The goal of the dissertation is to find new method of solving two-dimensional Schrödinger equation in such cases, when the separation of the variables is not applicable. The results are applied to the two-dimensional Schrödinger equation with the potentials of the form of the quartic polynomial, of the sextic polynomial and of the quartic Morse potential. For these cases, the analytical formulae for the ground state wave functions and the corresponding energies have been found. For the specific class of the potentials of the form of the quartic polynomial, analytical formula for one of the excited states and for the corresponding energy have been found.