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Bakalářská práce

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Real Economic Convergence of the Czech Republic and Germany

Bakalářská práce

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Abstrakt

Občané České republiky vidí Německo nejen jako svého největšího obchodního partnera, ale také jako vyspělou zemi západní Evropy, na jejíž úroveň by chtěli svou zemi dostat. Srovnání s naším největším sousedem je pro nás relevantnější než například s průměrem Evropské unie. Snažím se hledat odpovědi na často kladené otázky, zda Česká republika dožene Německo a kdy se tak stane. Konvergenční teorii vysvětluji pomocí neoklasického modelu růstu v teoretické části práce. Soustředím se na determinanty růstu a rovnovážného stavu. Dále diskutuji argumenty předkládané kritiky neoklasického modelu růstu. Vysvětluji různé definice konvergence a poukazuji na jejich klady i zápory a předkládám výsledky významných konvergenčních studií. V empirické části práce zkoumám beta a sigma konvergenci hrubého národního produktu mezi regiony obou zemí v rozmezí let 1995-2009. Používám k tomu jak průřezové tak panelové modely. Výsledky této analýzy pak porovnávám s teorií.

Abstract

Citizens of the Czech Republic do not view Germany solely as their biggest trade partner, but also as a benchmark of advanced economy in Western Europe. This is the status that the Czech Republic would like to achieve. Comparison with Germany is more relevant than with the average of the European Union. In this paper, I search for answers to frequently asked questions such as whether the Czech Republic is catching up to Germany's level or how long it would take to do so. In the theoretical part of the thesis, I explain the convergence theory using the neoclassical growth model. I focus on the determinants of economic growth as well as the steady state position. Next, I discuss the arguments brought forward by the critics of the neoclassical growth model. The pros and cons of distinct types of convergence are explained in this work, and results of some influential convergence analyses are mentioned. In the empirical part of my thesis, I estimate beta and sigma convergence of the gross domestic product among the regions of both countries between years 1995 and 2009. Cross-sectional and panel data models were used for the estimation of the convergence coefficient. The results of my analysis are then confronted with the theory.

Klíčová slova

beta konvergence, sigma konvergence, neoklasický model růstu, Česká republika, Německo

Keywords

beta convergence, sigma convergence, neoclassical growth model, Czech Republic, Germany

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Předběžný obsah práce:

Německo je náš úspěšný soused. Často je nám dáváno za vzor, za příklad, jak by jednou naše země mohla vypadat. Hodně Čechů si při srovnání životní úrovně obou zemí klade otázky jako: „Zmenšuje se rozdíl mezi Českou republikou a Německem?" nebo „Kdy Česko dožene Německo?"

To jsou otázky, na které se snaží odpovídat teorie ekonomické konvergence. Cílem této práce je čtenáři na ně poskytnout možné odpovědi. V teoretické části práce bych začal uvedením podmínek, které se pokládají ke konvergenci států za nutné a přiblížením determinantů dlouhodobého ekonomického růstu. Dále se pokusím objasnit neoklasický model ekonomického růstu a nastínit, jak dlouhá by doba konvergence mohla být.

V empirické části se budu snažit teorii na reálných datech ověřit. Pokusím se vysvětlit případný nesoulad teorií s reálným světem a vyvodit z toho závěry. To vše především s důrazem na srovnání České republiky a Německa.

Předběžný obsah práce v anglickém jazyce:

Germany is our successful neighbour. It is frequently given to us as an ideal, an example, how our state could look like some day. A lot of Czechs is asking yourselves questions comparing the living conditions of both states such as: “Is the gap between Czech Republic and Germany getting smaller?“ or “When is the Czech Republic going to catch up Germany?“

These are the questions, for which is the theory of economic convergence trying to find answers. The goal of this work is to offer potential reader answers for them. In the theoretical part of work I would like to begin with stating the assumptions, which are required for the convergence of the states and introducing determinants of the long-term economic growth. Then I will try to enlighten the neoclassical model of economic growth and sketch how much time the convergence could take.

In the empirical part I will try to test the theory on the real data. I will try to clarify eventual disagreement of theory and real world and to draw a conclusion. All of this especially with stress of comparing the Czech Republic and Germany.

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V Praze dne

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Introduction

Journalists in the Czech Republic love to publish distinct predictions of time needed for the Czech Republic to catch up with Germany either in terms of gross domestic product (GDP) per capita or some measure of living standards. Some of the researches are more sophisticated some are rather simple. If we want to investigate whether is the Czech Republic getting closer to Germany, we should at first take a look at the theory which is dealing with the topic of economic growth and convergence. This will give us helpful points of view of the problem before we will use empiricism.

The aim of theoretical part of the paper is not to fully describe the theory that is behind convergence or economic growth. We try to present the concept and the main ideas of models with stressing the importance of the assumptions made so we can use the theory in empirical part of the work and compare the outcomes with theory.

We will start with introducing the neoclassical growth model[1], stating its assumptions, explaining the concept of steady state and showing dynamics of it. Important for this work are especially determinants of the steady state position because if countries share the steady state, then they should converge in terms of per capita income according to the neoclassical growth theory. We will specify possible problems that may arise from the simplifications made in the neoclassical model. Then we will shortly present the model of endogenous growth and compare it to the neoclassical model.

At the end of theoretical part of the paper, we will define the most common measures of convergence and present results of some influential works connected to the theory of economic convergence. Last but not least, we will derive formula for estimating the convergence time. Theoretical part is written mainly on the basis of publications by Solow (1956), Barro & Sala-i-Martin (1992) and by Tondl (2001).

In the empirical segment of the work we will use the theory described in previous section in praxis. We decided to use the Eurostat regional database as our source of data for it contains a lot of measurements at the regional level since 1995 on. At first, we will use average past growth rate of GDP per capita of both economies to estimate possible future development of both countries. Then we will measure income dispersion and σ-convergence on both country and regional level. We will employ

[1] The neoclassical growth model is sometimes called exogenous growth model or the Solow-Swan growth model. In this text we will always use the first name.

cross-sectional models to estimate absolute β-convergence. To make a statement about convergence implications of the neoclassical growth model, we need to test conditional β-convergence and to be able to measure conditional β-convergence, fixing of additional variables is needed. So we will include a discussion which variables shall we hold constant in our model. Because of the criticism that was put on the estimating of convergence using cross-sectional data, we will also include panel data model to compare the results of both approaches.

1. Neoclassical Growth Model

1.1 Historical Introduction

The cornerstone of the modern economic growth theory was laid by Ramsey (1928) by introducing his view of consumer optimization over time. Harrod (1939) and Domar (1946) used production function where production factors have fixed proportions (no substitutability among inputs) in their model. It supported Keynesian argument of instability of the capitalist system but it was criticized for insufficient explanation of maintaining of the full employment. Robert Solow (1956) and independently Trevor Swan (1956) introduced their models using the neoclassical production function – hence name neoclassical growth model. This model predicts conditional convergence and has a big influence on current growth theory.

1.2 Capital Accumulation and Output Development

Households are owners of all the firms in the neoclassical model. Firms produce just one homogenous good – output *Y*. Output is made by two production factors – labor *L* and capital *K*. Output can be either consumed or saved and reinvested back into companies. The model assumes constant saving rate *s* and closed economy. It means that all the money saved will be transformed into capital. The assumption of full employment of capital says that the capital already accumulated is inelastically supplied. Then the increase of capital by investment equals:

$$I = s \cdot Y \text{ . (1)}$$

If we take into account depreciation rate of capital δ, the change of quantity of capital $\dot{K}$ is equal to:

$$\dot{K} = I - \delta \cdot K = s \cdot Y - \delta \cdot K \text{ . (2)}$$

Solow assumes full employment. In such theoretical situation, labor force consists of every citizen of an economy and everyone is employed. Population growth rate is also constant and it is exogenously determined. Relative population growth is denoted *n*. Thus labor supply at time *t* equals:

$$L(t) = L_0 \cdot e^{n \cdot t} \text{ .(3)}$$

Equation (3) states that labor is supplied completely inelastically – quantity of laborers changes according to the change of population. The real wages adjust so that all the laborers are employed.

Now we can describe simple process of the capital accumulation and the development of output. Labor force is given by Eq. (3). Quantity of capital is given since we have the assumptions of full capital employment and we know what the current amount of capital is. Thus we know quantity of both inputs and we can use the production function to determine the level of output. Because of the constant saving rate, we can compute the fraction of output that will be saved and invested again. New quantity of capital is equal to the amount of capital one period ago plus investment minus depreciated capital. So we know the exact level of capital one period ahead. Quantity of labor supply in that time is given by Eq. (3) again. We can repeat the process with the inputs from period "*t+1*" to find the output in two periods ahead from now and so on.

Production of output in time is specified by production function. As we stated earlier, there are only two inputs used in the neoclassical growth model – labor and capital. We can write production function in general as:

$$Y(t) = F(K(t), L(t)) . (4)$$

Technology level *A(t)* is exogenously given. Every economy has free access to the technology. This assumes fast diffusion of technology which may be possible in trade-open highly developed countries (just like the Czech Republic and Germany) but is less justifiable in developing economies. Technology improvement has got the same effect as the increase of labor force in the neoclassical model - we say that the technology is labor augmenting and we denote $A \cdot L$ effective labor. When we write production function time subscript is for lucidity usually dropped out. General production function including technology is:

$$Y = F(K, A \cdot L) . (5)$$

For properties of the neoclassical production function please see Appendix 1.

1.3. The Steady State

Let's define the capital/labor ratio $k = K / (A \cdot L)$. We can interpret the ratio as a quantity of capital that approximately uses one worker. Output per capita is defined as $y = Y / (A \cdot L)$. If we divide the production function by the effective labor, we get:

$$y = F\left(\frac{K}{A \cdot L}, \frac{A \cdot L}{A \cdot L}\right) = f(k,1) = f(k) \ . \ (6)$$

We can look at *F(k,1)=f(k)* as a production function with changing quantity of capital per one unit of effective labor. Production function expressed in terms of per capita is said to be in its intensive form. The Cobb-Douglas production function in the intensive form is:

$$y = k^{\alpha} \ . \ (7)$$

If we divide Eq. (2) by the effective labor, put all the expressions in per capita terms and do some rearrangement, we can get:

$$\dot{k} = s \cdot f(k) - (n + x + \delta) \cdot k \ . \ (8)$$

This is called the equation of capital accumulation and it is one of the most important equations of the neoclassical growth model. We can see that the change of capital in per capita terms is dependent on two expressions. The first one shows gross investment. Because the saving rate is constant and the production function must have diminishing marginal product with respect to capital according to the assumptions of neoclassical production function, increment of this term will be smaller with every additional unit of capital. The term *(n + x + δ)* is called the effective depreciation rate of capital. We suppose stable technological growth rate *x*. Since all the three variables of the effective depreciation rate of capital are constant, the second term will grow linearly with the quantity of k.

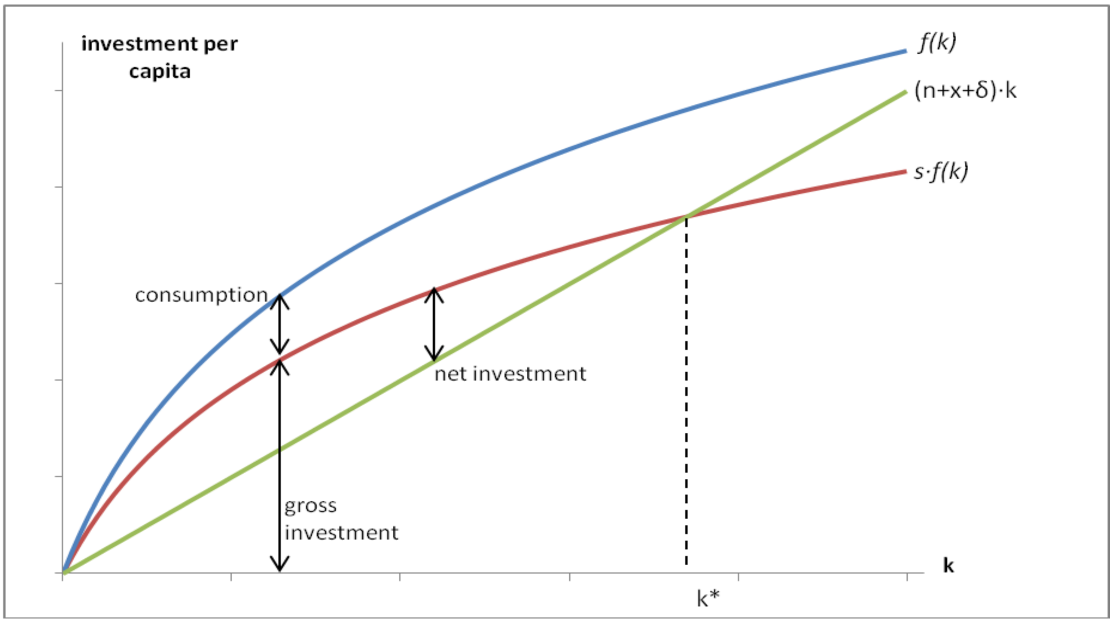

Figure 1: Investment Path of the Neoclassical Model (Based on Barro & Xala-i-Martin (1992))

You can see both right hand terms of the equation of capital accumulation mentioned above and production function *f(k)* in the Figure 1. Gross investment function is only less sloped production function thanks to multiplication by the saving rate. The difference between the production function and the gross investment function is equal to consumption per capita. The gap among right hand side expressions of Eq. (8) is equal to the net investment per capita. Net investment has to decrease from some point of capital per capita on. In point k^*, depreciation will equalize gross investment and there will be no increment in k. Situation when $k=k^*$ (or $s \cdot f(k^*)=(n+x+\delta)\cdot k$) is called (the) steady state. When economy reaches its steady state, all the variables grow at the constant rate. If there is no change in the level of technology, k and y stay unchanged and K and Y grow accordingly to the population growth n.

1.4 Transition to the Steady State

Now we will show how the economy moves toward its steady state. In order to obtain the growth rate of output per capita γ_y, we need to define the growth rate of capital per person γ_k first. We will receive γ_k by dividing Eq. (8) by k:

$$\gamma_k = \frac{s \cdot f(k)}{k} - (n + x + \delta). \quad (9)$$

According to this equation, capital will grow faster the higher is the difference between the terms on the right hand side. We plot these terms into the graph (Figure 2) with the growth rate of capital on the vertical axis and the amount of capital per capita on the horizontal axis. The first term on the right side will be downward-sloping curve (recall the Inada condition – Appendix 1). It approaches infinity as k goes to zero and with high values of k it asymptotes to zero. The second term is a horizontal line since neither n, x nor δ does not change with the level of k. Both curves will intercept only in one point – in the steady state. We can see that the growth rate of capital is high with low quantities of k and that the growth is getting smaller until it reaches the steady state value k^* in the Figure 2. On the right side from k*, growth rate of capital per capita is negative so k will decline in time until it reaches k^*. This means that the steady state amount of capital per capita is stable. It does not matter whether economy is initially rich ($k_{rich}>k^*$) or poor ($k_{poor}<k^*$), capital per capita will converge towards its steady state value k^*.

If we change the value of the saving rate, steady state position shifts. This is illustrated by the green curve in the Figure 2 ($s_2>s_1$). As a consequence of the increase of the saving rate, new steady state position k_2^* will have higher capital per capita.

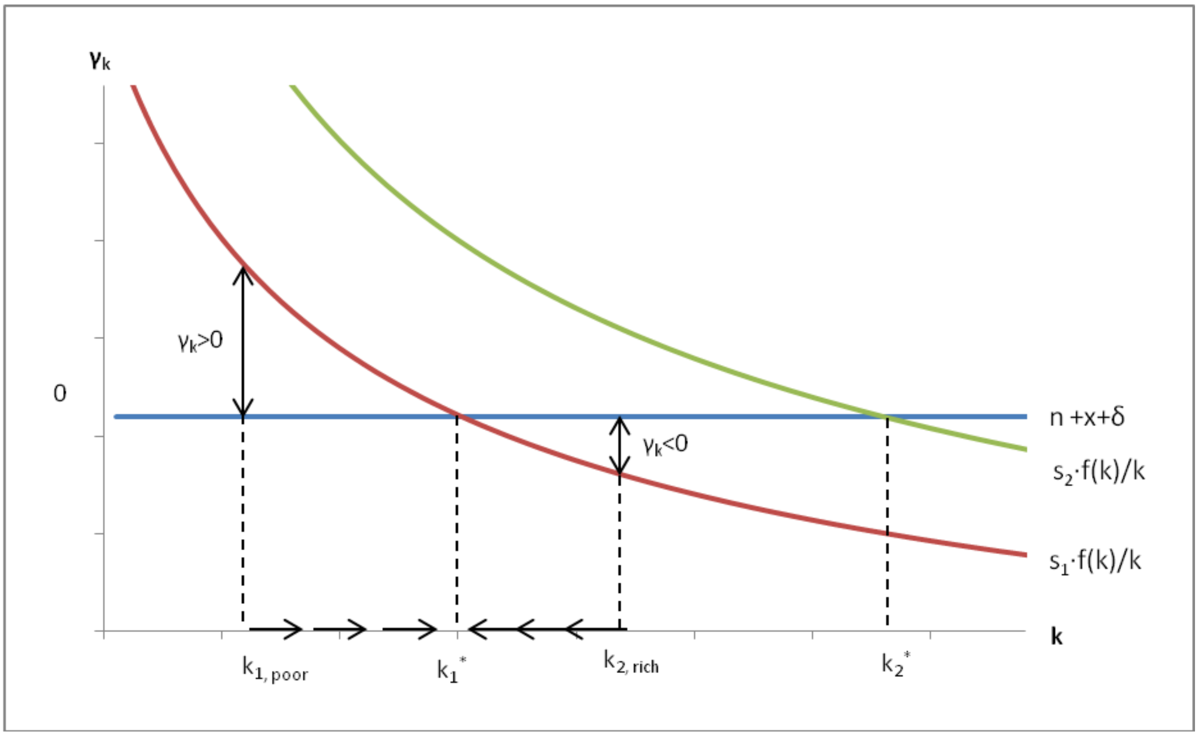

Figure 2: Dynamics of the Neoclassical Model (Based on Barro & Xala-i-Martin (1992))

Growth rate of output per capita γ_y during transition is closely linked to the growth rate of capital per person:

$$\gamma_y = \frac{\dot{y}}{y} = \left[\frac{k \cdot f'(k)}{f(k)}\right] \cdot \gamma_k . \quad (10)$$

The term in brackets on the right hand side is called the capital share – share of rental income on capital in total income. If we use the Cobb-Douglas production function, then the capital share is equal to the constant α. This means that the outcome per capita grows and decreases with amount of per capita capital and its position towards equilibrium value of capital per person k^*.

1.5 Growth in the Steady State, Prediction of the Neoclassical Model

Since economy reaches the steady state, output and capital will grow at the rate of population and technological growth *(n + x)*. In per capita terms, growth of capital and output will be driven only by the technological change x. This means that without any technological improvement, there is no growth in the output per capita and the economy stagnates.

Steady state position can be altered by permanent change of variables such as the saving rate, population growth rate, technological change or change in the depreciation rate. We have shown example of the change of the saving rate in the Figure 2. When the steady state position changes, economy reaches it again after some time because we have shown that the steady state is stable. This gives the economy temporal growth bonus during its transition to the new steady state position. Similar effect has got increasing of the technological level or decreasing of the effective depreciation of capital.

If the change of variables mentioned above is only temporal, then the economy growth change is also only short-term because the steady state shifts back to its original spot.

The neoclassical growth model says that if a bunch of economies has the same production function and similar characteristics such as the population growth rate, saving rate or depreciation rate and the difference is only in the original level of capital, then the economies should converge toward the same level of output per capita given by the steady state position. This means that relatively poorer countries should be able to grow faster than the rich ones and to converge to one common level of output per capita. This statement is important for our work – we will try to analyze:

1) If it is probable for the Czech Republic to have the same steady state as Germany or not.
2) If the economies are converging in terms of GDP per capita.

But before we do so, let us briefly look at the theory that represents different opinion

1.6) AK Model as an Alternative to the Neoclassical Growth Model

Important convergence implication resulting from the neoclassical model of economic growth has been heavily tested by many researches. The hypothesis of constant saving rates for both poor and rich countries have been discussed (poor cannot afford to save the same percent of their income as the rich ones) and the hypothesis of diminishing returns to capital. The neoclassical model is also criticized for the exogenousness of its key growth determinants.

After the oil shocks in the 1970s came the period of regional divergence which the neoclassical model could not explain. Economists started to introduce models where the growth determinants are included in the model, hence name endogenous growth models. One of the simplest versions of the endogenous growth model is called the AK model. It was invented by Paul Romer (1986). The name comes from inputs used for production by this model – *A* for the technology level and *K* for the capital. Capital in the AK model includes also human capital, not only physical capital as in the neoclassical growth model and therefore it does not predict diminishing returns to capital. Human knowledge can accumulate, lead to new ideas arising from existing knowledge and to spillovers. Let us assume that the saving rate *s* and the technology level *A* are constant and given. The equation of the model in the intensive form is:

$$y = A \cdot k \text{ . (11)}$$

If we derivate this equation with respect to capital, we find out that the marginal product of capital is equal to *A* and since we assume technology level to be constant, the marginal product of capital is also constant and not diminishing as in the neoclassical growth model.

The growth rate of capital in the AK model is equal to:

$$\gamma_k = s \cdot A - (n + \delta) \text{ . (12)}$$

Both expressions on the right side of Eq. (12) are constant. This means that the capital growth will be still the same and not dependant on the level of capital per capita *k*. If $s \cdot A > n + \delta$, then the capital per person grows by a constant rate even without any technological change. Moreover, the growth rate of capital is equal to the growth rate of output per capita and the growth rate of consumption per capita as we can see in the Figure 3.

Another interesting property of the AK model is that the change of the saving rate will permanently shift the growth rate and the growth will not diminish after some time as in the neoclassical model. There is no space for convergence across different economies because the AK model predicts constant growth rates.

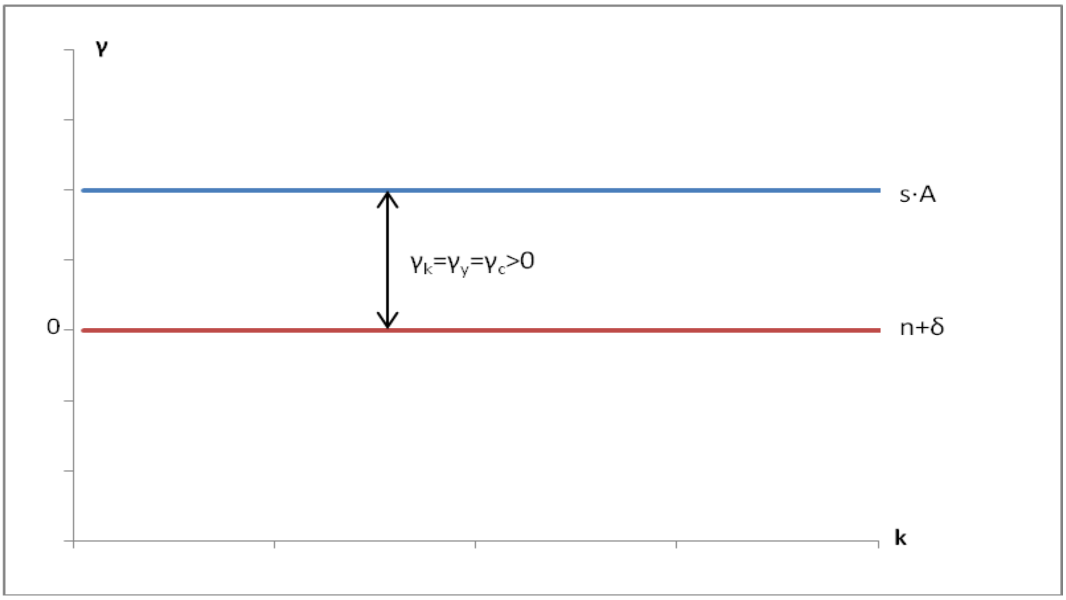

Figure 3: Dynamics of the AK model (Based on Barro & Xala-i-Martin (1995))

We have shown with help of two basic models of economic growth outcomes both in accordance with and against convergence. The neoclassical model predicts convergence if economies are similar (have the same steady state). The AK model on the other hand predicts no convergence at all.

2. Theory of Economic Convergence

2.1 Definitions of Convergence

We will introduce both heavily used concepts of convergence: β-convergence and σ-convergence. Then we will derive time needed for achieving convergence and we will take a look at the overview of the convergence estimates of some chosen empirical convergence papers.

The concept of β-convergence can be further divided into absolute and conditional β-convergence. Absolute β-convergence occurs when initially poorer economies grow faster than rich ones. This assumes only one steady state for all the economies in the sample. The economies should have similar properties such as the saving rate, population growth rate, depreciation rate, technology level and they should have the same production function to have the same steady state. The economies should differ only in an initial endowment of capital. Let $\gamma_{y,i}$ be the average growth rate of an economy *i* during observed period and $log(y_{i,0})$ is a logarithm of economy's GDP per capita in the beginning of the period. We can estimate the regression equation:

$$\gamma_{y,i} = \alpha - \beta \cdot \log(y_{i,0}) + \varepsilon_i \,. \quad (13)$$

Similar equation was used for the first time by Baumol (1986) who estimated negative relation of GDP growth rate and productivity of labor. Eq. (13) is stated in Barro & Sala-i-Martin (1992). If coefficient β>0, then there is absolute β-convergence in the data set. In the real world, properties of economies more or less differ. This implies that there must be more than one steady state to which the economies will converge. The phenomenon when economies split into several subgroups and converge towards the subgroup steady states is called a club convergence. In this situation richer countries converge to their high income steady states and overall absolute convergence does not occur.

Conditional β-convergence appears when economies further from their steady states grow faster than those closer to their ones. This is identical to the concept of absolute convergence only if there is just one steady state for all the economies, otherwise it says something different. It may happen that the relatively richer countries grow faster than the poorer ones if they are more distant from their steady states. The neoclassical growth model does not predict absolute β-convergence but conditional β-convergence. This is important to realize. If we want to test the convergence hypothesis of the neoclassical model, we should use conditional β-convergence. Absence of absolute β-convergence in the sample cannot be treated as evidence against the neoclassical model. In order to test the conditional β-convergence, we need to hold constant steady states of economies either by introducing proxy variables for them or by restricting the set of economies in such way that we can assume that they can have only one common steady state. Sala-i-Martin (1996) states that it is not unrealistic to assume that OECD countries[2] will have the same steady state but we will fix additional variables to test the conditional steady state.

The last measure of convergence we will mention here is not connected directly to growth but rather to comparing the income differences. The concept of σ-convergence says that the dispersion of real GDP per capita of economies has to decline over time. If σ_t denotes standard deviation of logarithmized GDP per capita across a group of economies at time *t*, then:

$$\sigma_t > \sigma_{t+x}, x \in \mathbb{N} \quad (14)$$

must hold to achieve σ-convergence.

Sala-i-Martin (1990) was the first who introduced this terminology. In the simple case of only two economies, the poorer one has to grow relatively faster than the rich one to achieve σ-convergence. This shows that absolute β-convergence is necessary condition for σ-convergence. If we go back to the two-economies example: the poorer one can overgrow the initially richer economy so much that in the end of measurement period its GDP per capita will be more distant to the initially rich economy than it was in the beginning. Therefore, it is clear that the β-convergence is necessary condition for σ-convergence but it is not the sufficient condition. The difference between β and σ-convergence is that β-convergence is aimed at the movement of the particular economy in relation with the others and σ-convergence is concerning overall sample income distribution over time.

There are also many problems connected to the measuring of convergence. Convergence estimation can be only as good as the data used are. Long time intervals are preferable for estimating convergence and they do not have to be always available. Another problem is that the data have to be reliable – this problem arises mainly from work with developing countries or from work with older data that have to be estimated ex-post. We are measuring convergence in terms of GDP per capita so we have to be aware of limited explanatory value of such indicator (we can also calculate convergence of labor productivity, wage levels etc.). Models measuring β-convergence such as Eq. (13) has to satisfy certain assumptions so we can make statistical inference. In this case, violation of zero conditional mean assumption is often criticized (Tondl 2001). The neoclassical growth model predicts that the growth rate of GDP depends mainly of the initial level of GDP when the steady state is given. In reality, the growth rate may depend on more variables and if we do not include such variables in the model, our estimation will suffer from a bias. Islam (1995) states that we should use panel data models in order to avoid bias created by omitting unobservable factors that determine the steady state income. Another plus of panel data models is that they work with more measurements than the cross-sectional models

2.2 Convergence Time

We need to know how long does transition period to the steady state last. If it is relatively short, we should put more stress to the steady state behavior and not to transition period and vice versa. Let us assume that economy has got the Cobb-

[2] Both Germany and the Czech Republic are members of the OECD.

Douglass production function and that the economy is not too far from its steady state so we can use approximation. The convergence coefficient β tells us how fast does the gap between current position and the steady state vanish. The convergence coefficient derived by Barro & Sala-i-Martin (1995) is:

$$\beta = (1-\alpha)\cdot(x+n+\delta)\,. \quad (15)$$

Economy moves towards its steady state exponentially. Economy reveals β-convergence if the convergence coefficient is positive. If e.g. $\beta = 0.02$, then the economy closes 2 per cent of the gap to the steady state per one time period. Convergence coefficient value of 2% is generally accepted as a benchmark value of β-convergence (Barro & Sala-i-Martin (1995), Tondl (2011)).The size of the convergence coefficient is positively related to the growth rate of the effective labor *(x + n)* and to the depreciation rate δ and negatively to the capital share ratio α. The growth rate of output per capita can be approximated like:

$$\gamma_y \cong -\beta \cdot \left[\log\left(\frac{y}{y^*}\right)\right]. \quad (16)$$

The higher is β coefficient, the higher the growth of the economy is. The closer gets the output per capita to its steady state, the slower the growth rate is. Eq. (16) is a differential equation with solution:

$$\log\left[y(t)\right] = (1-e^{-\beta\cdot t})\cdot\log(y^*) + e^{-\beta\cdot t}\cdot\log\left[y(0)\right], \quad (17)$$

where *log[y(0)]* is logarithmized output in the beginning of the period. If we want to estimate time needed to close one half of the gap between current income and steady state income, we have to fulfill the condition:

$$1-e^{-\beta\cdot t} = e^{-\beta\cdot t} = \frac{1}{2}\,. \quad (18)$$

Now it is easy to find solution:

$$t_{half} = \frac{\log(2)}{\beta}\,. \quad (19)$$

Equations 16 – 19 and more detailed explanation can be found in Tondl (2001). Similarly, we can derive time needed to close every possible part of gap to the steady state income. We show results for chosen betas in the Table 1:

		Convergence coefficient β:					
		0.001	0.005	0.01	0.02	0.05	0.1
Fraction of distance to y* to close:	1/4	288	58	29	14	6	3
	1/2	693	139	69	35	14	7
	3/4	1386	277	139	69	28	14
	9/10	2303	461	230	115	46	23

Table 1: Estimated Convergence Time in Years for Chosen Convergence Coefficients

2.3 Results of Convergence Analyses of Countries and Regions

In his famous work, Baumol (1986) finds a convergence in productivity of 16 developed countries. He used a long run data by Maddison (1870-1979). The problem of the work was, as Romer (1986) pointed out, that such data included only ex-post chosen countries which were all successful during the measurement period and hence the analysis suffered from the selection bias. After the dataset was enlarged, the convergence disappeared (see Romer (1986)). Barro & Sala-i-Martin (1995) tested absolute β-convergence on a sample of 97 countries during years 1965-1985 and found out slight positive correlation between the initial GDP per capita and its growth rate. This not only proved lack of absolute β-convergence but it was an evidence of slight divergence among countries. Iancu (2006) investigated absolute β-convergence on the sample of 93 countries in years 1980-2003 and also found rather divergence than convergence. When we restrict a sample only to the countries that appear to be more similar, the results change. Sala-i-Martin (1996) tested β-convergence on the data set of OECD countries (1960-90) and estimated convergence coefficient β=1.4%. When he added additional variables, he discovered conditional β-convergence of β=2.9%. He finds lack of β-convergence in the world scale. It means that the initially rich countries remain rich.

To find out whether the difference in countries' income is shrinking or not, σ-convergence is needed. Sala-i-Martin (1996) discovered decreasing dispersion of GDP per capita in OECD countries from σ=0.5 in 1960 to σ=0.4 in 1990 and increasing dispersion of the worlds' (110 countries) income from σ=0.9 to σ=1.1 during the same time.

Let us move from the country level to the regional one to see the results of the work of Barro & Sala-i-Martin (1995). They estimated convergence of the US states, European regions and Japanese prefectures. The US states converged in the rate of β=1.7% in between 1880-1990, Japanese prefectures in β=2.7% during 1930-90 and finally regions of eight European countries converged at β=1.9% from 1950 to 1990. When the regional dummies were added to test the conditional β-convergence, the convergence speed increased slightly. σ-convergence was found in all three locations. The authors point out that income convergence in the EU regions is slower than in the American states in the last 40 years of the previous century. Moreover, convergence rate is getting smaller after 1975 and some peripheral regions seem to be excluded from the convergence process.

We can spot interesting events when observing results of the convergence analyses. Concerning the whole world, income disparities rose in the second half of the twentieth century and initially rich countries stayed at the top. If we focus on the more homogenous groups such as OECD countries, the US states or the regions of the EU, we can find β convergence at approximate rate β=2%.At this speed, it would take 35 years just to close half of the income gap. The convergence process seemed to slow or even stop for a short time after mid-seventies and then continued at the slower pace.

When we want to check the convergence prediction of the neoclassical growth model, we have to aim to the conditional convergence. The conditional convergence at the rate β=1.3% was found among 110 world countries in the work of Sala-i-Martin (1996). In the more homogenous groups such as OECD countries, Western European regions or even the regions of Germany, the convergence coefficient was higher during the second half of the twentieth century.

We may forecast that there is absolute β-convergence among regions of the Czech Republic and Germany from the results stated above for it appeared in the groups of developed countries. We may dare to anticipate σ-convergence in the regions of both countries from the same reason as well. Conditional β-convergence may be around 2% if our results will be similar to those presented above.

3. Comparison of the Steady State Determinants of the Czech Republic and Germany

Let us remind that the steady position in the neoclassical growth model can be derived from the equation of capital accumulation (Eq. 8). The steady state quantity of capital per capita is:

$$k^* = \frac{s \cdot f(k)}{n + x + \delta}. \quad (20)$$

The higher is k*, the richer the country is. We assume that the Czech Republic and Germany have the same production function, depreciation rate δ and technological growth rate *x*. It is reasonable to predict the same technology growth rate in both the Czech Republic and Germany because of the great interconnectedness of both economies and fast technology diffusion of innovations thanks to modern technologies.

On the other hand, the saving rate of households of both countries is quite different. In the Chapter 1.6, we stated that the households in the rich countries can afford to safe higher ratio of their income. The data acquired from the Eurostat database support this idea. Saving rate in Germany was 16.6% in 1995 and it rose to 17.4% in 2009. The trend in the Czech Republic was just different: saving rate of 13.6% in 1995 declined to 10.3% in 2009. Interesting is that even though the Czech Republic got richer, the saving rate decreased. The average saving rate in the period was 10.4% in the case of Czech Republic and 16.2% in Germany. The saving rate of households in Germany is significantly higher than in the Czech Republic. This means equivalent difference in the steady state quantity of *k**. The evolution of the saving rates is illustrated in the Figure 4.

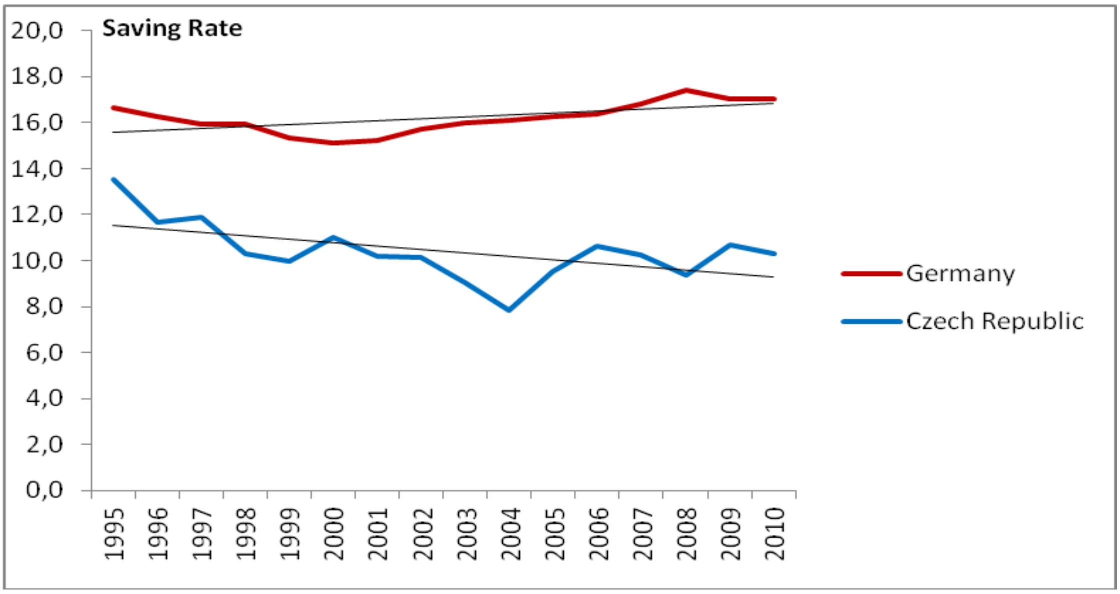

Figure 4: Gross Household Saving Rate of the Czech Republic and Germany

Population growth rate of both countries seems to be exactly opposite in both states during last twenty years, only the deviations are higher in the Czech Republic. Higher volatility may be explained by lower country population and consequently higher sensitivity to shocks. In the Czech Republic, population boom during last ten years was caused by entering of strong population of so called Husák's childrem into fertile age and high immigration during pre-crisis period which already ceased. Germany has got problems with low fertility rate since reunification of the country. Average population growth rate between 1999 and 2010 was 2‰ in the Czech Republic and -0.3‰ in Germany. This supports hypothesis of different steady states for both countries according to Eq. (20). If the steady states of both countries are different, then the absolute convergence is different from conditional one. The neoclassical model predicts conditional convergence and not absolute convergence and if these two are not the same, the model does not say that the initially poorer economy has to grow faster than the richer one. It is clear that all the variables determining the steady state cannot be exactly the same but the examples of the saving rate and population grow seem to be almost negatively correlated. The neoclassical model further assumes these variables to be constant and we can see that they change a lot in time.

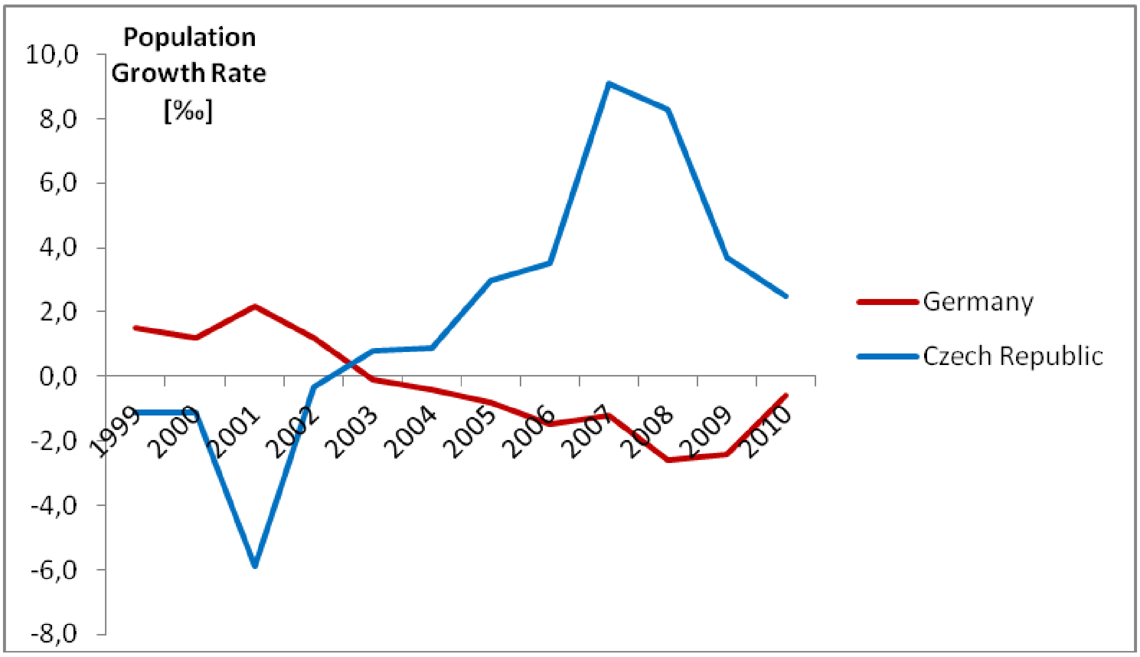

Figure 5: Population Growth Rate Comparison

4. Comparison of GDP Growth and its Outlook

Now let us focus on the GDP per capita of both countries and its development. We will compare the GDP per capita in purchasing power standards (PPS) expressed in relation to the EU-27 average set equal to 100. In 1995, the Czech Republic was at the 77% of the EU-27 average. Czech economy was in the mild recession for two years after the political and financial crisis in 1997. GDP per capita in PPS fell to 71% in 2000. Then the economy was booming and reached 83% of the EU-27 average in 2007. The current financial crisis and austerity measures ended the period of fast growth (real

GDP/capita growth of 7% in 2006) and country went to recession with -4.7% GDP per capita in 2009. In 2010, the Czech Republic was at the 80% of the EU-27 average.

Germany was at the 129% of EU-27 in 1995. Because of difficult transformation of the GDR from the central planned economy, the economic growth of reunified Germany was slow and country fell down to 114% of EU-27. German government as a reaction on poor economic effort introduced series of reforms called Agenda 2010 which had to reform the social system, labor market (famous Hartz I-IV reforms) and boost the economic growth. Since then, Germany's GDP per capita relative to EU-27 stayed at almost constant level. The crisis hit hard export-oriented German economy but thanks to the flexible labor market and high competitiveness of its industry, Germany became the engine of European recovery with high real GDP/capita growth of 3.7% in 2010 and 3% in 2011.

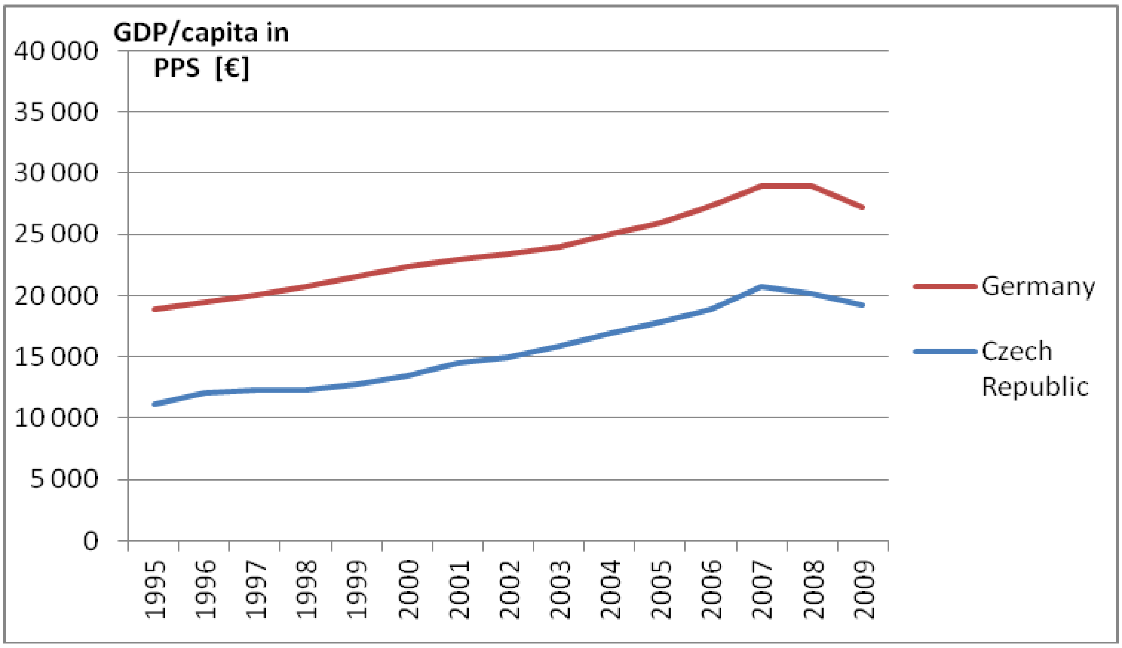

Figure 6: Development of GDP per Capita in the Czech Republic and Germany, 1995-2009

Average GDP per capita growth rate in PPS of Germany between 1996 and 2010 was 1.4%. In the same time period, average growth of the Czech Republic was 2.8%. From 16 measures, the growth rate of the Czech economy was higher in eleven cases. Germany's GDP per capita in PPS in 2010 was €28,800 and the Czech Republic reached €19,400 which is 67% of the Germany's level. With the approximate grow of 1996-2011 it would take Czech economy 14 years just to reach the level of Germany in 2010. If we want to forecast time *t* needed for the Czech Republic to catch up Germany in terms of GDP per capita on basis of past growth rate, we can use formula:

$$t = \frac{\log(y_{G,2010}) - \log(y_{C,2010})}{\log(1+g_C) - \log(1+g_G)}, \ (21)$$

where $y_{G,2010}$ denotes GDP per capita of Germany in 2010. $y_{C,2010}$ is the same variable for the Czech Republic. We assume stable GDP growth of both countries g_G and g_C respectively. Estimated catch up time is very sensitive to the selection of the growth rate of both countries. Table 2 shows that the estimated convergence time changes a lot with the change of the growth rate. If the growth is equal to the growth rate of both countries in 1996-2010, then it takes 29 years to equalize countries GDP per capita.

		Czech Republic Growth Rate		
		2,3%	2,8%	3,3%
Germany	0,9%	29	21	17
Growth	1,4%	45	29	21
Rate	1,9%	101	45	29

Table 2: Forecasted Time in Years for the Czech Republic to Catch Up Germany

We have illustrated possible development of the GDP per capita of the Czech Republic and Germany in the Figure 7. The exponential curves show how much can growth of the countries differ if the long term growth of the GDP is changed by 0.5 of the percentage point. There are three variants for every country given by the initial GDP level and growth rates from the Table 2. This should help to realize how much can convergence time differ given by a little change in the long term growth. If the Czech Republic grows by 2.3%, it will reach €60,000 GDP/capita in fifty years. If the growth rate is higher by one percentage point, country's GDP per capita will be €98,000. The second number is by 62% higher than the first one.

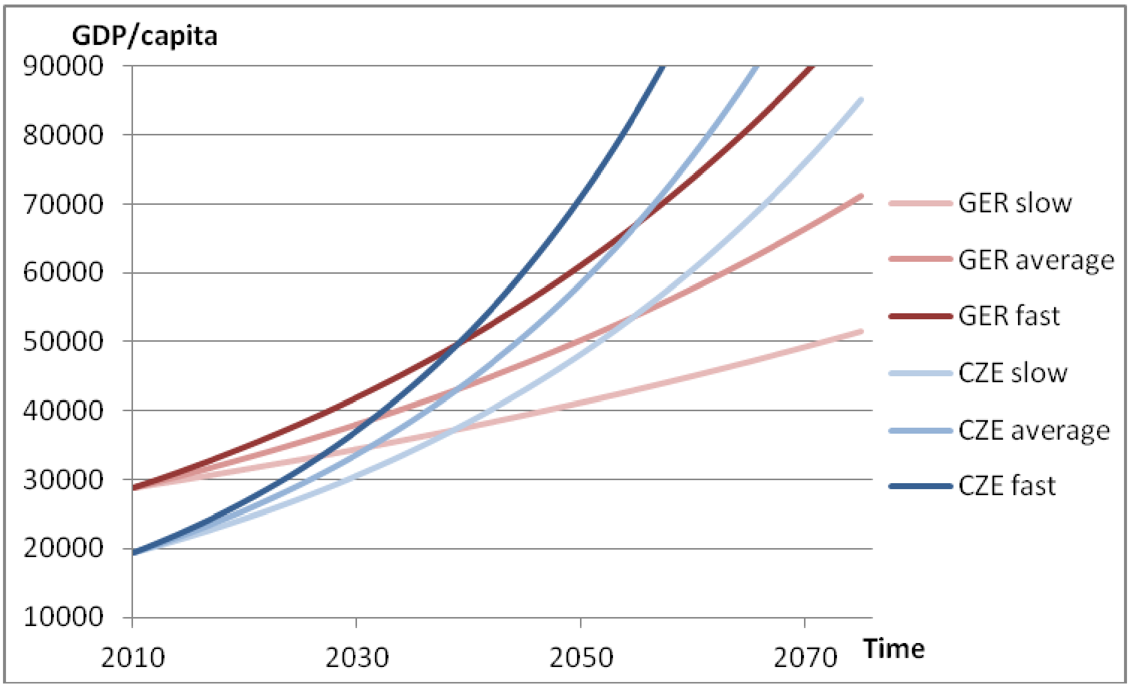

Figure 7: Impact of Different Growth Rates on the Level of GDP per Capita

This model is really simple and the constant growth rate regardless of the country's GDP contradicts the neoclassical growth model because it states that the growth rate of output decreases with the quantity of capital per capita *k* or (output per capita *y;* recall properties of the neoclassical production function). The only way how to achieve constant per capita growth and be in accordance with the neoclassical growth model is a corresponding change of the steady state position given e.g. by the change of the technology level.

5. Sigma Convergence and Income Distribution in the Czech Republic and Germany

5.1 Sigma Convergence at the State Level

We will take a quick look at the σ-convergence of the both countries at the state level at first and then we will do more detailed analysis at the regional level. There are more possibilities how to calculate dispersion of GDP per capita. We will compute variation coefficient of the GDP per capita σ in time *t* with standard deviation formula:

$$\sigma_t = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\log(y_{it}) - \log(\bar{y}_t))^2}\ , (22)$$

where n is a number of regions and $\log(\bar{y}_t)$ is average GDP per capita in all regions in time t. We will use data from the Eurostat database that are available for period 1995-2009 for both countries and their regions.

In case of comparing income variation of observed countries, we can find declining income dispersion over time. Dispersion rose only during recessions, throughout expansion it declined steadily. During the Czech economic crisis of 1997-1998, the dispersion rose from 0.239 in 1996 to 0.262 in 1999. The strong growth of the Czech Republic in the beginning of the new millennium depressed the GDP dispersion to 0.167 in 2007. Dispersion rose a little in another recession year of 2008 but after year 2009 dispersion dropped almost to the pre-crisis level. Trend line in the Figure 8 shows clear sign of σ-convergence between years 1995 and 2009. The dispersion seems to decline at the constant rate and deviations from the trend caused by crises seem not to have the long-run impact on the decrease of dispersion of GDP per capita in both countries.

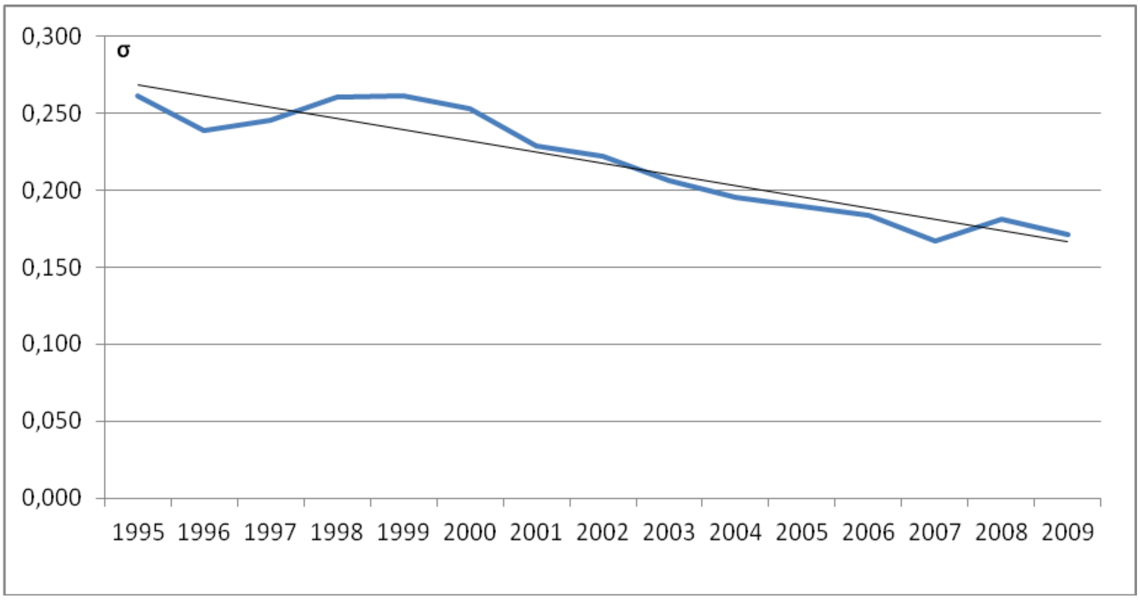

Figure 8: Income Dispersion of the Czech Republic and Germany

5.2 Sigma Convergence at the Regional Level

If we want to analyze σ-convergence of Czech and German regions, situation is slightly more complicated. We will divide countries into NUTS 2 regions. NUTS is a standard developed by the European Union for statistical purposes. Population of the NUTS 2 region should be in between 800 000 and 3 million inhabitants but this is no strict rule. There are 39 NUTS 2 regions in Germany (8 of them in former GDR[3]) and 8 regions in the Czech Republic. We obtained data for the regions in years 1995-2009 from the Eurostat regional database. Let us take a look at the data before we will examine σ-convergence among NUTS 2 regions.

The richest region in terms of the GDP per capita in PPS in 1995 was Hamburg reaching 176% of the average GDP per capita in PPS of all the regions from both countries. Hamburg was followed by 14 regions of former West Germany, 16th was Praha region reaching 105% of the average. Beneath Prague were the rest of former FRG regions. Former GDR regions ordered behind the West Germen regions. Their GDP/capita varied from 78% (Leipzig) to 64% (Thüringen) of sample average. The poorest regions in the beginning of the measurement period were the Czech ones (with the exception of Prague), the richest Severozápad reached 60% and the poorest Střední Morava 53% of average GDP/capita in PPS of the Czech Republic and Germany Regions.

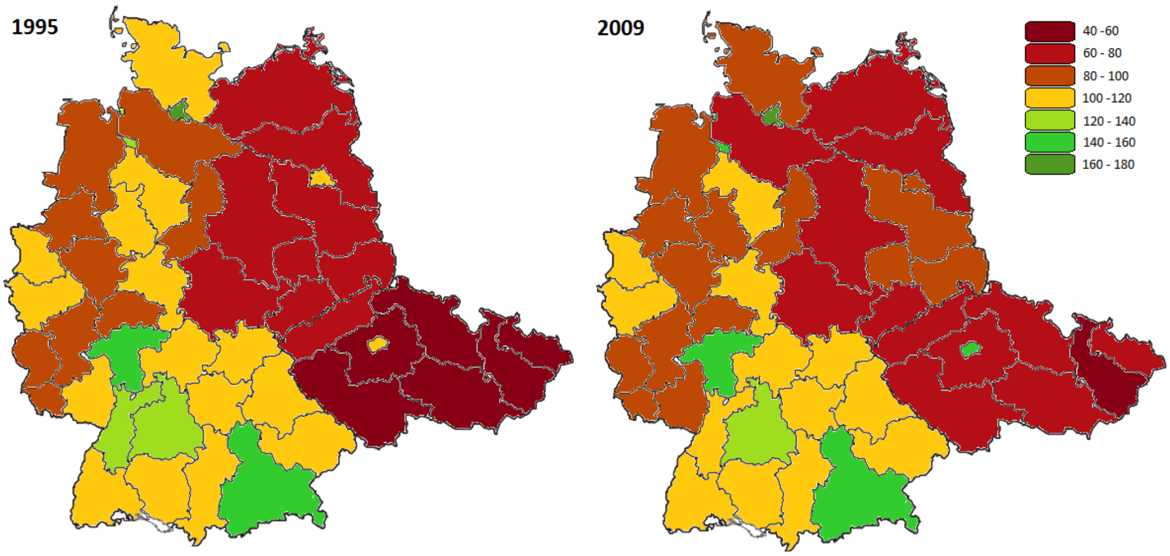

Figure 9: Relative Position of Czech and German NUTS 2 Regions in Terms of Percent of Average GDP per Capita in PPS, 1995 and 2009; (map is based on the map of NUTS 2 regions of the EU by Eurostat)

Hamburg was still the richest region at the end of the measurement period but Praha climbed up to the second spot. Hamburg's position declined to 168% of the average GDP/capita but Praha rose from 105% in 1995 to 157% in 2009. This was the highest increase of all the regions. There were not many changes in a relative position of the

[3] We treat Berlin region for simplification as former FRG region because economically more significant

regions between – former FRG regions are still richer than former FDR regions and that are richer than the Czech ones. Only Lüneburg region (former FRG) relatively weaken its position and is somewhere in the middle of the East German regions in terms of GDP per capita.

We calculate the variation coefficient with the formula from Eq. (22). We can spot the big difference between income dispersion at the country and at the regional level. Dispersion of all Czech and German NUTS 2 regions grew a lot because of Czech crisis in 1997. Afterwards, it more or less stagnated and decreased from 2006 on. The dispersion in 2009 (0.22) was still higher than it was in 1995 (0.2). Regional dispersion was at the lowest level in 1997 – it was 0.18.

We can observe interesting results if we alter the area where we measure σ-convergence. The data for the Czech regions only show σ-divergence because the variation coefficient of GDP per capita is increasing over time. This is due to the strong economic growth of Praha region, because if we measure σ-convergence in the Czech regions without Prague, the dispersion is stagnating around 0.05 which is really low level.

In the case of German regions, dispersion is declining steadily and we can spot σ-convergence. In 1995, variation coefficient of GDP per capita between German regions was 0.23, in 2009, it diminished to 0.20. Income dispersion was very stable in the former FRG regions. It rose from 0.16 (1995) to 0.17 (2009). The GDP dispersion in former East German regions was slightly more volatile and much lower than in the FRG regions. It oscillated around same value as the Czech regions without Prague.

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part of it belonged to the West Germany.

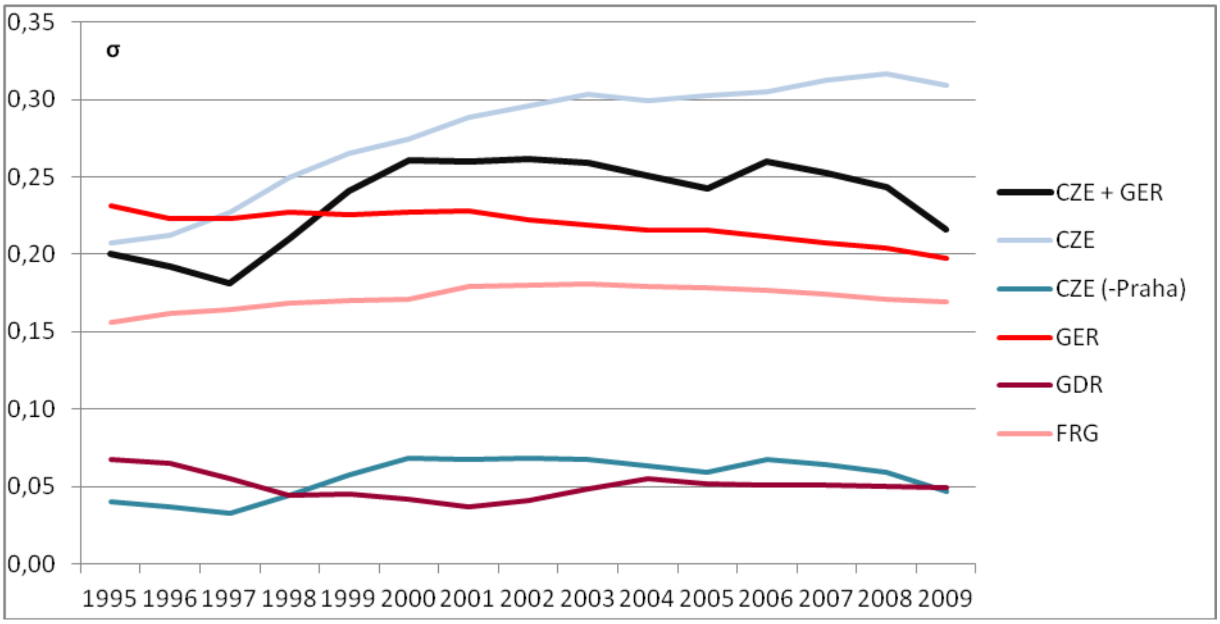

Figure 10: Variation Coefficient of the GDP per capita of Czech and German Regions

Comparing σ-convergence on both country and regional level, we can spot interesting differences. Although income dispersion is decreasing at the country level, the regional level of GDP dispersion was lower in 1995 than in 2009. Income dispersion at the national level is slightly lower than at the NUTS 2 level.

If we compare income distribution of NUTS 2 regions expressed in percents of sample's average GDP/capita in PPS in 1995 and 2009 (see Figures 11 and 12), we can notice that the number of regions that are close to the average GDP per capita grew. Czech regions with the exception of Střední Morava moved up to the group that has got 60% - 80% of average GDP/capita. Some of the East German regions improved to 80% - 100% group, others stayed one group below. So there is a clear sign of catching up of regions from below.

Situation on the above-average part of income distribution is more interesting. It seems that a group of initially rich regions has separated from the rest and created its own rich group. If we take a closer look at those regions, they all contain a big city (or are mainly formed by one city)[4]. Praha shows that even initially poorer region can climb up into such group almost beating Hamburg in 2009. Praha also demonstrates that the rich region does not have to be automatically from the West Germany. That the presence of a big city is not a sufficient condition for a membership of a region in a rich club proves Berlin reaching only 94% and Köln with 104% of average GDP/capita.

[4] Hamburg region, Praha, Oberbayern (München), Bremen and Darmstadt (Frankfurt am Main)

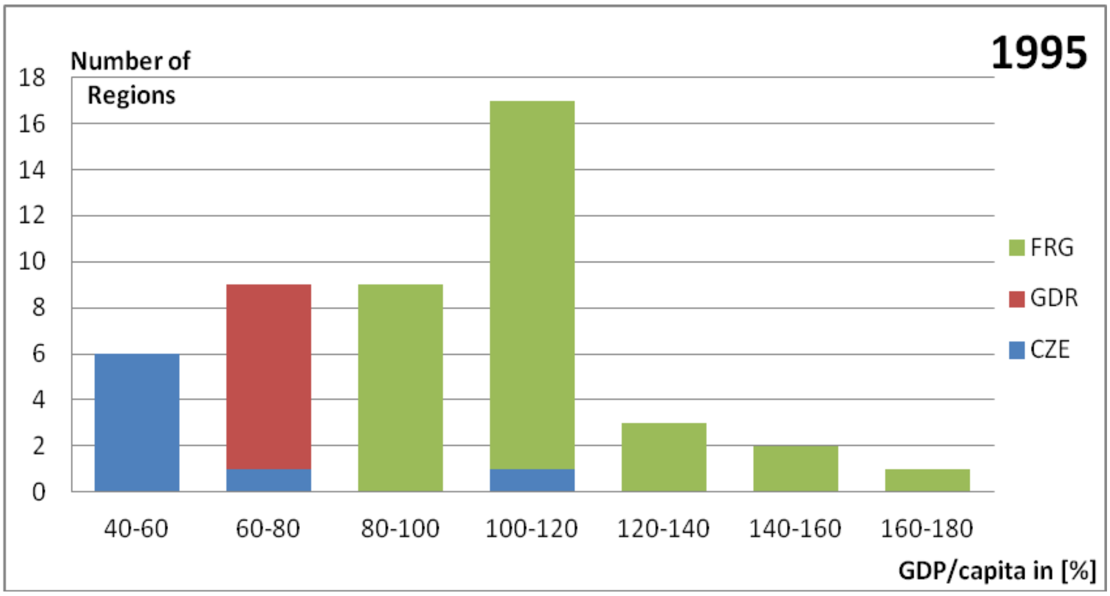

Figure 11: Distribution of GDP per Capita in PPS in NUTS 2 Regions in Percents of Average of the Czech Republic and Germany, 1995

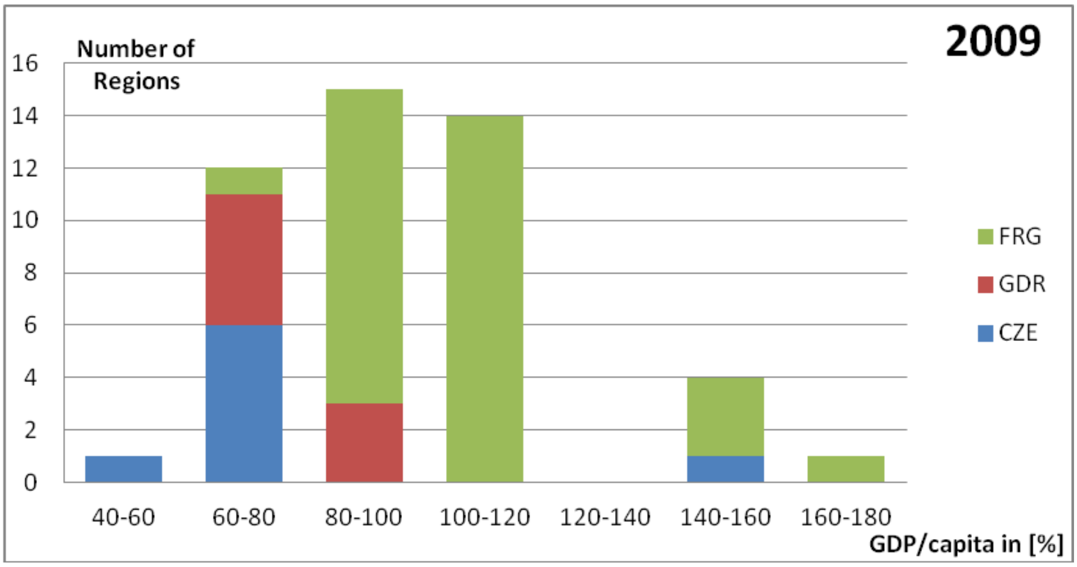

Figure 12: Distribution of GDP per Capita in PPS in NUTS 2 Regions in Percents of Average of the Czech Republic and Germany, 2009

Let us take a closer look at the relative change of position of individual regions. Figure 13 and Table 3 show change of position of the regional GDP per capita in PPS with relation to the average value in between years 1995 and 2009. That the West German regions are slowly losing their positions can be seen from the Figure 13. On the other side, Czech and GDR regions position is getting better (with the exception of -0.2 percentage point loss of Severozápad region). Regions of the Czech Republic reached average gain of 11.3 percentage point per region. This may be a bit misleading because the only region that reached at least average gain was Praha region with tremendous increase of relative GDP per capita by 51.9 percentage points. The average change of relative position of the Czech regions excluding Prague was 5.5 percentage points which was surpassed by change of position of GDR regions (7.7).

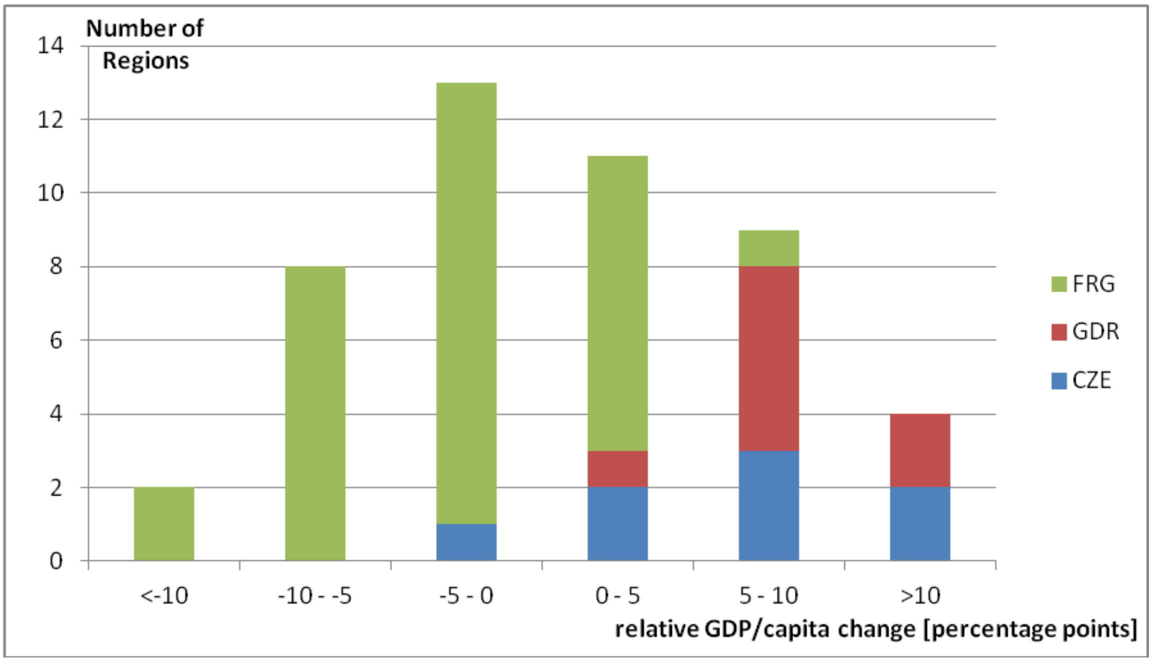

Figure 13: Relative Change in Regional GDP/capita 1995-2009

Change in Relative GDP position 1995 - 2009				
[perentage points]		Lowest	Average	Highest
Czech regions		-0,2	11,3	51,9
German regions		-13,4	-1,1	11,0
	Former FRG regions	-13,4	-3,4	11,0
	Former GDR regions	3,9	7,7	5,6

Table 3: Relative Change in Regional GDP/capita 1995-2009

6. Beta Convergence in the Czech Republic and Germany

6.1 Estimating Absolute Beta Convergence

The easiest way how to test the hypothesis of absolute β-convergence is to estimate the cross-section regression equation:

$$\frac{\log(\frac{y_{i,t+T}}{y_{i,t}})}{T} = a - b \cdot \log(y_{i,t}) + u_i, i = (1,..., N) \qquad (23)$$

as suggested by Sala-i-Martin (1996). Explained variable $\frac{\log(\frac{y_{i,2009}}{y_{i,1995}})}{T}$ denotes annualized growth rate of region *i* between years 1995 and 2009. According to the neoclassical growth model, growth of the economy should be inversely related to its initial GDP per capita (explanatory variable $\log(y_{i,1995})$). *a* and *b* are constants and u_i is a disturbance term. We will estimate this equation using the ordinary least squares (OLS) method and we assume that the simple regression model assumptions hold. Constant *b* is important for us because it shows whether there is absolute β-convergence or not. In this model, we assume existence of only one steady state and hence it measures absolute β-convergence[5]. In case $b \leq 0$, economies are diverging. If $b > 0$, we are dealing with absolute β-convergence and the higher is *b*, the stronger the convergence is. It may happen that $b > 1$, then the initially poorer economies overgrew initially richer ones during measurement period. We will use the data of Czech and German NUTS 2 regions of the Eurostat regional database again.

We obtained estimate $\hat{b} = 0.011$ from the regression of 47 Czech and German NUTS 2 regions. 95% confidence interval predicts value of *b* between 0.005 and 0.016. P-value of explanatory variable is really close to zero so we can strongly reject the hypothesis that initial GDP per capita does not have any effect at the GDP growth. R-squared of this simple model is around 25%. This means that initial GDP per capita explains only about one quarter of variance of the growth rate. The neoclassical growth model states that the growth rate of output during transition to the steady state is from great part explained by amount of capital per capita *k* which determines the GDP (see Eq. 9) so it is rather a surprise that initial GDP explains only so little of the growth rate variance. Sala-i-Martin (1996) further advises to transform *b* into β to be able to compute the speed of convergence:

$$b = \frac{1 - e^{\beta \cdot T}}{T} . \quad (24)$$

T in the Eq. (24) is a number of time periods between the beginning and the end of a measurement. We observed regions in 1995 and 2009 so *T=15*. Rearranging and

[5] As we stated above, the model can also capture conditional convergence in the case that there is in reality also only one common steady state for all the regions. In such rare case, conditional β-convergence equals absolute β-convergence.

inserting into Eq. (24) we receive convergence coefficient $\beta = 0.012$. This means that the economies are closing about 1.2% of the income gap to the steady state each year. In this case it would take about 58 years to close one half of the distance to the steady state level. The complete results of the regressions can be found in Appendix 2 of this paper.

Figure 14 shows regression graphically. OLS regression line is in black. The regions that are above the regression line are growing faster than average – this is the case of East German regions. On the other hand, the West German regions are mostly underperformers. Praha region is a leverage point. If we compute β-convergence only among the Czech regions, then there is a divergence because of Praha region. If we left Praha region out, then there is β-convergence among the Czech NUTS 2 regions.

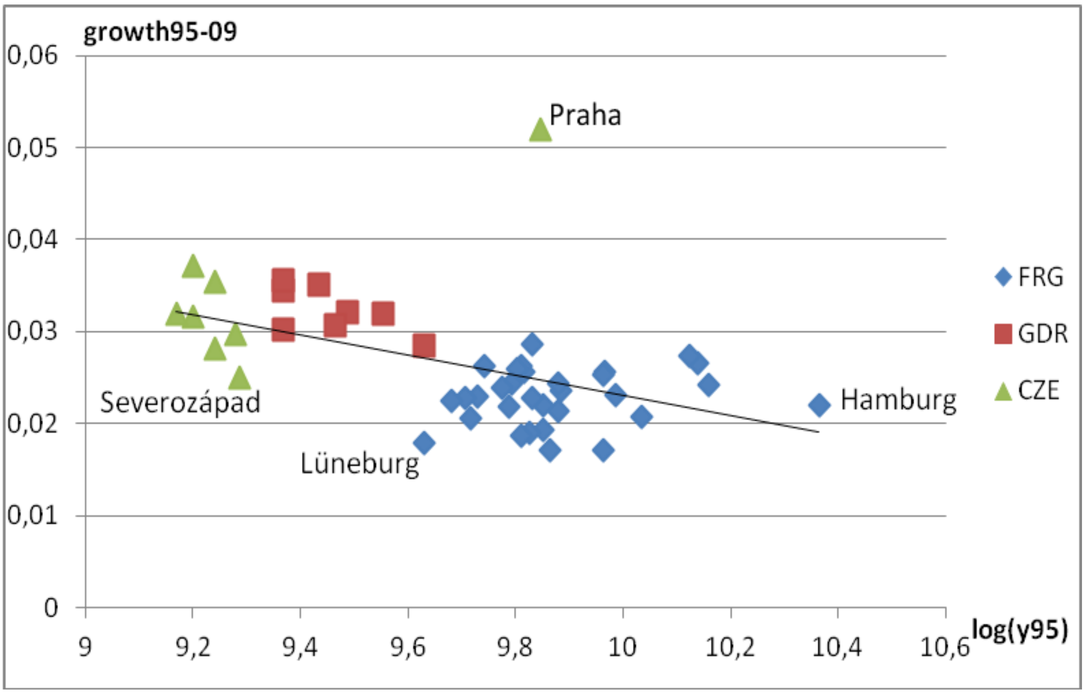

Figure 14: Convergence of GDP per Capita, 1995-2009

We can divide observed time interval into subperiods and watch how does the convergence coefficient change over time. We split up the 1995-2009 period into three 5-year-long intervals (1995-1999, 2000-2004, 2005-2009) and use adapted Eq. (23) for everyone of them. Our estimate of b is: $\hat{b} = -0.004$ during the first five years. This means no absolute β-convergence at that time with $\beta = -0.004$. The initial GDP per capita is insignificant variable (p-value is 0.383) and the R-squared is only 0.02. The divergence is caused not only by depression that hit the Czech lands in 1997 and 1998 but there is also divergence among the West German regions as you can see in Figure 15.

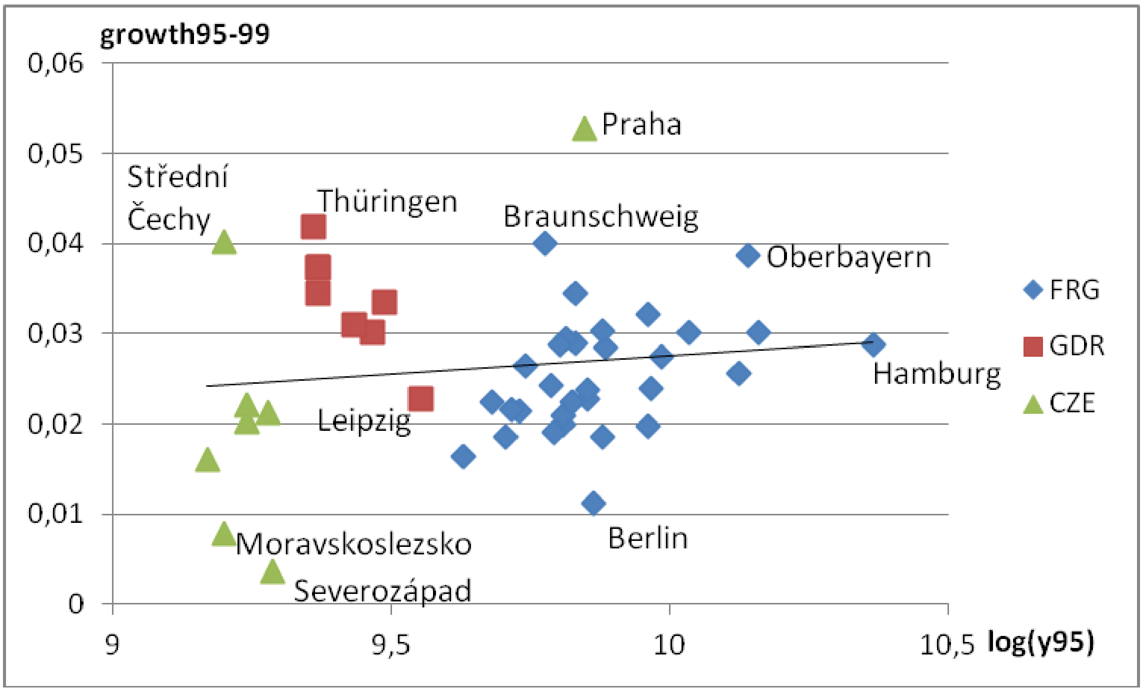

Figure 15: Convergence of GDP per Capita, 1995-1999

The years 2000-2004 show strong evidence for β-convergence. We estimate $\hat{b} \doteq 0.020$ which is equal to $\beta = 0.020$. P-value of initial GDP/capita is close to zero and R-squared is equal to the 29%. The years of slow growth of German economy caused good conditions for the convergence of the Czech regions. There is also quite significant increase of the growth rate of the Czech region between periods 1995-1999 and 2000-2005.

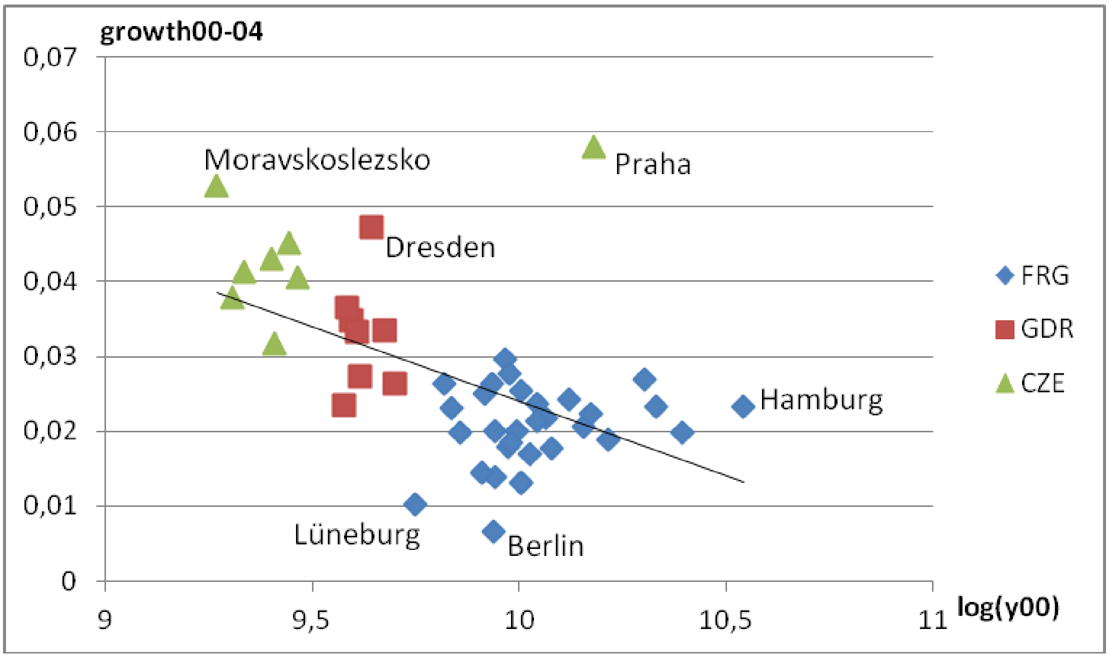

Figure 16: Convergence of GDP per Capita, 2000-2004

We estimate $\hat{b} = 0.012(\beta \doteq 0.013)$ in the last period. Initial GDP per capita is very significant again. About 31% of variance of growth is explained by the model. Speed of growth of the Czech regions declined with comparison to previous period but underperformers are again rather West Germany regions. The GDR regions are all growing above the trend.

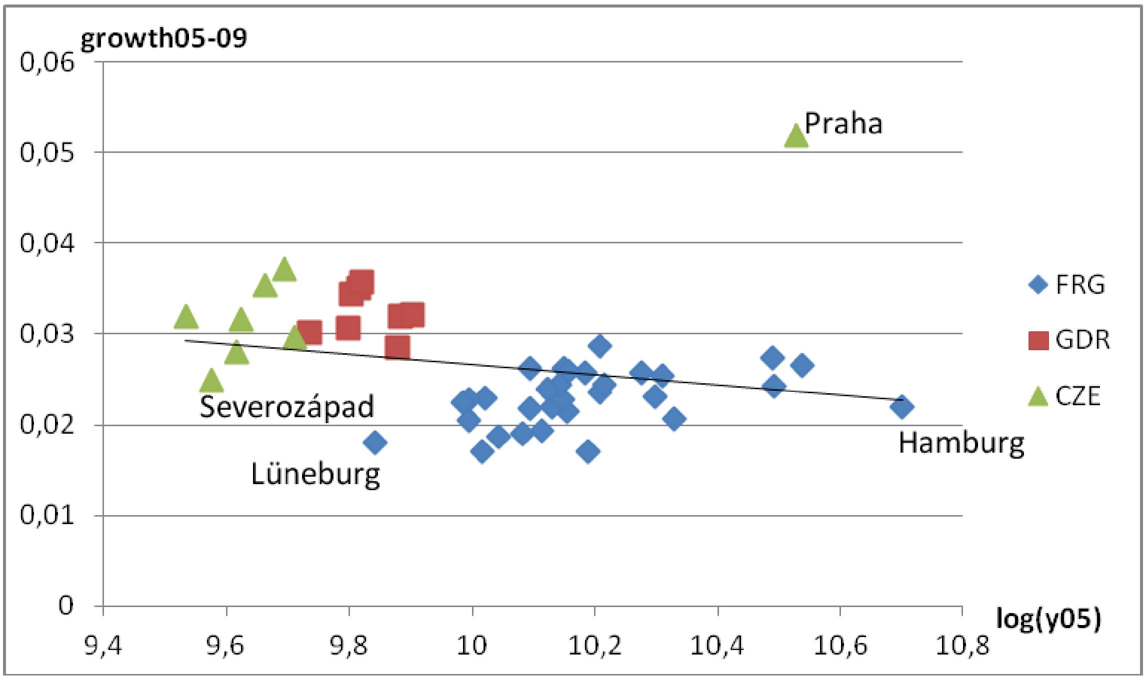

Figure 17: Convergence of GDP per Capita, 2005-2009

We can see that convergence coefficient β is not stable in time from the results of regressions above. The first period (1995-1999) was affected by the transformation of Czech economy from centrally planned to market economy. There was a clear sign of convergence among Czech and German regions in the first decade of 21^{st} century but the convergence speed was not impressive. The convergence benchmark of 2% was reached only in the period 2000-2004. East German regions and especially Praha region were performing well throughout all the measurement periods. Some of the West German regions also performed well (Oberbayern, Hamburg) but the most of them was underperforming (Lüneburg, Berlin, Köln).

We can use slightly more sophisticated model to obtain estimates of β directly without any additional computation:

$$1/T \cdot \log\left(\frac{y_{i,2009}}{y_{i,1995}}\right) = a - \frac{1 - e^{-\beta \cdot T}}{T} \cdot \log(y_{i,1995}) + u_i, i \in 1,\dots,N \tag{25}$$

This model is estimated with non-linear least squares and is used for example by Barro & Sala-i-Martin (1995), Tondl (2001) or Iancu (2009). The advantage of this model is that we receive β and its standard errors directly without any further computation.

6.2 Estimating Conditional β-convergence

The neoclassical growth model does not predict that the initially poorer economies will grow faster than richer ones but that the economies further from their steady states will grow at faster rate than economies closer to it. To estimate conditional β-convergence, we need to hold constant the steady state of every economy if we suspect that the economies in the database can have different steady state. We can do so by adding variables that proxy the steady state position. Theory stated earlier says that the steady state position is affected by the saving rate, level of technology and its growth rate, population growth rate and depreciation rate of capital. It is not always easy to find these variables and we often have to look for proxy variables. For example we choose percentage of human resources in science and technology (*HRST*) in total population as a proxy for the level of technology. Interesting is that the saving rate is insignificant in all the models we tried. We added population density variable (*density*) and dummy variable that distinguish between regions that are formed only by one city (*justcity*). This variable proved to be more significant than including variable for regions that include city with at least million inhabitants. We estimated equation:

$$\frac{\log(\frac{y_{i,2009}}{y_{i,1995}})}{T} = a - b \cdot \log(y_{i,1995}) + c \cdot HRST + d \cdot density + e \cdot justcity + u_i, i = (1,...,N) \quad (26)$$

All the variables included are significant at 5% significance level. Initial income variable has p-value really close to zero. Estimated value of *b* is 0.016 which refers to the $\beta = 0.018$. Such convergence rate is quite close to the benchmark value of 2%. Higher percentage of population in science and technology has a positive effect on the growth rate of GDP per capita and the positive effect has also the fact that the region is formed by one big city. Interesting in this case is slightly negative effect of population density on the growth rate. R-squared of the model is around 50% (adjusted R-squared is 46%) which is quite high number in comparison with the models presented above.

6.3 Panel Data Estimation of β-convergence

Criticism of the cross-sectional convergence analysis is often based on the fact that it violates the assumption of zero conditional mean and so the estimates are biased. The cross-sectional analysis omits the variables that lead to the different steady state incomes. Islam (1995) showed that omitting variables leads to the downward bias of the convergence coefficient β. Another benefit of panel data analysis is that we use more measurements and thus have more information in the model. We will estimate panel data equation:

$$\log y_{i,t} - \log y_{i,t-1} = a_i - b \cdot \log y_{i,t-1} + \lambda_t + u_{i,t} \quad (27)$$

described in Tondl (2001). Explained variable is annual growth of output per capita. This should have negative relation to the GDP per capita one period ago. a_i is a regional fixed effect and λ_t is a time specific effect. Regional fixed effect determines the steady state for every region. This is the difference between estimating conditional convergence using dummy variables where the groups of regions have their steady state not every region alone. We use dummy variables for individual years instead of using λ_t. We estimate Eq. (27) using fixed effects model.

As in the cross-sectional model we are interested in the coefficient b which can be transformed into correlation coefficient through relation:

$$\beta = \frac{-\log(1-b)}{T} \quad (28)$$

again stated in Tondl (2001). We estimated $\hat{b} = 0.138$ which is equal to $\beta = 0.011$ using the data for NUTS 2 regions in 1995-2009 by Eurostat. All the variables are significant at the 5% significance level. R-squared of the regression is 31% which is similar to the models estimated earlier. On contradiction to the finding of Islam (1995), estimated convergence using panel data ($\beta = 0.011$) was slightly lower than using cross-sectional data ($\beta = 0.012$).

cross-sectional analysis						
	absolute convergence			conditional convergence		
period	β	p-value	R^2	β	p-value	R^2
1995-2009	1.2%	0	25%	1.8%	0	51%
1995-1999	-0.4%	0.383	2%	0.2%	0.672	38%
2000-2004	2%	0	29%	2.4%	0	44%
2005-2009	1.3%	0	31%	2.0%	0	52%
panel data analysis						
period	β	p-value	R^2			
1995-2009	1.1%	0	31%			

Table 4: Summary of Estimated Models

Conclusion

According to the neoclassical theory of economic growth, the steady state position is important for determination of growth of economies. If the economies have the same steady state position, then they should converge in terms of GDP per capita. We observed two of the determinants of the steady state position: population growth and saving rate of both countries and noticed that development of those variables is substantially different. It even seems to be negatively correlated. We can assume that the steady state position of both countries is different from our finding.

We tried to estimate time needed for the Czech Republic to catch-up with Germany using average growth rate form years 1995-2010. If the both countries continue to grow at that rate, GDP per capita will even up in 29 years. This simple analysis is in conflict with the neoclassical growth theory that says that the growth rate should diminish as an economy approaches steady state income.

Income dispersion on the national level declined steadily during 1995-2009 period. We can say that both countries converged in the sense of σ-convergence. If we are interested in σ-convergence of the Czech Republic and Germany on the regional level, situation is slightly more complicated. Income dispersion of the regions is slightly higher in 2009 than it was in 1995. This is caused by sharp increase in income dispersion during Czech economic crisis in the late 1990s. Income dispersion was stable in the beginning of new millennium and we can observe σ-convergence in the regional level since 2005. Income dispersion in the Czech regions is growing because of the fast growth of Praha region that is not followed by the rest of the Czech Republic. Czech regions without Praha have small stable income dispersion. Income dispersion of German regions is slowly decreasing although it is increasing in the former FRG regions.

We used cross-sectional model to estimate β-convergence among the regions of both countries. We found out that initially poorer regions tend to grow faster than the rich ones. This event is called absolute β-convergence. Estimated value of convergence coefficient β is 1.2% which means that an economy shifts by 1.2% of the gap between current position and the steady state position. It would take almost 58 years to close just one half of the gap with such speed. We divided the 15 years long observation period into three intervals and observed β-convergence on every of them. The first period (1995-1999) shows negative value of convergence coefficient and initial income variable appears to be insignificant. We accredit such results to the economic crisis of the Czech Republic connected to the end of transformation from the central planned economy. Initial income is very significant and there is clear sign of absolute β-convergence in another two periods. We can see from the results of analyses of individual periods that the convergence speed is not stable in time.

We tried to verify validity of the neoclassical growth model by testing the hypothesis of conditional β-convergence. We added additional variables into our cross-sectional model to differentiate between the steady state of observed regions. Interestingly, the population growth rate appeared to be insignificant in all the models and so we decided to exclude the population growth rate variable from the model. Neoclassical growth model assumes population growth rate to be one of the key determinants of the steady state position. Convergence coefficient was higher than in the case without any additional explanatory variables. It reached 1.8% which is slightly lower than 2% benchmark value reported by Barro & Sala-i-Martin (1995). Divided into three periods, analysis showed similar results as the analysis of absolute β-convergence: insignificant result in the 1995-1999 period and significant positive estimates of convergence coefficient in later years. We estimated panel data model to enrich the dataset by adding the measurements for every region in every year. Estimated convergence coefficient was slightly lower than the one we received from the cross-sectional analysis of β-convergence which contradicts the work of Islam (1995) that states that using cross-sectional analysis instead of panel data analysis brings the downward bias for the convergence coefficient.

To analyze convergence of both countries, we used measurements from 15 years long period. This is quite short time if we take into consideration that we estimated that the half of the gap to the steady state will be closed in the period at least twice as long but thanks to the young age of both countries in current arrangement , we cannot find more relevant data. Despite this, we obtained some interesting results from our analysis of the real economic convergence of the Czech Republic and Germany.

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All the data used in this work were acquired from the Eurostat database. Available online:
http://epp.eurostat.ec.europa.eu/portal/page/portal/statistics/themes

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Appendix 1: Properties of the Neoclassical Production Function

We take into account the neoclassical production function which has to meet following requirements. For any K>0 and L>0:

a) diminishing marginal product with respect to both capital and labor

If we increase amount of an input, output must increase as well but the increment will get smaller with rising amount of the input:

$$\frac{\partial F(K, L\cdot A)}{\partial K} > 0, \frac{\partial F(K, L\cdot A)}{\partial L} > 0$$

$$\frac{\partial^2 F(K, L\cdot A)}{\partial^2 K} > 0, \frac{\partial^2 F(K, L\cdot A)}{\partial^2 L} > 0 \quad .$$

b) production function is homogenous of the first degree

In other words, production shows constant returns to scale. We assume here that no scarce resource which quality will decrease with amount used (e.g. land), is employed:

$$F(a\cdot K, a\cdot L\cdot A) = a\cdot F(K, L\cdot A), a > 0 \, .$$

c) Inada condition

Marginal product of an input approaches infinity as its amount goes to zero and as its amount goes to infinity its marginal product approaches zero:

$$\lim_{K\to 0}(MPK) = \lim_{L\to 0}(MPL) = \infty$$

$$\lim_{K\to \infty}(MPK) = \lim_{L\to \infty}(MPL) = 0 \, .$$

Simple equation that satisfies the assumptions of the neoclassical function is the Cobb-Douglas production function:

$$Y = F(K, A\cdot L) = K^{\alpha}(A\cdot L)^{1-\alpha}, 0 < \alpha < 1 \, ,$$

where α is the output elasticity of capital and $1-\alpha$ is the output elasticity of labor. For proof that this function satisfies the neoclassical conditions, please see Barro & Sala-i-Martin (1992).

Appendix 2: Regression Results

Following results are ordered by page of appearance in the text. Program STATA/SE 11.0 was used to estimate regression equations.

```
. regress growth9509 logy95

      Source |       SS       df       MS              Number of obs =      47
-------------+------------------------------           F(  1,    45) =   15.09
       Model |  .00047875      1   .00047875           Prob > F      =  0.0003
    Residual |  .001427983    45  .000031733           R-squared     =  0.2511
-------------+------------------------------           Adj R-squared =  0.2344
       Total |  .001906733    46  .000041451           Root MSE      =  .00563

------------------------------------------------------------------------------
  growth9509 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      logy95 |   .0110294   .0028396     3.88   0.000     .0053102    .0167486
       _cons |    .133385   .0275649     4.84   0.000     .0778665    .1889035
------------------------------------------------------------------------------
. regress growth9599 logy95

      Source |       SS       df       MS              Number of obs =      47
-------------+------------------------------           F(  1,    45) =    0.78
       Model |  .000064689     1  .000064689           Prob > F      =  0.3830
    Residual |  .003750358    45  .000083341           R-squared     =  0.0170
-------------+------------------------------           Adj R-squared = -0.0049
       Total |  .003815047    46  .000082936           Root MSE      =  .00913

------------------------------------------------------------------------------
  growth9599 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      logy95 |  -.0040543   .0046018    -0.88   0.383    -.0133228    .0052142
       _cons |  -.0129657   .0446715    -0.29   0.773    -.1029388    .0770074
------------------------------------------------------------------------------
. regress growth0004 logy00

      Source |       SS       df       MS              Number of obs =      47
-------------+------------------------------           F(  1,    45) =   18.45
       Model |  .001630555     1  .001630555           Prob > F      =  0.0001
    Residual |  .003977487    45  .000088389           R-squared     =  0.2908
-------------+------------------------------           Adj R-squared =  0.2750
       Total |  .005608042    46  .000121914           Root MSE      =   .0094

------------------------------------------------------------------------------
  growth0004 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      logy00 |   .0198362   .0046184     4.30   0.000     .0105343    .0291381
       _cons |     .22236   .0456173     4.87   0.000      .130482     .314238
------------------------------------------------------------------------------
. regress growth0509 logy05

      Source |       SS       df       MS              Number of obs =      47
-------------+------------------------------           F(  1,    45) =   20.24
       Model |  .000524926     1  .000524926           Prob > F      =  0.0000
    Residual |  .001166992    45  .000025933           R-squared     =  0.3103
-------------+------------------------------           Adj R-squared =  0.2949
       Total |  .001691918    46  .000036781           Root MSE      =  .00509

------------------------------------------------------------------------------
  growth0509 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      logy05 |   .0123434   .0027436     4.50   0.000     .0068176    .0178692
       _cons |   .1339784   .0275795     4.86   0.000     .0784303    .1895264
------------------------------------------------------------------------------
```

. regress growth9599 logy95 hrst density justcity

Source	SS	df	MS
Model	.001441547	4	.000360387
Residual	.0023735	42	.000056512
Total	.003815047	46	.000082936

Number of obs =	47
F(4, 42) =	6.38
Prob > F =	0.0004
R-squared =	0.3779
Adj R-squared =	0.3186
Root MSE =	.00752

growth9599	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
logy95	.001864	.0043778	0.43	0.672	-.0069709	.0106988
hrst	.0011748	.0002652	4.43	0.000	.0006395	.0017101
density	-.0000125	3.99e-06	-3.12	0.003	-.0000205	-4.41e-06
justcity	.025727	.0098385	2.61	0.012	.0058722	.0455818
_cons	.0171408	.0404972	0.42	0.674	-.0645858	.0988675

. regress growth0004 logy00 hrst density justcity

Source	SS	df	MS
Model	.002463027	4	.000615757
Residual	.003145014	42	.000074881
Total	.005608042	46	.000121914

Number of obs =	47
F(4, 42) =	8.22
Prob > F =	0.0001
R-squared =	0.4392
Adj R-squared =	0.3858
Root MSE =	.00865

growth0004	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
logy00	.0241745	.0050477	4.79	0.000	.0139878	.0343612
hrst	.0001527	.0003145	0.49	0.630	-.000482	.0007875
density	-.0000101	4.59e-06	-2.21	0.033	-.0000194	-8.83e-07
justcity	.0349345	.0114006	3.06	0.004	.0119271	.0579419
_cons	.2626189	.0469998	5.59	0.000	.1677694	.3574684

. regress growth9509 logy95 hrst density justcity

Source	SS	df	MS
Model	.000966927	4	.000241732
Residual	.000939806	42	.000022376
Total	.001906733	46	.000041451

Number of obs =	47
F(4, 42) =	10.80
Prob > F =	0.0000
R-squared =	0.5071
Adj R-squared =	0.4602
Root MSE =	.00473

growth9509	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
logy95	.0156757	.0027548	5.69	0.000	.0101164	.0212351
hrst	.0004271	.0001669	2.56	0.014	.0000902	.0007639
density	-6.51e-06	2.51e-06	-2.59	0.013	-.0000116	-1.44e-06
justcity	.0219048	.0061909	3.54	0.001	.0094111	.0343985
_cons	.1683206	.0254829	6.61	0.000	.116894	.2197473

```
. regress growth0509 logy05 hrst density justcity

      Source |       SS       df       MS              Number of obs =      47
-------------+------------------------------           F(  4,    42) =   11.50
       Model |  .000884493     4  .000221123           Prob > F      =  0.0000
    Residual |  .000807425    42  .000019224           R-squared     =  0.5228
-------------+------------------------------           Adj R-squared =  0.4773
       Total |  .001691918    46  .000036781           Root MSE      =  .00438

------------------------------------------------------------------------------
  growth0509 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      logy05 |   .0189276   .0028676     6.60   0.000     .0131405    .0247147
        hrst |   .0003412   .0001592     2.14   0.038     .0000199    .0006626
     density |   1.70e-06   2.33e-06     0.73   0.471    -3.01e-06    6.40e-06
    justcity |   .0037792   .0058647     0.64   0.523    -.0080564    .0156147
       _cons |   .1902334   .0273699     6.95   0.000     .1349987    .2454681
------------------------------------------------------------------------------
. xtreg logylogy1 loggdpcap1 dummy97 dummy98 ..., fe
Fixed-effects (within) regression               Number of obs      =       658
Group variable: region                          Number of groups   =        47

R-sq:  within  = 0.7734                         Obs per group: min =        14
       between = 0.1129                                        avg =      14.0
       overall = 0.3171                                        max =        14

                                                F(14,597)          =    145.52
corr(u_i, Xb)  = -0.7959                        Prob > F           =    0.0000

------------------------------------------------------------------------------
   logylogy1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  loggdpcap1 |   .1382719    .017476     7.91   0.000     .1039499    .1725938
     dummy97 |  -.0099253    .003373    -2.94   0.003    -.0165496    -.003301
     dummy98 |  -.0110534   .0035026    -3.16   0.002    -.0179324   -.0041744
     dummy99 |   .0120695    .003649     3.31   0.001     .0049031     .019236
     dummy00 |   .0140784   .0040181     3.50   0.000      .006187    .0219698
     dummy01 |   .0112491   .0044323     2.54   0.011     .0025444    .0199539
     dummy02 |   .0125728   .0047979     2.62   0.009       .00315    .0219956
     dummy03 |   .0162892   .0051526     3.16   0.002     .0061698    .0264085
     dummy04 |   .0415224   .0055246     7.52   0.000     .0306723    .0523725
     dummy05 |   .0430216   .0062266     6.91   0.000     .0307928    .0552504
     dummy06 |   .0561826   .0068812     8.16   0.000     .0426684    .0696969
     dummy07 |   .0662329   .0076679     8.64   0.000     .0511735    .0812922
     dummy08 |   .0264423    .008523     3.10   0.002     .0097037    .0431809
     dummy09 |   -.039795   .0086258    -4.61   0.000    -.0567357   -.0228544
       _cons |   1.383794   .1695868     8.16   0.000     1.050735    1.716854
-------------+----------------------------------------------------------------
     sigma_u |   .0372113
     sigma_e |   .0159566
         rho |  .84468131   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(46, 597) =     3.51             Prob > F = 0.0000
```