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BACHELOR THESIS

Strategic Behaviour upon Market Entry

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently, using only the listed resources and literature.

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Signature

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Abstract

This thesis focuses on the behaviour of firms when choosing among multiple markets one to enter. This situation occurs frequently during expansions of firms or introduction of new products. The model used is similar to the model of a static entry game by Bresnahan and Reiss (1990), which was applied on single markets under various assumption to model the impact of market structure. In the thesis a similar discrete-choice model is formulated and studied under multiple markets, to determine the market and output a firm chooses. An entry model with heterogeneous products, 2 firms and N markets is studied and compared to findings about single market entry. To test various characteristics of the structure of markets a simple computer program is used to simulate market entry.

JEL Classification	L11, L12, L13, L22
Keywords	market entry, market structure, oligopoly markets, firm strategy

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Abstrakt

Bakalářská práce se zabývá chováním firem při možnosti vstupu na více trhů. Tato situace nastává často při expanzi firem nebo při zavedení nového výrobku. Použitý model je podobný modelu Bresnahana a Reisse (1990), který byl použit pro modelování vstupu na jeden trh za různých okolností. V této práci formulujeme podobný diskrétní model pro více trhů, abychom zjistili trh a množství výstupu, které si firma vybere. Model vstupu na trh s heterogenním produktem, dvěma firmami a N trhy je studován a porovnán se závěry pro jeden trh. Pro testování vstupu na více trhů je použit jednoduchý program na simulování různých struktur trhů.

Klasifikace JEL	L11, L12, L13, L22
Klíčová slova	vstup na trh, struktura trhu, oligopoly, strategie firem

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Contents

List of Tables

List of Figures

1 Introduction

In the field of industrial organization a game-theoretic approach is very useful to model relationships where individuals affect each others decisions. A large number of market procedures is determined by strategic interactions of multiple companies, therefore many papers have focused on oligopolies in the presence of one market with game-theoretic discrete models of entry. Furthermore the equilibrium number of companies, that can be profitable in the market has been studied.

The idea behind these models is that entry occurs in two steps. The first step is the decision of a firm to enter or not to enter a market. Followed by the second step determining the output. The first step serves as a benchmark for the firm. If the firm expects no profits from the market, no entry will occur. In the second step the company estimates its optimal output, using the information they have available. As each firm influences the decisions of its competitors, these strategic interactions become complicated, especially with a growing number of contestants.

The topic of market entry and other aspects of the competitors and markets have been studied under various assumptions. For single market entry the model by Bresnahan and Reiss (1990) has been widely used and has even been applied in recent works on the topic for example by Abbring and Campbell (2007). The model has various applications, which are not only limited to market entry, but can be used to market dynamics and market structure in general. The extensions of this model are used for estimating sunk costs, entry barriers and others.

For example, Seim (2006) uses a discrete choice location model to model product differentiation in a single market.

The motivation behind the study of market entry is to be able to find equilibria and assess how these equilibria are affected by the characteristics of the market, because market characteristics determine the market structure. So, we want to know how entry barriers, economies of scale, first-mover advantages and other characteristics influence the market structure. This information can subsequently be used to determine the impact of regulations and the number of competitors on market effectiveness and competition. This provides useful information for regulators and firms as well in the decision-making process.

1.1 Why Study Entry into Multiple Markets

To extend the work on single market entry to multiple markets can be seen as redundant, because some may argue that we can get all necessary information from examining a single market. However this is not true in most cases, because it is a frequently occurring economic situation to expand to new markets. The situation where a company has more options available in the form of multiple markets is quite common and certainly more frequent than the option of entering only one market at a time.

In the case where we have no restrictions on the size of the market, we can consider establishments from cities to countries as markets. In this case almost any expanding firm faces the decision making process of multiple market entry at some time. It is reasonable to assume that a company compares various markets in order to enter the first best market, according to the information and expectations the firm has, in order to maximize its profit.

Furthermore the introduction of multiple markets into the decision making process, causes huge differences in the order of entry. For example we know that any company if entering one specific market, that we have chosen, would be profitable. However no entry occurs. Therefore we either estimated the market profit incorrectly or that an alternative market with higher profit exists. This can result in the fact that markets that are profitable are empty or monopolistic for a long time even though another firm would be profitable. Without comparing the markets among each other we cannot give reliable answers on the order of entry.

So far economists were able to determine how single market characteristics influence the market structure. Moreover with the extension to multiple markets we can determine which factors are judged as most important by companies. For example if the market size, the distance of a market or the opportunity to expand from one market to other ones in the future play a key role in the decision-making process.

1.2 Behaviour of Firms

The behaviour of firms upon entering multiple markets is difficult to assess since we do not know much about the decisions the company makes and the methods the company uses to estimate the profitability of the markets.

We have no information about what market characteristics companies prefer, for example market size, market distance, trade barriers or other characteristics. Neither do we know which of these aspects are considered as most important. Some of these data can be found ex post after a company enters a market. However other characteristics, that play an important role for the company, can not be observed at all. An example of this would be special knowledge about the market or some sort of preferential treatment compared to other companies.

All the information available is the information about which market the company chose, however we have no information about the rank of the other markets. If the company then expands further, we get additional information in the form of a second market. So with every new market entry of a company we get more data. Nonetheless, we can not say that in the previous decision-making period (let's call it t) and the current period(t+1), that the market chosen in period t+1 was ranked second in the period t. The market conditions might have changed without us being able to observe it from the decisions of the company. This makes the observation of multiple market entry very difficult.

Compared to single market entry, multiple market entry gives us significantly less information about the market, because of insufficient information about the not entered markets. In this case the amount of unobserved data is even higher than for single market entry.

As a result markets that are not chosen appear to us as identical, because of a lack of information. So the first step to grasp the decision-making process companies use is to set up a simple model with identical markets, so that the only variable is the number of opponents in the market. To see how companies react to opponents in a varying number of markets.

1.3 Outline of the Thesis

In the first part of the thesis we introduce the model and justify the methods we are using. Then, in order to be able to use the results from previous papers on the topic, which have used the Bresnahan and Reiss model, we compare the results from a single market to the results of the Bresnahan and Reiss model. This ensures that we do not have to prove that an equilibrium output and an equilibrium number of competitors exist. Then we use the model on multiple markets to see the effect of varying number of markets on the entry strategy companies choose.

We further extend our model to more realistically suit the needs of market entry and apply the results to other situations of multiple market entry. Finally to test the model under various structures of markets a computer program is used to simulate market entry into multiple markets to quantify the findings. At the end we conclude our findings.

2 Model for Market Entry

For industries established in one country it is natural to expand to another country, if there is a possibility to be profitable. However as companies have to make decisions about foreign market entry, they are limited by the costs this expansion incurs on them. So, as a result the expansion of a company, can be seen as a series of entries based on information and expectations about these available profitable markets. Thanks to some simplifications, we can treat the expansion process of a company as a sequential moves game. To make the sequential moves easier, there is a limitation that companies can access only one market at a time.

Further, we use a similar model as in Bresnahan and Reiss(1990) to model a simple duopoly with heterogeneous products in a number of markets (n). We model a symmetric duopoly in order to keep the calculations simple, because the same findings apply for a higher number of players and for players with different cost functions. The model works with heterogeneous products to make the findings more realistic. In the BR model the decision process was divided into two steps, however we omit the decision if the market is profitable or not, because this only changes the total number of markets available. This means that in the model we work only with markets, that are estimated by the firms to be profitable.

In order for our model to make entry possible, we assume that at least one profitable market exists ($n \geq 1$). The result is that some of the markets available are entered, in our case a maximum of two markets at once, because each player can enter one market. However the result might as well be that both players enter the same market.

First we assume all markets to be identical and the two players to have the same cost function. So, we consider a cost function with fixed and variable costs $TC = cq + FC$,where the marginal costs are constant. We have knowledge about the market demand $p = a - bq_1 - hq_2$. The own production quantity is considered more important than the opponents quantity in setting the price $b > h$ and $a > c$. Under these assumptions we are able to calculate equilibria in which companies produce. Furthermore, we require the company to stay in a market when it at least covers its variable costs while producing.

We restrict the quantity outputs to pure strategies developed as best response in oligopoly markets. In the model, companies select markets to enter simultaneously. Under the assumption of sequential moves companies wouldn't behave differently for a single market. However this behavior is not very interesting for multiple markets as a player would, if possible, choose an empty market and monopoly output.

2.1 Possible Strategies

From classical game-theoretic models of oligopolies where we maximize profit, we get equilibrium prices and outputs. The results differ according to the approach used to optimalize the companies output. So, each of the following is an equilibrium strategy where the players have no tendency to do any changes to their strategy, because there is no strategy with a higher payoff. However, we are only interested in the quantities produced as the prices will change according to the quantities of the opponent. The players can choose Cournot output, Bertrand output, Stackelberg leader output or Stackelberg follower output. Furthermore, we add the optimal monopoly output, where marginal revenues are equal to marginal costs.

The monopoly output is added, because as stated above it is possible for only a single player to enter a market, in which case the monopoly output is optimal and naturally, when one player does not enter a market his output is zero. The discrete choice model is restricted to pure strategies only. We allow for any combination of the above stated strategies, so for example one player can choose Cournot output while the other one chooses monopoly output.

A 6x6 payoff matrix is created, when we calculate profits under various combinations using optimal output.

The approach using different oligopoly models is not intuitive, because these strategies can be seen as independent and are not able to occur at the same time. However, the setting of the model with the uncertainty about which market will be entered, makes this approach quite correct. The idea behind it is that players try to maximalize their profit in every setting, using all information available. When information is only available about possible output and the number of available markets the players estimate their profit using probability from the number of markets.

It is intuitive to imagine that the optimal output will increase with an increasing number of markets and decrease with an increasing number of players. This results in a situation where each of the strategies is directly preferred to any other under a certain setting of markets and players.

Another reason for choosing these strategies is their simplicity and because they are all optimal solutions for oligopoly markets under different approaches, so the optimal solution for a number of markets from which we are selecting one market, should be one of these strategies. The extension to n identical and independent markets only alters the expected payoff. Without this simplification the model would become very complicated in an incorrect way.

The model can be made more difficult, however this is better done by including other characteristics, such as different markets or varying costs of production. These reflect the real situation better, on the other hand expanding the space of strategies does not give us much additional information. We want to observe the impact of player interactions and priority characteristics that are looked for in a market, not output itself.

2.2 Calculations

In the following paragraphs we calculate the prices and profits, using optimal output for our strategies under the assumptions mentioned above in our model. However, first we summarize optimal output quantities[1]

2.2.1 Summary of Quantities

So far we have calculated optimal output for each strategy. Since in our model any two strategies can occur at the same time, we have to calculate all combinations and the profit players have in these situations. This results in a 6x6 matrix of profits for each player[2]

[1]Calculations for optimal output are included in Appendix A

[2]Profits are calculated without fixed costs as they are identical for each strategy

Monopoly quantity	$q^* = \frac{a-c}{2b}$
Cournot quantity	$q^* = \frac{a-c}{2b+h}$
Bertrand quantity	$q^* = \frac{b(a-c)}{(2b-h)(b+h)}$
Stackelberg Leader quantity	$q^* = \frac{(a-c)(2b-h)}{4b^2-2h^2}$
Stackelberg Follower quantity	$q^* = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

Profit with No Opponent

In this situation when only one firm enters a market, we compare profits for each strategy without the entry of an opponent. We calculate monopoly profit for all possible outputs.

Monopoly $q_m = \frac{a-c}{2b}$ $p_m = \frac{a+c}{2}$
$\Pi_m = (p_m - c)\, q = \frac{(a-c)^2}{4b}$

Cournot $q_c = \frac{a-c}{2b+h}$ $p_m = \frac{2ab+ah-ab+bc}{2b+h}$
$\Pi_c = (p_c - c)\, q = \frac{(b+h)(a-c)^2}{(2b+h)^2}$

Bertrand $q_b = \frac{b(a-c)}{(2b-h)(b+h)}$ $p_b = \frac{2ab^2-abh-ah^2+bch+2b^2c}{4b^2-h^2}$
$\Pi_b = (p_b - c)\, q = \frac{b\left(b^2+bh-h^2\right)(a-c)^2}{(2b-h)^2(b+h)^2}$

Stackelberg Leader $q_{sl} = \frac{(a-c)(2b-h)}{4b^2-2h^2}$ $p^*_{sl} = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2}$
$\Pi_{sl} = (p_{sl} - c)\, q = \frac{(2b-h)\left(2b^2+bh-h^2\right)(a-c)^2}{(4b^2-2b^2)^2}$

Stackelberg Follower $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$ $p^*_{sf} = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$
$\Pi_{sf} = (p_{sf} - c)\, q = \frac{\left(16b^4-20b^2h^2+4bh^3+3h^4\right)(a-c)^2}{4b(4b^2-2h^2)}$

Profit with Monopoly Opponent

A monopoly firm with optimal output is compared to other firms entering the market with all possible strategies.
Monopoly output $q_m = \frac{a-c}{2b}$

Monopoly $q_m = \frac{a-c}{2b} \qquad p_m = a - \frac{a-c}{2} - h\frac{a-c}{2b}$
$\Pi_m = (p_m - c)\, q_m = \frac{(b-h)(a-c)^2}{4b^2}$

Cournot $q_c = \frac{a-c}{2b+h} \qquad p_c = a - b\frac{a-c}{2b+h} - h\frac{a-c}{2b}$
$\Pi_c = (p_c - c)\, q_c = \frac{\left(2b^2-h^2\right)(a-c)^2}{2b(2b+h)^2}$

Bertrand $q_b = \frac{b(a-c)}{(2b-h)(b+h)} \qquad p_b = a - b^2\frac{a-c}{(2b-h)(b+h)} - h\frac{a-c}{2b}$
$\Pi_b = (p_b - c)\, q_b = \frac{\left(2b^3-3bh^2+h^3\right)(a-c)^2}{2(2b-h)^2(b+h)^2}$

Stackelberg Leader $q_{sl} = \frac{(a-c)(2b-h)}{4b^2-2h^2} \qquad p_{sl} = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{a-c}{2b}$
$\Pi_{sl} = (p_{sl} - c)\, q_{sl} = \frac{\left(b^2-h^2\right)(2b-h)^2(a-c)^2}{b(4b^2-2h^2)^2}$

stackelberg Follower $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} \qquad p_{sf} = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} - h\frac{a-c}{2b}$
$\Pi_{sf} = (p_{sf} - c)\, q_{sf} = \frac{\left(16^5-16b^4h-12b^3h^2+16b^2h^3-bh^4-2h^5\right)(a-c)^2}{4b^2(4b^2-2h^2)^2}$

Profit with Cournot Opponent

A Cournot firm with optimal output is compared to other firms entering the market with all possible strategies.
Cornout output $q_c = \frac{a-c}{2b+h}$

Monopoly $q_m = \frac{a-c}{2b} \qquad p_m = a - \frac{a-c}{2} - h\frac{a-c}{2b+h}$
$\Pi_m = (p_m - c)\, q_m = \frac{(2b-h)(a-c)^2}{4b(2b+h)}$

Cournot from previous calculations $\Pi_c = (p_c - c)\, q_c = \frac{b(a-c)^2}{(2b+h)^2}$

Bertrand $q_b = \frac{b(a-c)}{(2b-h)(b+h)}$ $p_b = a - b^2 \frac{a-c}{(2b-h)(b+h)} - h\frac{a-c}{2b+h}$
$\Pi_b = (p_b - c)\, q_b = \frac{\left(2b^4 - b^3h - 2b^2h^2\right)(a-c)^2}{(2b-h)^2(b+h)^2(2b+h)}$

Stackelberg Leader $q_{sl} = \frac{(a-c)(2b-h)}{4b^2-2h^2}$ $p_{sl} = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{a-c}{2b+h}$
$\Pi_{sl} = (p_{sl} - c)\, q_{sl} = \frac{\left(4b^3 - 3bh^2\right)(2b-h)(a-c)^2}{(2b+h)(4b^2-2h^2)^2}$

Stackelberg Follower $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$ $p_{sf} = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} - h\frac{a-c}{2b+h}$
$\Pi_{sf} = (p_{sf} - c)\, q_{sf} = \frac{\left(32b^5 - 16b^4h - 24b^3h^2 + 12b^2h^3 + 2bh^4 - h^5\right)(a-c)^2}{4b(4b^2-2h^2)^2(2b+h)}$

Profit with Bertrand Opponent

A Bertrand firm with optimal output is compared to other firms entering the market with all possible strategies.
Bertrand output $q_b = \frac{b(a-c)}{(2b-h)(b+h)}$

Monopoly $q_m = \frac{a-c}{2b}$ $p_m = a - \frac{a-c}{2} - h\frac{b(a-c)}{(2b-h)(b+h)}$
$\Pi_m = (p_m - c)\, q_m = \frac{\left(2b^2 - bh - h^2\right)(a-c)^2}{4b(2b-h)(b+h)}$

Cournot $q_c = \frac{a-c}{2b+h}$ $p_c = a - b\frac{a-c}{2b+h} - h\frac{b(a-c)}{(2b-h)(b+h)}$
$\Pi_c = (p_c - c)\, q_c = \frac{\left(2b^3 + b^2h - bh^2 - h^3\right)(a-c)^2}{(2b+h)^2(2b-h)(b+h)}$

Bertrand from previous calculations $\Pi_b = (p_b - c)\, q_b = \frac{b(a-c)^2(b-h)}{(2b-h)^2(b+h)}$

Stackelberg Leader $q_{sl} = \frac{(a-c)(2b-h)}{4b^2-2h^2}$ $p_{sl} = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{b(a-c)}{(2b-h)(b+h)}$
$\Pi_{sl} = (p_{sl} - c)\, q_{sl} = \frac{\left(4b^4 - 5b^2h^2 - bh^3 + 2h^4\right)(a-c)^2}{(4b^2-2h^2)^2(b+h)}$

Stackelberg Follower $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$ $p_{sf} = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} - h\frac{b(a-c)}{(2b-h)(b+h)}$

$\Pi_{sf} = (p_{sf} - c)\, q_{sf} = \frac{\left(32b^6-16b^5h-40b^4h^2+24b^3h^3+4b^2h^4-8bh^5-3h^6\right)(a-c)^2}{4b(2b-h)(b+h)(4b^2-2h^2)^2}$

Profit with Stackelberg Leader Opponent

A Stackelberg leader firm with optimal output is compared to other firms entering the market with all possible strategies.
Stackelberg leader output $q_{sv} = \frac{(a-c)(2b-h)}{4b^2-2h^2}$

Monopoly $q_m = \frac{a-c}{2b}$ $p_m = a - \frac{a-c}{2} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$

$\Pi_m = (p_m - c)\, q_m = \frac{(b-h)(a-c)^2}{4b^2-2h^2}$

Cournot $q_c = \frac{a-c}{2b+h}$ $p_c = a - b\frac{a-c}{2b+h} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$

$\Pi_c = (p_c - c)\, q_c = \frac{\left(4b^3-2bh^2-h^3\right)(a-c)^2}{(2b+h)^2(4b^2-2h^2)}$

Bertrand $q_b = \frac{b(a-c)}{(2b-h)(b+h)}$ $p_b = a - b^2\frac{a-c}{(2b-h)(b+h)} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$

$\Pi_b = (p_b - c)\, q_b = \frac{\left(4b^5-6b^3h^2+b^2h^3+h^5\right)(a-c)^2}{(2b-h)^2(b+h)^2(4b^2-2h^2)}$

Stackelberg Leader $q_{sl} = \frac{(a-c)(2b-h)}{4b^2-2h^2}$ $p_{sl} = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$

$\Pi_{sl} = (p_{sl} - c)\, q_{sl} = \frac{\left(4b^2-h^2\right)(b-h)(a-c)^2}{(4b^2-2h^2)^2}$

Stackelberg Follower from previous calculations $\Pi_{sf} = (p_{sf} - c)\, q_{sf} = \frac{\left(4b^2-2bh-h^2\right)^2(a-c)^2}{4b(4b^2-2h^2)^2}$

Profit with Stackelberg Follower Opponent

A Stackelberg follower firm with optimal output is compared to other firms entering the market with all possible strategies.
Stackelberg follower output $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

Monopoly $q_m = \frac{a-c}{2b}$ $\quad p_m = a - \frac{a-c}{2} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

$\Pi_m = (p_m - c)\, q_m = \frac{\left(4b^3-4bh^2+h^3\right)(a-c)^2}{4b^2(4b^2-2h^2)}$

Cournot $q_c = \frac{a-c}{2b+h}$ $\quad p_c = a - b\frac{a-c}{2b+h} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

$\Pi_c = (p_c - c)\, q_c = \frac{\left(8b^4-4b^2h^2+h^4\right)(a-c)^2}{2b(2b+h)^2(4b^2-2h^2)}$

Bertrand $q_b = \frac{b(a-c)}{(2b-h)(b+h)}$ $\quad p_b = a - b^2\frac{a-c}{(2b-h)(b+h)} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

$\Pi_b = (p_b - c)\, q_b = \frac{\left(8b^5+10b^3h^2+4b^2h^3+3bh^4-h^5\right)(a-c)^2}{2(2b-h)^2(b+h)^2(4b^2-2h^2)}$

Stackelberg Leader from previous calculations

$\Pi_{sl} = (p_{sl} - c)\, q_{sl} = (p - c)\, q = \frac{\left(8b^4-8b^3h-2b^2h^2+4bh^3-h^4\right)(a-c)^2}{2b(4b^2-2h^2)^2}$

Stackelberg Follower $q_{sf} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$ $\quad p_{sf} = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$

$\Pi_{sf} = (p_{sf} - c)\, q_{sf} = \frac{\left(16b^5-16b^4h-4b^3h^2+8b^2h^3-bh^4-h^5\right)(a-c)^2}{4b^2(4b^2-2h^2)^2}$

2.3 Results for One Market

From the calculations for one market, we know that an unique equilibrium exists. Therefore even when players enter the market with different outputs after a certain period of time they end up producing the equilibrium output. So in our model for multiple markets we know that the market interactions result in an equilibrium solution. Under our assumption that two players can be profitable in a market, no market exit will occur once the players enter a market. The market dynamics in these type of models are determined by threshold rules. We simplified the setting to only two players and used the assumption that both players are profitable. However if we would either relax this assumption or increase the number of players the equilibrium might no longer be unique. This problem can be solved by using sequential entry instead of simultaneous entry. Nonetheless the sequential move model does not suit multiple market entry.

Individual profits of players in duopoly markets are lower compared to monopoly markets as shown in a similar model by Bresnahan and Reiss. This finding is important in the following decision-making for multiple markets, because it ensures that players try to avoid competition and behave as monopolists. They also state that the level of market demand at which a new firm enters is equal to the ratio of unobserved fixed costs to per capita variable profits. By using this information we would be able to estimate the thresholds for each market. [3]

2.4 Model for Multiple Markets

2.4.1 Expected Profit

We calculate the expected profit of each strategy in regard to the number of markets available, the model uses expected profit, because we have to adjust the model for the uncertainty of entry of another player.

The values are calculated as follows:

$$\text{E(profit)}=\frac{1}{n^2}(\text{profit with other player})+\left(1-\frac{1}{n^2}\right)(\text{profit as only player})$$

The reason, why these expected profits are calculated as shown above is, because we have identical and independent markets, so the choice is determined by probabilities. As all markets are identical, the probability is the same for each market. The term $(a-c)^2$ is left out for simplification as it is included in all profits and fixed costs as well.

[3] The table for these duopoly results is shown in the appendix B

Profit for various strategies(because players are symmetric we analyze only identical strategies):

$$\begin{aligned}
\text{Monopoly:} \quad & E(m) = \tfrac{1}{4} - \tfrac{1}{n^2}\left(\tfrac{h}{4b}\right) \\
\text{Cournot:} \quad & E(c) = \tfrac{b^2+bh}{(2b+h)^2} - \tfrac{1}{n^2}\left(\tfrac{bh}{(2b+h)^2}\right) \\
\text{Bertrand:} \quad & E(b) = \tfrac{b^4+b^3h-b^2h^2}{(2b-h)^2(b+h)^2} - \tfrac{1}{n^2}\left(\tfrac{b^3h}{(2b-h)^2(b+h)^2}\right) \\
\text{Stackelberg leader:} \quad & E(sl) = \tfrac{\left(2b^3+b^2h-2bh^2\right)(2b-h)}{(4b^2-2h^2)^2} - \tfrac{1}{n^2}\left(\tfrac{4b^3h-4b^2h^2+bh^3}{(4b^2-2h^2)^2}\right) \\
\text{Stackelberg follower:} \quad & E(sf) = \tfrac{\left(16b^4-20b^2h^2+4bh^3+3h^4\right)}{4(4b^2-2h^2)^2} - \tfrac{1}{n^2}\tfrac{\left(16b^4-16b^3h^2-8b^2h^3+4bh^4+h^5\right)}{4b(4b^2-2h^2)^2}
\end{aligned}$$

For a large number of markets the situation becomes similar to a single player entry and the other player can be neglected, because the probability that they choose the same market is minimal. Therefore the profits are ordered as follows: $\Pi_m > \Pi_b > \Pi_{sl} > \Pi_c > \Pi_{sf}$

Which results in players choosing monopoly output for a large number of markets and increasing output for less markets. The prices and profits decline.

2.4.2 Break-even Point for Expected Profits

We compare points for which the number of markets and the expected profit are equal to zero. We are interested in the fact, if all strategies become positive for a single market. Else some strategies would not be selected.

$$\begin{aligned}
\text{Monopoly:} \quad & n = \sqrt{\tfrac{h}{b}} \\
\text{Cournot:} \quad & n = \sqrt{\tfrac{h}{b+h}} \\
\text{Bertrand:} \quad & n = \sqrt{\tfrac{bh}{b^2+bh-h^2}} \\
\text{Stackelberg leader:} \quad & n = \sqrt{\tfrac{h(2b-h)^2}{4b^3-5bh^2+2h^3}} \\
\text{Stackelberg follower:} \quad & n = \sqrt{\tfrac{16b^4h-16b^3h^2-8b^2h^3+4bh^4+h^5}{16b^5-20b^3h^2+4b^2h^3+3h^4}}
\end{aligned}$$

All results have in common that profit becomes positive before n=1, which means that all strategies can be selected for an arbitrary number of markets with positive profit.

If we compare the results, we get the following order of breakeven points:

$$n_m > n_b > n_{sl} > n_c > n_{sf}$$

From the breakeven points and the behaviour of the expected profit for a large number of markets, we get the following intervals based on the values of b and h.

$$\begin{aligned}
&\text{Monopoly:} && n \in \left(\sqrt{\tfrac{4b^3-3b^2h-2bh^2-h^3}{(b^3-2b^2h+bh^2)}}; +infinity\right)\\
&\text{Bertrand:} && n \in \left(\sqrt{\tfrac{8b^4-8b^3h-5b^2h^2+6bh^3-h^4}{4b^4-7b^3h+5bh^3-2h^4}}; \sqrt{\tfrac{4b^3-3b^2h-2bh^2-h^3}{(b^3-2b^2h+bh^2)}}\right)\\
&\text{Stackelberg leader:} && n \in \left(\sqrt{\tfrac{8b^2-3h^2}{4b^2-bh-2h^2}}; \sqrt{\tfrac{8b^4-8b^3h-5b^2h^2+6bh^3-h^4}{4b^4-7b^3h+5bh^3-2h^4}}\right)\\
&\text{Cournot:} && n \in \left(\sqrt{\tfrac{16b^3-8bh^2-h^3}{8b^3-3bh^2}}; \sqrt{\tfrac{8b^2-3h^2}{4b^2-bh-2h^2}}\right)\\
&\text{Stackelberg follower:} && n \in \left(0; \sqrt{\tfrac{16b^3-8bh^2-h^3}{8b^3-3bh^2}}\right)
\end{aligned}$$

We see that it is sufficient to know the values of the parameters b and h, to be able to calculate the intervals for which companies choose each strategy.

2.4.3 Results for Multiple Markets

From the results of this model, we see that the strategic output chosen by a player depends on the number of markets available and approximately on the ratio between b and h. The results are quite logical, that as the number of markets increases each player prefers a lower quantity in order to behave as a monopolist in the absence of another player in the market.

Since all the markets are identical, the probability that two players choose the same market is $\frac{1}{n^2}$. As the number of players increases the output decreases, because the probability of more players entering the same market becomes higher.

In a graph of the number of markets in the x axis and profit on the y axis the optimal strategy chosen for each number of markets forms an envelope with the highest profit in each point out of all strategies, because all profits for various strategies have a higher bound and are increasing and concave with the increasing number of markets. Also this gives us a unique solution for any number of markets. Except for the points where two strategies are equal, there players are indifferent to which strategy they choose.

As for the relationship between parameters b and h, which gives us an estimation of product differentiation. As the ratio b/h gets higher, which means that the influence of our own product is higher compared to the output of the competitor, the output decreases. However as we are only interested in whole numbers the changes are not very sensitive. This means, that the number of market is relatively stable to ratio changes.

In this setting of the model all markets are identical, this is very much simplified to be realistic. However the assumption of identical markets can still be applied for some situations. An example of this is when there is a number of markets with similar characteristics and a high cost of acquiring additional information. Instead of companies spending money on market research, they can treat the markets as similar and consider entry into any of those markets as equal. So, for example when we divide our markets with similar characteristics into groups and then treat each market in a group as identical, we can still use this model.

2.4.4 Extending the Model to Varying Market Sizes

To consider markets identical as we did above is a simplification that makes our model very unrealistic. Even when we consider that market sizes are identical companies still prefer one market to another based on other characteristics of the market. The result is that they can have preferences about markets and do not consider them as identical.

So we introduce the concept of a variable $\alpha \in (0, 1)$ in order to create differences among the markets. Although, we only extend the model by using one variable the difference is quite substantial, because this one variable can not only be considered as market size, but as any other characteristic about the market we want. However as market size is an important factor and information about market size can be easily obtained by all players equally, we treat the variable from now on as market size only.

The concept of equal knowledge about a market for all players is realistic, because without entering a market we have only limited information. We consider that no company invests additional funds to get more information. The knowledge about other players creates problems, similar to the problems in a game-theoretic guessing game.

The idea behind the guessing game is to guess $\frac{2}{3}$ of the average from a number between 0 and 100 that a group of people have guessed. The results depend on the level of rationality we assume that the people use. If no person thinks about the reasoning of other players they simply choose $\frac{2}{3} * 100$. However, when we anticipate this behaviour, we guess that the average is $\left(\frac{2}{3}\right)^2 * 100$ and so on for any level of rationality we expect. A perfectly rational player, who expects all other players to be perfectly rational as well, chooses 0 as the outcome of the guessing game.

This is similar to our situation, because we do not know how rational our opponents are. When we are only interested in the market size we order all available markets by size and enter the biggest market. However as for every player a bigger market meaning a higher value of α is preffered to a lower one. So anticipating the choice of their opponents, players enter the second best market. Then when we anticipate that all players enter the second market as their next choice, we can either decide to enter the first market or continue this process with the third market.

No matter which option we use this option is not stable. One of the players always wants to change their reaction. This is not only restricted to identical players as every player prefers larger markets. To solve this problem we can for example use the following method.

The idea is that players have to decide by chance in order to eliminate their identical choices. The model without varying markets introduced above is used in the second part. However first we have to choose the number of markets we want to include in the model. So, we rank all markets decreasingly by α and we restrict ourselves to a number of highest ranked markets. The higher the number of markets, the lower the expected payoff from each market, but we manage to decrease the probability that a similar market is entered by more players. The aim is to choose an optimal number of markets to maximize our expected profit.

This largely depends on our assumptions about the distribution of the markets. We assume that markets are distributed evenly and show a possible solution for this situation. Since our choice is by chance all markets have the same probability of a player entering. The expected value is as follows:

$$E(v) = \frac{1}{m^2}\frac{1}{m}\frac{S}{2} + \left(1 - \frac{1}{m^2}\right)\frac{1}{m}S$$

,where m is the number of markets we choose and S is the sum of market values. The expression $\frac{1}{m^2}$ is the probability of both entering the same market, when all markets are considered identical and $\frac{1}{m}$ is the probability of choosing a market.

For the case of even distribution $S = \frac{n-m}{n-1}$, where n is the total number of markets. For simplicity, we assume that when both players enter the same market their individual market value is halved. This is not correct because it depends on parameters from our multiple identical markets model, but it does not largely affect our results.
After substituting for S in our equation and some minor editing of the expressions we arrive at the following expected value:

$$E(v) = \frac{2n-m-1}{2(n-1)} - \frac{2n-m-1}{2m^2(n-1)}$$

and the optimal value is obtained in the point where

$$\frac{\partial E(v)}{\partial m} = m^3 + m - 4n + 2 = 0$$

This leads to a cubic equation in the form $x^3 + px = q$ With the approximate result of $m = \sqrt[3]{4n-2}$. This is sufficient as m has to be a whole number. [4]
After this first step of determining the set of markets we are working with, we use the same method to determine our output as in our model with identical markets. This results in higher output with an increasing number of markets (m).

The above mentioned method is a possible solution of the problem of similar preferences of players. However as it is very clumsy and not very realistic, we will not use in further parts of the thesis. We have only shown that this

[4] The solution of this equation is shown in detail in the appendix

problem can by solved by randomizing the market out of a subset of most profitable markets. In reality when deciding without sufficient information about the opponent, we simply disregard them in our decision-making process completely. So it is reasonable to assume that companies decide only based on their expectations of the market without including other companies and access the most profitable market.

3 Alternative Applications of the Results of the Model

3.1 Model with Restricted Access to Markets

From the behavior observed in our model, we can make assumptions about decisions outside the framework established for the model. In the model we assumed that we have identical markets and that a player can enter any market. However in reality for some products a company can only access the adjacent market and then continue to the desired market. If we place this restriction it alters our model in various ways.

First the number of markets a player can choose depends on how many markets are connected to the markets, we have already entered. However the decision making process is somehow different than in our original model, because we have much more information about the possibilities and moves of our opponents. Due to the restriction on entering only adjacent markets, we are able to observe how the other player enters available markets and we only face a direct confrontation when, from markets the two players entered, both can enter the same market at once.

In this situation however as can be seen from the model it is more advantageous for both to choose a market which they both can not enter at once. Then both can produce monopoly output and avoid competing for a market, because this would only lower their profits. Only in the situation where both have only options to enter the same markets the decision making process from our original model applies and they decide based on the number of markets and parameters b and h.

The assumptions made above, about the behaviour upon the possibility to move only to adjacent markets, are not always valid and we can not come to any conclusions from our model, because in each setting of markets they are different and general rules don't apply. The simplest situation, we can consider is where the players have entered all the markets they can access, without the threat of entry from the other players in the same period and thus they have to access a market where players can clash, in order to be able to expand further. In this case the markets can't be considered as identical, because a certain market can have a strategic advantage compared to the others.

In another setting one market can have a higher strategic value for a company, because it enables the player to access many markets from this one point. However treating a market as strategic depends on the number of entries into markets that we think ahead. A market is not strategic if we consider only one market entry at a time, because then we only select a market with highest profit.

The same market is strategic on the other hand when we calculate many market entries ahead. In this case, we can choose a market with lower profits in the first few periods in exchange for higher profits in later periods. All decisions depend on the other player as well so securing markets, where we can behave as monopolist players for a longer period are preferred to the ones where we face a threat of entry. Nonetheless companies usually do not make predictions this far ahead, because their actions depend on results that are ex ante only estimated, so the higher the number of steps we think ahead the higher the uncertainty.

Moreover information regarding the actions of the opponents and their behaviour in the future are difficult to obtain. The existence of a strategic market is not limited to the assumption of identical markets. We see that the model gets quite difficult especially when we consider not only identical markets, however the value of a market is influenced by the amount of connections it allows the player in further moves, access from other players and other factors.

3.1.1 Model with Restricted Access to Markets and Varying Market Sizes

When we include varying market sizes as in the previous chapter, we are able to make much more realistic assumptions than in the model where we can access all markets. This is due to the fact, that the number of markets that can be accessed by more players at once is significantly lower and that the options of our opponents are visible beforehand.

The importance of strategic markets is not influenced much by varying market sizes, because these markets are valued for their access to other markets and not by the size of the strategic market. These markets are again dependent on the setting, which can be depicted in the form of a graph with dots for markets and lines for access among markets.

In the current setting the movements of players become more interesting, as it is no longer true that markets that can be entered by more players, are preferred to markets where we can be sure of a monopoly for the player. The reason for competing in a market rather than entering a market with monopoly output is the market size. Since it can be more profitable to be a duopolist in a large market than have a monopoly in a small one.

The decision-making process is altered. Now we have to count the number of markets where multiple entry is possible in the next move. From this number we determine our output and expected profit for each market. Then we choose the most profitable duopoly and compare to the most profitable monopoly market, out of all possible monopoly markets we can enter in the next move. Furthermore, we have to take into account possible moves where we can enter monopoly markets controlled by the other player.

Only then we can say which option is optimal. Here we already expect the players to decide for the most profitable out of all the duopoly markets that have not been entered in the previous moves, because our results from the previous model have proven not to be realistic nor elegant. In this setting players actions show much more variation than in the model with identical markets and they are more influenced by the structure of all markets.

3.2 Model with Monopolists in all Markets

Each player prefers to enter a market which has not been occupied yet, because we can directly compare that given any output the profit in a duopoly is always lower than in a monopoly market. This approach of avoiding the opponent is not possible infinitely, because the number of markets is limited, so at some point entry into monopoly markets is the only option.

As all markets are occupied by at least one company the threat of entry becomes more probable and companies have to adjust accordingly. Their reaction depends on the number of companies they expect to enter and on the number of markets there are. We estimate the ratio for which companies decide to change their output using the model from chapter 2. Furthermore, we assume that entering players choose Cournot output and that the players controlling the monopoly markets have this knowledge as well. The Cournot output is used because it is the optimal strategy for both players in one market.

So when players compete in one market, after some time the market should become stable with both producing Cournot output. The reaction of companies here is considered as ex ante. This means that companies change their output or make the decision before some actual entry occurs.

It is natural that companies change their output ex post, but that is a result of the fact that two players can be profitable, which means that we have no way to push the opponent out of the market. As a result Cournot output is produced. The expected profit for the player not changing the monopoly output is as follows:

$$E(\Pi) = \tfrac{e}{n}\Pi_{MC} + \left(1 - \tfrac{e}{n}\right)\Pi_M$$

where Π_{MC} is the profit when behaving as a monopolist with a Cournot opponent and Π_M is monopoly profit with no opponent in the market.

The expected profit of a player who changes the output under the threat of entry is:

$$E(\Pi) = \tfrac{e}{n}\Pi_{CC} + \left(1 - \tfrac{e}{n}\right)\Pi_C$$

where Π_{CC} is the profit of a Cournot duopolist facing another Cournot duopolist in a market and Π_M is the profit of a Cournot duopolist with no opponent, n is the number of markets and e is the number of entrants. In the case where expected profit under Cournot output is higher than under monopoly output the player will change the output ex ante. Therefore:

$$\tfrac{e}{n}\Pi_{MC} + \left(1 - \tfrac{e}{n}\right)\Pi_M < \tfrac{e}{n}\Pi_{CC} + \left(1 - \tfrac{e}{n}\right)\Pi_C$$

this can be rewritten as:

$$0 < \tfrac{e}{n}\left(\Pi_{CC} - \Pi_{MC}\right) - \left(1 - \tfrac{e}{n}\right)\left(\Pi_M - \Pi_C\right)$$
$$\text{or } 1 < \tfrac{e}{n-e}\left(\tfrac{\Pi_{CC}-\Pi_{MC}}{\Pi_M-\Pi_C}\right)$$

This means that the threat of entry is only dependent on the number of markets and the number of entrants. On the other hand it does not matter how many monopoly markets a certain player controls, as it does not affect the behaviour. The impact of a high number of entrants is that players change their strategy and switch to Cournot output, because the probability that an entrant enters a market under their control increases.

The higher the difference between the number of markets and the number of entrants, the higher the stability of monopoly output. So monopolists do not change output when there is a high number of markets compared to a low number of entrants.

3.3 Cooperative Behaviour Among Players

So far all our calculations were based on non-cooperative behaviour, however as it is more profitable for companies to cooperate, especially when there is low competition, we examine the results of cooperative behaviour as well. This can be divided into cooperation in one market or across many markets. The case for a single market has been widely studied and therefore we concern ourselfs with multiple markets. To make a clear division with our model, we consider two cases one according to the model in part 2 with the possibility to enter all markets and a second with only adjacent market entry.

First we mention some characteristics similar for both models. The basic idea is similar to collusion for single markets. The higher the costs of controlling that the other players are behaving as agreed on, the lower the probability of a collusion. The same applies to entry of new players, when new entry is likely the probability of collusion decreases. The only exception is when players agree to keep another player out of the market by cooperating.

However the entry in a new market for companies is a credible signal for other players and because of that it can be easily controlled, which makes co-operation more likely. Furthermore it is very difficult to prove some sort of cooperation across markets, so the risk for companies is relatively low. So as long as there are enough markets and no threat of entry, companies divide the market with no interest in competing with each other. When we consider varying sizes of markets the situation changes only slightly. Companies want to control large markets, however when they can agree on similar markets to enter, so that no company is much better of than the other, cooperation can still occur.

In contrast to models with a single market, as companies encounter each other in more markets and cooperate the cooperation becomes more stable. This is logical because the punishment from one player can be much more severe. The difference between cooperative and uncooperative behaviour is increased by each market they share.

Π(for all markets)= $m\left(\Pi_{cop} - \Pi_{non}\right)$, where m is the number of markets they share, Π_{cop} is their profit under cooperation and Π_{non} is their profit without cooperation.

3.3.1 Cooperative Behaviour with all Market Entry Option

When players can enter any market as in our model in chapter 2 the probability that two players end up in the same market is very low, so cooperation can occur for the last couple of markets, because here the difference between the expected value without cooperation and the value while cooperating increases. So when we consider some risk of entering as a constant, where the profit is adjusted by $r \in (0,1)$, then companies cooperate if

$$r\Pi_M > \tfrac{1}{n^2}\Pi_D + \left(1 - \tfrac{1}{n^2}\right)\Pi_M$$

, where Π_M is monopoly profit and Π_D is duopoly profit.
In other words a player chooses to cooperate when the gains outweigh the risk.

For the case where markets vary in size the decision about cooperation largely depends on the market sizes available. If companies are limited to entering one market and this number does not increase under no circumstances. Such as, for example that the two markets we want to enter are smaller in total then another one. So for markets where the sizes differ significantly, cooperation takes place in a single market in the form of a negotiated output, because they have difficulties in dividing the markets.

On the other hand when companies are able to negotiate some kind of division of the markets it is more profitable for them to do so, even if this includes sharing some markets, because it solves their problem of entering the same market due to the same preferences.

3.3.2 Cooperative Behaviour with Adjacent Market Entry

The cooperation with adjacent market entry has many similarities, so only the differences are discussed. The number of markets, where two players can meet is lower, because we are able to anticipate the movements better. Since it is more profitable to avoid competing for a market players choose markets where they can not end up both in the next move. Only under the circumstances that markets both can enter have a big size or that there are no empty markets, where the other player can not move to cooperation occurs. Players try to avoid each other, but if they are to meet they cooperate, because the number of markets is lower than when we can enter any market.

In this setting it is easier to cooperate for a higher number of players, because when each player controls a few markets two competing players can agree to cooperate in only a few markets. Or rather they can divide markets among themselves only in parts they control and not for the whole structure of markets. As a result more small scale cooperations can occur.

4 Modeling Market Entry Using a Computer Program

To obtain data for heterogeneous products and for multiple markets has proven to be very difficult, because of that no reasonable data could be obtained. With the little amount of data we found, which was mainly for homogeneous products, no significant conclusions could be made. So in order to avoid having no significant results from our model, we turn to computer modeling instead. This gives us many advantages compared to acquiring data. First the decision-making process is completely controlled by us and second all results we require are observable. Finally the amount of data we have can be expanded by making more simulations of the model.

The setting is based on the models we have created so far. So we use the restriction on adjacent market entry and model various situations for two identical players and heteregenoues products. We model a structure of markets, represented by a graph where markets have varying market sizes. The structure of the markets as well as the market sizes are created randomly, then for this created setting we can change the following properties:

- The starting position of players
- The cost function, represented by the equation $TC = cq + FC$, where c and FC are variables we control
- The market specifications, represented by the equation $p = a - bq_1 - hq_2$, where a,b and h are variables we control

We are interested in how changing these characteristics influences the stability of the process, output, prices, the choice of markets and other characteristics.

All decisions are modeled under the assumption that players are non-cooperative and that they only decide based on their profits and that they move to the markets they have chosen simultaneously. Players have perfect knowledge about the opponent and to make the decision-making more realistic players optimize their decision only one move ahead. The game ends either after a set number of moves or when all markets are entered. As it is profitable for players to enter new markets in our setting each player enters a new market each move. These are the basic ideas behind the modeling further requirements and assumptions are given in more detail in the following part.

4.1 Specifications of the Program

The computer program is programmed for two players on a 8x8 grid, where a set number of markets are randomly generated. Each market is represented by a dot. These markets are then connected by lines in order to show market access from various points. The restriction we make is that there has to be at least one connection from each point and a maximum of 5 connections is imposed.

Another restriction is that the markets are accessible from any point, this means that starting from any point we can always get to any other point, we choose, by a series of moves. The reason behind this restriction is quite simple, because if the generated market structure was not perfectly accessible either some markets would not be entered or players would have no threat of entry from other players. To set up such structure algorithms from graph theory can be successfully used. The application to this model is very simple compared to all the various graph settings programs specializing in graph theory are capable of.

After generating a structure of markets each market is given a value $\alpha \in (0, 1)$ in order to vary the market sizes. The players then multiply the profit obtained by optimizing their output by the α from each market to get the final profit for each market. As a result from the above described process we arrive at the point where we have a random structure of markets with random market sizes.

The next step is to determine the output and the market we choose. The possible outputs are restricted to the same options as in our basic model so 5 quantities are available. These are monopoly, Stackelberg leader, Stackelberg follower, Counrnot and Bertrand output. From these options we choose the output that we expect to have the highest profit. We enable players to access any market that is connected to a market that they have already entered. So we take all the markets that we can access and divide them into 3 groups.

The first group are the markets that can be accessed in the next move by only one player and that have not been entered so far. Here we can automatically choose monopoly output. In the second group only markets that we can access in the next move and that have already been entered by the other player are chosen. Here we choose Cournot output, because we know that the other player will optimize their output after our entry. So the equilibrium solution is Cournot output for both.

The third group consists of markets that can be entered by both players at the same time. Here we once again use our calculations from our basic model and using the parameters b,h and the number of markets we determine the output. However as a simplification to our previous model to make the decision-making here more realistic we use the output from our calculations and enter the largest market.

As a result we obtain profits for all markets we can access from all 3 groups, which we multiply by the α of each market and choose the most profitable market to enter. The described process is made for each market entry choice.

4.2 Factors Analyzed

The program used for modeling enables us to control for many variables and to quantify the results of these changes. We are no longer limited to only information about prices, output and profit. There are many other factors we can analyze that give us information about the influence of each factor on competition. We test the following characteristics:

- **The distance between players**. To measure the distance between players effectively, we define it as the minimal distance between the two starting points of players and examine the effects the changing distance has on market structure in individual markets.
- **Stability, position of the players and the outcome**. To determine

whether the structure of markets is stable we change the starting positions and compare the outcome under different situations.

- **Market access**. We define market access as the total number of connections among markets and study if access increases the speed of entry.
- **Prices and profits**. We examine how profits and prices change for an increasing number of moves, in order to examine the influence of competition.
- **Structure of markets**. How many moves are necessary under different structures of markets to occupy all markets by at least one player.

4.2.1 Results for distance between players and stability

We examine how the starting distance between players affects their behaviour for entry. For a random structure of markets we position players, so that the distance between them grows and analyze how this changes the final distribution of markets. The final distribution means that all markets have been entered by at least one player. We examine differences using the number of moves it takes the players to occupy all markets.

For the simulation we generate 16 markets with 2 of them occupied by players so a maximum of 15 moves and a minimum of 7 moves is necessary to occupy all markets. This gives sufficient insight on how the markets are occupied and the level of competition. The higher the number of moves necessary the higher the competition. Furthermore we examine how changing the starting conditions influences the stability of markets

trials	n=50	distance when moves change						average moves
p	0.54	1	2	3	4	5	6	with monopoly =10.96
1-P	0.46	0.22	0.15	0.19	0.04	0.22	0.19	overall =13.67

Table 4.1: Distance between players results

From the results we see that the probability that the number of moves neccesary to occupy all is below 15 moves is about 0.54. This means that not all markets are occupied by both players, so at least two monopoly markets exist. So about half of the structures have two stable solutions. The first stable solution is that players behave almost identically. After a certain number of moves they start to enter the same markets and this results in the fact that they both enter the smallest market last and the simulation lasts 15 moves.

The second stable solution is that a barrier in the form of a very small market is created, so both players do not move in the direction of the opponent.Only when they have no more better markets left they enter this market. Then the simulation lasts on average 11 moves and we end up with more monopoly markets. Logically the shorter the simulation in terms of moves the lower the competition.

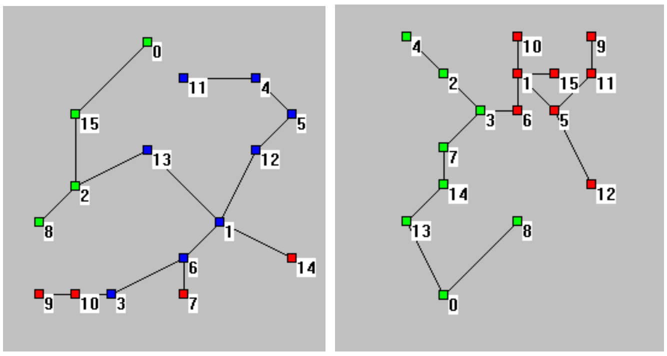

Figure 4.1: Stable solution with divided market

Two structures of markets with monopolies are shown above. In the left simulation players started at markets number 1 and 2. Markets occupied by player 1 are marked red markets occupied by player 2 are marked green. Shared markets are marked blue. In the right simulation all markets are monopoly markets. The simulation ended after 7 moves that is the shortest time to finish. Players started at markets 1 and 2. Market 6 has a very low alpha value and acts as barrier. The solution is stable for any starting position, where both players are on opposite sides of the market number 6.

Occasionally a system with three stable solutions can be encountered, but that is very rare. In general when we move both players to the same side of the barrier market, no matter how far away from each other they are, the system results in the solution with identical market entry. However when players are randomly distributed and divided by a barrier, the second stable solution with less moves applies.

Because of the stability of the system an increase in distance between players does not suffice to decrease the number of moves to occupy all markets.

So distance alone does not play a crucial role, which can be seen in the table, where the distance when the system changes is roughly equally divided. Instead of the distance the market sizes influence the stability of the solution and the second most important factor is the number of markets each player can access, without competition from the other player, because as long as a player has enough alternatives the barrier in the form of a small market functions.

The findings about how effective a small market is as a barrier are very interesting when we think about them as barriers imposed by some player as entry deterrence. In that case we can see that this form of protection is even more effective then in single market entry, because entry deterrence in one market provides a shield for other markets as well.

In general market entry in this setting for consumers is either higher competition with less markets occupied or more markets occupied with higher monopoly prices. It is questionable to say which is more desirable for consumers.

4.2.2 Results for Market Access and Speed of Entry

For market access we modify our generated graph to have more connections thus better access to markets. We work with 16 markets so the number of connections is 15 at least. To simulate market access we add connections until they add up to a maximum of 19 and examine the market situation after 7 moves, because that is the minimum number of moves neccesary to occupy all markets.

As we have seen in the previous part players tend to copy the players moves resulting in a movement to the same markets at the same time. The elimination of this problem is another reason for imposing the restriction of seven moves. A higher number of markets occupied after 7 moves could suggests that market access leads to avoiding competition.

number of connections	15	16	17	18	19
average number of markets	13.38	12.68	13.48	13.03	13.05
average number with restriction	14.1	13.75	14.25	13.95	14

Table 4.2: Market access

The results show that an increase in market access does not mean that companies avoid competition. However the results are influenced by two factors. The first is the fact that an increase of 4 connections may not be sufficient. The second is that when players start to close to each other they can not avoid competition. In order to control for the second factor which we think has a higher influence we set a rule of minimal distance, where the minimal distance is four moves between starting markets.

After the distance restriction increasing the number of connections does not decrease competition for markets. So no relation between the market access and competition can be found for a small increase in connections. This can be connected to the fact that the probability that a connection from one market to another, that we added, proves useful is low, when the connection are chosen randomly.

However the average number of markets entered after seven moves has increased approximately by one market. This means that although increasing the distance does not change the strength of competition, the speed of access is influenced. We can conclude that as the players start further away from each other, they enter more markets as monopolists first and afterwards start competing in markets. This is caused by the fact that they gain a higher market share without competing for the markets with the opponent.

The speed of entry into new markets rapidly decreases and is substituted by competition for larger occupied markets approximately after the seventh move. This is shown in our picture, where we examined the number of markets occupied after up to 15 moves.

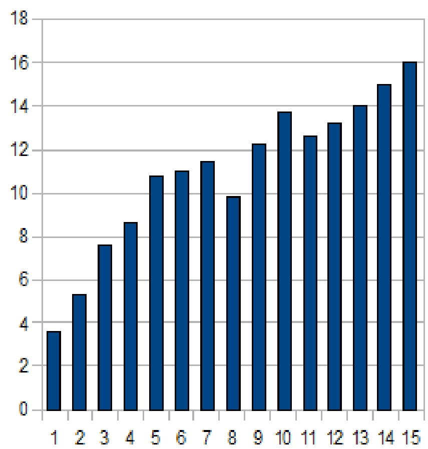

Figure 4.2: Speed of entry

For random structures of markets we restrict ourselves two only the number of moves. So the simulation stops after a certain number of moves. However we disregard the number of moves it takes to enter all markets. The consequence is that various structures are examined only at one point in time. The result is that the number of markets entered in one move decreases from two markets per move to one market per move.

So in the first part of the simulation players enter different markets, while in the second part of the simulation they either enter the same market or one enters a previously not occupied market and the other enters a previously occupied one.

4.2.3 Estimating Speed of Market Entry

Predicting market access is connected to predicting, how quickly markets are entered and to predict the number of moves it takes to enter all markets. Once again we divide this situation into two parts. The first part is estimating the number of moves to enter all markets and the second part estimating the number of markets accessed after 7 moves.

In our predictions we avoid using α values of the markets, because we use the fact that players do not have specific market information for other markets than the ones one move ahead. To estimate these models we use the method of ordinary least squares. The data used are generated from our program.

model	moves to finish=β_1+$\beta_2$market share+$\beta_3$long dist			
values	const=19.60	market share=-0.84	longest distance=-0.41	R^2=0.5

Table 4.3: Estimating moves to finish

This is the model where we already removed insignificant variables, such as distance between players and other not connected to the structure of markets. All we are left with are variables that give us some information about the structure of the graphs generated. Market share refers to the number of markets a player can access without competing for it with the other player and longest distance is the longest line of markets given in the graph. The value of R^2 is only 0.5 so a lot of variation is still not explained. This is related to the fact that we do not include α values and that we can not describe the structure of the generated market structure.

The model confirms that the distance between players has insignificant influence on the number of moves it takes to enter all markets. Competition for markets is limited by the share of markets each player has, when players have enough markets to enter they do not compete for them and all markets are entered in less moves. This fact can be used by players entering later to avoid competition, if they occupy the correct market.

Finally the longest distance decreases the number of moves necessary as well. This variable is similarly to the market share related to the structure of the markets. Even though it is very useful for a limited number of markets it gives us less information about the structure with each increasing number of markets, because describing the structure by this one characteristic is very limited. To enhance our model we would need additional information about the structure and α values.

Markets Entered After 7 Moves

model	markets=β_1+$\beta_2$market share		
values	const=8.27	market share=0.99	R^2=0.74

Table 4.4: Estimating markets entered after 7 moves

The only factor significantly influencing the number of markets entered after 7 moves is the market share of each player at the beginning. Distance between players is excluded for two reasons, the first reason is that there is a relationship between distance and the market share and the second reason is that market share is simply a better indicator.

Changing the distance can change the market share significantly, however we are not able to say exactly how many markets have been added or removed. Nonetheless we are able to explain 74% of the variation of data, this value is quite high. Especially considering that we leave out information about market sizes and the structure of markets.

4.2.4 Prices, Profits and the Structure of Markets

The results for prices and profits confirm our findings about competition. As the players decide only one move ahead their average profit from each following move is approximately equal. The only exception can be found in the last market to enter, where the average market profit is lower compared to the previous markets.

Generally market profits and prices have a tendency to decline with an increasing number of moves. This reflects the fact that players often face competition in later moves and so the prices and profits are lower. For simulations where players have equal market shares the profits and prices are higher, while for models with higher competition profits are lower.

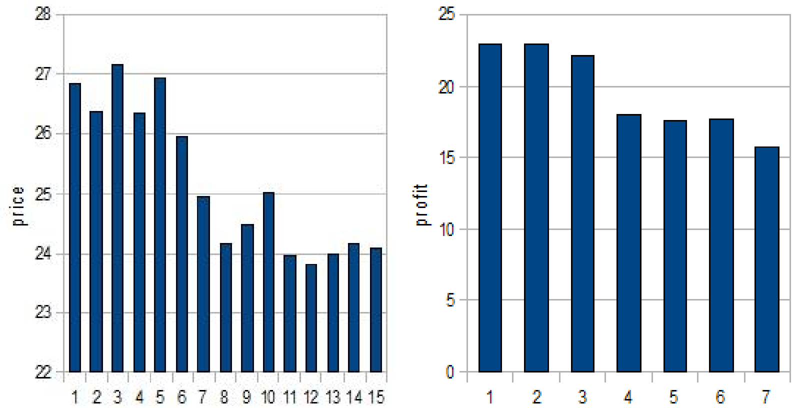

Figure 4.3: Profits and prices for increasing moves

The cumulative profit, which is defined as the sum of profits from each market for each round, has the highest variability. Even when players end up competing in all markets, their cumulative profit can be significantly different. The reason for this variability is, that the impact of the first few markets is by far higher then for the following ones for the cumulative profit. So the starting position has the highest influence on cumulative profit and players can decide their first entry accordingly.

The structure of markets is quite difficult to describe by only a few variables. However it has a significant impact on the decision of the players on market entry and on the market structure. As described before a market share, without competing for the market, of a player plays a key role in the outcome of the market structure.

This characteristic is connected to the position of the players and to the structure of markets itself. However as it is hard to imagine for companies in the process of market entry, making predictions by using information about the structure of markets, we examine this situation no further.

4.3 Comments on Findings and Further Extensions

The whole simulation of the program confirms that the starting position and structure of markets have a significant impact on the resulting market structure in each market. Overall the conclusion is that the entry process is a trade-off between speed and competition. We are either interested to enter all markets as fast as possible and place players accordingly or we can place them so that they compete from the very beginning for each market and thus decrease prices.

The main factor, which influences the nature of competition, is the share of markets each player can have without competition from the opponent. With this information we are able to predict successfully, how markets are occupied several moves ahead. However the prediction of the number of moves it takes to enter all markets has proven to be more difficult.

The simulation could be extended in the future. One possibility to make the movements of the players more realistic and to apply the fact that with more time players gather more information, is that the players could decide not only one move ahead, but more moves ahead. However as their information would be limited each further move would be more biased.

To achieve this setting each alpha value would be randomly distorted for further moves. So for example players would have perfect information about the market size of the markets they can enter in the next move, but it would get more and more biased for each move farther in the future.

Another possibility would be to introduce entry barriers, enabling players to create barriers for the opponent. As entry barriers in the setting of multiple markets are more important than in single market entry, the result could be a significant change in the entry process of each player. Or to simulate cooperative behaviour players would cooperate in the way that each of them would have the strating move in a round and the starting player would change, giving the second player more information and less uncertainty.

5 Conclusion

This thesis studies entry into markets when companies can decide to enter a market from multiple markets. We constructed a theoretical model with identical markets, heterogeneous products and identical players to model players entry decisions. We were able to find a relationship between the number of markets and the output companies choose from this model. The result was that companies decrease the output and behave as monopolists in the case of a high number of markets. However when the number of players increases or the number of markets decreases players become more competitive. As a result prices became lower and output increased.

We then extended our model to account for varying market sizes. The similarity of preferences in the form of preferring bigger markets to smaller ones, resulted in companies having better alternative choices when analyzed ex post. We showed a way to solve this problem by randomly entering a market from a subset of most profitable markets.

The setting of the model was then applied to the situation when all markets are entered by at least one player and threat of entry from other players exists. We found that if the threat of entry from players is judged as significantly dangerous by monopolists, players increase their output even before actual entry occurs.

We also compared cooperative behaviour with single market cooperation. The behaviour of companies suggested that rather than cooperate companies avoid to compete with each other by giving information about their future market entry. On the other hand cooperation becomes more stable as players share markets, because the losses from uncooperative behaviour increase with each market they share.

Finally we used a computer program to simulate market entry when we are restricted to entering only adjacent markets. For random structures of markets and random market sizes we analyzed the market entry of two players. The results were that market entry is a trade-off between the speed of entry and competition among players. As a consequence prices and profits decrease with time, because competition becomes tougher.

Although the model uses many simplifications, we were able to get some insight about multiple market entry. The fact, that the influence of some characteristics can either be amplified or reduced compared to single markets, shows that it is worth to study the topic further. As only one variable was studied an extension with more variables could lead to more findings.

Since we failed to gather sufficient data to test the model, we had to simulate it instead. So a verification of the findings on real data still remains as a future step to understanding multiple market entry.

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A Appendix

Optimal quantities for all strategies.

Cournot

Cournot competitors decide on their output based on reaction functions dependent on the output of the opponent. Therefore the equilibrium output is calculated in the point where both reaction functions meat. The profit function is as follows:

$$\Pi_1 = p_1 q_1 - cq_1 - FC \quad \Rightarrow \quad \Pi_1 = (a - bq_1 - hq_2)\, q_1 - cq_1 - FC$$

$$\Pi_2 = p_2 q_1 - cq_2 - FC \quad \Rightarrow \quad \Pi_2 = (a - bq_2 - hq_1)\, q_2 - cq_2 - FC$$

$$\tfrac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - hq_2 - c = 0 \quad \Rightarrow \quad q_1 = \tfrac{a - hq_2 - c}{2b}$$

$$\tfrac{\partial \Pi_2}{\partial q_2} = a - 2bq_2 - hq_1 - c = 0$$

$$a - 2bq_2 - h\left(\tfrac{a - hq_2 - c}{2b}\right) = 0 \Rightarrow q^*_{1,2} = \tfrac{a-c}{2b+h}$$

with the following price for both competitors:

$$p^*_{1,2} = a - b\tfrac{(a-c)}{(2b+h)} - h\tfrac{(a-c)}{(2b+h)} = \tfrac{ab+cb+ch}{2b+h}$$

and the following profits: $\Pi_{1,2} = (p_i - c)\, q_i = \left(\tfrac{ab+cb+ch}{2b+h} - c\right) \tfrac{a-c}{2b+h} = \tfrac{b(a-c)^2}{(2b+h)^2}$

Monopoly

A monopolist under our conditions chooses monopoly output independently on the fact if another player will enter the market or not, so the monopoly output under the assumption that no other player is in the market is similar to monopoly profit under homogeneous products.

$$\Pi_m = pq - cq = (a - bq)\,q - cq - FC$$

$$\frac{\partial \Pi_m}{\partial q} = a - 2b - c = 0 \Rightarrow q^* = \frac{a-c}{2b}$$

$$p^* = a - \frac{a-c}{2} = \frac{a+c}{2}$$

$$\Pi^* = \left(\frac{a+c}{2} - c\right) q = \frac{a-c}{2}\frac{a-c}{2b} = \frac{(a-c)^2}{4b}$$

Stackelberg

The Stackelberg leader maximizes profit according to the reaction function of the follower, because the output of the leader is known beforehand and so the follower uses his reaction function to calculate the optimal quantity.

$$\Pi_{sl} = \left(a - bq_{sl} - h\left(\frac{a-hq_{sl}-c}{2b}\right)\right) q_{sl} - cq_{sl} - FC$$

$$\frac{\partial \Pi sl}{\partial q_{sl}} = 2ab - 4b^2 q_{sl} - ah + 2h^2 q_{sl} + ch - 2bc = 0$$

$$q_{sl}^* = \frac{(a-c)(2b-h)}{4b^2-2h^2}$$

$$q_{sf}^* = \frac{a - \frac{h(a-c)(2b-h)}{4b^2-2h^2} - c}{2b} = \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$$

$$p_{sl}^* = a - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$$

$$p_{sl}^* - c = (a-c) - b\frac{(a-c)(2b-h)}{4b^2-2h^2} - h\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)}$$

$$\Pi_{sf}^* = (p-c)\,q = \frac{\left(8b^4-8b^3h-2b^2h^2+4bh^3-h^4\right)}{2b(4b^2-2h^2)^2}$$

$$p_{sf}^* = a - b\frac{(a-c)\left(4b^2-2bh-h^2\right)}{2b(4b^2-2h^2)} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$$

$$p_{sf}^* - c = (a-c) - \frac{(a-c)\left(4b^2-2bh-h^2\right)}{2(4b^2-2h^2)} - h\frac{(a-c)(2b-h)}{4b^2-2h^2}$$

$$\Pi_{sf}^* = (p-c)\,q = \frac{\left(4b^2-2bh-h^2\right)^2(a-c)^2}{4b(4b^2-2h^2)^2}$$

Bertrand

For Bertrand oligopolists the players decide optimal output based on the reaction function dependent on the price of the opponent.

$p_1 = a - bq_1 - hq_2 \Rightarrow q_1 = \frac{ab-ah+hp_2-bp_1}{b^2-h^2}$

$p_2 = a - bq_2 - hq_1 \Rightarrow q_2 = \frac{ab-ah+hp_1-bp_2}{b^2-h^2}$

$\Pi_1 = p_1q_1 - cq_1 = p_1\frac{ab-ah+hp_2-bp_1}{b^2-h^2} - c\frac{ab-ah+hp_2-bp_1}{b^2-h^2}$

$\Pi_2 = p_2q_1 - cq_2 = p_2\frac{ab-ah+hp_1-bp_2}{b^2-h^2} - c\frac{ab-ah+hp_1-bp_2}{b^2-h^2}$

$\frac{\partial\Pi_1}{\partial p_1} = ab - ah + hp_2 - 2bp_1 + cb = 0 \Rightarrow p_1 = \frac{ab-ah+hp_2+cb}{2b}$

$\frac{\partial\Pi_2}{\partial p_2} = ab - ah + hp_1 - 2bp_2 + cb = 0$

$ab - ah + h\left(\frac{ab-ah+hp_2+cb}{2b}\right) - 2bp_2 + cb = 0 \Rightarrow p^*_{1,2} = \frac{2ab^2-abh-ah^2+bch+2b^2c}{4b^2-h^2}$

$q^*_{1,2} = \frac{b(a-c)}{(2b-h)(b+h)}$

$\Pi_b = (p-c)\,q = \left(\frac{(2b+h)(b-h)(a-c)}{4b^2-h^2}\right)\left(\frac{b(a-c)}{(2b-h)(b+h)}\right) = \frac{b(a-c)^2(b-h)}{(2b-h)^2(b+h)}$

B Appendix

Table B.1: Table of summary of profits for one market

	monopol	cournot	bertrand	s.leader	s.follower
monopol	$\frac{b-h}{4b}, \frac{b-h}{4b}$	$\frac{(2b-h)}{4(2b+h)}, \frac{\left(2b^2-h^2\right)}{2(2b+h)^2}$	$\frac{\left(2b^2-bh-h^2\right)}{4(2b-h)(b+h)}, \frac{b\left(2b^3-3bh^2+h^3\right)}{2(2b-h)^2(b+h)^2}$	$\frac{b(b-h)}{4b^2-2h^2}, \frac{\left(b^2-h^2\right)(2b-h)^2}{\left(4b^2-2h^2\right)^2}$	$\frac{\left(4b^3-4bh^2+h^3\right)}{4b\left(4b^2-2h^2\right)}, \frac{\left(16^5-16b^4h-12b^3h^2+16b^2h^3-bh^4-2h^5\right)}{4b\left(4b^2-2h^2\right)^2}$
cournot	$\frac{\left(2b^2-h^2\right)}{2(2b+h)^2}, \frac{(2b-h)}{4(2b+h)}$	$\frac{b^2}{(2b+h)^2}, \frac{b^2}{(2b+h)^2}$	$\frac{b\left(2b^3+b^2h-bh^2-h^3\right)}{(2b+h)^2(2b-h)(b+h)}, \frac{b\left(2b^4-b^3h-2b^2h^2\right)}{(2b-h)^2(b+h)^2(2b+h)}$	$\frac{b\left(4b^3-2bh^2-h^3\right)}{(2b+h)^2\left(4b^2-2h^2\right)}, \frac{b\left(4b^3-3bh^2\right)(2b-h)}{(2b+h)\left(4b^2-2h^2\right)^2}$	$\frac{\left(8b^4-4b^2h^2+h^4\right)}{2(2b+h)^2\left(4b^2-2h^2\right)}, \frac{\left(32b^5-16b^4h-24b^3h^2+12b^2h^3+2bh^4-h^5\right)}{4\left(4b^2-2h^2\right)^2(2b+h)}$
bertrand	$\frac{b\left(2b^3-3bh^2+h^3\right)}{2(2b-h)^2(b+h)^2}, \frac{\left(2b^2-bh-h^2\right)}{4(2b-h)(b+h)}$	$\frac{b\left(2b^4-b^3h-2b^2h^2\right)}{(2b-h)^2(b+h)^2(2b+h)}, \frac{b\left(2b^3+b^2h-bh^2-h^3\right)}{(2b+h)^2(2b-h)(b+h)}$	$\frac{b^2(b-h)}{(2b-h)^2(b+h)}, \frac{b^2(b-h)}{(2b-h)^2(b+h)}$	$\frac{b\left(4b^5-6b^3h^2+b^2h^3+h^5\right)}{(2b-h)^2(b+h)^2\left(4b^2-2h^2\right)}, \frac{b\left(4b^4-5b^2h^2-bh^3+2h^4\right)}{\left(4b^2-2h^2\right)^2(b+h)}$	$\frac{b\left(8b^5+10b^3h^2+4b^2h^3+3bh^4-h^5\right)}{2(2b-h)^2(b+h)^2\left(4b^2-2h^2\right)}, \frac{\left(32b^6-16b^5h-40b^4h^2+24b^3h^3+4b^2h^4-8bh^5-3h^6\right)}{4(2b-h)(b+h)\left(4b^2-2h^2\right)^2}$
s.leader	$\frac{\left(b^2-h^2\right)(2b-h)^2}{\left(4b^2-2h^2\right)^2}, \frac{b(b-h)}{4b^2-2h^2}$	$\frac{b\left(4b^3-3bh^2\right)(2b-h)}{(2b+h)\left(4b^2-2h^2\right)^2}, \frac{b\left(4b^3-2bh^2-h^3\right)}{(2b+h)^2\left(4b^2-2h^2\right)}$	$\frac{b\left(4b^4-5b^2h^2-bh^3+2h^4\right)}{\left(4b^2-2h^2\right)^2(b+h)}, \frac{b\left(4b^5-6b^3h^2+b^2h^3+h^5\right)}{(2b-h)^2(b+h)^2\left(4b^2-2h^2\right)}$	$\frac{b\left(4b^2-h^2\right)(b-h)}{\left(4b^2-2h^2\right)^2}, \frac{b\left(4b^2-h^2\right)(b-h)}{\left(4b^2-2h^2\right)^2}$	$\frac{\left(8b^4-8b^3h-2b^2h^2+4bh^3-h^4\right)}{2\left(4b^2-2h^2\right)^2}, \frac{\left(4b^2-2bh-h^2\right)^2}{4\left(4b^2-2h^2\right)^2}$
s.follower	$\frac{\left(16^5-16b^4h-12b^3h^2+16b^2h^3-bh^4-2h^5\right)}{4b\left(4b^2-2h^2\right)^2}, \frac{\left(4b^3-4bh^2+h^3\right)}{4b\left(4b^2-2h^2\right)}$	$\frac{\left(32b^5-16b^4h-24b^3h^2+12b^2h^3+2bh^4-h^5\right)}{4\left(4b^2-2h^2\right)^2(2b+h)}, \frac{\left(8b^4-4b^2h^2+h^4\right)}{2(2b+h)^2\left(4b^2-2h^2\right)}$	$\frac{\left(32b^6-16b^5h-40b^4h^2+24b^3h^3+4b^2h^4-8bh^5-3h^6\right)}{4(2b-h)(b+h)\left(4b^2-2h^2\right)^2}, \frac{b\left(8b^5+10b^3h^2+4b^2h^3+3bh^4-h^5\right)}{2(2b-h)^2(b+h)^2\left(4b^2-2h^2\right)}$	$\frac{\left(4b^2-2bh-h^2\right)^2}{4\left(4b^2-2h^2\right)^2}, \frac{\left(8b^4-8b^3h-2b^2h^2+4bh^3-h^4\right)}{2\left(4b^2-2h^2\right)^2}$	$\frac{\left(16b^5-16b^4h-4b^3h^2+8b^2h^3-bh^4-h^5\right)}{4b\left(4b^2-2h^2\right)^2}, \frac{\left(16b^5-16b^4h-4b^3h^2+8b^2h^3-bh^4-h^5\right)}{4b\left(4b^2-2h^2\right)^2}$

C Appendix

Cubic Equation We have a cubic equation in the form $x^3 + px = q$. The solution to these types of equations was found by Scipione del Ferro. However as the solution was published by Cardano it is called Cardano's formula.
Its general form is $x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$
In our case with the cubic equation $m^3 + m = 4n - 2$ we arrive at

$$m = \sqrt[3]{\frac{(4n-2)}{2} + \sqrt{\frac{(4n-2)^2}{4} + \frac{1}{27}}} + \sqrt[3]{\frac{(4n-2)}{2} - \sqrt{\frac{(4n-2)^2}{4} + \frac{1}{27}}}$$

As the second term is almost zero and $pm = m$ so $p = 1$ we can simplify the equation to $m = \sqrt[3]{4n - 2}$

D Appendix

Program Manual and Enclosed CD with .exe File MarketSimulator.exe is a program designed to simulate structures of markets and market entry with 2 identical players and heterogeneous products. The interface looks as follows:

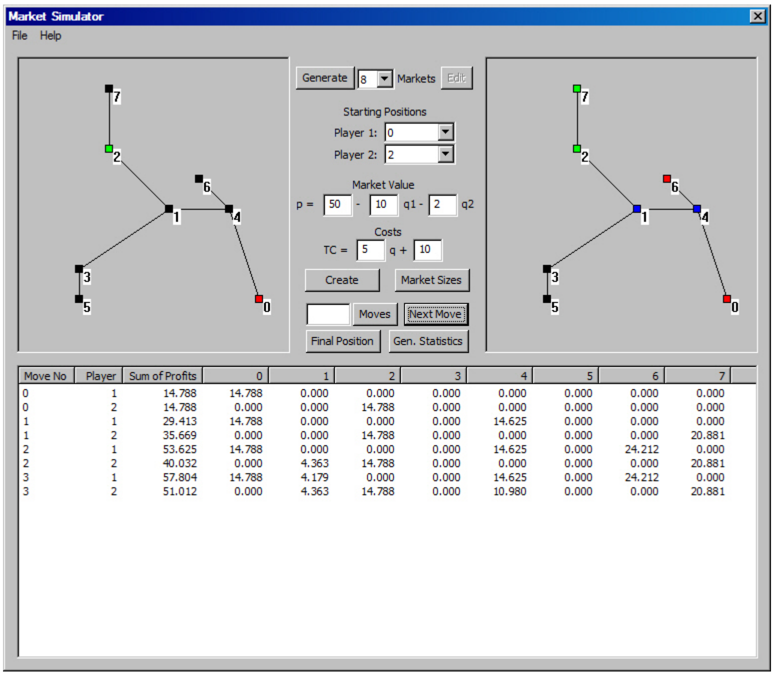

Figure D.1: MarketSimulator illustration

The starting position of the program is in the window on the left and the position we are currently at is shown in the right window.

- **Generate**-the generate button creates a random structure of markets, where the number of markets is controlled next to the generate button, with a maximum of 16 markets.

- **Edit**-enables to change the generated structure by removing or adding market connections.
- **Starting Position**-determines the starting position of both players from the available markets. Markets entered by player 1 are red, markets entered by player 2 are green. If both players occupy a market then the color is blue.
- **Market Value and Costs**-the values for actual profit, prices and output are entered. Market value is given by the equation $p = a - bq_1 - hq_2$. Costs are calculated using equation $TC = cq + FC$, where a>c, b>h and b>0 for companies to produce.
- **Create and Market Sizes**-create uses the entered data to simulate the starting situation and randomly determines market sizes. These values are shown after pressing the Market Sizes button.
- **Moves and Next Move**-the number of moves is either controlled by the Next Move button advancing one move forward or by the Moves button, where we enter the number of moves the players make. However when we enter more moves than are necessary for the simulation to finish, the simulation terminates when all markets are entered.
- **Final Position and Gen. Statistics**-the Final Position button creates the situation where all markets are entered. Generate Statistics button opens a window where we can see information about profits, quantities and prices.
- **Moves and Market information**-the table at the bottom gives us a summary of moves of each player that have taken place with according profits from the entered markets.

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Akademický rok 2008/2009

TEZE BAKALÁŘSKÉ PRÁCE

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Garant studijního programu Vám dle zákona č. 111/1998 Sb. o vysokých školách a Studijního a zkušebního řádu UK v Praze určuje následující bakalářskou práci

Předpokládaný název BP:

Strategic behavior upon market entry based on game theory

Charakteristika tématu, současný stav poznání, případné zvláštní metody zpracování tématu:

The aim of the bachelor thesis is to apply game theory to the problem of market entry, to evaluate which strategy is best for a company, while choosing a market to enter. Comparing the decisions of the companies with the models, to see which factors are most important for companies. Using empirical evidence and comparing the evidence to the models, to see if companies make decisions according to the models.

Struktura BP:

1. introduction
2. theoretical background
3. applying models
4. comparing the decisions of companies to the models
5. using empirical evidence to compare the results of the models with the real situation
6. conclusion

Seznam základních pramenů a odborné literatury:

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Carlsson, van Damme: ***Global Games and Equilibrium Selection,*** Econometrica, Vol. 61, No. 5 (Sep., 1993), pp. 989-1018
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