the result is almost same ($\sigma = \mathbf{3.13}\%$) we will stay with the original approach.
In the case of risk-free interest rate the procedure is similar:

$$r = \frac{0,0062}{26} = \mathbf{0.237}\% \tag{73}$$

Estimation of binomial model parameters

An important part of pricing options using binomial tree is an estimation of three unknown parameters:

- probability, p
- growth coefficient, u
- drop coefficient, d

For the estimation itself we use a moment method[37]. We choose these parameters so that the tree gives correct values for the mean and standard deviation of the underlying price changes, i.e. it has to satisfy following two equations:

$$e^{r(T-t)} = pu + (1-p)d \tag{74}$$

and

$$\sigma^2(T-t) = pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 \tag{75}$$

This gives us two conditions for three unknown variables it means that for unique solution of this equation system we are going to need one more, which is:

$$u = \frac{1}{d} \tag{76}$$

[37] See Hull (2009)

If we solve equation system of (74), (75) and (76) we obtain searched parameters:

$$u = e^{\sigma\sqrt{\Delta t}} \tag{77}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{78}$$

$$p = \frac{1-d}{u-d} \tag{79}$$

where $\Delta t = \frac{T-t}{k}$ and k denotes a number of periods we are going to split the time to maturity into.

As the length of one period is 14 days and also all variables are adjusted to this fact we can put $\Delta t = 1$, after that the estimated parameters are:

$$u = e^{\sigma} = e^{0,0308} = 1,0312 \tag{80}$$

$$d = e^{-\sigma} = e^{-0,0308} = 0,9697 \tag{81}$$

$$p = \frac{1-0,9697}{1,0312-0,9697} = 0,4923 \tag{82}$$

Now we can finally sum up all estimated parameters to write the final form of pricing formula which is

$$C_{binomial}(F,K,k) = \frac{1}{(1.0024)^k}\sum_{j=a}^{k}\binom{k}{j}(0.4923)^j(0.5077)^{k-j}((1.0312)^j(0.9697)^{k-j}F_t - K) \tag{83}$$

for call, resp.

$$P_{binomial}(F,K,k) = \frac{1}{(1.0024)^k}\sum_{j=a}^{k}\binom{k}{j}(0.4923)^j(0.5077)^{k-j}(K-(1.0312)^j(0.9697)^{k-j}F_t) \tag{84}$$

for put option, where

$$a \in Z \text{ and } a > \frac{\ln\left(\frac{K}{(0,9697)^k * F_t}\right)}{0,0614} \tag{85}$$

and $k = 2,3, \dots ,21$ according to time to expiration.

Table 8: Market & model premium comparison - Binomial model

	Average	Maximum	Minimum	Median	SD
Option premium - market	2,84	12.75	0.1	2.18	2.48
Option premium – model	2.93	12.73	0.00	2.24	2.45

Source: Author

As we can see overall statistics of the binomial model are slightly worse than the previous ones. An interesting thing is that only binomial model estimated zero value of some options.

3.5.3 Monte Carlo simulation

The situation is very similar in the case of Monte Carlo. We again use different number of periods simulated for different times to expiration and again chose 14-day length of one period (number of simulated periods differs from 3 to 21). It means that used values of volatility and interest rate are the same as in binomial model and we can directly write the form of one-period-later equation according to which the development of underlying asset price will be simulated as:

$$F_{t+1} = F_t * \exp(0{,}002 + 0{,}0308\varepsilon) \text{ where } \varepsilon \in N(0{,}1) \qquad (86)$$

The resulting discounted price of single observation is:

$$C_{Monte\ Carlo}(F, K, k) = \frac{max\{0, F_{t+k} - K\}}{(1.0024)^k} \qquad (87)$$

resp.

$$P_{Monte\ Carlo}(F, K, k) = \frac{max\{0, K - F_{t+k}\}}{(1.0024)^k} \qquad (88)$$

The searching price is than a mean value of appropriate number of random processes. How many simulations should be performed in order to obtain reliable results is the last question we have to solve. When choosing a right number of simulations a compromise between two factors, accuracy and time of simulation, has to be made.

The error of result obtained by Monte Carlo method is usually estimated as the standard deviation of arithmetic average. The resulting error of result obtained by performing n simulations is than $\frac{1}{\sqrt{n}}$. So if want to improve our result about one decimal place, we have to increase number of simulations by two orders. With respect to the lower bound for option price that is €0,1 we decided to perform 10 000 of simulations, which should ensure an error smaller than 10 % of the option price (€0,01) and also keep time of simulation around a half of an hour (for all 2643 observations).

Table 9: Market & model premium comparison - Monte Carlo

	Average	Maximum	Minimum	Median	SD
Option premium - market	2,84	12.75	0.1	2.18	2.48
Option premium – model	2.92	12.74	0.02	2.26	2.44

Source: Author

3.6 Model evaluation

3.6.1 Evaluation criteria

We have two different possibilities for precision evaluation – we can use either the absolute or the percentage difference between estimated and real market price. As the option premiums differ a lot (from €0,01 to €11) the absolute difference would not tell us much and so we will rely on the percentage one, which is defined as difference between estimated (or model) and market price one divided by the market one, i.e.:

$$\delta_C = \frac{C_{model} - C_{market}}{C_{market}} \quad \text{resp.} \quad \delta_P = \frac{P_{model} - P_{market}}{P_{market}} \tag{89}$$

The main criterion for evaluation will be then the average of absolute values of single percentage differences defined as:

$$\Delta = \frac{\sum_{i=1}^{n} |\delta_i|}{n} \tag{90}$$

where n is a number of examined options, i.e. $n = 2643$.

Besides that we will also analyze the standard deviation of percentage differences, its maximum over- and underestimating values and take those as the auxiliary criteria.

The evaluation of second and third criterion – ease of use and maintenance – will be, as already mentioned, a subjective opinion of the author based on the experiences gained during the pricing process.

3.6.2 Accuracy evaluation

As already mentioned the most important clue for finding the most precise model will be average percentage difference. An overview of this criterion together with the auxiliary ones can be found in table 10. As the pricing formulas are different for put and call options we obtained separate result for each group. As the results are quite interesting, especially in the case of Monte Carlo we state them here as well:

Table 10: Accuracy evaluation

	Δ	**Maximal overestimation**	**Maximal underestimation**	**SD**
Black model	**17.34 %**	**281.98 %**	**85.41 %**	**33.96 %**
Black - call	19.96 %	278.44 %	85.41 %	33.85 %
Black - put	16.73 %	281.98 %	56.33 %	34.07 %
Binomial model	**17.78 %**	**376.24 %**	**61.68 %**	**35.65 %**
Binomial – call	18.54 %	376.24 %	49.91 %	37.46 %
Binomial – put	17.04 %	309.19 %	61.68 %	33.75 %
Monte Carlo	**18.44 %**	**281.11 %**	**65.21 %**	**35.09 %**
Monte Carlo - call	20.39 %	272.31 %	65.21 %	36.64 %
Monte Carlo - put	16.48 %	281.11 %	62.86 %	32.65 %

Source: Author

On the first look all models seem to be almost even as the difference between best and worst is only 1.1 percentage point. The fact that results are truly very similar can be seen from spikes charts attached to discussion of single models results.

We should also mention here that almost all overestimating differences higher than 100 % but also underestimating ones higher than 50 % (which make approximately 5 % of all observations) are caused by reaching values that are very close to restrictive criteria (with the exception of time to expiration – despite the fact that all options are far from 14-day criterion, all of those with high differences lie under the border of 100 days to expiration and the difference converges to zero as time to expiration rises). The underlying/expiration price difference of these options is generally higher than 10 % and its price is under €0.5. Nevertheless the conditions are same for all models and so we have no reason to drop these observations.

Next general fact that can be seen from attached charts is that all models mostly overprice the options. This can be caused for example by to high volatility estimate which does not correspond with reality and is an example of “outdated” estimate as we discussed it in chapter devoted to volatility. However as we do not have any better way how to estimate volatility from historical data[38] we have settle for the current one.

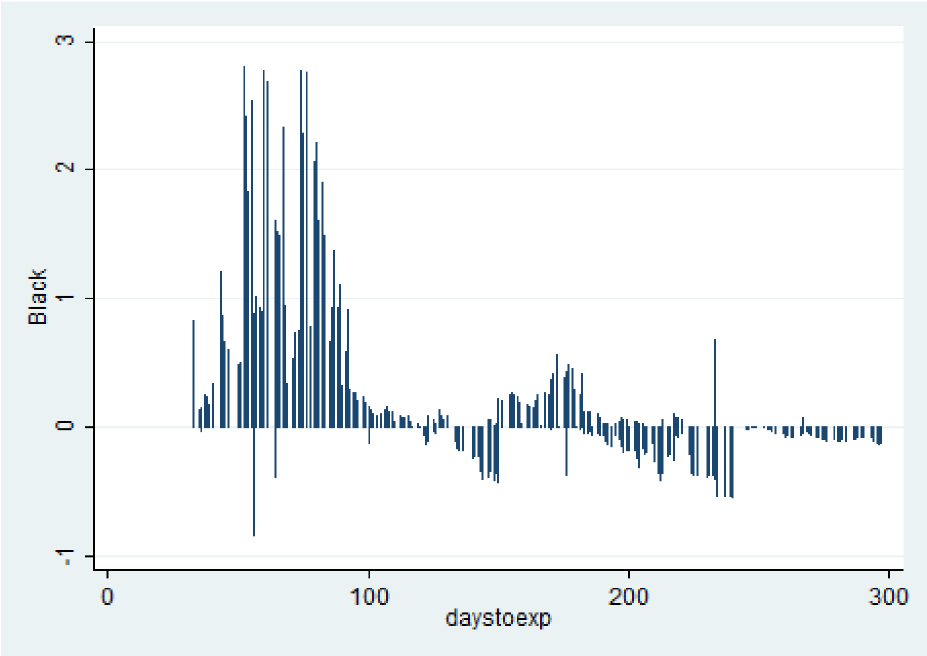
Substantially better results in pricing of put options are the last observed common feature among all models. Especially high is the difference between put and call in the case of Black model and Monte Carlo simulation where its average is about three resp. four percentage points. Unfortunately we were not able to find a clear explanation for this phenomenon, the only substantial difference between put and call options is the height of option premiums. This may indicate similar absolute differences between both groups which are then, on the put side, “reduced” by higher premiums.

Black model

From the table we can see that Black model fulfilled our expectations and is the most accurate model among all others. It excels not only in average difference but also the standard deviation and maximal difference belong among the best values reached. The only one sphere where Black model lag behind other models is maximal underestimation but the difference is not dramatic.

[38] The only possibility would be change of examined period to make the estimate more actual

Figure 14: Spikes chart – Black model



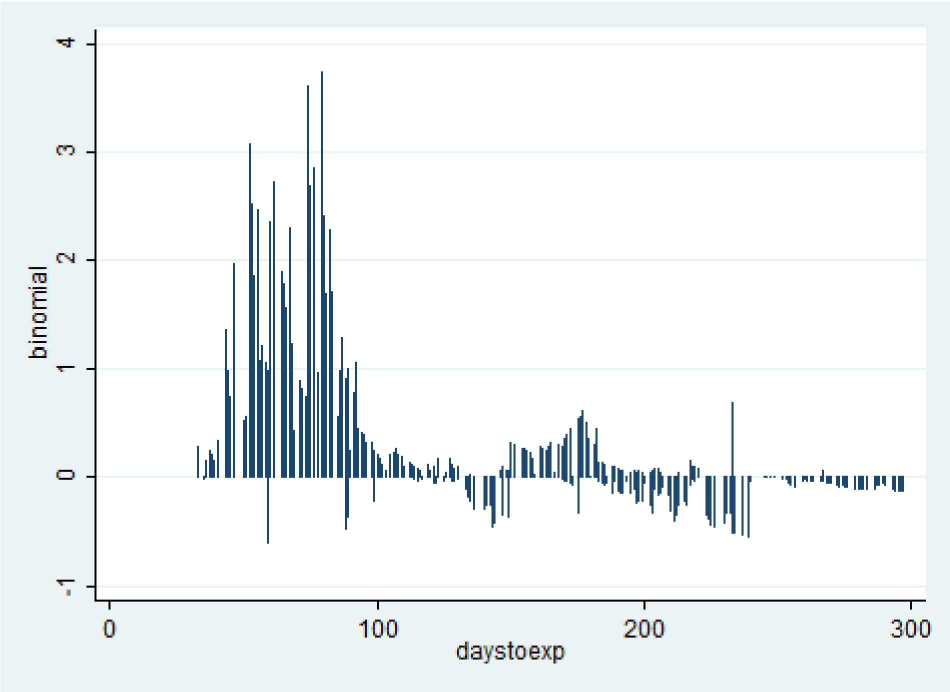
Source: Author

As already mentioned in previous chapter the use and maintenance is also easiest in Black model case. For pricing was in our thesis used Microsoft Excel and this simple tool was more than sufficient. A simple formula which provides us with immediate and precise results seems to be the best possible combination.

Binomial model

From the first look on the chart below we can see that binomial model provides us with worse results than the Black one. And it is not only due to a higher magnitude of overestimations which reach even levels higher than 350 % but also variance rate is higher than in the case of both remaining models. Nevertheless we should point out that the overall accuracy is still better than in the case of Monte Carlo simulation. Binomial model also produce most balanced results between put and call options.

Figure 15: Spikes chart – Binomial model



Source: Author

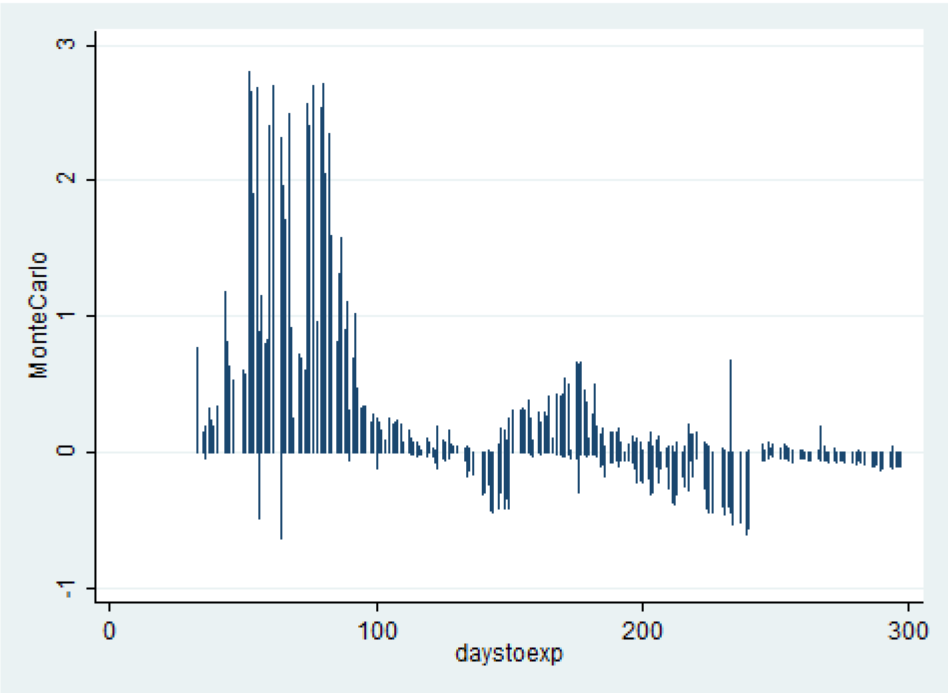
The pricing by binomial model was performed with the use of Microsoft Excel as well. Its use is slightly more complicated when compared to Black model due to need of different lengths of pricing trees for different times to expiration. Nevertheless it is only a small complication which consumes only several minutes to solve. There are of course other programmes, specially designed for option pricing, available but they are not free of charge and author did not have the possibility to use them. With these the option pricing will not be more complicated than the one with Black model anymore and so the only one weakness of binomial model is its inaccuracy. To sum it up the binomial model is losing with the Black one in all three criteria.

Monte Carlo simulation

On a first glance, according to average difference, we would say that Monte Carlo is most imprecise method among all examined ones. However there are several circumstances influencing the results which need further discussion. First is the number of simulations performed, despite the fact that 10,000 seems to be sufficient number, the possible error is still quite high. In practise the usual amount range between 100,000 and 1,000,000 simulations. In our thesis this number was not possible due very high number of options that has to be priced.

Second thing that should be mentioned is a huge difference between put and call option pricing. While the results from put are best among all others (average difference 16.85 %), the ones from call options are significantly worse (20.03 %). If we compare results from put and call at two remaining models the difference is maximal in case of Monte Carlo (3.2 %). Reason for this remains unclear but we think it should be mentioned here.

Figure 16: Spikes chart – Monte Carlo simulation



Source: Author

The Monte Carlo was performed using RiskAMP Monte Carlo, an add-in for Microsoft Excel[39]. With this software accomplishing of the simulation is not a difficult task, the only one problem is so the time-demandingness of this method. As already said performing of 10,000 simulations for each of 2643 options took over a half of an hour, but this is not a relevant information that should be assessed. For practical purposes maximally tens of options are priced and so much more simulations in significantly shorter time can be performed. The time of simulation also heavily depends on software and hardware used. According to Gregor (2005), with proper equipment, pricing of one option, even with 1,000,000 simulations, can be done up to one minute. It means that not even time-demandingness should be problem, the only one drawback is than a need of special software.

[39] See: http://www.riskamp.com/

3.6.3 Summary

The overall results may on the first look seem not surprisingly. The most accurate and most "user friendly" method, resp. model shows to be an analytical method, namely Black model, this finding confirms conclusions of the most of papers dealing with option pricing models testing.

The binomial model confirmed our expectations about its performance as well. As the approximation of Black-Scholes, resp. Black model provided us with slightly worse results. Unfortunately our anticipation that this weakness will be offset by much easier use was not fulfilled. Quite surprising may be higher accuracy of binomial model in comparison with Monte Carlo simulation, the fact is that binomial model produced unusually good results even compared to findings of other works (e.g. Filáček (1998)). Reasons of such a good performance can be found in more precise volatility or risk-free rate estimates or different approach to binomial parameters estimation.

Probably the most interesting findings can be found in the case of Monte Carlo simulation. As this is quite sophisticated and complicated approach to option pricing, our expectations about obtained results were much higher, however, as already mentioned, this may be caused by relatively small number of simulations performed. The reason for a high difference between put and call options also remains an unanswered question. Nevertheless in our analysis Monte Carlo evinces the worst results among all methods which together with nontrivial use cause the last place of this method in our imaginary ranking. Monte Carlo simulation can be recommended as an alternative option pricing tool, but its advantages will show when pricing more complicated and exotic options.

Our results show that the impact of complicated power price development does not influence performance of single pricing models much. As in the case of stock options the most precise results are provided by analytical methods (in case of futures options by Black model). Surprising are substantially better results of binomial model, again compared to stock options, however the statement that this is caused by different underlying asset would be only a speculation, clear reason of this fact is discoverable only with difficulty. Impact of underlying asset on performance of Monte Carlo simulation should be, thanks to character of this method, minimal and so relatively worse results should be only caused by above mentioned factors.

Conclusion

This thesis, which dealt with pricing of options written on a futures contract for electricity delivery, can be divided into three main parts.

In the first one we introduced general characteristics of options as a derivative instrument. We mentioned basic classification of options according to various features, reasons to trade options and also briefly discuss option markets. After that we moved from general fundamentals to specifics of options written on futures contract and reasons for their popularity. We derived an important relationship between prices of put and call option and also bounds for futures option price, both these relationships showed to be necessary for deriving of single pricing formulas. Last field we covered in the first part are specifics of electricity and electricity options. We described influence of non-standard characteristics of electric energy which cause trouble during pricing derivatives written on it and also summed up the difference between "usual commodity" and electricity pricing.

In the second part we already focused on option pricing. We started with review of factors influencing option price and briefly discussed direction and intensity of its influence. After that we presented our criteria which led to choice of models that will be tested and which will be also used for overall evaluation of chosen models. In the second half of this part we introduced three basic approaches to option pricing (analytical, numerical and simulation one) and based on above mentioned criteria chose one representative for each approach – analytical Black model, numerical binomial tree and finally Monte Carlo simulation. These models were then briefly introduced – we mentioned assumptions which models are based on and in the derivation of single models characterized their basic approach to option pricing. Resulting pricing formulas were later used in testing of models accuracy.

Last third part can be considered as the biggest contribution of this thesis. Here the data obtained from European Energy Exchange in Leipzig are applied to pricing models and based on above mentioned criteria (model accuracy and ease of its use and maintenance) the best model that can be recommended for futures option pricing in the power sector was chosen.

We started with description of the used dataset that was at first restricted by three criteria eliminating observation with "extreme" values of some of variables (deeply out-of-money or in-the-money, short time to expiration and very low option premium) and summarized important characteristics of single observed variables. As not only variables directly observable on the exchange are necessary for option pricing, we approached then to estimation of volatility from daily historical data and also calculated average EURIBOR rate over past half of an year which served in as a proxy for risk-free rate. Last step before the pricing itself was discussion of model assumptions with respect to our dataset. Surprisingly we did not find any severe violation of some of crucial assumptions which should indicate possible good results in accuracy testing. Last step in our effort to find best futures option pricing model was application of the data and comparison of estimated model prices with the real, market ones. For this purpose was used a percentage difference, as the main criterion its mean value.

Order of examined models was, according to above mentioned criterion:

1. Black model
2. Binomial model
3. Monte Carlo simulation

This result does not seem surprisingly as it confirms conclusions of the most of works dealing with similar topic. From this we can also conclude that pricing approaches (analytical and numerical) provide similar results regardless of type of underlying asset (stock or futures) and not even complicated power price development causes any significant deviations from usual model performance. Comparison to simulation approach performance, as already mentioned, is missing. However in our thesis Monte Carlo simulation produced the worst results among all models.

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This thesis was written using following software: Microsoft Word, Microsoft Excel, Stata and RiskAMP Monte Carlo.