Charles University in Prague

Faculty of Social Sciences
Institute of Economic Studies

MASTER THESIS

Comovement of Stock Markets and Commodities: A Wavelet Analysis

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Prague, July 29, 2012

Signature

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Abstract

The thesis applies the wavelet analysis to four stock market indices (USA, UK, Germany and Japan) and four commodities (Gold, Crude oil, Heating oil and Natural gas) and it aims to reveal how they comoved in the period of the Global financial crisis, which began in the USA as the Subprime mortgage crisis. Also the potential presence of contagion caused by the bankruptcy of Lehman Brothers bank is investigated. In addition the Granger causality test is applied to give a different perspective and to extend the analysis.

Empirical results revealed that stock markets comoved during the whole period with each other, but much less with commodities. Also, the wavelet correlation of stock markets and commodities differ significantly when talking about the short-term and the long-term horizon. This information can be utilized in the portfolio analysis. The wavelet analysis revealed contagion coming from the USA to the German stock market, Crude oil and Heating oil market after the bankruptcy of Lehman Brothers. The Granger causality test indicates that there is a very strong causal relationship between stock markets and commodities and it differs at different scales.

JEL Classification C22, C40, E32, F30, G15
Keywords comovement, contagion, wavelet analysis, wavelet correlation, wavelet coherence

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Abstrakt

Práce aplikuje Waveletovou analýzu na čtyři akciové indexy (USA, Velká Británie, Německo a Japonsko) a čtyři komodity (zlato, ropa, topný olej, zemní plyn). Předmětem výzkumu je snaha odkrýt vzájemné vztahy a pohyby mezi zvolenými časovými řadami v době Světové finanční krize, která začala jako Hypoteční krize v USA. Práce se dále zabývá přítomností a šířením nákazy mezi finančními trhy v důsledku bankrotu banky Lehman Brothers. V neposlední řadě aplikujeme Grangerův test kauzality na vybrané časové řady a porovnáváme kauzalitu mezi jednotlivými škálami.

Výstupy modelů ukazují, že vzájemný pohyb akciových trhů je velmi silný v celém sledovaném období, což kontrastuje s obecně slabým vzájemným pohybem mezi akciovými trhy a komoditami. Výstupy Waveletové korelace napovídají, že korelace se významně liší při porovnání krátkodobého a dlouhodobého časového horizontu, což může být využito v případné analýze portfolia. Waveletová analýza zaznamenala, že nákaza se po bankrotu Lehman Brothers přenesla z USA na německý akciový trh a dále na trhy s ropou a topným olejem. Výstup Grangerova testu kauzality ukazuje na provázanost akciových trhů a dále na rozdíly mezi jednotlivými škálami.

Klasifikace JEL	C22, C40, E32, F30, G15
Klíčová slova	vzájemné pohyby, nákaza, waveletová analýza, waveletová korelace, waveletová koherence

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Acronyms

CWT Continuous Wavelet Transform

DAX Stock market index for the Frankfurt stock exchange

DWT Discrete Wavelet Transform

FFT Fast Fourier Transform

FTSE100 Stock market index for the London stock exchange

GC Granger causality

LB Lehman Brothers bank

MODWT Maximum Overlap Discrete Wavelet Transform

MRA Multiresolution analysis

NIKKEI Stock market index for the Tokyo stock exchange

OLS Ordinary Least Squares

SP500 Stock market index for the New York stock exchange and NASDAQ

WTC Wavelet Coherence

XWT Cross Wavelet Transform

XWP Wavelet Power Spectrum

Master Thesis Proposal

Author	Bc. Marek Vavřina
Supervisor	Mgr. Lukáš Vácha, Ph.D.
Proposed topic	Comovement of Stock Markets and Commodities: A Wavelet Analysis

Topic characteristics In my thesis I would like to focus on commodity markets and their role during the recent financial crisis. The understanding of relationship between commodities and stock markets is crucial, especially during the crisis, when investors are looking for alternative investment opportunities. Modern methods of data storage give me a chance to analyze intra - day price changes and they should reveal new characteristics of commodity futures and also how was their development connected to major world indexes during the recent crisis. In addition I would like to investigate if the crisis was contagious to commodity markets and how much they were affected. For purposes of my thesis I would like to use wavelet coherence analysis, rolling correlation and causality tests.

Hypotheses

1. Significant increase of comovement between commodity futures and major indexes during the crisis

2. Significant changes in correlations between commodity futures and major indexes will reveal contagion caused by the financial crisis

3. Indexes Granger-cause commodity futures prices

Methodology

1. Wavelet analysis will be used to recognize comovements between commodity futures and major indexes. Thanks to wavelet analysis I will be

able to analyze data from two different perspectives at the same time from frequency domain and from time domain. Especially for my purposes I would like to use wavelet coherence analysis, which I expect to reveal significant coherence between commodity futures and indexes.

2. Detection of contagion will be achieved by rolling wavelet correlations

3. After the recognition of high levels of coherence, I would like to use Granger causality test to determine causality between chosen variables. This should offer an explanation of comovements recognized by Wavelet Coherence.

Outline

1. Introduction
2. Survey of literature concerning modeling of commodity futures
3. Wavelets
   - Theoretical backround
4. Granger Causality test
   - Theoretical backround
5. Description of the data, basic descriptive analysis and testing
6. Empirical part
   - Results of rolling wavelet correlations
   - Wavelet coherence maps - Commodity futures and major indexes co-movement
   - Results of Granger Causality test
7. Conclusion

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Author   Supervisor

Chapter 1

Introduction

Many financial crises were preceded by bubbles, which were caused by excessive investors' interest in one market sector. The Global financial crisis in the late 2000s was not an exception, it started as a housing bubble. Every crisis is specific in some way, but still they have something in common, it is increased volatility of markets. They also have many different consequences, some of them can be even positive, but mostly every crisis is followed by extreme financial losses, downturn of economic activity, unemployment and many other consequencies that are not generally desired. This is also the reason why it is vital to understand how financial markets comove, how interdependent they are, how contagion is spread and if their comovement can be considered causal or not. Most of the research and investigation revolves around stock markets and their historical and potential future development, but recently commodities came to the foreground and they are playing bigger and bigger role. This is also the focus of this thesis, to analyze mainly comovement and in addition also contagion and causality between stock markets and commodity markets.

There are few ways how the comovement can be modeled and studied. The very basic method is correlation coefficient, another more advanced are Vector Autoregressive models, cointegration analysis, family of GARCH models and last but not least wavelets. This quite new method became popular in finance lately, because it has something what others are missing. Usually, an analysis of financial data is conducted in the frequency domain or the time domain. Wavelets combine both of them and provide results that seem to be more comprehensive than those acquired by other methods. The thesis applies the wavelet correlation and the wavelet coherence to examined time series. To obtain even more detailed results, the Granger causality test is applied to discover if there were any causal relations at different scales.

Given the basic idea and methods of the thesis, we turn to data, which are several stock market index returns and commodity returns, namely S&P500 (USA), FTSE100 (UK), DAX (Germany), NIKKEI (Japan) and Gold, Crude oil, Heating oil, Natural gas in the period from 1.1.2007 until 29.11.2011.

The thesis begins with the introduction to the wavelet methodology in Chapter 2, where we present the methodology of models that are later applied to data. Chapter 3 describes data and also provides the basic analysis of data. Empirical results of the application of the wavelet correlation and comments to these results are in Chapter 4. We present the concept of contagion, empirical results acquired by the wavelet correlation and comments in Chapter 5. In Chapter 6 we study comovement of examined time series by using the wavelet coherence, all results are commented. Chapter 7 introduces the Granger causality test, basic theory behind and empirical results at different scales. Last chapter concludes.

Chapter 2

Theoretical Background

2.1 A Brief History of Wavelets

When we look back to the history, we can trace the origin of wavelets back to Joseph Fourier. In 1807 he presented a paper in which he proposed a new way how we can look at time series, so called Fourier series. In general, there are two different ways how one can look at time series, first one is called the frequency domain, normally represented by Fourier series and second one is the time domain. Nevertheless, the biggest deficiency of both of them is that by analyzing one we exclude the other from the analysis. Simply there was no way how to analyze the frequency domain and the time domain at the same time. This all changed with the introduction of wavelets. The first step forward was made by Alfred Haar (Haar (1910)), where he firstly mentioned wavelets, it was in an appendix to his thesis. He proposed an orthogonal system of functions defined on [0,1] and basically he found the simplest possible wavelet, which is now called Haar wavelet. However, it is not continuously differentiable, so its application is limited. Littlewood & Paley (1931) conducted investigation on localization of energy in Fourier series, they used dyadic blocks to decompose a time series and after that they applied the Fourier series on them. Their results indicated that energy is not conserved and that results vary when the energy is concentrated around few points or distributed over a larger interval. Coifman & Weiss (1977) later interpreted Hardy spaces in terms of atoms and their decomposition and it became one of the cornerstones in the wavelet theory. Goupillaud *et al.* (1984) formulated continuous wavelet transform. Mallat (1989) unified the wavelet theory and introduced the multiresolution analysis. Later, Daubechies (1992) built up on discoveries of Mallat (1989) and constructed a family of orthogonal wavelets with compact support. Nowadays wavelets are a tool used in many different fields of science and finance is one of them, for more details see for example Ramsey (2002).

In Grasps (1995) there are mentioned few dissimilarities between the Fourier transform and the wavelet transform. The most important one is that individual wavelet functions are localized in space, while the Fouriers' are not. When we look at Figure 2.1 that shows different transforms, we notice that the most detailed is the wavelet transform. Its windows vary in comparison to the windowed Fourier transform and that makes it more powerful tool in the analysis of time series since it can react to sudden changes in the time series and to nonstationary behavior.

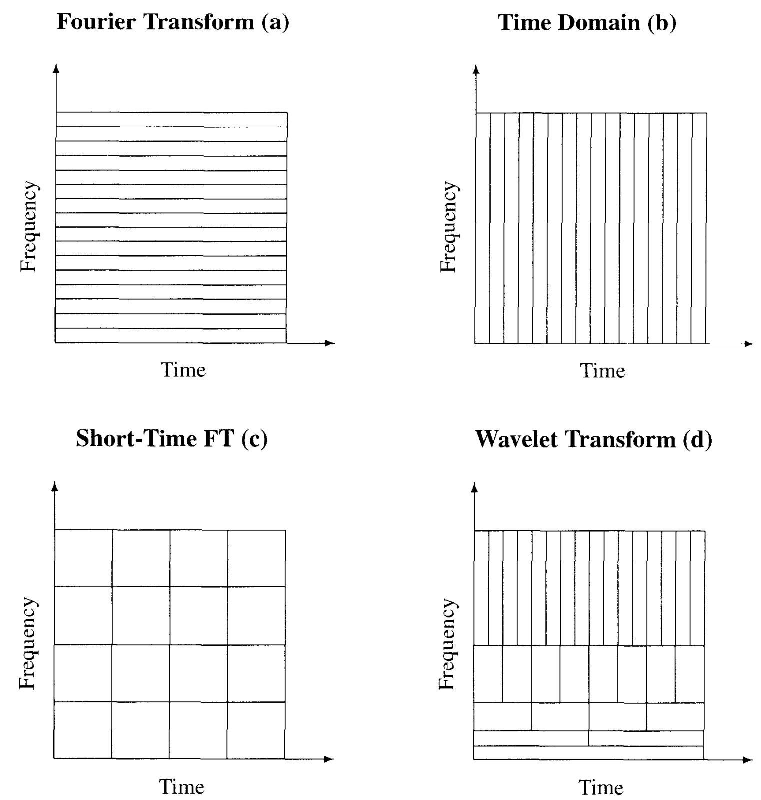


Figure 2.1: A comparison of different approaches to time series analysis (Gencay, Selchuk, Whicher (2002))

2.2 The Continuous Wavelet Transform

The continuous wavelet transform (CWT) is a function $W(\tau, s)$, which projects time series onto particular wavelet Ψ. The derivation we use in this part of the thesis comes from Gencay *et al.* (2002), for more detailed methodology introduction see Daubechies (1992) or Adisson (2002). As we mentioned before, the biggest advantage of the CWT in comparison to Fourier transform is that we look at the time series from two different points of view, we analyze the frequency domain, represented by scale in the wavelet methodology, and the time domain at the same time (Crowley & Lee (2005)). For this reason the function $W(\tau, s)$ has two parameters. Parameter τ represents the time domain (translation parameter) and s is a frequency parameter (scale parameter). Before we derive function $W(\tau, s)$, we have to define the general wavelet function, which is dependent on so called mother wavelet described as

$$\Psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\Psi\left(\frac{t-\tau}{s}\right), \tag{2.1}$$

where $\frac{1}{\sqrt{s}}$ is a normalization factor, which allows us to compare wavelets in different scales.

There are three conditions that mother wavelets have to satisfy (Daubechies (1992), Gencay *et al.* (2002)):

1. Its mean has to be 0

$$\int_{-\infty}^{\infty} \Psi(t)dt = 0 \tag{2.2}$$

2. Integral of a square mother wavelet is equal to 1

$$\int_{-\infty}^{\infty} \Psi^2(t)dt = 1 \tag{2.3}$$

3. Admissibility condition is defined as

$$0 < C_\Psi = \int_{0}^{\infty} \frac{|\hat{\Psi}(w)|^2}{w} dw < +\infty, \tag{2.4}$$

where $\hat{\Psi}$ is a Fourier transform, a function of frequency w, of Ψ. This condition is very important, because it ensures that the original time series can be obtained from its CWT using the inverse transform.

Finally we arrive to the continuous wavelet transform $W(\tau, s)$, which is given by

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\Psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} x(t)\Psi^* \left(\frac{t-\tau}{s}\right) dt, \tag{2.5}$$

where $*$ denotes a complex conjugate (Daubechies (1992)). For our following analysis we also need to define the wavelet power spectrum, in our case we start with a local version of this spectrum. Following Adisson (2002) we define the wavelet power spectrum as

$$(WPS)_x(\tau, s) = |W_x(\tau, s)|^2 \tag{2.6}$$

In case we would like to compare derived wavelet power spectrum with the Fourier power spectrum, we generally use so called the global wavelet power spectrum. It is basically integrated the WPS over all scales, so we get the overall energy of the time series and it can be written as

$$(GWPS)_x(s) = \int_{-\infty}^{\infty} |W_x(\tau, s)|^2 d\tau \tag{2.7}$$

The power spectrum basically depicts the local variance of the particular time series.

The Morlet wavelet

The Morlet wavelet, depicted in Figure 2.2, is the most common complex wavelet used in the wavelet analysis. Complex wavelets are such wavelets that have both real and imaginary part and their Fourier transforms are zero for negative frequencies (Adisson (2002)). Moreover by using the Morlet wavelet we can separate the phase and amplitude components within the signal, which we utilize especially when we talk about the wavelet coherence and the wavelet phase. The Morlet wavelet has simple structure and it is very easy to use. Its mother wavelet is defined in the following way:

$$\Psi(t) = \pi^{\frac{-1}{4}} e^{iw_0 t} e^{\frac{-t^2}{2}} \tag{2.8}$$

and its Fourier transform is defined as

$$\hat{\Psi}(t) = \pi^{\frac{1}{4}} \sqrt{2} e^{\frac{-1}{2}(w-w_0)^2} \tag{2.9}$$

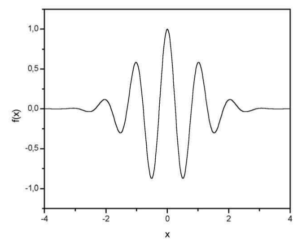


Figure 2.2: The Morlet Wavelet

In our analysis, the central frequency of the mother wavelet denoted by w_0 equals to 6 , which is the most common choice in the economic literature. The Morlet wavelet has four properties, which made it the most popular and at the same time the most used of all the wavelets in the research. Starting with the fact that the Morlet wavelet can be treated as an analytic wavelet, despite the fact that it is complex. Secondly all frequencies like peak, energy and central are equal, $w_\Psi^P = w_\Psi^E = w_\Psi^I = w_0$. Third, it has the best results when speaking about the Heisenberg rule[1] and that means $\sigma_{t,\psi_0}\sigma_{w,\psi_0} = 1/2$. Finally, the Morlet wavelet is the best compromise between a time and a frequency concentration, because a time radius and a frequency radius are equal to $1/\sqrt{2}$ (Aguiar-Conraria & Soares (2011)) .

2.3 The Wavelet Coherence

The wavelet coherence (WTC) is a powerful tool that allows us to depict a relationship of two time series and analyze their comovement from the frequency and the time domain at the same time. We follow Liu (1994) that defines the cross wavelet transform (XWT), which is the first step in deriving the wavelet coherence, which is built on it. The XWT is defined as

$$W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s) \tag{2.10}$$

In this case W_x and W_y are wavelet transforms of the original time series x and y. Symbol * indicates complex conjugate. Liu (1994) defines the cross wavelet

[1]Heisenberg uncertainty principle comes from quantum physics and states that there is limit on the accuracy of the certain pairs of physical properties, such as position and momentum. In simple words the more precisely we measure one property, the less we can measure the other one (Mallat (1998)).

power (XWP) as

$$(XWP)_{xy} = |W_{xy}(\tau, s)| \tag{2.11}$$

The result we get by using the XWP is basically the local covariance of examined time series.

Having the XWT defined we can proceed to the wavelet coherence. We define the squared wavelet coherence coefficient in the following way

$$R_n^2(s) = \frac{|S(s^{-1}W_{xy}(s))|^2}{S(s^{-1}|W_x(s)|^2)S(s^{-1}|W_y(s)|^2)} \tag{2.12}$$

where S is a smoothing operator[2], the WTC coefficient is in the range $0 \leq R_n^2(s) \leq 1$ and because of that we can see certain similarity between the correlation coefficient and the WTC. We can consider the WTC as a local correlation coefficient between two time series with respect to the time domain and the frequency domain. Similarly as the correlation coefficient, as close the WTC to 1 as strong comovement is between two time series. On the other hand as close the result is to 0 as weak co - movement is.

Since the method never shows the negative correlation, $R_n^2(s)$ is never less than 0, we use phase differences, they will help us to see in detail how cycles of the time series changed during the observed period. Based on Torrence & Webster (1999) we define phase differences in the following way

$$\phi_{xy}(u, s) = \tan^{-1}\left(\frac{\Im\{S(s^{-1}W_{xy}(u, s))\}}{\Re\{S(s^{-1}W_{xy}(u, s))\}}\right) \tag{2.13}$$

The phase differences are represented by arrows in our figures, if the arrows are pointing to the right that means that our time series are in phase, opposite direction means anti-phase. If they are pointing down then the first one is leading the second one and if they are pointing up then the second one is leading the first one.

2.4 The Discrete Wavelet Transform

In this chapter we focus on the discrete wavelet transform (DWT) and the multiresolution analysis (MRA), we provide the basic methodology and general

[2]Smoothing operator is $S(W) = S_{scale}(S_{time}(W_n(s)))$, S_{time} stands for smoothing in time and S_{scale} is smoothing along the wavelet scale (Grinsted *et al.* (2004)).

properties of them. For a more detailed treatment of DWT and MRA see Gencay *et al.* (2002). Before we derive the transform let us denote $h_0, ..., h_{L-1}$ and $g_0, ..., g_{L-1}$ where h_l are wavelet filters and g_l are scaling filters.

The DWT is implemented practically via a pyramid algorithm derived by Mallat (1989). As described in Gencay *et al.* (2002) the analysis begins with data X_t, which is filtered by h_l and g_l. It subsamples[3] both filter outputs to half of their original length, keeps the subsampled output from the h_l as wavelet coefficients and then repeats the process described above on the subsampled output of the scaling filter g_l.

In addition there are three conditions that have to be satisfied:

1. Its mean has to be 0
$$\sum_{l=0}^{L-1} h_l = 0 \tag{2.14}$$

2. It has a unit energy
$$\sum_{l=0}^{L-1} h_l^2 = 1 \tag{2.15}$$

3. The wavelet filter h_l is orthogonal
$$\sum_{l=0}^{L-1} h_l h_{l+2n} = 0 \tag{2.16}$$

Now we can continue with the first part of the derivation of the DWT via a pyramid algorithm, which we have already described above.

$$w_{1,t} = \sum_{l=0}^{L-1} h_l X_{2t+1-lmodN} \qquad t = 0, 1, ..., N/2 - 1, \tag{2.17}$$

$$v_{1,t} = \sum_{l=0}^{L-1} g_l X_{2t+1-lmodN} \qquad t = 0, 1, ..., N/2 - 1, \tag{2.18}$$

where w_j and v_j denote the vector of discrete wavelet coefficients $w = (w_1, w_2, w_3, ...w_j, v_j)$,

[3]to subsample means to create a sample of the original sample

where w_j extracts high frequency and v_j extracts low frequency. The length of vectors is $w_j \in \mathbb{R}^{\frac{N}{2^j}}$ and $v_J \in \mathbb{R}^{\frac{N}{2^J}}$.

We continue with the next step,

$$w_{2,t} = \sum_{l=0}^{L-1} h_l v_{1,(2t+1-lmodN)} \qquad t = 0, 1, ..., N/4 - 1 \tag{2.19}$$

$$v_{2,t} = \sum_{l=0}^{L-1} g_l v_{1,(2t+1-lmodN)} \qquad t = 0, 1, ..., N/4 - 1 \tag{2.20}$$

After two steps described above we have $w = (w_1, w_2, v_2)$, of course, we can repeat the procedure and obtain more wavelet coefficients. The major limitation of the method is that data must have a dyadic length.

The multiresolution analysis of data is obtained by reconstructing wavelet coefficients at each scale independently. The pyramid algorithm reveals $w = (w_1, w_2, ..., w_J, v_J)$ and based on this we define the wavelet detail as:

$$d_j = \mathcal{W}_j^T w_j \tag{2.21}$$

Moreover if time series length is $N = 2^J$ the last vector is equal to the time series mean

$$s_j = \mathcal{V}_j^T v_j, \tag{2.22}$$

where $\mathcal{W}$ and $\mathcal{V}$ are N x N orthonormal matrices defining the DWT. Furthermore following Mallat (1989) we define the mutiresolution analysis as

$$X = \sum_{j=1}^{J} d_j + s_j \tag{2.23}$$

2.5 The Maximum Overlap Discrete Wavelet Transform

The maximum overlap discrete wavelet transform (MODWT) is a natural step in the theory of wavelets after the DWT. Although the DWT seems to be very useful in our journey in the world of time series, it is not perfect. There are two very important deficiencies (Crowley & Lee (2005)):

- time series have to have the dyadic length, otherwise they can not be transformed
- DWT is non shift invariant

Both deficiencies were solved by the introduction of the MODWT in Shensa (1992) and later on with the phase - corrected MODWT in Walden & Cristian (1998).

Hence, switching from the DWT to the MODWT brings certain benefits, which are described in Gencay *et al.* (2002):

- We do not have to worry about the length of our time series. The MODWT can transform both dyadic and non - dyadic time series.
- The information in the original time series is connected to the information in the multiresolution analysis. This is achieved by the fact that detail and smooth coefficients of the MODWT multiresolution analysis are associated with the zero phase filter.
- By circularly shifting the original time series we do not change MODWT coefficients in other words the MODWT is shift invariant.
- Both the DWT and the MODWT can be used for the variance analysis, despite that the MODWT wavelet variance estimator is asymptotically more efficient than the one produced by the DWT.

The difference between the MODWT and the DWT lies in a fact that in a MODWT output signal is not subsampled as a DWT's, filters in the MODWT are upsampled at each level, so all wavelet coefficients have a same length on the contrary to the DWT where every additional wavelet coefficient is shorter. In case of the MODWT we obtain wavelet coefficients $\tilde{h}_{j,l}$ and scaling coefficients $\tilde{g}_{j,l}$ by simple rescaling in the following way

$$\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}} \tag{2.24}$$

$$\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}} \tag{2.25}$$

$\tilde{h}_{j,l}$ must satisfy following conditions:

1. Its mean has to be 0

$$\sum_{l=0}^{L-1} \tilde{h}_l = 0 \tag{2.26}$$

2. The value of the energy is 1/2

$$\sum_{l=0}^{L-1} \tilde{h}_l^2 = \frac{1}{2} \tag{2.27}$$

3. Wavelet filter h_l is orthogonal

$$\sum_{l=0}^{L-1} \tilde{h}_l \tilde{h}_{l+2n} = 0 \tag{2.28}$$

The procedure of obtaining the MODWT via the pyramid algorithm is same as in the case of DWT.

$$\tilde{w}_{1,t} = \sum_{l=0}^{L-1} \tilde{h}_{j,l} X_{t-lmodN} \qquad t = 0, 1, ..., N-1 \tag{2.29}$$

$$\tilde{v}_{1,t} = \sum_{l=0}^{L-1} \tilde{g}_{j,l} X_{t-lmodN} \qquad t = 0, 1, ..., N-1 \tag{2.30}$$

and we continue with the second step

$$\tilde{w}_{2,t} = \sum_{l=0}^{L-1} \tilde{h}_l \tilde{v}_{1,(t-lmodN)} \qquad t = 0, 1, ..., N-1 \tag{2.31}$$

$$\tilde{v}_{2,t} = \sum_{l=0}^{L-1} \tilde{g}_l \tilde{v}_{1,(t-lmodN)} \qquad t = 0, 1, ..., N-1 \tag{2.32}$$

After two steps described above we have $\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \tilde{v}_2)$, of course, we can repeat the procedure and obtain more wavelet coefficients. Also the MODWT multiresolution analysis is analogous to the one we presented in the previous chapter, when we were talking about the DWT multiresolution analysis.

For purposes of the analysis of the wavelet correlation, contagion and the analysis of Granger causality at different scales we use filter denoted by LA(8) of length $L = 8$, this filter is commonly used in the literature as can be found in Percival & Walden (2000), for more details see Gencay *et al.* (2002) or Daubechies (1992).

2.6 The Wavelet Correlation

Before we derive the estimator for the wavelet correlation, we have to mention the wavelet variance and the wavelet covariance. The basic idea of the MODWT variance is to detect variability between different scales. The very first use of the MODWT variance is mentioned in Percival & Mofjeld (1997), another useful example of the usage of the MODWT variance can be found in Kim & In (2005). They used the MODWT variance in their analysis of the relationship between stock returns and inflation.

Based on Gencay *et al.* (2002) we define the MODWT variance as:

$$\tilde{\sigma}_l^2(j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^l, \tag{2.33}$$

where $\tilde{d}_{j,t}^l$ is the coefficient at scale j of variables l and $\tilde{N}$ is the number of non - boundary coefficients.

Moreover, following Gencay *et al.* (2002) we define the MODWT covariance as:

$$Cov_{XY}(j) = \frac{1}{\tilde{N}_j} \sum_{t=L_j-1}^{N-1} \tilde{d}_{j,t}^X \tilde{d}_{j,t}^Y \tag{2.34}$$

Since we have defined the MODWT variance and the MODWT covariance, we can also define the MODWT correlation coefficient as

$$\tilde{\rho}_{XY}(j) = \frac{Cov_{XY}(j)}{\tilde{\sigma}_X^2(j)\tilde{\sigma}_Y^2(j)} \tag{2.35}$$

This correlation coefficient behaves in the same way as any other, so there is a condition that $|\tilde{\rho}_{XY}(j)| < 1$.

Since we have defined the wavelet correlation estimator, the very last step is a computation of confidence intervals. We use those mentioned in Whitcher *et al.* (1999)

$$\left[\tanh\left(h\left[\rho_{XY}(j)\right]-\frac{\Phi^{-1}(1-p)}{\sqrt{N_j-3}}\right),\tanh\left(h\left[\rho_{XY}(j)\right]+\frac{\Phi^{-1}(1-p)}{\sqrt{N_j-3}}\right)\right] \quad (2.36)$$

The interval provides a $100(1-2p)$ certain scale, which is calculated by using the DWT. It is because of Fisher's transformation and its assumption of uncorrelated observations and the DWT also approximately decorrelates a range of power-law processes.

2.7 A Comparison of Two Synthetic Time Series

In this section we demonstrate why the wavelet analysis can provide more accurate information than analyzing data only in the frequency domain by the Fourier transform. It is going to turn out that two synthetic time series have similar Fourier spectrum, but when we add the time domain, which means we use wavelets, it is going to give us two absolutely different power spectrums and that means absolutely different results. This characteristic of the wavelet analysis can be very useful especially in a crisis when there many time localized breaks caused by a turmoil on financial markets and at the same time it can help us to understand what impact different events had on financial markets.

2.7.1 The Description of Two Synthetic Time Series

For purposes of our motivating example we are going to use two synthetic time series. Both of them are consisted of two same periodic signals, but they differ as we show in following figures. The difference is made by the presence of signals in different periods. Signals have several components. They include μ, which is the mean, in our simulations we use a particular one ($\mu = 5$) and it is same for both series. Second component is a periodic one, in this case we use two of them, $p_1 = 2$ a $p_2 = 10$. Next component is t and represents time, last component ε is the noise.
In the first one, both signals are present for the whole period.

$$y(t) = \mu + \frac{1}{2}\cos\left(\frac{2\pi t}{p_1}\right) + \frac{1}{2}\cos\left(\frac{2\pi t}{p_2}\right) + \varepsilon \quad (2.37)$$

In the second one, we can see that signals are present only for a certain part of the period. Since we assume that time has no negative values, we can see that firstly we use the first signal and when passes t_s, we switch to the second signal.

$$y(t) = \mu + \frac{1}{2} \cos\left(\frac{2\pi t}{p_1}\right) + \varepsilon \qquad \text{if } t < t_s \tag{2.38}$$

$$y(t) = \mu + \frac{1}{2} \cos\left(\frac{2\pi t}{p_2}\right) + \varepsilon \qquad \text{if } t > t_s \tag{2.39}$$

The analysis of them should reveal the weakness of the Fourier spectrum, which does not notice the break, this is also the strength of the wavelet power spectrum, which notices the break.

2.7.2 The Analysis of Two Synthetic Time Series

Now we are going to compare differences between graphical represantion of our two cases.

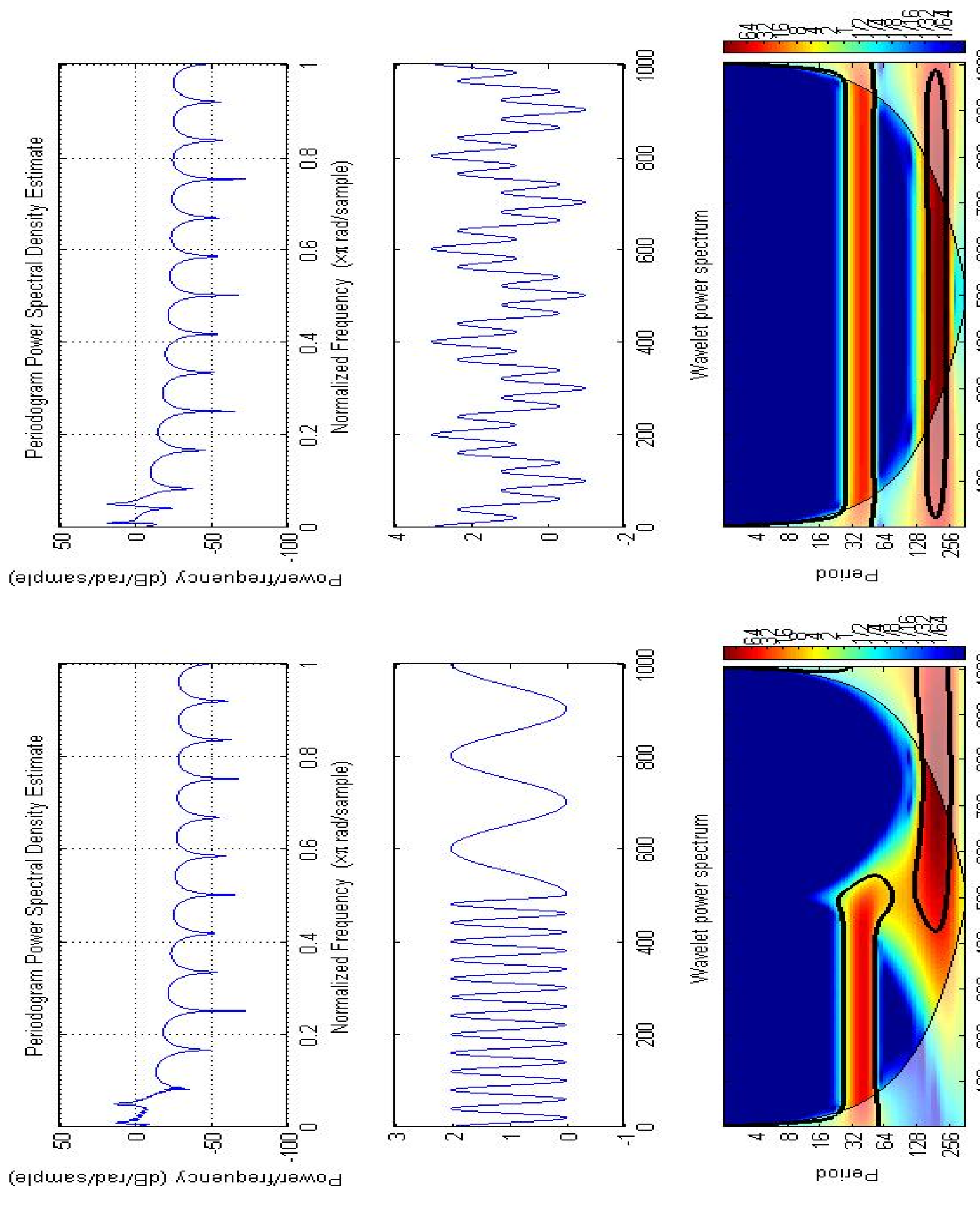


Figure 2.3: Comparison of two synthetic signals

So as we can see in the Figure 2.3, they have a very similar Fourier spectrum even though it is obvious that the time series differ. This is exactly the case, why Fourier analysis reaches its limits in economics. The reason is that in economics and particularly in finance, the time domain holds crucial information. When we arrive to the third part of our figures, which is the wavelet power spectrum, we can see the difference between both synthetic time series very clearly. The wavelet power spectrum noticed the change in the second case and changed as a response to that.

Chapter 3

Data

Throughout the whole thesis, we are going to use one set of data. We are going to analyze eight time series, more precisely four stock market indices (S&P500, FTSE100, DAX and NIKKEI) and four commodities 3 - month futures (Gold, Crude Oil, Heating Oil and Natural Gas). Data was collected by the company TickData[4].

3.1 The Data Description

The analysis requires certain adjustments of original data. For the sake of the consistence we use only data from days in which all stock markets and commodity markets were opened, from 1.1. 2007 until 29.11.2011 it makes 1140 days in total, this allows us to compare results among each other. Then we calculate the first differences of logarithms (ΔR_t), where P_t is the closing price in time t and P_{t-1} is the closing price in time $t-1$.

$$\Delta R_t = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{3.1}$$

Also in the preliminary analysis we use the augmented Dickey-Fuller test and the Jarque-Bera test to find out if our data can be considered stationary and normally distributed.

[4] http://www.tickdata.com/

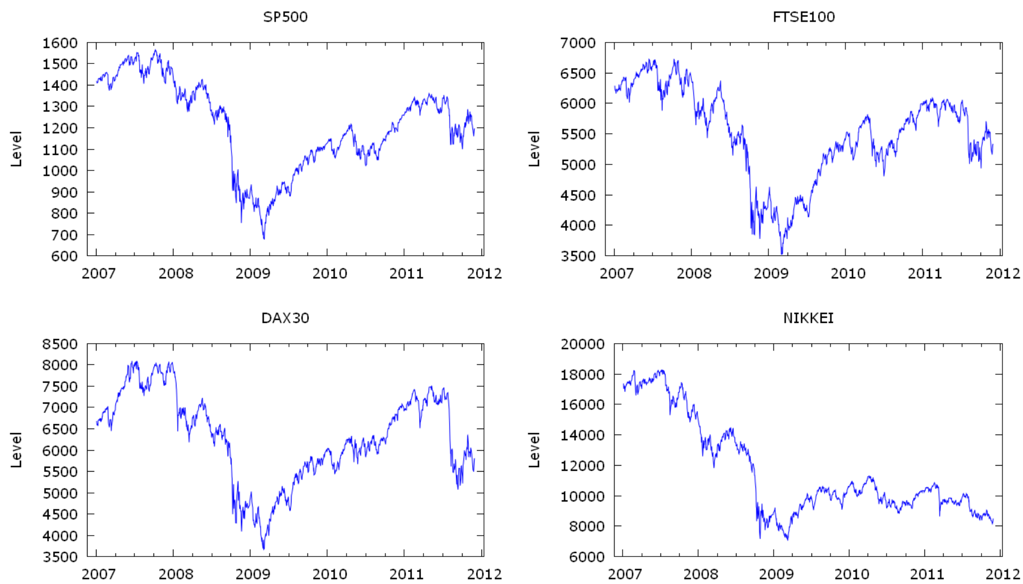
3.2 Basic Characteristics of Data

The analysis begins with basic characteristics of data. This preliminary results should give us a basic idea what happened with stock market indices and commodities in last five years.

Stock markets

The analysis is based on the following stock market indices: S&P500, FTSE100, DAX and NIKKEI. We can see in Figure 3.1 that all of them share similar pattern, which is a huge fall in year 2008, which was caused by the Subprime mortgage crisis in the USA. Later on the development differs slightly. We can see very similar development of S&P500, FTSE100 and DAX, but on the other hand the development of DAX does not show such a strong slump in the first half of 2010. Also the development of NIKKEI suggests that comovement with other indices should be weaker, because the recovery after the beginning of the financial crisis was much slower than in another economies.

Figure 3.1: Stock market indeces



When we take a look at returns of stock market indices in Figure 3.2, we can conclude that all of them became very volatile in the second half of 2008. There was no exception, the crisis was obviously global. In addition, there is a recent increase of volatility in the second half of 2011 and the possible explanation can be a tension on financial markets caused by the EU sovereign debt crisis. At the same time we can see that NIKKEI does not seem to be that volatile in the same period. There is a slump in the first half of 2011, which is probably caused by the fact that Japan faced a natural disaster in terms of tsunami. Considering descriptive statistics, we obtained negative mean, which means that in general stocks included in indices produced negative returns in the observed period. In addition the Jarque-Bera test of normality suggests that we have to reject null hypothesis for all our indices. On the other hand the augmented Dickey Fuller test indicates that all time series are stationary, which is a crucial assumption.

Figure 3.2: Returns of stock markets

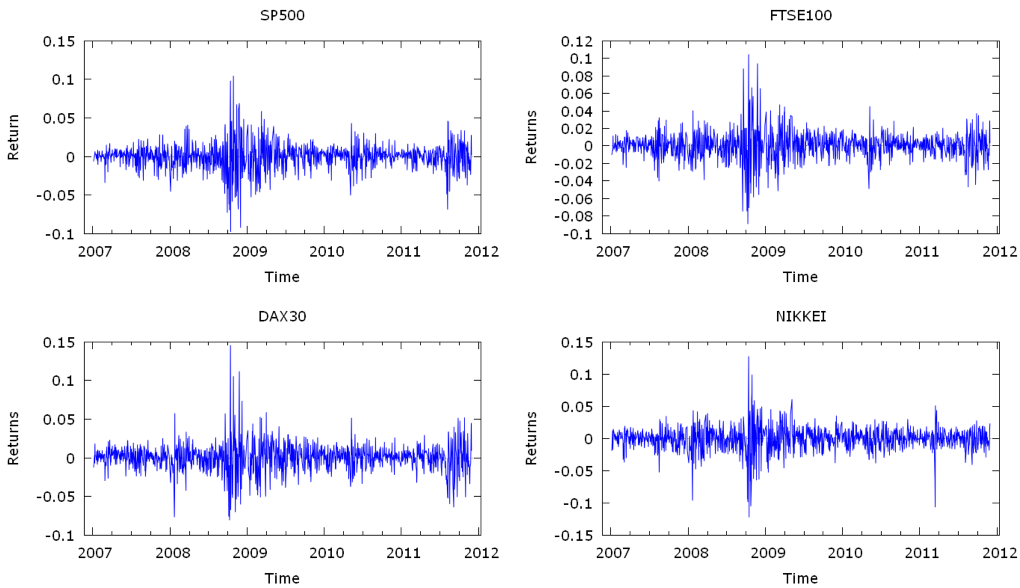


Table 3.1: Descriptive statistics of returns of stock markets

Level	S&P500	FTSE100	DAX	NIKKEI
Mean	-0.000150092	-0.000144406	-0.000123190	-0.000628378
Median	0.000940637	0.000193656	0.000654683	0.000404357
Minimum	-0.0978939	-0.0892567	-0.0808954	-0.122617
Maximum	0.104667	0.104655	0.145846	0.127714
Standard Deviation	0.0171386	0.0158566	0.0180374	0.0187909
Skewness	-0.257431	0.0827446	0.382197	-0.561124
Kurtosis	5.53372	6.44919	7.80516	7.66902
JB test statistics	1467.14	1976.93	2921.48	2853.48
JB null hypothesis	Reject	Reject	Reject	Reject
ADF test statistics	-9.1913	-9.75517	-8.56027	-7.92889
ADF null hypothesis	Reject	Reject	Reject	Reject

Source: author's computations.

Commodity markets

In our analysis we focus on 4 commodities: Gold, Crude oil, Heating oil, Natural gas. We can see in Figure 3.3 big differences in the price development, there is an obvious increase of the price of Gold, that can be caused by the crisis and that Gold served as a safe haven for investors during the economic turmoil. The price increased three times in the examined period. On the other hand commodities that represent a necessary part of the economy - energies went through a huge decrease in 2008. Crude oil and Heating oil seem to follow very similar development, but based on time series, we can conclude that the price of Heating oil started going down earlier than the price of Crude oil. The reason of such drop in the prices can be probably explained by a lower industry production during the crisis and correspondingly lower demand for fuels. The development of the price of Natural gas differs from Crude oil and Heating oil, even it is considered to be the energy commodity too.

Figure 3.3: Commodity prices

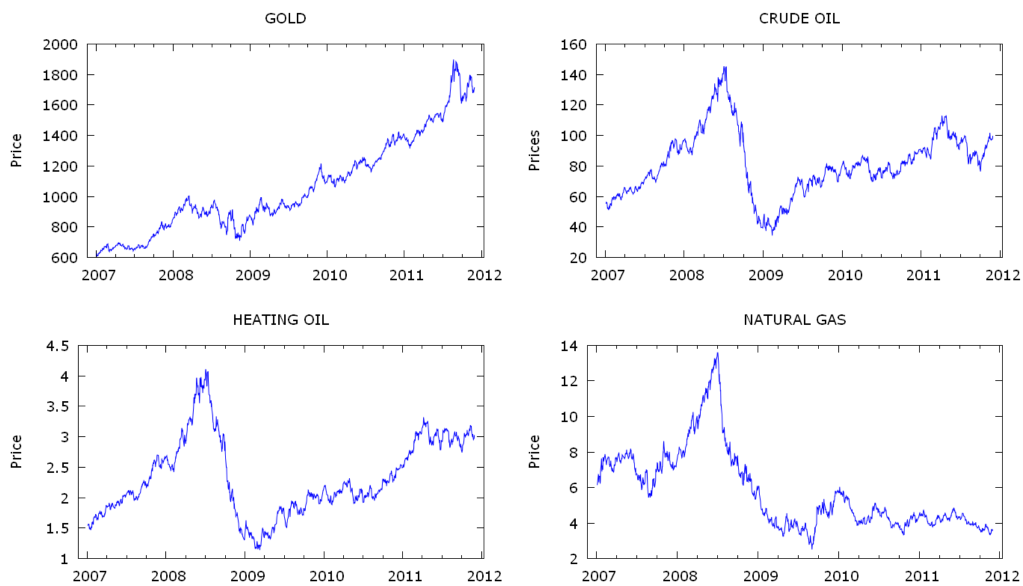


Figure 3.4, depicting commodity returns, indicates that there was also a higher volatility, which began in the second half of 2008, they also suggest that first commodity which was affected was Gold, followed by Crude oil and Heating oil and later by Natural gas. The interesting conclusion of descriptive statistics is that all commodities have a positive mean. Results of Skewness and Kurtosis indicate that our time series are not normally distributed. This is confirmed by obtained result of the Jarque-Bera test. On the other hand

the augmented Dickey Fuller test rejects null hypothesis of the unit root, so we conclude that data are stationary.

Figure 3.4: Returns of commodities

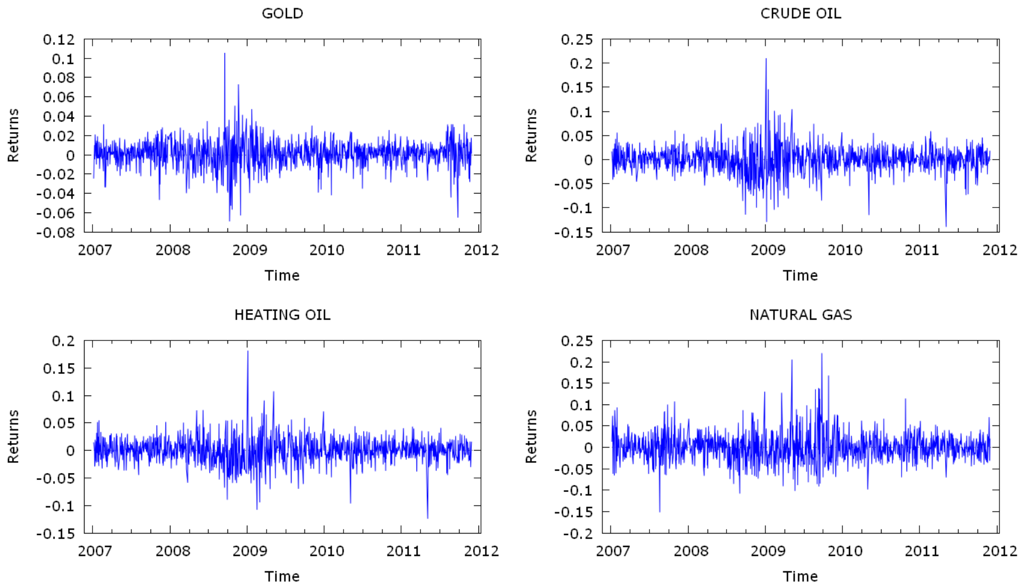


Table 3.2: Descriptive statistics of returns of commodities

Level	Gold	Crude Oil	Heating Oil	Natural Gas
Mean	0.000885618	0.000513489	0.000589003	-0.000465649
Median	0.00138496	0.000822441	0.000641788	-0.00187123
Minimum	-0.0691879	-0.139682	-0.124176	-0.151599
Maximum	0.105566	0.210225	0.181365	0.220109
Standard Deviation	0.0140809	0.0277429	0.0232689	0.0344744
Skewness	-0.0945135	0.0433878	0.111387	0.778753
Kurtosis	4.88323	5.43098	5.14167	3.98529
JB test statistics	1134.38	1401.4	1258.11	869.647
JB null hypothesis	Reject	Reject	Reject	Reject
ADF test statistics	-8.02568	-6.2306	-8.5241	-7.93762
ADF null hypothesis	Reject	Reject	Reject	Reject

Source: author's computations.

The Correlation of returns of stock markets indices and commodities

The unconditional correlation of examined time series is another part of the basic analysis of data. Empirical results in Table 3.3 revealed that FTSE100 and DAX have the highest level of correlation among stock market indices. On the other hand the lowest level of correlation was observed between S&P500

and NIKKEI. A possible explanation of such differences can be that S&P500 and NIKKEI are located in different time zones. Among stock market indices and commodities we observe quite low levels of correlation, Gold and Natural gas have the lower levels of correlation with all indices than Crude oil and Heating oil. Between each pair of commodities we conclude that there is a very low level of correlation with one exception, which is Crude oil and Heating oil.

Table 3.3: The Pearson's correlation coefficients of stock markets and commodities

Level	S&P500	FTSE100	NIKKEI	DAX	Gold	Crude Oil	Heating Oil	Natural Gas
S&P500	1							
FTSE100	0.6332	1						
NIKKEI	0.2187	0.4534	1					
DAX	0.6720	0.8916	0.4437	1				
Gold	0.0418	0.0880	0.0730	0.0625	1			
Crude Oil	0.4436	0.3897	0.1512	0.3578	0.2874	1		
Heating Oil	0.4080	0.3733	0.1732	0.3365	0.2804	0.8514	1	
Natural Gas	0.1299	0.0824	0.0555	0.0855	0.1041	0.2731	0.2875	1

Source: author's computations.

Chapter 4

The Wavelet Correlation of Stock Markets and Commodity Markets

In this chapter, we focus on the correlation of examined time series. We are going to use the wavelet correlation of the MODWT wavelet coefficients. Results will show us how time series are correlated at different scales. The analysis can be very helpful especially for potential investors. Since there are different kinds of investors, from those who trade on the short-term horizon to those who trade on the long-term. Previous studies like Gallegati (2005) or Ranta (2010) suggest that correlations of stock markets differ when we take into account different time horizon.

First, we would like to review some of the available literature. Fernandez-Macho (2011) studied 11 Eurozone stock markets and their correlations within 2454 trading days, his results indicate that Eurozone stock markets returns are highly correlated, the lowest is a daily scale, but still has a correlation coefficient approximately 0.95. Another interesting paper was written by Gallegati (2005). He is using the MODWT correlation estimator on five major MENA equity markets (Egypt, Israel, Jordan, Morocco and Turkey). His analysis provided conclusions that the correlation between MENA markets increases with increasing scale and also that at high frequencies, there is the smallest number of significant comovement. Ranta (2010) analyzes correlations of the world major stock markets, more precisely major world indices S&P500, FTSE100, DAX, NIKKEI. His research led to the conclusion that correlation among these indices increases with increasing scale. Kim & In (2005) analyzed monthly data covering the period from January 1926 to December 2000 and provided new evidence of the relationship between stock returns and inflation. Their results suggest that there is a negative relationship between stock returns on intermediate scales. On the other hand on high (1 month) and low scales (128 months) they observed a positive relationship. In & Brown (2007) studied international links between the dollar, euro and yen interest rate swap markets and their finding besides others is that correlation between swap markets in general is very high, but varies over time. They also concluded that yen swap market is relatively less integrated with euro and dollar swap markets.

4.1 Empirical Results

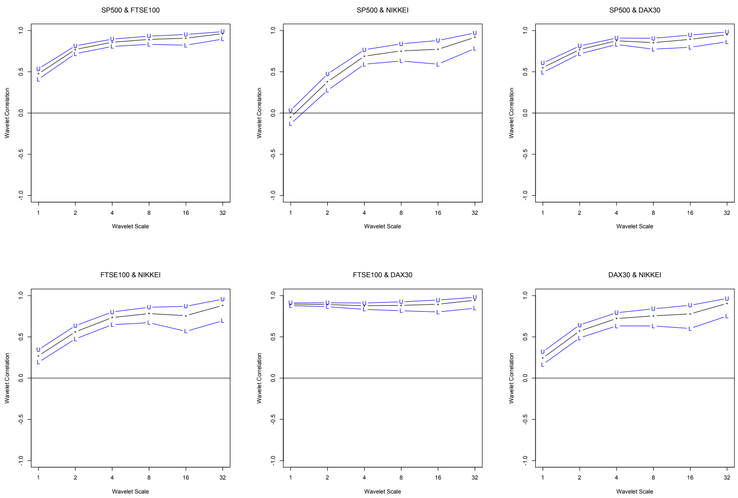
Our empirical analysis follows the approach of Gencay *et al.* (2002) and we use filter denoted by LA(8), which has a length $L = 8$, this filter is commonly used in literature as can be found in Percival & Walden (2000). In our figures x-axis represents different scales a y-axis represents levels of the correlation between examined time series. Wavelet scales, based on the length of the time series, are ranging from the scale 1 to the scale 6 and are associated to changes of 1-2, 2-4, 4-8, 8-16, 16-32, 32-64 days, respectively. Symbol "U" represents upper bound and "L" represents lower bound of the estimate, for the approximate 95% confidence interval. Our results were acquired by using R 2.15.1 and package Waveslim, which was written by Whitcher (2012)[5].

4.1.1 The Wavelet Correlation of Stock Markets

We begin our analysis with correlations of stock market indices, see Figure 4.1. Based on our results we can observe that the correlation at daily scale is lowest in all cases except in the case of FTSE100 and DAX. Results suggest that correlations of S&P500 with FTSE100 and DAX have very similar development. It starts approximately at 0.5 at first scale and then increases with every scale. S&P500 and NIKKEI have the weakest relationship on the daily basis from all the examined pairs. FTSE100 and NIKKEI start with a very low correlation on first scale, but on higher scales the correlation tends to increase. On the other hand, FTSE100 and DAX have the strongest relationship, which is very close to 1 at low and also at high scales. The development of the correlation of DAX and NIKKEI is very similar to FTSE100 and NIKKEI.

[5]http://cran.r-project.org/web/packages/waveslim/index.html

Figure 4.1: The wavelet correlation of stock market indices



Source: author's computations.

4.1.2 The Wavelet Correlation of Stock Markets and Commodities

Correlations of S&P500 and Gold are most of the time very close to 0, but with respect to confidence intervals, which seem to be very wide, we can only say that in general correlation is very low, especially on scales 1,2,3 (i.e., high frequencies). Considering the correlation of S&P500 and Crude oil, we observe that correlation on scales 1,2,3 is very close 0.5. Further, S&P500 and Heating oil seem to suggest very similar results as S&P500 and Crude oil. In the case of S&P500 and Natural gas, based on the confidence interval, we conclude that there is a quite low level of the correlation, especially on scales 1,2,3. For more details see Figure 4.2.

Correlations of FTSE100 and commodities, see Figure 4.3, give us very similar results as S&P500 and commodities. FTSE100 and Gold do not seem to be correlated, at least on time scales 1,2,3. The correlation of FTSE100 and Crude oil is lower in comparison to S&P500 and Crude oil. FTSE100 and Heating oil correlate in a similar manner as FTSE100 and Crude oil. FTSE100 and Natural gas start close to 0 at scale 1, which suggests no correlation on the daily basis.

Since DAX is strongly and positively correlated to FTSE100. Results of FTSE100 we obtained above are very similar and can be applied to DAX too, see Figure 4.4. The correlation of DAX and Gold is very close to 0 at scales

1,2,3, then confidence intervals become too wide to conclude anything. The correlation of DAX and Crude oil is increasing on the daily basis. DAX and Heating oil seem to give us very similar results as DAX and Crude oil at time scales 1,2,3. The correlation with Natural gas is increasing on scales 1,2,3, but then again confidence intervals become too wide.

We continue with Figure 4.5 considering NIKKEI and commodities. Starting with the correlation of NIKKEI and Gold, which is moving close to 0 at first scales. Daily scale of NIKKEI and Crude oil is around zero and that means no correlation or very low correlation if we take into account confidence intervals. NIKKEI has very similar correlation with Heating oil as with Crude oil, in general lower than the other indices at scales 1,2,3. Last figure depicts a relationship of NIKKEI and Natural gas and again we acquired daily correlation very close to 0.

Figure 4.2: The wavelet correlation of S&P500 and commodities

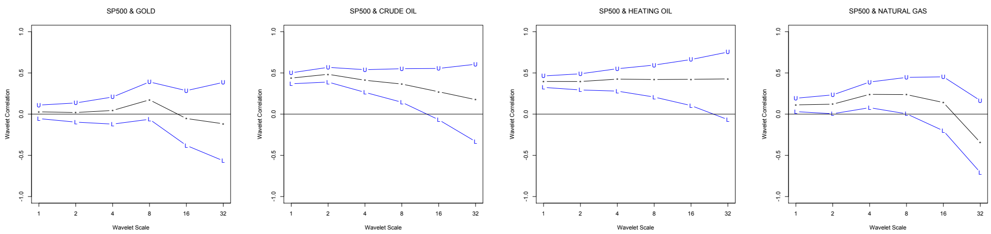


Figure 4.3: The wavelet correlation of FTSE100 and commodities

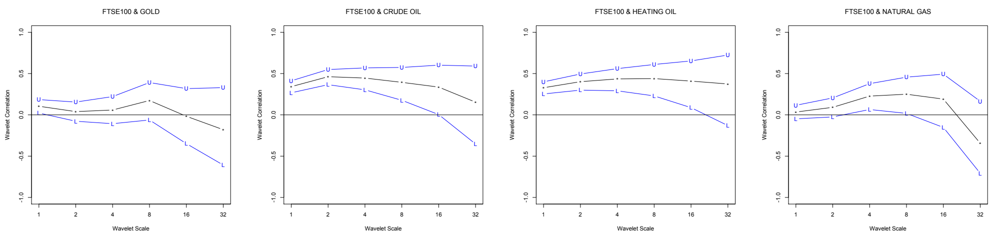


Figure 4.4: The wavelet correlation of DAX and commodities

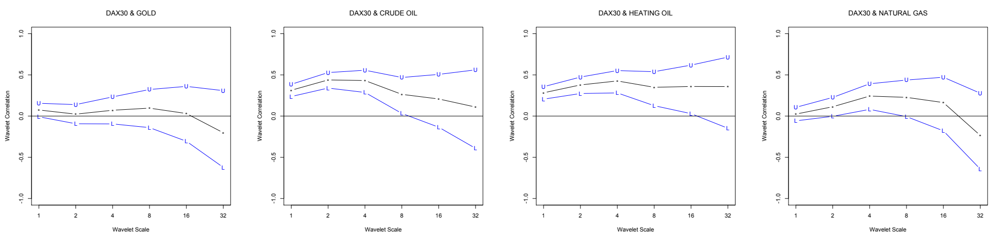
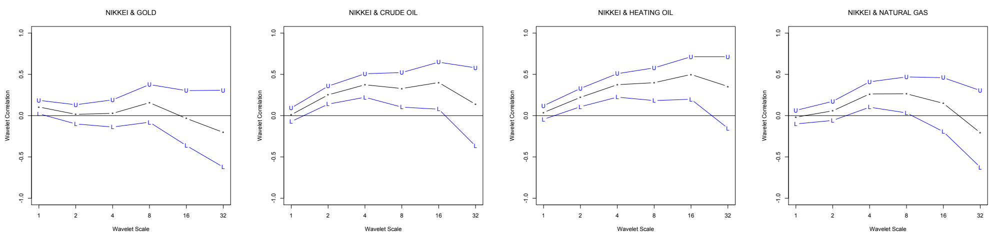


Figure 4.5: The wavelet correlation of NIKKEI and commodities

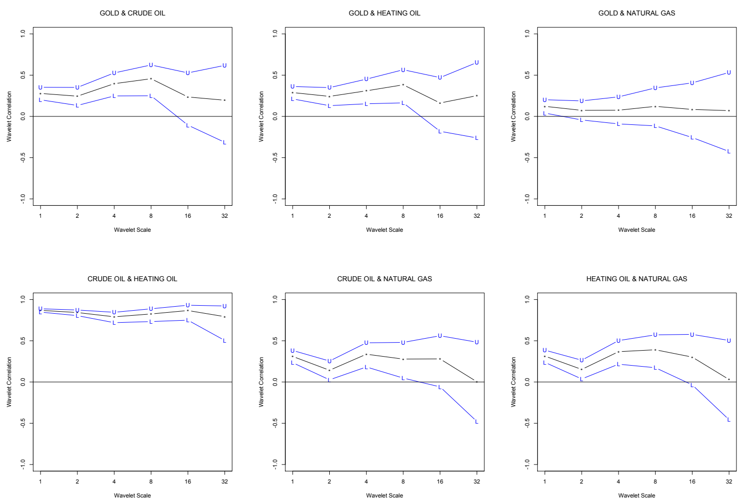


Source: author's computations.

4.1.3 The Wavelet Correlation of Commodities

We begin this part, which discusses Figure 4.6 with Gold and Crude oil. Their daily correlation seems to be positive, around 0.25 at scales 1 and 2. Gold and Heating oil provide very similar result. The correlation of Gold and Natural gas suggests very week relationship, at least on scales 1,2,3. Results of Crude oil and Heating oil correlation lead us to the conclusion that they are strongly correlated at all scales, which was expected, also confidence intervals are not wide as in other cases. The correlation of Crude oil and Natural gas starts around 0.3, but then decreases at scale 2 to rise again at scale 3. Heating oil and Natural gas have very similar results as Crude oil and Natural gas.

Figure 4.6: The wavelet correlation of commodities



Source: author's computations.

In this chapter we focused on the wavelet correlation that gives us more detailed results than the Pearson's correlation coefficient. We can observe that the wavelet correlation of examined time series differs at different scales. As a result of that the wavelet correlation has a potential to become a very useful tool especially in the portfolio analysis, since it shows differences in the correlation between scales, it can serve both short term horizon and long term horizon investors. We also observed that stock markets are in general more correlated with Crude oil and Heating oil than with Gold and Natural gas.

Chapter 5

Contagion among Stock Markets and Commodity Markets

Almost every decade witnessed some crisis, some of them became global and it causes a rising interest, how and why they are transmitted from countries they started to the whole world. There is an obvious increase of interrelations between financial markets in general all around the world. It is not unusual that a crisis is exported from one country to another. As a result of that, we can not anymore focus on one country, but we have to look for a bigger picture and focus on the world as a whole. Otherwise there can be unnecessary losses caused by an assumption that a crisis is happening in a foreign country or even on another continent and that it does not have any impact on others.

Gallegati (2010) distinguishes two major types of contagion, which are "fundamentals - based" and "pure" contagion. The definition of "fundamentals - based" describes shocks that are transmitted through channels, which are already established between economies and that means that we are talking more about the interdependence than contagion. On the other hand "pure" version of contagion is a transmission of a crisis above the expectations, which can be hardly explained by fundamentals. It refers to a human behavior of investors like panicking, collective irrationality, etc. Dornbusch *et al.* (2010) consider 'fundamental – based' contagion to have three distinct components, which are Common shocks, Trade links and competitive devaluations and Financial links. Another source of contagion, which they consider, is Investors' behavior and it can be described by Liquidity and incentives problems, Information asymmetries and coordination problems, Multiple equilibriums, Changes it the rules of the game. All the mentioned above suggests that there are many different ways of the transmission of contagion. At the same time there are various methods that allow us to study contagion and its effects. In this study we are going analyze the wavelet correlation between stock market index returns and commodity returns. As a result of this we are not looking for the channel of transmission, but we focus on the presence of contagion after an event. The Global financial crisis had many significant events, which shaped its development, but we chose to analyze wavelet correlations before and after the bankruptcy of Lehman Brothers Bank. We consider it the most important event, because it was followed by the enormous increase of volatility on financial markets (Chong (2011)).

Before we present empirical results we are going to review recent literature considering two different methods how to uncover contagion. First method is known as the rolling wavelet correlation and it was proposed by Ranta (2010). This method is based on simple rolling correlation, but instead of the correlation coefficient it is using wavelet correlation coefficients, more precisely MODWT correlation coefficients. The study covers several crisis and incidents, which might have a potential impact on stock market indices (DAX, FTSE100, S&P500, NIKKEI) in last 25 years. Their results revealed that some of them like, the financial crisis in 1987, the Gulf War and the Subprime mortgage crisis caused, with some exceptions, a significant increase of the correlation between almost all examined indices. Another paper, which is using the rolling wavelet correlation is Dajcman *et al.* (2012), the paper focuses on Central european stock markets, LJSEX(Slovenia), PX (Czech republic), BUX (Hungary), and their relations to Western european stock markets like DAX (Germany), CAC40 (France), FTSE100 (Great Britain) and ATX (Austria) between years 1997 and 2010. Their results suggested that Czech and Hungarian stock markets are more connected to Western Europe than the Slovenian stock market, financial market crises covered by this paper are the Russian financial crisis, dot-com crisis and the Subprime mortgage crisis, had a short - lasting effect on stock market comovements.

The second method, which was proposed by Gallegati (2010), is based on division of the sample into two subsamples, where the first one covers the period before the event and the second one after the event, and subsequent estimation of the wavelet correlation and its confidence intervals. Contagion is detected if confidence intervals of subsamples are not overlapping. The analysis conducted in Gallegati (2010) focused on S&P 500 (US), S&P TSX (Canada), NIKKEI 225 (Japan), FTSE100 (UK), CAC 40 (France), DAX (Germany), FTSE MIB

(Italy), BVSP (Brazil), and HSI (Hong Kong) in the beginning of the Subprime mortgage crisis revealed that there is an evidence of contagion and that it is actually scale dependent. This is also the method we are going to use in this chapter.

5.1 Empirical results

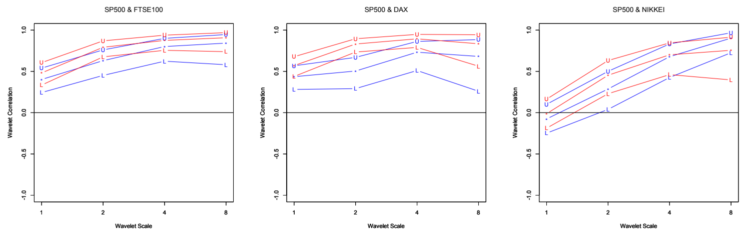
The empirical analysis focuses on analyzing the wavelet correlation before and after the bankruptcy of Lehman Brothers. All pairs of examined time series consist from S&P500 index and other time series. This is based on the assumption that the effect of the bankruptcy had an impact firstly on the stock market in the USA and then it was transmitted to others. Both windows contain 250 observations, more precisely first window contains observations starting 20.8.2007 and ending 15.9.2008 and the second window begins 16.9.2008 and ends 15.10.2009. We estimate their wavelet correlation separately for both windows and then we compare them. If 95% confidence intervals of the wavelet correlation estimates are not overlapping we conclude that contagion was detected. We are going to use the LA(8) wavelet filter with the filter length L=8, this filter was also used by Ranta (2010) and Gallegati (2010). The dots represents estimates of the wavelet correlation, whereas blue color represents the period before and red color the period after the event. Symbol "U" represents upper bound and "L" represents lower bound, for the approximate 95% confidence interval. Our results were acquired by using software R 2.15.1 and package Waveslim, which was written by Whitcher (2012)[6].

5.1.1 The bankruptcy of Lehman Brothers and its impact on stock market indices

Based on our results in Figure 5.1 we can conclude that the bankruptcy of Lehman Brothers did not have contagious effect on other examined indices. There is only one exception, when confidence intervals are not overlapping, which is scale 2 in the figure depicting S&P500 and DAX. A possible explanation of the lack of contagion could be that the crisis was already present, fundamental changes already happened in 2007 and contagion was already spread from some previous event.

[6]http://cran.r-project.org/web/packages/waveslim/index.html

Figure 5.1: The bankruptcy of Lehman Brothers and its impact on indices

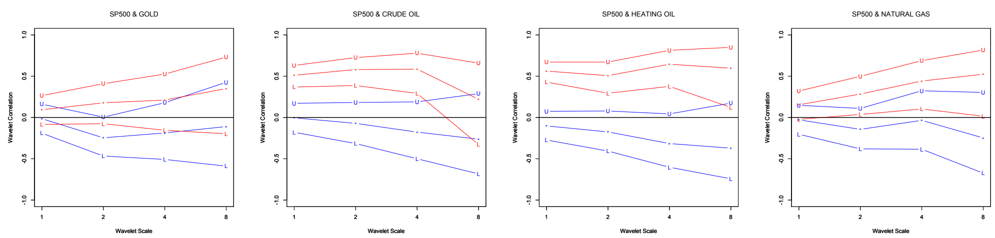


Source: author's computations.

5.1.2 The bankruptcy of Lehman Brothers and its impact on commodities

The wavelet correlation in Figure 5.2 did not reveal any sign of contagion coming from S&P500 to Gold and Natural gas markets, a 95% confidence intervals of wavelet correlations are overlapping. On the other hand, it revealed that the fall of Lehman Brothers was contagious to Crude Oil and also to Heating Oil market. We conclude that contagion affected scales 1,2 and 3 in both cases and that means that contagion affected especially high frequencies.

Figure 5.2: The bankruptcy of Lehman Brothers and its impact on commodities



Source: author's computations.

5.1.3 The bankruptcy of Lehman Brothers: The Pearson's correlation coefficient analysis

Wavelet correlation revealed that the bankruptcy of Lehman Brothers was contagious to DAX, Crude oil and Heating oil markets, we would like to confirm it with a simple analysis of the Pearson's correlation coefficients. To keep the methodology consistent, we are going to use the same approach as before. We compare correlation coefficients' 95% confidence intervals of the period before and after the bankruptcy of Lehman Brothers. If they are overlapping, we conclude that there is no sign of contagion. On the other hand, if they do not intersect, we conclude that contagion is present. We construct confidence intervals by using Fisher transformation[7]. Results suggest that there was contagion coming from the USA in four cases: German stock market, Crude oil, Heating oil and Natural gas market. These results are slightly different than results of the wavelet correlation. The Pearson's correlation coefficient revealed even one additional case of contagion, Natural gas market.

Table 5.1: The analysis of contagion - The Pearson's correlation coefficient and confidence intervals

	Lower CI	before LB	Upper CI	Lower CI	after LB	Upper CI	Contagion
S&P500 - FTSE100	0.47802	0.53899	0.72744	0.62021	0.63211	0.86963	No
S&P500 - DAX	0.45265	0.52074	0.70207	0.73883	0.69808	0.98825	Yes
S&P500 - NIKKEI	0.052592	0.17546	0.30201	0.13555	0.25454	0.38498	No
S&P500 - Gold	-0.23129	-0.10618	0.01812	-0.05964	0.06497	0.18977	No
S&P500 - Crude oil	-0.12563	-0.00092	0.12378	0.43923	0.51089	0.68865	Yes
S&P500 - Heating oil	-0.21965	-0.09465	0.02977	0.48423	0.54338	0.73366	Yes
S&P500 - Natural gas	-0.17701	-0.05225	0.07240	0.08366	0.20541	0.33309	Yes

Source: author's computations.

To sum up, we can comment that the analysis of contagion using the wavelet correlation revealed that the correlation differs at different scales when comparing periods before and after the Bankruptcy of Lehman Brothers. Also contagion was present on only some of scales that means that we have got more detailed picture than in the case of the Pearson's correlation coefficient, which revealed that the event was contagious to same stock markets and commodities with one additional (Natural gas), but results are overall and we do not see differences between scales.

[7]Firstly we transform correlation coefficients in the following way $z = \frac{1}{2}\ln(\frac{1+r}{1-r})$, where r are correlation coefficients. The approximate variance of z is $\sigma^2 = \frac{1}{n-3}$. Next step is to construct confidence intervals, the lower confidence interval is defined $\zeta_{lower} = z_r - z_{(\frac{1-\alpha}{2})}\sqrt{\frac{1}{n-3}}$ and the upper confidence interval $\zeta_{upper} = z_r + z_{(\frac{1-\alpha}{2})}\sqrt{\frac{1}{n-3}}$

Chapter 6

Comovement of Stock Markets and Commodity Markets: Wavelet Coherence Analysis

Results of studying the comovement of different stock markets, commodity markets, exchange rates and many other variables can be achieved by different methods as can be found in Dajcman *et al.* (2012). These methods are the linear correlation (the Pearson's correlation coefficient), Vector autoregressive models, the cointegration, the family of GARCH models, regime switching models and the wavelet analysis. We focus on the wavelet analysis and more precisely on the wavelet coherence (WTC). Wavelets have recently become a very frequent method in finance. Despite the fact that in other fields like climatology, geology, medicine and many others, it has already very strong foundations, recent research suggests that results given by wavelets will continue enrich our knowledge about financial markets too. Before we take a closer look at our results, we would like to review literature that covers the usage of the WTC on time series representing financial markets and commodities.

There are already several papers about the comovement among stock markets. We choose only some of them to demonstrate the contribution of the method. Rua & Nunes (2009) analyzed monthly returns in the period 1973 - 2007 among stock markets of four developed countries, namely USA, UK, Germany and Japan. Their analysis led to a discovery that the comovement among these stock markets is stronger on higher frequencies, from which they concluded that international diversification of portfolio might play a key role especially for short term investors. Baruník *et al.* (2011) did research on the comovement between Central European Economies, more precisely, they analyzed the comovement of stock market index returns between Czech Republic, Poland, Hungary and Germany, which was used as a benchmark. Their results based on high frequency data revealed that the comovement differed in time and also in frequency between economies during the period 2008 - 2009. Ranta (2010) used the WTC for an analysis of contagion among stock markets like USA, UK, Japan and Germany between years 1984 and 2009. Results indicate that after a crisis the comovement between stock markets increased, especially on high frequencies and this suggests the existence of contagion.

The comovement of commodities and stock markets was a subject of several papers too. Starting with Aguiar-Conraria & Soares (2011), they used the WTC to analyze the comovement between S&P500 and Oil prices. Their dataset included monthly returns for the period starting in July 1954 and ending in December 2010. By using the wavelet partial coherence with controlling variables they concluded that there was a significant comovement in mid-1970s and mid-1980s and also in the early 1990s. Another paper written by Vácha & Baruník (2012) is studying the comovement between Crude oil, Gasoline, Heating oil and Natural Gas. Based on their results they concluded that comovement varied a lot during the analyzed period, which started in 1993 and ended in 2010. Moreover, the comovement did not vary only in time, but also in terms of frequencies, which provides a completely new information about the development of studied returns.

6.1 Empirical Results

The wavelet coherence is a very efficient tool how we can study when and at what scales examined time series comove. Our results are acquired by using Matlab package, which was written by Grinsted et al. (2004)[8]. Following figures depict the wavelet coherence into a contour plot. The time domain is represented by x-axis and the frequency by y-axis. In addition, the frequency is represented by the period, i.e. the higher frequency the lower the period. The interpretation of our figures is based on the color of regions, blue color means that there is low or even no comovement. On the other hand, red regions with a thick black outline mean that there is a significant comovement between time series. As a result of this we can obtain very detailed results based on the

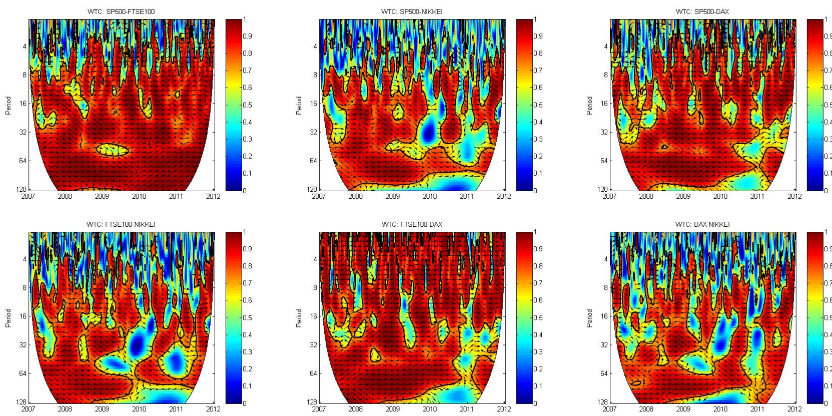
[8]http://www.pol.ac.uk/home/research/waveletcoherence

time domain and the frequency domain at the same time. Another thing that helps us to interpret results are so called phase arrows, which show the relative phasing of time series at given scale. If arrows are pointing to the right that means that time series are in phase, opposite direction means anti-phase. If they are pointing down then the first variable is leading the second one and if they are pointing up then the second variable is leading the first one.

6.1.1 Comovement of Stock Markets

Based on our results acquired from the wavelet coherence, we can conclude that major world indices seem to comove significantly during the period, see Figure 6.1. We observe the strongest comovement between S&P500 and FTSE100. Most of the time they are in phase (phase arrows pointing to the right) at all frequencies for the whole period and that means that there is not a leading market, but two markets with returns that evolve the same direction over time. Also both markets S&P500 and FTSE100 seem to comove with DAX significantly too at all frequencies and phase arrows are pointing to the right that means that DAX is in phase with S&P500 and FTSE100. At the same time when we look at the end of the observed period we can see that the comovement is getting weaker. When we focus on NIKKEI index, there is a strong comovement in 2008 with others, which can be interpreted as a result of the beginning of the crisis in the USA. Also results suggest that starting in 2009, the comovement is getting weaker with all the other indices.

Figure 6.1: Comovement of stock market indices



Source: author's computations.

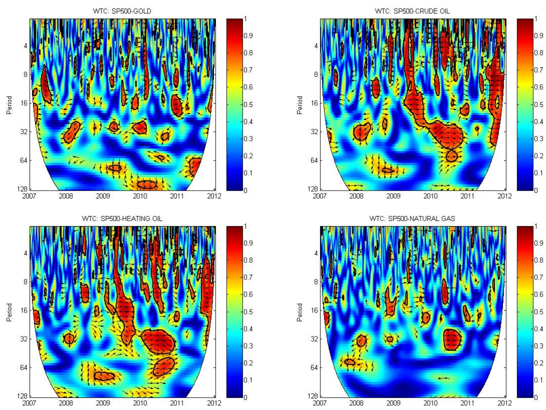
6.1.2 Comovement of Stock Markets and Commodities

We focus on the comovement between each of stock indices and commodities. This part of analysis should reveal how interdependent are stock markets and commodity markets. We can observe how specific commodities behaved in the crisis, whether they comoved with stock markets or not.

Comovement of S&P500 and commodities

Starting with S&P500 and its comovement with Gold, Crude oil, Heating oil and Natural gas, see Figure 6.2. We observe that S&P500 did not comove with Gold significantly in the studied period. Crude oil and Heating oil comoved with S&P500 in the second half of 2009 and also in 2010 at certain frequencies. In addition we observed a very strong comovement at almost all frequencies starting in the second half of 2011. Lowest rate of significant comovement was observed between S&P500 and Natural gas.

Figure 6.2: Comovement of S&P500 and commodities

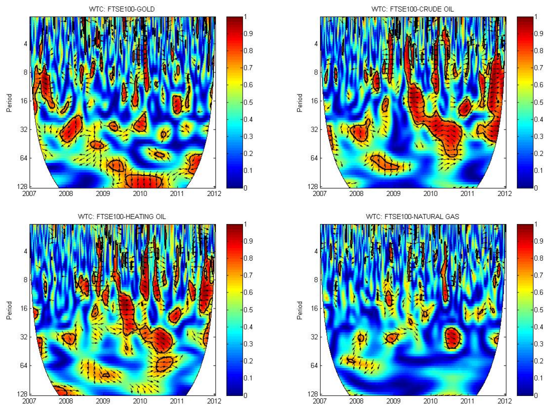


Source: author's computations.

Comovement of FTSE100 and commodities

We continue with FTSE100 and commodities, see Figure 6.3. The wavelet coherence revealed very similar patterns as in case of S&P500. We observe that Gold comoved with the stock market index in different periods and only on certain frequencies, there are three significant regions. The first one is at high frequencies in 2007, second appears at the beginning of 2008 around 32 day period that represents low frequency and last but not least there is a significant region at very low frequencies in 2010. More significant comovement is in the case of Crude oil and Heating oil, in the second half 2009 there was a strong comovement on 12-32 day period. In 2010 we observed a comovement at quite low frequencies and last one in 2011 at almost all frequencies. The coherence between FTSE100 and Heating oil reveals very similar results as FTSE100 and Crude oil. Last figure displays that the comovement between FTSE100 and Natural gas is almost insignificant.

Figure 6.3: Comovement of FTSE100 and commodities

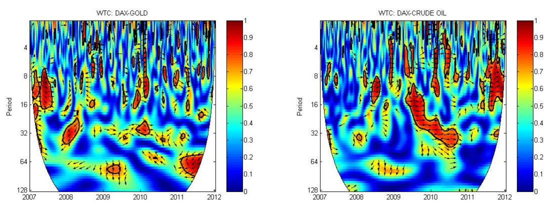


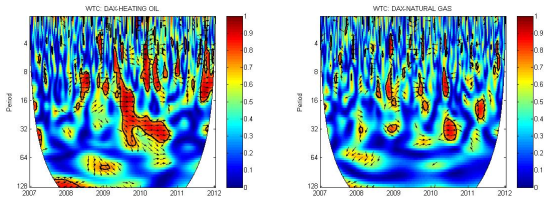
Source: author's computations.

Comovement of DAX and commodities

We do observe some significant comovement among DAX and Gold. There are islands filled with red color, but in general they are too small. At the same time we observe a very significant comovement with Crude oil in the second half of 2009 on 10 - 35 day period and also in 2010 on 30 - 40 day period. Both time series tend to be in phase. DAX and Heating oil provide very similar results. Latter figure regarding the comovement of DAX and Natural gas in general do not seem to comove. For further details see Figure 6.4.

Figure 6.4: Comovement of DAX and commodities



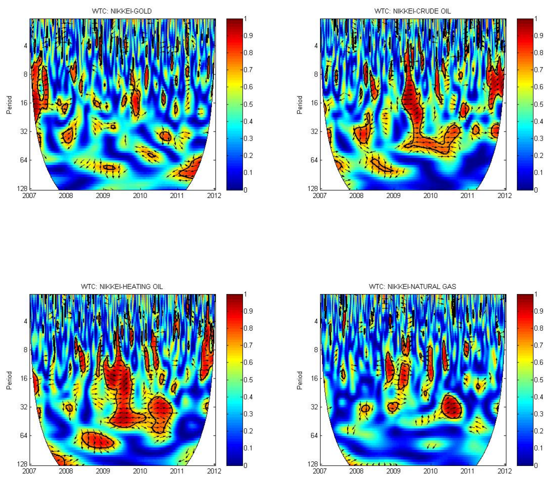


Source: author's computations.

Comovement of NIKKEI and commodities

In the case of NIKKEI and commodities we can observe a significant comovement between NIKKEI and Gold in 2007 at high frequencies, but that is all, see Figure 6.5. Considering NIKKEI and Crude oil there is a very significant comovement on 8 - 64 day period in 2009. Time series seem to be in phase, because arrows point to the right. Also in the second half of 2011 there is a significant comovement at high frequencies. A figure, which depicts the comovement between NIKKEI and Heating oil reveals even bigger red area in 2009 on 8 - 64 day period and then continues in 2010 on 30 - 62 day period. We can conclude that there is any significant comovement between NIKKEI and Natural gas in 2010 at 32 day period.

Figure 6.5: Comovement of NIKKEI and commodities

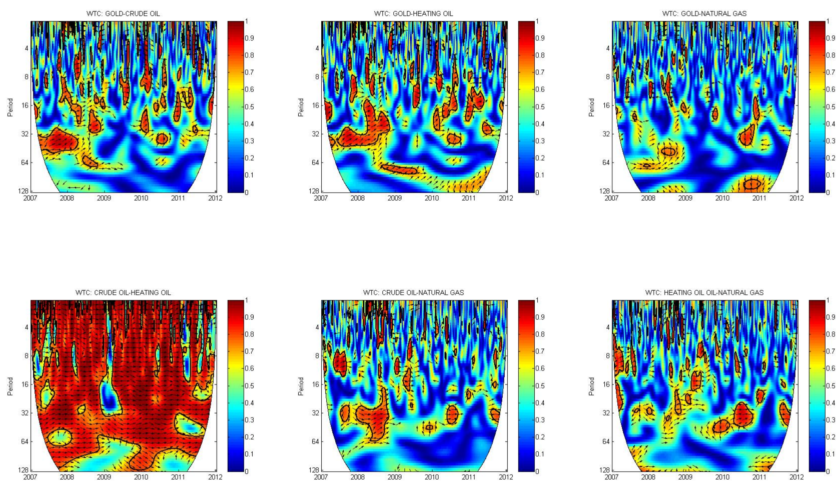


Source: author's computations.

6.1.3 Comovement of Commodities

In the last part of this chapter we are going to focus on the comovement between commodities, see Figure 6.6. Starting with Gold and Crude oil the figure revealed that there was a strong comovement at low frequencies in the second half of 2007 and at the beginning 2008. Also we observe many small islands, but which can not be considered significant, because they are too small. Looking at Gold and Heating oil, the figure suggests that there was a significant comovement in the second half of 2007 at medium frequency and it continues in 2008 at higher frequencies. In the case of Gold and Natural gas we did not acquire results that would suggest any comovement. Crude oil and Heating oil revealed a significant comovement almost in the whole period at all frequencies. Comovement is getting weaker at very low frequencies. Most of the time they are in phase. The figure of the coherence between Crude oil and Natural gas depicts a significant comovement in 2008 at 32 - 64 day period and Crude oil seems to be the leader. Heating oil and Natural gas figure suggests that they comoved strongly in 2010 at 25-40 day period.

Figure 6.6: Comovement of commodities



Source: author's computations.

In this chapter we arrive to the conclusion that the comovement between stock markets is very strong and that in general there is not a leading market, most of the time they seem to be in phase. On the other hand, when we took a closer look at comovement of stock markets with commodity markets, we observed much weaker comovement during the whole period and that means that commodity markets are not that tightly connected to stock markets as stock markets to each other. Also the comovement between commodities, with one exception, which is Crude oil and Heating oil, seem to have a low level of comovement. Since we analyzed the period when the Global financial crisis took place, it might be a good motivation for a further research to analyze, whether the Global financial crisis with its weak comovement between stock markets and commodities was an exception in comparison to other crises or if it is a rule that even in the crisis or only in the crisis stock markets do not comove with commodities.

Chapter 7

Causal Relations between Stock Markets and Commodity Markets

In the last chapter we are going to study causal relations between time series. For that purpose we are using the Granger causality test. The reason why we chose such a different method to study relations in comparison to what we were using until now is that it is never advisable to stick to only one approach. Each method has its pros and cons and we believe that using different methods can help us to avoid potential misleading results and conclusions.

We are going to briefly review some of available literature, which is connected to our analysis. Nazlioglu & Soytas (2011) examined the dynamic relationship between oil prices and twenty four world agricultural commodity prices on the monthly basis between January 1980 and February 2010. Besides other tests they used the Granger causality test and their results provided an evidence that there is actually a strong impact of changes of oil prices on agricultural prices. Jang & Sul (2002) studied comovement of East Asian stock markets during the Asian crisis (1997). Their results of the Granger causality test revealed that there was almost no Granger causality of seven Asian stock markets before the crisis, but the situation changed when the crisis began. Their observations suggest that Granger causality increased during and even after the crisis significantly. Zhang & Wei (2010) searched for causal relations between Gold and Crude oil in the period 2000 - 2007 and they found out, based on the Granger causality test, that there is a unilateral linear Granger causality, that Crude oil market Granger-causes Gold market. Their explanation of such results is that Crude oil is a necessary commodity in the industry and when its price is going up, so the inflation is going and this leads us to Gold, which plays an important role in hedging against the inflation. Gencay *et al.* (2002) analyzed the MODWT MRA coefficients of unadjusted monthly percentage changes in the money supply and the percentage change in the price level of 6 different countries (Argentina, Brazil, Chile, Israel, Mexico and Turkey). Their results revealed that causality differs at different scales. On scale 1, which is associated to the period 2 - 4 months, changes in the money supply Granger-caused change in the price level in all countries except Brazil, but at scale 2, 3, 4, which represents 4 - 8, 8 - 16 and 16 - 32 months, causality changes and in the majority of cases goes both ways, in other words, changes in money supply Granger-causes and is Granger-caused by changes in price level. Hacker *et al.* (2010) conducted an investigation of causal relations between exchange rates and interest rate differentials by using the MODWT MRA coefficients.

7.1 Granger Causality

We introduce Granger causality and the Granger causality test very briefly, for more detailed information see Granger (1969), Sims (1972) or Sargent (1976). Granger causality assumes that we work with weakly stationary data. As Kirchgässner & Wolters (2008) we assume I_t to be a whole information set available at time t. This set contains two time series x and y. Also $\bar{x}_t := (x_t, x_{t-1}, ..., x_{t-k}, ...)$ is set that is consisted from past and present values of x and analogously for y. Last thing that we need to define is $\sigma^2(.)$, which is the variance of the corresponding forecast error. Based on these assumptions Granger (1969) proposed following:

Granger causality - x is said to Granger - cause y, $\iff$ the application of an optimal linear prediction function leads to

$$\sigma^2(y_{t+1}|I_t) < \sigma^2(y_{t+1}|I_t - \bar{x}_t) \tag{7.1}$$

The logic behind this is very intuitive. If we wish to forecast future values of y and previous equation holds, we use also past values of x to get prediction with smaller forecast error variance.

7.1.1 The Granger Causality Test

Before we can apply the Granger causality test we have to find out a right number of lags, we are going to use Akaike criterion, which is defined by the following equation

$$AIC = 2k - 2\ln(L), \tag{7.2}$$

where k is the number of parameters in the model and L is the maximized value of the likelihood function for the estimated model.

We follow Kirchgässner & Wolters (2008), so let x and y be stationary and to test for simple causality from x to y, we have to examine whether lagged values of x in the OLS model of y on lagged values of x and y significantly reduce the error variance. Based on this we estimate the following equation:

$$y_t = \alpha_0 + \sum_{k=1}^{k_1} \alpha_{11}^k y_{t-k} + \sum_{k=k_0}^{k_2} \alpha_{12}^k x_{t-k} + u_{1,t}, \tag{7.3}$$

where $k_0 = 1$. An F test is applied to test the null hypothesis, $H_0 : \alpha_{12}^1 = \alpha_{12}^2 = ... = \alpha_{12}^{k_2} = 0$. By changing the model we can also test whether there is a simple causal relation from y to x. Since this is not the main method of this thesis, we are not going to explain all the theoretical background, but it can be found for example in Kirchgässner & Wolters (2008), Sims (1972) or Sargent (1976).

7.2 Empirical results

Empirical results, acquired by using software JMulti[9], suggest that in most of cases when we tested Granger causality led to a rejection of null hypothesis that there is no causality. Let's take a closer at each of tables.

7.2.1 The Granger causality test of stock market returns and commodities

Starting with stock markets, see Table 7.1, we acquired results that suggest that only in three cases we did not reject null hypothesis. Two of them lead to the information that NIKKEI does not Granger-cause FTSE100 and DAX and that DAX does not Granger - cause FTSE100. In the rest of tests among indices we rejected null hypothesis and this leads to the conclusion that stock markets

[9] JMulti 4.24 - http://www.jmulti.de/

are interconnected based on the Granger causality test. This also confirms our results from previous chapters.

Table 7.1: Results of Granger causality tests between indices

Direction of causality	# of lags	F - Value (GC)	H_0 of GC	Granger Causality
S&P500 → FTSE100	10	21.4065	Reject	YES
FTSE100 → S&P500	10	4.2966	Reject	YES
S&P500 → NIKKEI	4	136.3160	Reject	YES
NIKKEI → S&P500	4	2.4023	Reject	YES
S&P500 → DAX	10	14.9850	Reject	YES
DAX → S&P500	10	4.5502	Reject	YES
FTSE100 → NIKKEI	5	51.3445	Reject	YES
NIKKEI → FTSE100	5	1.0247	Do not reject	NO
FTSE100 → DAX	10	1.9784	Reject	YES
DAX → FTSE100	10	1.5770	Do not reject	NO
NIKKEI → DAX	5	1.8730	Do not reject	NO
DAX → NIKKEI	5	58.7020	Reject	YES

Source: author's computations.

Based on testing stock indices and commodities, see Table 7.2, we conclude that in the case of S&P500 and commodities, we have to reject the null hypothesis that S&P500 Granger-causes Gold and Natural gas. On the other hand S&P500 is not Granger-caused by Heating oil and Natural gas. In comparison to the causality testing between stock markets, there is a weaker relationship of S&P500 and analyzed commodities. We continue with FTSE100 and commodities and obtained results suggest that there is only one relationship that does not ended with the rejection of the null hypothesis and it is that FTSE100 Granger-causes Natural gas. The analysis of Granger causality between DAX and commodities revealed same result as FTSE100. In all cases except one we reject null hypothesis of the Granger causality test. Relationships of NIKKEI with commodities suggest different results than results of previous indices. We reject the null hypothesis only in cases that NIKKEI is Granger-caused by Crude oil, Heating oil and Natural gas. In the rest of cases the test suggests that there is no causal relationship. Basically changes of NIKKEI does not have impact on commodities.

Table 7.2: Results of Granger causality tests between stock markets and commodities

Direction of causality	# of lags	F - Value (GC)	H_0 of GC	Granger Causality
S&P500 → Gold	2	1.9368	Do not reject	NO
Gold → S&P500	2	5.4405	Reject	YES
S&P500 → Crude Oil	7	2.4727	Reject	YES
Crude Oil → S&P500	7	2.6790	Reject	YES
S&P500 → Heating Oil	2	2.6267	Reject (7%)	YES
Heating Oil → S&P500	2	1.4535	Do not reject	NO
S&P500 → Natural Gas	4	1.4647	Do not reject	NO
Natural Gas → S&P500	4	1.2192	Do not reject	NO
FTSE100 → Gold	10	3.1221	Reject	YES
Gold → FTSE100	10	2.2092	Reject	YES
FTSE100 → Crude Oil	7	3.8661	Reject	YES
Crude Oil → FTSE100	7	2.1861	Reject	YES
FTSE100 → Heating Oil	5	3.4879	Reject	YES
Heating Oil → FTSE100	5	2.3831	Reject	YES
FTSE100 → Natural Gas	5	1.4836	Do not reject	NO
Natural Gas → FTSE100	5	2.1682	Reject (5.5%)	YES
DAX → Gold	10	3.0580	Reject	YES
Gold → DAX	10	2.1835	Reject	YES
DAX → Crude Oil	7	4.1672	Reject	YES
Crude Oil → DAX	7	2.0209	Reject	YES
DAX → Heating Oil	5	3.3905	Reject	YES
Heating Oil → DAX	5	2.7596	Reject	YES
DAX → Natural Gas	5	1.9545	Do not reject	NO
Natural Gas → DAX	5	2.4304	Reject	YES
NIKKEI → Gold	2	1.8922	Do not reject	NO
Gold → NIKKEI	2	0.6133	Do not reject	NO
NIKKEI → Crude Oil	2	1.3036	Do not reject	NO
Crude Oil → NIKKEI	2	31.4725	Reject	YES
NIKKEI → Heating Oil	1	2.7220	Do not reject	NO
Heating Oil → NIKKEI	1	47.5014	Reject	YES
NIKKEI → Natural Gas	4	1.3269	Do not reject	NO
Natural Gas → NIKKEI	4	2.6602	Reject	YES

Source: author's computations.

In the Table 7.3 we tested commodities between each other and results are that Gold Granger-causes Crude oil, but it seems that except this case, Gold does not Granger-cause others and is not Granger-caused by others. When we take a look at energy commodities, we conclude that we reject null hypothesis in all cases except one, which is that Heating oil Granger-causes Crude oil.

Table 7.3: Results of Granger causality tests between commodities

Direction of causality	# of lags	F - Value (GC)	H_0 of GC	Granger Causality
Gold → Crude Oil	2	3.7014	Reject	YES
Crude Oil → Gold	2	0.3109	Do not reject	NO
Gold → Heating Oil	1	0.2549	Do not reject	NO
Heating Oil → Gold	1	1.0382	Do not reject	NO
Gold → Natural Gas	4	1.3412	Do not reject	NO
Natural Gas → Gold	4	0.9190	Do not reject	NO
Crude Oil → Heating Oil	7	2.9672	Reject	YES
Heating Oil → Crude Oil	7	1.3615	Do not reject	NO
Crude Oil → Natural Gas	7	3.5002	Reject	YES
Natural Gas → Crude Oil	7	3.6442	Reject	YES
Heating Oil → Natural Gas	7	1.9141	Reject (6.5%)	YES
Natural Gas → Heating Oil	7	5.3848	Reject	YES

Source: author's computations.

7.2.2 The Granger causality test of MODWT MRA coefficients of stock markets and commodities

The MODWT multiresolution analysis decompose examined time series into wavelet scales that are associated to changes in 1 -2, 2 - 4, 4 - 8, 8 - 16 days, respectively. We are going to use a filter denoted by LA(8) of the length $L = 8$. Results were acquired by using R 2.15.1 and the package Waveslim, which was written by Whitcher (2012)[10]. We test causality of all time series at each scale separately, so we will be able to compare scales between each other. Detailed results can be found in the Appendix C, in this chapter we present final results. All results were compared to 5% significance level. Every time the null hypothesis is rejected, the relationship is considered Granger causal and we denote it with YES and green color, on the other hand, NO and red color mean that we did not reject the null hypothesis and there is no sign of causality in that particular relationship. All examined time series were tested with the augmented Dickey-Fuller test and the null hypothesis was rejected in all cases. We begin with causality of stock markets, then we continue with the relationship of stock markets and commodities and we end with commodities.

The Granger causality test of the MODWT MRA coefficients of stock markets

Results of testing relationships of stock markets indicate that there are strong causal relations at all scales, even though at scales 1 and 2, we observed several exceptions that indicate that the null hypothesis of Granger causality test could

[10] http://cran.r-project.org/web/packages/waveslim/index.html

not be rejected. We conclude that at scales associated to 4 - 8 and 8 - 16 days all of the stock markets have causal relations, see Table 7.4. Testing the Granger causality confirms results of the wavelet correlation, which also suggested that relations of stock markets are strong especially on low frequencies.

Table 7.4: Results of Granger causality tests between stock markets - MODWT MRA coefficients

Direction of causality	Scale 1	Scale 2	Scale 3	Scale 4
S&P500 → FTSE100	YES	YES	YES	YES
FTSE100 → S&P500	YES	NO	YES	YES
S&P500 → DAX	YES	YES	YES	YES
DAX → S&P500	NO	YES	YES	YES
S&P500 → NIKKEI	YES	YES	YES	YES
NIKKEI → S&P500	YES	YES	YES	YES
FTSE100 → DAX	YES	YES	YES	YES
DAX → FTSE100	YES	NO	YES	YES
FTSE100 → NIKKEI	YES	YES	YES	YES
NIKKEI → FTSE100	NO	NO	YES	YES
DAX → NIKKEI	YES	YES	YES	YES
NIKKEI → DAX	YES	YES	YES	YES

Source: author's computations.

The Granger causality test of MODWT MRA coefficients of stock markets and commodities

We are going to analyze relations of stock markets and commodities twice. Firstly we analyze results from the point of view of stock markets and secondly from the point of view of commodities. Both tables contain same results, but the only thing that differs is the order, which should make results clearer.

We begin with Table 7.5, which provides results of stock markets and their causal relations to commodities,. If we take a look at results in general, we conclude that the lowest number of causal relations is at scale 1, on the other hand the highest number of causal relations can be found at scale 3. Hence, we observe that causal relations differ at each scale and in general are higher at low frequencies. S&P 500 tends to have stronger causal relations with commodities with every additional scale. The weakest causal relations are observed with Natural gas and the strongest one is with Gold, it has a causal relation at all scales. FTSE 100 has stronger causal relations to commodities in comparison to S&P 500. Also causality increases with every additional scale and Gold Granger-causes and is Granger-caused at all scales. Results of DAX and commodities are very similar to results of FTSE 100. They indicate that at scale 1 it Granger-causes and is Granger-caused only by Gold. Starting with scale 2 causality rapidly increases. Testing NIKKEI and commodities revealed that results differ from the rest of stock markets. The main difference is observable at scale 1 where NIKKEI Granger-causes all commodities except Natural gas, also all commodities except Natural gas Granger-cause NIKKEI. On the other hand at the scale 4 causality changes and only Natural gas Granger-causes NIKKEI, the rest does not.

We continue with Table 7.6, which provides same results but from the point of view of commodities. Results indicate Gold Granger-causes and is Granger-caused by all stock markets at all scales. The only exception is NIKKEI at scale 4. Crude oil has lowest number of causal relations with stock markets at scale 1 and at scales 2, 3, 4 has much stronger causal relations with stock markets. Heating oil is not Granger-caused and does not Granger-cause any stock market at scale 1 except NIKKEI and in general has lower number of causal relations with stock markets than Crude oil. The lowest number of causal relations of all examined commodities has Natural gas, there is no causality present at scale 1 and at scale 2 it is only Granger-caused by FTSE 100 and DAX.

Table 7.5: Results of Granger causality tests of stock markets and commodities - MODWT MRA coefficients

Direction of causality	Scale 1	Scale 2	Scale 3	Scale 4
S&P500 → Gold	YES	YES	YES	YES
Gold → S&P500	YES	YES	YES	YES
S&P500 → Crude oil	NO	YES	YES	YES
Crude oil → S&P500	NO	YES	YES	YES
S&P500 → Heating oil	NO	NO	YES	NO
Heating oil → S&P500	NO	NO	YES	YES
S&P500 → Natural gas	NO	NO	NO	NO
Natural gas → S&P500	NO	NO	NO	YES
FTSE100 → Gold	YES	YES	YES	YES
Gold → FTSE100	YES	YES	YES	YES
FTSE100 → Crude oil	NO	NO	YES	YES
Crude oil → FTSE100	YES	YES	YES	YES
FTSE100 → Heating oil	NO	YES	YES	YES
Heating oil → FTSE100	NO	YES	YES	YES
FTSE100 → Natural gas	NO	YES	YES	NO
Natural gas → FTSE100	NO	NO	YES	YES
DAX → Gold	YES	YES	YES	YES
Gold → DAX	YES	YES	YES	YES
DAX → Crude oil	NO	NO	YES	YES
Crude oil → DAX	NO	YES	YES	YES
DAX → Heating oil	NO	YES	YES	NO
Heating oil → DAX	NO	YES	YES	YES
DAX → Natural gas	NO	YES	YES	YES
Natural gas → DAX	NO	NO	NO	NO
NIKKEI → Gold	YES	YES	YES	NO
Gold → NIKKEI	YES	YES	YES	NO
NIKKEI → Crude oil	YES	YES	YES	YES
Crude oil → NIKKEI	YES	YES	YES	NO
NIKKEI → Heating oil	YES	YES	NO	YES
Heating oil → NIKKEI	YES	YES	YES	NO
NIKKEI → Natural gas	NO	NO	YES	YES
Natural gas → NIKKEI	NO	NO	YES	YES

Source: author's computations.

Table 7.6: Results of Granger causality tests of commodities and stock markets - MODWT MRA coefficients

Direction of causality	Scale 1	Scale 2	Scale 3	Scale 4
Gold → S&P500	YES	YES	YES	YES
S&P500 → Gold	YES	YES	YES	YES
Gold → FTSE100	YES	YES	YES	YES
FTSE100 → Gold	YES	YES	YES	YES
Gold → DAX	YES	YES	YES	YES
DAX → Gold	YES	YES	YES	YES
Gold → NIKKEI	YES	YES	YES	NO
NIKKEI → Gold	YES	YES	YES	NO
Crude oil → S&P500	NO	YES	YES	YES
S&P500 → Crude oil	NO	YES	YES	YES
Crude oil → FTSE100	YES	YES	YES	YES
FTSE100 → Crude oil	NO	NO	YES	YES
Crude oil → DAX	NO	YES	YES	YES
DAX → Crude oil	NO	NO	YES	YES
Crude oil → NIKKEI	YES	YES	YES	NO
NIKKEI → Crude oil	YES	YES	YES	YES
Heating oil → S&P500	NO	NO	YES	YES
S&P500 → Heating oil	NO	NO	YES	NO
Heating oil → FTSE100	NO	YES	YES	YES
FTSE100 → Heating oil	NO	YES	YES	YES
Heating oil → DAX	NO	YES	YES	YES
DAX → Heating oil	NO	YES	YES	NO
Heating oil → NIKKEI	YES	YES	YES	NO
NIKKEI → Heating oil	YES	YES	NO	YES
Natural gas → S&P500	NO	NO	NO	YES
S&P500 → Natural gas	NO	NO	NO	NO
Natural gas → FTSE100	NO	NO	YES	YES
FTSE100 → Natural gas	NO	YES	YES	NO
Natural gas → DAX	NO	NO	NO	NO
DAX → Natural gas	NO	YES	YES	YES
Natural gas → NIKKEI	NO	NO	YES	YES
NIKKEI → Natural gas	NO	NO	YES	YES

Source: author's computations.

The Granger causality test of MODWT MRA coefficients of commodities

Empirical results of testing Granger causality of commodities revealed that causality in general increases with every additional scale, see Table 7.7. The strongest causal relations between commodities are at scale 4. An important observation is that Crude oil and Heating oil have much stronger causal relationship with Gold, than with each other at scale 1, we rejected that Crude oil Granger-causes Heating oil and vice versa at scale 1, also Crude oil Granger-causes Natural gas and vice versa at scale 1, but not at scale 2. In general we observe that commodities have stronger causal relations at low frequencies than at high frequencies.

Table 7.7: Results of Granger causality tests of commodities - MODWT MRA coefficients

Direction of causality	Scale 1	Scale 2	Scale 3	Scale 4
Gold → Crude oil	YES	NO	YES	YES
Crude oil → Gold	YES	YES	YES	YES
Gold → Heating oil	YES	YES	YES	YES
Heating oil → Gold	YES	YES	YES	YES
Gold → Natural gas	NO	YES	YES	YES
Natural gas → Gold	NO	YES	NO	YES
Crude oil → Heating oil	NO	YES	YES	YES
Heating oil → Crude oil	NO	NO	YES	YES
Crude oil → Natural gas	YES	NO	YES	YES
Natural gas → Crude oil	YES	NO	YES	YES
Heating oil → Natural gas	YES	YES	NO	YES
Natural gas → Heating oil	NO	YES	YES	YES

Source: author's computations.

To sum up, in this chapter we analyzed causal relations between the examined time series and the Granger causality test revealed that majority of pairs of stock markets have causal relations, which leads us to the conclusion that stock markets are highly interconnected. We also searched for causality between pairs of stock market indices and commodities, based on that we can conclude that strongest causal relations with commodities have DAX and FTSE100. In the case of causal relations between commodities, we conclude that Gold has very weak causal relations with other examined commodities with one exception, which is Crude oil. The other interesting result is that Crude oil Granger-causes Heating oil, but Heating oil does not Granger-cause Crude oil, which is on the contrary with results from previous chapters, where we noticed strong comovement and the wavelet correlation of Crude oil and Heating oil.

We also analyzed Granger causality of the MODWT MRA coefficients and we conclude that causality depends on scale we decide to analyze. In general, the level of causality is low at low scales (high frequencies) and the number of causal relations is increasing with additional scales. We observed that Gold has very strong causal relations with stock markets indices at all scales. On the other hand, Natural gas shows very weak causal relations with stock markets. Also results of the relation of Gold and the rest of commodities show us that when we focus on scales we observe much stronger causal relations than in the case of returns.

Chapter 8

Conclusion

The thesis revolves around the topic of wavelets and their application to stock markets and commodity markets in the time of the Global financial crisis. We analyze relations of four stock market indices: S&P500 (USA), FTSE100 (UK), DAX (Germany) and NIKKEI (Japan) and four commodities: Gold, Crude oil, Heating oil and Natural gas. First part describes the theoretical background of wavelets and the motivation why wavelets can be such a useful tool in the analysis of time series. The analysis is conducted on dataset of daily returns, which includes days from 1.1. 2007 until 29.11.2011, in total we have 1140 daily returns. In the last chapter we analyze the dataset from a slightly different point of view, we use the Granger causality test to reveal causal relations between examined time series.

Acquired results of the wavelet correlation indicate that stock market indices are highly correlated, especially on low frequencies. The wavelet correlation tends to grow with every additional scale between all pairs of stock market indices, except DAX and FTSE100, where the lowest correlation was observed at the scale 3. The analysis of the wavelet correlation can be very useful, especially for investors and potential diversification of their portfolios. On the one hand, short term investor can quite well diversify his portfolio, because in many cases the wavelet correlation is low, on the other hand, long term investor is in a more complicated situation, because the wavelet correlation is much higher. In the analysis of stock market indices and commodities, results suggest that the wavelet correlation is low. It is hard to make the conclusion, especially at high scales (low frequencies), when confidence intervals are very wide. We can conclude that the lowest correlation have stock market indices with Gold. This indicates that the wavelet correlation confirmed something what is generally known, Gold is a safe haven for investors during the crisis. We also observed a strong positive correlation between Crude oil and Heating oil.

Testing contagion based on the wavelet correlation of two time windows, which are situated before and after the bankruptcy of Lehman Brothers bank revealed that there was no contagion coming from the US stock market to other examined stock markets with one exception, which is German stock market. The sign of contagion can be found in the relation of the US stock market and commodity markets in case of Crude oil and Heating oil market. To confirm the results, we also analyzed Pearson's correlation coefficient, which revealed same results, but with one additional infected market, which was the Natural gas market.

In the following chapter we analyzed the wavelet coherence of examined time series, which is an excellent tool that allows us to see their comovement in the frequency and the time domain at the same time. Stock market indices comove strongly in the whole period. When we looked at comovement of stock market indices and commodities, we could see that there is most of the time no comovement in the case of Gold and Natural gas. On the other hand, Crude oil and Heating oil market comoved with stock markets especially in the second half of 2008 and at almost all frequencies. So even during the crisis, when markets become volatile, commodities were not comoving with indices in the way that indices with each other. Between commodities we observed a very strong comovement only between Crude oil and Heating oil, other pairs show weak, temporary or even no comovement.

In the last chapter we were studying causal relations between stock market indices and commodities. Obtained results suggest that the majority of stock markets have causal relations with others. This means that if we involve them in a model, which is predicting a development of one stock market index, we can acquire more precise prediction. Very strong relations with commodities have FTSE100 and DAX. In the case of commodities we observed that causal relations are mostly between Crude oil, Heating oil and Natural gas, which are considered to be energy commodities. On the other hand Gold seems not to have causal relations with any other commodity except Crude oil.

We also analyzed Granger causality of MODWT multiresolution analysis coefficients and acquired results suggest that causal relations differ at different scales. Results of the analysis of stock markets showed that the majority of pairs at scales 1 and 2 have causal relations and at scales 3 and 4 even all of them. In the analysis of causal relations of stock markets and commodities we observed that causality depends on which scale we are analyzing data. In general, the lowest number of causal relations from the point of view of stock markets is at scale 1 and the highest number of them is at scale 3. By analyzing same results, but from the point of view of commodities, we observed that Gold with exception of NIKKEI at scale 4 Granger-causes and is Granger-caused by all stock market indices. Crude oil and Heating have weak causal relations with stock markets at scale 1. The weakest causal relation with stock markets has in general Natural gas. In the case of commodities and their causal relations we conclude that causality depends on scale on which we are analyzing data, in general causality grows with every additional scale.

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Appendix A

Results of the Wavelet Correlation

Table A.1: The wavelet correlation of stock markets

	Scale	Wavelet Correlation	Lower CI	Upper CI
S&P500 - FTSE100	d1	0.476821743	0.410782266	0.537884023
	d2	0.771006749	0.719250909	0.81425227
	d3	0.857800575	0.807122212	0.895930103
	d4	0.892431346	0.83245	0.931739804
	d5	0.907046398	0.822381032	0.952408385
	d6	0.96239347	0.896392372	0.986646002
	s6	0.939825119	0.696271331	0.98930903
S&P500 - DAX	d1	0.550868562	0.490954098	0.605596568
	d2	0.769571434	0.717542931	0.813059404
	d3	0.875776233	0.830921161	0.90932101
	d4	0.853229904	0.773999929	0.906153218
	d5	0.893192623	0.797250307	0.945120792
	d6	0.949956105	0.863643609	0.982156589
	s6	0.96996466	0.83822528	0.994731091
S&P500 - NIKKEI	d1	-0.04910241	-0.130700718	0.033156662
	d2	0.378180845	0.274034796	0.473559819
	d3	0.68953865	0.592061125	0.767130748
	d4	0.75315768	0.630672195	0.839031427
	d5	0.772873552	0.592136055	0.87957819
	d6	0.917730608	0.782049973	0.970351143
	s6	0.90785698	0.563996318	0.983404509
FTSE100 - DAX	d1	0.896693421	0.879322484	0.911681406
	d2	0.891839258	0.865316444	0.913381556
	d3	0.877676454	0.833446597	0.910732525
	d4	0.881748423	0.816388137	0.92480559
	d5	0.895940721	0.802209051	0.946570513
	d6	0.943672479	0.847370682	0.9798744
	s6	0.901707788	0.540427209	0.982250098
FTSE100 - NIKKEI	d1	0.271128375	0.193307289	0.345559338
	d2	0.560348642	0.475092609	0.635182012
	d3	0.734479096	0.648173519	0.802154686
	d4	0.782213471	0.671443737	0.858793183
	d5	0.757182554	0.567008362	0.87072132
	d6	0.881665865	0.695936979	0.956834647
	s6	0.80910185	0.242943931	0.964094313
DAX - NIKKEI	d1	0.244621937	0.165828	0.320312336
	d2	0.571437689	0.487625899	0.644813236
	d3	0.722025457	0.632529528	0.792495472
	d4	0.754596061	0.63267477	0.840015075
	d5	0.778760344	0.601654757	0.882882051
	d6	0.905721121	0.752787366	0.965886433
	s6	0.882618	0.470700626	0.978627003

Table A.2: The wavelet correlation of S&P500 and commodities

	Scale	Wavelet Correlation	Lower CI	Upper CI
S&P500 - Gold	d1	0.027998547	-0.054251565	0.109871276
	d2	0.019553525	-0.096853574	0.135432901
	d3	0.044768371	-0.120850121	0.207962015
	d4	0.173365008	-0.062465239	0.390859754
	d5	-0.053252033	-0.379759347	0.285053079
	d6	-0.117003073	-0.565827449	0.385310358
	s6	0.304694332	-0.509338612	0.830953386
S&P500 - Crude oil	d1	0.43738215	0.368493119	0.501493775
	d2	0.483588422	0.389273353	0.567868799
	d3	0.411135242	0.264308296	0.539335892
	d4	0.365089642	0.144056146	0.55142479
	d5	0.271454983	-0.067937264	0.554537289
	d6	0.17594821	-0.332844768	0.605395532
	s6	0.887965023	0.489725607	0.979647872
S&P500 - Heating oil	d1	0.396234309	0.324674127	0.463284002
	d2	0.395881345	0.293179451	0.489550882
	d3	0.424730142	0.279562346	0.550912988
	d4	0.420320243	0.207349693	0.595252189
	d5	0.422476134	0.103850579	0.662456309
	d6	0.427631002	-0.066729189	0.753419066
	s6	0.903833858	0.54851255	0.982649924
S&P500 - Natural gas	d1	0.112483162	0.030640847	0.192827244
	d2	0.120646044	0.004522376	0.233559187
	d3	0.239988648	0.078359119	0.389324869
	d4	0.237104493	0.004023318	0.445750485
	d5	0.140961387	-0.201762951	0.452931521
	d6	-0.342862576	-0.7069976	0.164968131
	s6	0.484620067	-0.334172221	0.886540588

Table A.3: The wavelet correlation of FTSE 100 and commodities

	Scale	Wavelet Correlation	Lower CI	Upper CI
FTSE100 - Gold	d1	0.104377395	0.022444423	0.184917641
	d2	0.039345781	-0.077194171	0.154825064
	d3	0.05877898	-0.106984169	0.221362908
	d4	0.173865728	-0.061950955	0.391297074
	d5	-0.01724111	-0.348485418	0.317831548
	d6	-0.179386253	-0.60763968	0.329684239
	s6	0.438212393	-0.385502245	0.873233845
FTSE100 - Crude oil	d1	0.341711456	0.267081238	0.412267291
	d2	0.462066313	0.365501619	0.548790916
	d3	0.445544871	0.303060213	0.56854472
	d4	0.393643664	0.17655443	0.574212557
	d5	0.336279873	0.003416157	0.602057394
	d6	0.153672114	-0.353049486	0.590687685
	s6	0.746385764	0.087993937	0.950916632
FTSE100 - Heating oil	d1	0.328340771	0.253038618	0.399688495
	d2	0.401783715	0.29958175	0.494869256
	d3	0.435149155	0.291302732	0.559752824
	d4	0.439240699	0.22945022	0.610030632
	d5	0.408768099	0.087431526	0.653055113
	d6	0.374013512	-0.130001277	0.724429329
	s6	0.782035877	0.17233107	0.958499416
FTSE100 - Natural gas	d1	0.033917429	-0.048342595	0.115720458
	d2	0.091267547	-0.025186596	0.205277819
	d3	0.227038497	0.064732169	0.377642169
	d4	0.249947513	0.01767444	0.456623827
	d5	0.190903264	-0.152014091	0.492798994
	d6	-0.343235854	-0.707209135	0.164556549
	s6	0.486323747	-0.332190641	0.88701676

Table A.4: The wavelet correlation of DAX and commodities

	Scale	Wavelet Correlation	Lower CI	Upper CI
DAX - Gold	d1	0.074150519	-0.008023709	0.155329982
	d2	0.023512153	-0.092928772	0.139318622
	d3	0.070987242	-0.094849154	0.23299004
	d4	0.096525491	-0.13992942	0.322565136
	d5	0.031217865	-0.30520379	0.360711835
	d6	-0.201651998	-0.622009729	0.30893574
	s6	0.324062128	-0.493247783	0.837488107
DAX - Crude oil	d1	0.31338128	0.237364868	0.385583068
	d2	0.438875276	0.340026732	0.528132025
	d3	0.431277551	0.286935054	0.556471335
	d4	0.263818947	0.032516608	0.468298909
	d5	0.207749529	-0.134831655	0.505967057
	d6	0.109718331	-0.391576431	0.560789556
	s6	0.854720333	0.377286285	0.973222827
DAX - Heating oil	d1	0.282750429	0.205394525	0.356595301
	d2	0.377966851	0.273803846	0.473366106
	d3	0.426370947	0.281408368	0.552306983
	d4	0.349349596	0.126343428	0.538743292
	d5	0.359360966	0.029667246	0.618534425
	d6	0.358243917	-0.147860696	0.715660489
	s6	0.868158365	0.421102097	0.975842422
DAX - Natural gas	d1	0.024297215	-0.057943773	0.106210647
	d2	0.113330892	-0.002894279	0.226535035
	d3	0.242808613	0.08133449	0.391862485
	d4	0.227227812	-0.006416057	0.437346456
	d5	0.164104907	-0.17892667	0.471562906
	d6	-0.234145765	-0.642484701	0.277758165
	s6	0.455094372	-0.367401543	0.878150773

Table A.5: The wavelet correlation of NIKKEI and commodities

	Scale	Wavelet Correlation	Lower CI	Upper CI
NIKKEI - Gold	d1	0.104977293	0.023050654	0.185503379
	d2	0.016192473	-0.100183068	0.132130949
	d3	0.027436729	-0.137913858	0.191299411
	d4	0.156934251	-0.079271893	0.376454718
	d5	-0.033569854	-0.362758209	0.303067114
	d6	-0.20292891	-0.622825336	0.307730921
	s6	0.181008381	-0.600224164	0.785490638
NIKKEI - Crude oil	d1	0.010906745	-0.07128247	0.092948855
	d2	0.251228302	0.139101542	0.356994869
	d3	0.37383285	0.22282739	0.507319173
	d4	0.328465406	0.103059336	0.521783704
	d5	0.401043862	0.078255949	0.647724329
	d6	0.136730848	-0.368105426	0.579304332
	s6	0.893826215	0.511025021	0.980761485
NIKKEI - Heating oil	d1	0.037963853	-0.044299631	0.119715969
	d2	0.221367705	0.107957313	0.329090537
	d3	0.374905038	0.224012097	0.508244607
	d4	0.39888561	0.182572331	0.578365758
	d5	0.498660662	0.198380359	0.713362763
	d6	0.3536332	-0.153020811	0.713075188
	s6	0.861306208	0.398496282	0.974510528
NIKKEI - Natural gas	d1	-0.018593361	-0.100565165	0.063629166
	d2	0.058487663	-0.058094178	0.173495732
	d3	0.262819886	0.102531573	0.409805216
	d4	0.264587458	0.033341894	0.468943656
	d5	0.148964274	-0.19390823	0.459405108
	d6	-0.206238149	-0.624934822	0.304600875
	s6	0.393263142	-0.430791557	0.859695601

Table A.6: The wavelet correlation of commodities

	Scale	Wavelet Correlation	Lower CI	Upper CI
Gold - Crude oil	d1	0.278711923	0.201191698	0.35276279
	d2	0.245055106	0.132644737	0.351241537
	d3	0.396732606	0.248227675	0.527017851
	d4	0.456675119	0.250008269	0.623543977
	d5	0.235586455	-0.105980442	0.527425669
	d6	0.196109568	-0.314146329	0.618458985
	s6	0.181867771	-0.599655367	0.785830756
Gold - Heating oil	d1	0.28902052	0.211925397	0.362540652
	d2	0.242528262	0.130004553	0.348884237
	d3	0.310075542	0.15317197	0.451729427
	d4	0.382841416	0.164204121	0.565624513
	d5	0.162510223	-0.180512131	0.470287872
	d6	0.251335407	-0.260816827	0.653085459
	s6	0.152844527	-0.618437455	0.774138552
Gold - Natural gas	d1	0.122091552	0.040370978	0.2021896
	d2	0.073395442	-0.043159668	0.187979489
	d3	0.07493106	-0.090918438	0.236736254
	d4	0.121888865	-0.114675923	0.345371233
	d5	0.084463585	-0.255988313	0.406274637
	d6	0.069828298	-0.425083374	0.532597697
	s6	0.127073717	-0.634407489	0.763388693
Crude oil - Heating oil	d1	0.869026401	0.847377622	0.887790918
	d2	0.84207227	0.804606191	0.872860492
	d3	0.790427493	0.719365131	0.845116413
	d4	0.825099613	0.732873084	0.887550855
	d5	0.866441803	0.749647544	0.930901131
	d6	0.792559649	0.503852138	0.921984113
	s6	0.963435474	0.805880112	0.99356814
Crude oil - Natural gas	d1	0.310167206	0.234002491	0.38254808
	d2	0.142398388	0.026652285	0.254376822
	d3	0.337010009	0.182409129	0.475348438
	d4	0.277977966	0.047772431	0.480142853
	d5	0.279911624	-0.058821793	0.560842971
	d6	0.002854285	-0.478446861	0.482836649
	s6	0.486529185	-0.331951204	0.887074122
Heating oil - Natural gas	d1	0.314310408	0.238337206	0.386460134
	d2	0.15261453	0.037085055	0.26411829
	d3	0.367822529	0.216194571	0.502125799
	d4	0.390747339	0.173236315	0.571913802
	d5	0.302201851	-0.03452125	0.577308928
	d6	0.030390485	-0.45692865	0.503678013
	s6	0.605168574	-0.173487307	0.918255289

Appendix B

Results of the Analysis of Contagion - The Wavelet Correlation

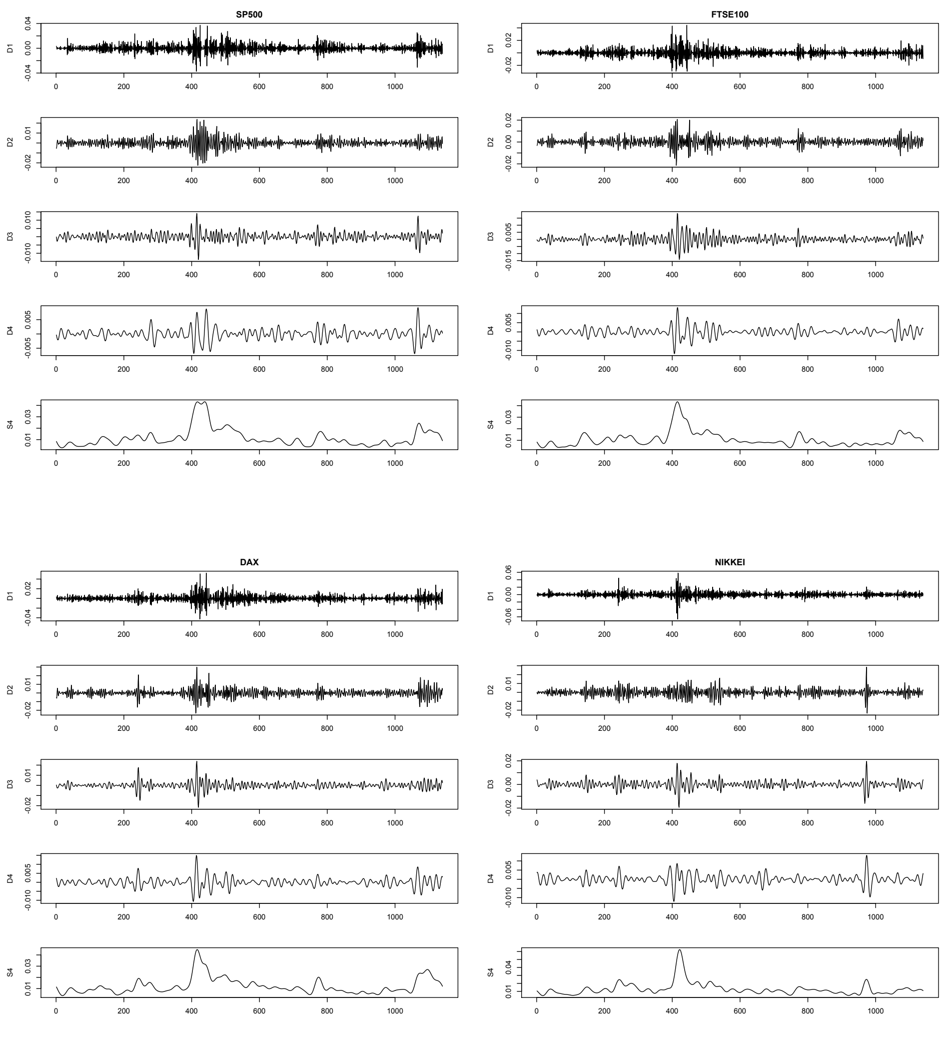
Table B.1: Results of the analysis of ontagion - the wavelet correlation

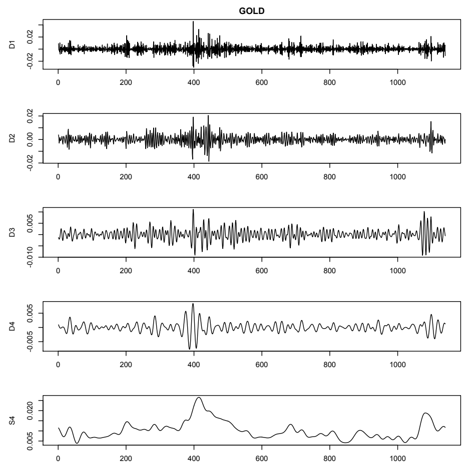
	Scale	Lower CI	WCor before LB	Upper CI		Scale	Lower CI	WCor after LB	Upper CI
S&P500 - FTSE100	d1	0.243732717	0.40212861	0.539629235	S&P500 - FTSE100	d1	0.333918869	0.4813034	0.605715542
	d2	0.451103316	0.629899901	0.760084755		d2	0.675758876	0.791872414	0.869639383
	d3	0.623448505	0.800866868	0.899848613		d3	0.755089599	0.875249296	0.938520483
	d4	0.580498134	0.842290926	0.946264789		d4	0.738680847	0.90753397	0.969213432
	s4	0.682087029	0.948134661	0.992527584		s4	0.587619433	0.929405047	0.989744754
S&P500 - DAX	d1	0.28142865	0.435512702	0.567702713	S&P500 - DAX	d1	0.438405506	0.570120951	0.677862549
	d2	0.291199688	0.504289681	0.669704855		d2	0.734192614	0.831484819	0.895315309
	d3	0.510403577	0.732302124	0.862767788		d3	0.790401042	0.894279568	0.94816575
	d4	0.259567555	0.681247137	0.884751522		d4	0.56260926	0.834395992	0.943414932
	s4	0.424947001	0.892421499	0.98411189		s4	0.746179531	0.959870515	0.994248067
S&P500 - NIKKEI	d1	-0.248196175	-0.075896618	0.101057802	S&P500 - NIKKEI	d1	-0.187716403	-0.012521652	0.163445267
	d2	0.039874085	0.286786105	0.500689867		d2	0.229538271	0.453317182	0.631575718
	d3	0.425990873	0.67799117	0.832364485		d3	0.460925047	0.700806483	0.845250694
	d4	0.725971013	0.902576507	0.967506514		d4	0.396684585	0.755437427	0.913998733
	s4	0.397736272	0.885554426	0.983045265		s4	0.359879171	0.875632248	0.981492364
S&P500 - Gold	d1	-0.190049902	-0.014941028	0.161089151	S&P500 - Gold	d1	-0.190049902	-0.014941028	0.161089151
	d2	-0.467036128	-0.245959264	0.004058248		d2	-0.467036128	-0.245959264	0.004058248
	d3	-0.507961514	-0.187341141	0.178873014		d3	-0.507961514	-0.187341141	0.178873014
	d4	-0.58917383	-0.110157967	0.426152597		d4	-0.58917383	-0.110157967	0.426152597
	s4	-0.972067783	-0.817272023	-0.166986007		s4	-0.972067783	-0.817272023	-0.166986007
S&P500 - Crude oil	d1	-0.179100413	-0.003606318	0.172110199	S&P500 - Crude oil	d1	0.368466223	0.5110087	0.630075182
	d2	-0.315566744	-0.071428426	0.181579251		d2	0.386952897	0.580606349	0.725207179
	d3	-0.500027892	-0.177059092	0.189148011		d3	0.291841538	0.585621823	0.778430392
	d4	-0.682918929	-0.262480786	0.288584945		d4	-0.32865796	0.220774279	0.658557875
	s4	-0.789834435	-0.090758958	0.710885907		s4	-0.550240614	0.346319562	0.871969192
S&P500 - Heating oil	d1	-0.272767584	-0.102048176	0.074901612	S&P500 - Heating oil	d1	0.428617645	0.561929579	0.671294398
	d2	-0.406567677	-0.174521753	0.078675932		d2	0.293880533	0.506472626	0.671318526
	d3	-0.602691811	-0.315791445	0.043405664		d3	0.376771131	0.645000391	0.813431203
	d4	-0.742475314	-0.371515677	0.173829251		d4	0.121900218	0.596890277	0.849428011
	s4	-0.737440915	0.035121327	0.76787049		s4	-0.105003431	0.70369873	0.952174684
S&P500 - Natural gas	d1	-0.203193632	-0.028606872	0.147742846	S&P500 - Natural gas	d1	-0.0223703	0.15384157	0.320782903
	d2	-0.379820215	-0.143682767	0.110034279		d2	0.03667259	0.28384091	0.498283581
	d3	-0.384237427	-0.034609402	0.323700418		d3	0.103390889	0.441554424	0.688215966
	d4	-0.674333602	-0.247601106	0.303106856		d4	0.01552215	0.523621459	0.816794129
	s4	-0.836954412	-0.226915605	0.634579926		s4	-0.564260737	0.328298222	0.867007993

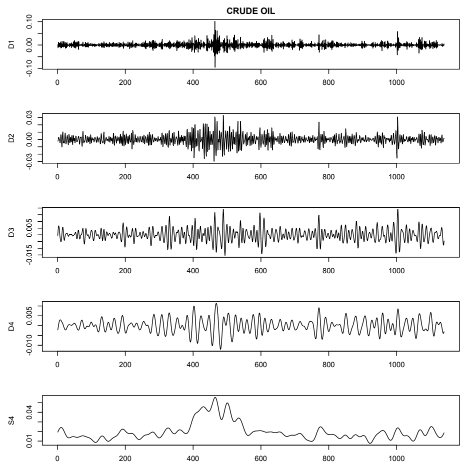
Appendix C

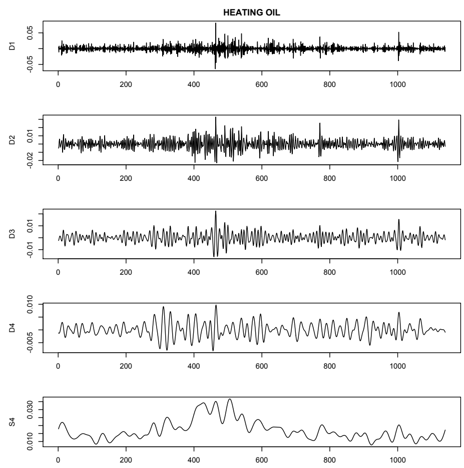
Results of the MODWT Multiresolution analysis

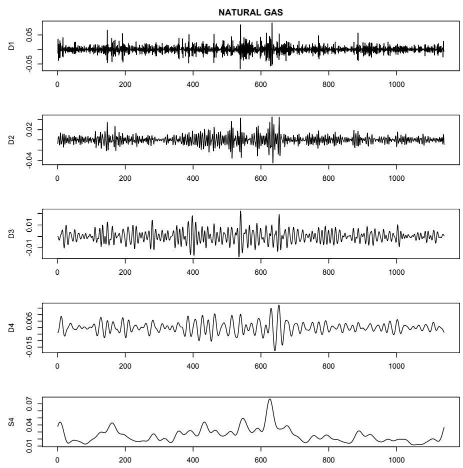
Figure C.1: Results of the MODWT Multiresolution analysis




GOLD
D1
D2
D3
D4
S4


CRUDE OIL
D1
D2
D3
D4
S4


HEATING OIL
D1
D2
D3
D4
S4


NATURAL GAS
D1
D2
D3
D4
S4

Appendix D

Results of the Granger causality test of MODWT Multiresolution analysis

Table D.1: The Granger causality test of the MODWT Multiresolution analysis - Stock markets

Scale		# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
1	S&P500 → FTSE100	10	2.5743	0.0043	Reject	YES
	FTSE100 → S&P500	10	2.1434	0.0187	Reject	YES
	S&P500 → DAX	10	4.5118	0	Reject	YES
	DAX → S&P500	10	1.1641	0.3105	Do not reject	NO
	S&P500 → NIKKEI	10	9.0622	0	Reject	YES
	NIKKEI → S&P500	10	4.2637	0	Reject	YES
	FTSE100 → DAX	10	2.6399	0.0034	Reject	YES
	DAX → FTSE100	10	2.4825	0.0059	Reject	YES
	FTSE100 → NIKKEI	10	4.6351	0	Reject	YES
	NIKKEI → FTSE100	10	1.5629	0.1116	Do not reject	NO
	DAX → NIKKEI	10	4.4668	0	Reject	YES
	NIKKEI → DAX	10	3.7834	0	Reject	YES
2	S&P500 → FTSE100	10	2.4659	0.0062	Reject	YES
	FTSE100 → S&P500	10	1.7899	0.0574	Do not reject	NO
	S&P500 → DAX	10	2.9047	0.0013	Reject	YES
	DAX → S&P500	10	1.9215	0.0382	Reject	YES
	S&P500 → NIKKEI	10	13.004	0	Reject	YES
	NIKKEI → S&P500	10	7.9358	0	Reject	YES
	FTSE100 → DAX	10	4.0239	0	Reject	YES
	DAX → FTSE100	10	1.7874	0.0578	Do not reject	NO
	FTSE100 → NIKKEI	10	3.4641	0.0002	Reject	YES
	NIKKEI → FTSE100	10	1.4825	0.1394	Do not reject	NO
	DAX → NIKKEI	10	6.3485	0	Reject	YES
	NIKKEI → DAX	10	3.6914	0.0001	Reject	YES
3	S&P500 → FTSE100	10	7.602	0	Reject	YES
	FTSE100 → S&P500	10	5.2972	0	Reject	YES
	S&P500 → DAX	10	7.2498	0	Reject	YES
	DAX → S&P500	10	5.1835	0	Reject	YES
	S&P500 → NIKKEI	10	12.353	0	Reject	YES
	NIKKEI → S&P500	10	12.5541	0	Reject	YES
	FTSE100 → DAX	9	4.3521	0	Reject	YES
	DAX → FTSE100	9	4.302	0	Reject	YES
	FTSE100 → NIKKEI	10	6.0342	0	Reject	YES
	NIKKEI → FTSE100	10	2.9548	0.0011	Reject	YES
	DAX → NIKKEI	10	6.7157	0	Reject	YES
	NIKKEI → DAX	10	3.493	0.0001	Reject	YES
4	S&P500 → FTSE100	10	9.561	0	Reject	YES
	FTSE100 → S&P500	10	2.8024	0.0019	Reject	YES
	S&P500 → DAX	10	3.8385	0	Reject	YES
	DAX → S&P500	10	2.5823	0.0041	Reject	YES
	S&P500 → NIKKEI	10	8.2582	0	Reject	YES
	NIKKEI → S&P500	10	5.3945	0	Reject	YES
	FTSE100 → DAX	10	7.1942	0	Reject	YES
	DAX → FTSE100	10	11.8019	0	Reject	YES
	FTSE100 → NIKKEI	10	8.7204	0	Reject	YES
	NIKKEI → FTSE100	10	2.6951	0.0028	Reject	YES
	DAX → NIKKEI	10	11.0609	0	Reject	YES
	NIKKEI → DAX	10	11.1431	0	Reject	YES

Table D.2: The Granger causality test of the MODWT Multiresolution analysis - Stock markets and commodities - scale 1

	# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
S&P500 → Gold	10	2.3799	0.0084	Reject	YES
Gold → S&P500	10	3.0403	0.0008	Reject	YES
S&P500 → Crude oil	10	1.2567	0.2497	Do not Reject	NO
Crude oil → S&P500	10	1.6722	0.0815	Do not Reject	NO
S&P500 → Heating oil	10	1.5647	0.111	Do not Reject	NO
Heating oil → S&P500	10	0.8548	0.5755	Do not Reject	NO
S&P500 → Natural gas	10	0.6458	0.7752	Do not Reject	NO
Natural gas → S&P500	10	0.554	0.8521	Do not Reject	NO
FTSE100 → Gold	10	2.1892	0.016	Reject	YES
Gold → FTSE100	10	3.9792	0	Reject	YES
FTSE100 → Crude oil	10	1.8334	0.0502	Do not Reject	NO
Crude oil → FTSE100	10	1.9472	0.0352	Reject	YES
FTSE100 → Heating oil	10	1.7477	0.0652	Do not Reject	NO
Heating oil → FTSE100	10	1.355	0.1954	Do not Reject	NO
FTSE100 → Natural gas	10	0.8837	0.5478	Do not Reject	NO
Natural gas → FTSE100	10	1.1759	0.3022	Do not Reject	NO
DAX → Gold	10	2.3006	0.011	Reject	YES
Gold → DAX	10	2.7309	0.0024	Reject	YES
DAX → Crude oil	10	1.6771	0.0804	Do not Reject	NO
Crude oil → DAX	10	1.6025	0.0997	Do not Reject	NO
DAX → Heating oil	10	1.041	0.4057	Do not Reject	NO
Heating oil → DAX	10	0.9422	0.4929	Do not Reject	NO
DAX → Natural gas	10	0.9445	0.4908	Do not Reject	NO
Natural gas → DAX	10	0.7687	0.6593	Do not Reject	NO
NIKKEI → Gold	10	2.398	0.0079	Reject	YES
Gold → NIKKEI	10	2.6751	0.003	Reject	YES
NIKKEI → Crude oil	10	2.3074	0.0108	Reject	YES
Crude oil → NIKKEI	10	4.8688	0	Reject	YES
NIKKEI → Heating oil	10	3.0714	0.0007	Reject	YES
Heating oil → NIKKEI	10	2.9048	0.0013	Reject	YES
NIKKEI → Natural gas	10	1.1172	0.3449	Do not Reject	NO
Natural gas → NIKKEI	10	1.1324	0.3335	Do not Reject	NO

Table D.3: The Granger causality test of the MODWT Multiresolution analysis - Stock markets and commodities - scale 2

	# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
S&P500 → Gold	10	2.7813	0.002	Reject	YES
Gold → S&P500	10	5.3803	0	Reject	YES
S&P500 → Crude oil	10	2.1461	0.0185	Reject	YES
Crude oil → S&P500	10	2.3782	0.0084	Reject	YES
S&P500 → Heating oil	10	0.8723	0.5587	Do not Reject	NO
Heating oil → S&P500	10	0.6681	0.755	Do not Reject	NO
S&P500 → Natural gas	10	0.9165	0.5168	Do not Reject	NO
Natural gas → S&P500	10	0.7199	0.7064	Do not Reject	NO
FTSE100 → Gold	10	4.2343	0	Reject	YES
Gold → FTSE100	10	4.4438	0	Reject	YES
FTSE100 → Crude oil	10	1.4759	0.142	Do not Reject	NO
Crude oil → FTSE100	10	2.0808	0.0229	Reject	YES
FTSE100 → Heating oil	10	2.8603	0.0015	Reject	YES
Heating oil → FTSE100	10	2.7918	0.0019	Reject	YES
FTSE100 → Natural gas	10	2.1044	0.0212	Reject	YES
Natural gas → FTSE100	10	0.8586	0.5719	Do not Reject	NO
DAX → Gold	10	3.7351	0.0001	Reject	YES
Gold → DAX	10	2.6384	0.0034	Reject	YES
DAX → Crude oil	10	1.7578	0.0632	Do not Reject	NO
Crude oil → DAX	10	2.3054	0.0108	Reject	YES
DAX → Heating oil	10	2.4506	0.0066	Reject	YES
Heating oil → DAX	10	2.5635	0.0044	Reject	YES
DAX → Natural gas	10	1.9691	0.0329	Reject	YES
Natural gas → DAX	10	1.2866	0.2321	Do not Reject	NO
NIKKEI → Gold	10	5.7389	0	Reject	YES
Gold → NIKKEI	10	2.4608	0.0063	Reject	YES
NIKKEI → Crude oil	10	2.6101	0.0037	Reject	YES
Crude oil → NIKKEI	10	3.1748	0.0005	Reject	YES
NIKKEI → Heating oil	10	3.1474	0.0005	Reject	YES
Heating oil → NIKKEI	10	2.0503	0.0253	Reject	YES
NIKKEI → Natural gas	10	1.6825	0.0791	Do not Reject	NO
Natural gas → NIKKEI	10	1.2584	0.2487	Do not Reject	NO

Table D.4: The Granger causality test of the MODWT Multiresolution analysis - Stock markets and commodities - scale 3

	# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
S&P500 → Gold	10	3.1629	0.0005	Reject	YES
Gold → S&P500	10	3.2088	0.0004	Reject	YES
S&P500 → Crude oil	10	2.6274	0.0035	Reject	YES
Crude oil → S&P500	10	3.3946	0.0002	Reject	YES
S&P500 → Heating oil	10	4.0529	0	Reject	YES
Heating oil → S&P500	10	3.0448	0.0008	Reject	YES
S&P500 → Natural gas	10	1.5285	0.1229	Do not Reject	NO
Natural gas → S&P500	10	1.117	0.345	Do not Reject	NO
FTSE100 → Gold	10	8.4149	0	Reject	YES
Gold → FTSE100	10	7.9738	0	Reject	YES
FTSE100 → Crude oil	10	3.4237	0.0002	Reject	YES
Crude oil → FTSE100	10	2.0017	0.0296	Reject	YES
FTSE100 → Heating oil	10	2.0533	0.0251	Reject	YES
Heating oil → FTSE100	10	2.0179	0.0281	Reject	YES
FTSE100 → Natural gas	10	2.5075	0.0054	Reject	YES
Natural gas → FTSE100	10	2.833	0.0017	Reject	YES
DAX → Gold	10	4.9059	0	Reject	YES
Gold → DAX	10	2.0434	0.0259	Reject	YES
DAX → Crude oil	10	4.0833	0	Reject	YES
Crude oil → DAX	10	3.6005	0.0001	Reject	YES
DAX → Heating oil	10	3.2861	0.0003	Reject	YES
Heating oil → DAX	10	2.2809	0.0118	Reject	YES
DAX → Natural gas	10	1.9128	0.0392	Reject	YES
Natural gas → DAX	10	1.3738	0.1862	Do not Reject	NO
NIKKEI → Gold	10	4.7727	0	Reject	YES
Gold → NIKKEI	10	5.6448	0	Reject	YES
NIKKEI → Crude oil	10	3.1391	0.0005	Reject	YES
Crude oil → NIKKEI	10	4.2145	0	Reject	YES
NIKKEI → Heating oil	10	1.5434	0.1179	Do not Reject	NO
Heating oil → NIKKEI	10	2.7127	0.0026	Reject	YES
NIKKEI → Natural gas	10	2.6058	0.0038	Reject	YES
Natural gas → NIKKEI	10	3.0785	0.0007	Reject	YES

Table D.5: The Granger causality test of the MODWT Multiresolution analysis - Stock markets and commodities - scale 4

	# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
S&P500 → Gold	10	6.9088	0	Reject	YES
Gold → S&P500	10	5.5505	0	Reject	YES
S&P500 → Crude oil	10	7.4243	0	Reject	YES
Crude oil → S&P500	10	3.3428	0.0002	Reject	YES
S&P500 → Heating oil	10	1.7631	0.0622	Do not Reject	NO
Heating oil → S&P500	10	2.0119	0.0286	Reject	YES
S&P500 → Natural gas	10	1.6122	0.097	Do not Reject	NO
Natural gas → S&P500	10	1.9764	0.0321	Reject	YES
FTSE100 → Gold	10	7.3735	0	Reject	YES
Gold → FTSE100	10	2.1566	0.0179	Reject	YES
FTSE100 → Crude oil	10	3.923	0	Reject	YES
Crude oil → FTSE100	10	2.5493	0.0046	Reject	YES
FTSE100 → Heating oil	10	3.7203	0.0001	Reject	YES
Heating oil → FTSE100	10	3.0453	0.0008	Reject	YES
FTSE100 → Natural gas	10	1.7153	0.0718	Do not Reject	NO
Natural gas → FTSE100	10	2.1867	0.0162	Reject	YES
DAX → Gold	10	8.6184	0	Reject	YES
Gold → DAX	10	7.1791	0	Reject	YES
DAX → Crude oil	10	2.0747	0.0234	Reject	YES
Crude oil → DAX	10	2.2712	0.0122	Reject	YES
DAX → Heating oil	10	1.3118	0.218	Do not Reject	NO
Heating oil → DAX	10	2.0364	0.0265	Reject	YES
DAX → Natural gas	10	3.5108	0.0001	Reject	YES
Natural gas → DAX	10	1.0817	0.3725	Do not Reject	NO
NIKKEI → Gold	10	1.7198	0.0708	Do not Reject	NO
Gold → NIKKEI	10	1.3821	0.1822	Do not Reject	NO
NIKKEI → Crude oil	10	6.0815	0	Reject	YES
Crude oil → NIKKEI	10	1.4698	0.1444	Do not Reject	NO
NIKKEI → Heating oil	10	4.3314	0	Reject	YES
Heating oil → NIKKEI	10	1.2522	0.2524	Do not Reject	NO
NIKKEI → Natural gas	10	1.8424	0.0489	Reject	YES
Natural gas → NIKKEI	10	1.9905	0.0307	Reject	YES

Table D.6: The Granger causality test of the MODWT Multiresolution analysis - Commodities

Scale		# of lags	F -Value (GC)	P - value	H0 of GC	Granger Causality
1	Gold → Crude oil	10	2.4312	0.007	Reject	YES
	Crude oil → Gold	10	3.3769	0.0002	Reject	YES
	Gold → Heating oil	10	2.3052	0.0108	Reject	YES
	Heating oil → Gold	10	2.6487	0.0033	Reject	YES
	Gold → Natural gas	10	1.2431	0.258	Do not reject	NO
	Natural gas → Gold	10	0.6577	0.7645	Do not reject	NO
	Crude oil → Heating oil	10	1.7133	0.0722	Do not reject	NO
	Heating oil → Crude oil	10	1.5534	0.1146	Do not reject	NO
	Crude oil → Natural gas	10	2.083	0.0228	Reject	YES
	Natural gas → Crude oil	10	1.8895	0.0422	Reject	YES
	Heating oil → Natural gas	10	2.1266	0.0197	Reject	YES
	Natural gas → Heating oil	10	1.3152	0.2161	Do not reject	NO
2	Gold → Crude oil	10	1.4935	0.1353	Do not reject	NO
	Crude oil → Gold	10	2.7647	0.0021	Reject	YES
	Gold → Heating oil	10	1.9995	0.0298	Reject	YES
	Heating oil → Gold	10	2.1626	0.0175	Reject	YES
	Gold → Natural gas	10	2.5925	0.004	Reject	YES
	Natural gas → Gold	10	2.5387	0.0048	Reject	YES
	Crude oil → Heating oil	10	1.9601	0.0338	Reject	YES
	Heating oil → Crude oil	10	1.1151	0.3465	Do not reject	NO
	Crude oil → Natural gas	10	0.6316	0.7878	Do not reject	NO
	Natural gas → Crude oil	10	1.3888	0.179	Do not reject	NO
	Heating oil → Natural gas	10	2.0196	0.0279	Reject	YES
	Natural gas → Heating oil	10	2.1858	0.0162	Reject	YES
3	Gold → Crude oil	10	4.2339	0	Reject	YES
	Crude oil → Gold	10	4.9763	0	Reject	YES
	Gold → Heating oil	10	3.6804	0.0001	Reject	YES
	Heating oil → Gold	10	5.9936	0	Reject	YES
	Gold → Natural gas	10	1.8559	0.0469	Reject	YES
	Natural gas → Gold	10	1.7441	0.0659	Do not reject	NO
	Crude oil → Heating oil	10	2.7874	0.002	Reject	YES
	Heating oil → Crude oil	10	3.6353	0.0001	Reject	YES
	Crude oil → Natural gas	10	2.3397	0.0096	Reject	YES
	Natural gas → Crude oil	10	2.6473	0.0033	Reject	YES
	Heating oil → Natural gas	10	1.6887	0.0777	Do not reject	NO
	Natural gas → Heating oil	10	1.9401	0.036	Reject	YES
4	Gold → Crude oil	10	2.4315	0.007	Reject	YES
	Crude oil → Gold	10	5.1615	0	Reject	YES
	Gold → Heating oil	10	2.0185	0.0281	Reject	YES
	Heating oil → Gold	10	4.0741	0	Reject	YES
	Gold → Natural gas	10	2.8257	0.0017	Reject	YES
	Natural gas → Gold	10	3.1196	0.0006	Reject	YES
	Crude oil → Heating oil	10	2.6459	0.0033	Reject	YES
	Heating oil → Crude oil	10	2.3709	0.0087	Reject	YES
	Crude oil → Natural gas	10	3.1529	0.0005	Reject	YES
	Natural gas → Crude oil	10	2.8026	0.0019	Reject	YES
	Heating oil → Natural gas	10	3.2572	0.0003	Reject	YES
	Natural gas → Heating oil	10	2.7577	0.0022	Reject	YES

Appendix E

Content of Enclosed DVD

There is a DVD enclosed to this thesis which contains empirical data and MatLab and R source codes.

- Folder 1: Source codes
- Folder 2: Empirical data