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REPORT ON DOCTORAL THESIS SUBMITTED BY ONDŘEJ SUCHÝ

This report concerns the thesis submitted in November 2010 for the doctoral degree at the Faculty of Mathematics and Physics of Charles University in Prague by Ondřej Suchý, with the title 'Parameterized Complexity: non-standard parametrizations of graph problems'.

The thesis concerns the parameterized computational complexity of graph problems and is divided into two main parts. The first part is an up-to-date overview of the field, with separate subsections discussing: the choice of parameter, the kernelization viewpoint, the main algorithmic methods, and intractability. The second part consists of a case study of three chapters on: Steiner problems, generalized domination problems, and equitable partition problems. Let me first say that I enjoyed reading this thesis, and let me then discuss the two parts separately.

The field of parameterized complexity has been developing very fast. In the first part of this thesis, a very good job has been done in both giving the historical background and also in highlighting recent developments. I would like to point out the chapter on kernelization and in particular the sections on the non-existence of polynomial kernels and on kernelization relaxed that do a very nice job of presenting and digesting recent results. Also the section on intractability shows a very good grasp of parameterized reductions. For this first part a few questions and comments come to mind.

- The definition used for parametrization (Def. 3.1) differs from the original by Downey-Fellows, following rather Flum-Grohe. I believe there are arguments in favor of both definitions. Why was this one chosen?
- The class XP. Is it the same as saying polynomial-time for bounded value of the parameter?
- On page 20 the Bodlaender algorithm for treewidth is mentioned and it is stated that it is completely impractical due to 'the huge function f(k)' involved in the running time. What is this function, and could it be improved?

- The notion of one graph parameter being 'more restrictive' than another reminds the definition in the master thesis of Robert Sasak (forwarded separately by email). The relation forms a quasi-order and this is useful to point out.
- The discussion of 'multivariate' approach on page 22. Is it the same usage of this concept as the one employed by Mike Fellows?
- The definition of kernelization on page 26 requires that $k' \leq g(k)$. What are the pros and cons of strengthening this to $k' \leq k$?

The second part of the thesis is based on two papers already accepted to reputable international journals: SIAM J.Discr.Math. and Applied Discr.Math. and on one LNCS paper presented at IWPEC'09. The chapter on Steiner problems considers three separate problems, DST, SCSS and DSN, and shows the connections between them. Table 8.1 showing an overview of results is very useful. The thesis both clarifyies the very natural connection between these three problems, and points out the difference in techniques applicable to the study of their time complexity.

The next chapter studies the parameterized complexity of (sigma, rho)-domination problems, with very general results, covering large infinite classes of problems, and thus quite strong. Much space is taken with reductions, say from problem A to C, and quite complex, but they are very elegant and provide a blueprint for reductions in this field. For example, a new intermediate problem B is introduced, reducing A to B and then B to C, with the intermediate problem itself being interesting. For example, the 'At most alpha-satisfiability' problem is very natural and useful and the community will benefit from being able to base later reductions on it. When construction of gadgets are involved an intuitive description of the need and use of the gadget is given before continuing with figures and formal definitions, in an exemplary way. This is important as the reductions are quite involved. The nice discussion along the way of how the various results reflect on each other gives the impression of a well-thought-out research programme. The detailed example in section 9.5 showing expressibility in first-order logic is very nice. It is seldom that one sees such things in print. The resulting algorithm for nowhere dense graphs follows from a 2009 paper by Dawar and Kreutzer, mentioned also in the first part of the thesis, but this constitutes a result that this reader has still not had the chance to understand.

The final chapter on equitable partitions considers two main problems, ECP and EC, and studies the parameterized complexity under various parametrizations. In the introduction it is stated that 'it is widely believed that almost every natural hard problem can be solved efficiently on graphs of bounded treewidth'. Is this not the same as saying that the problem is in XP when parameterized by treewidth?

Certainly, Courcelle's theorem gives a concrete class of such problems. But it could be a matter of discussion if it is widely believed that almost every natural hard problem not expressible in MSOL is solvable efficiently on graphs of bounded treewidth. The results presented in the chapter draw a quite precise line between hard and efficient parametrizations for the problems, and in particular for ECP. I believe we will see many more studies of this type in years to come, and consider this chapter to be somewhat pioneering in this respect. In this chapter I believe that the viewpoint of the 'more restrictive' notion as a quasi-order on graph parameters alluded to earlier, would have been beneficial, as the dividing line between W-hardness and FPT will respect this quasi-order. For example, it would be nice to see where the ml and fvs parameters fit into the diagram in the above-mentioned master thesis. A natural remaining question is if the parameterized complexity of ECP when parameterized by tree-depth is W[1]-hard or FPT.

Let me end by saying that on the basis of this thesis I can strongly recommend the candidate for acceptance to the doctoral degree.

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