Charles University in Prague

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MASTER THESIS

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Neutrino oscillations

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Prohlašuji, že jsem svou bakalářskou práci napsal samostatně a výhradně s použitím citovaných pramenů. Souhlasím se zapůjčováním práce a jejím zveřejňováním.

V Praze dne Bedřich Roskovec

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Název práce: Oscilace neutrin

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Abstrakt: Diplomová práce se zabývá možností zkoumání nestandardních jevů ve fyzice neutrin v experimentu Daya Bay.

Za prvé jsou zkoumány nestandardní interakce neutrin ve zdroji a detektoru. V práci je spočtena oscilační pravděpodobnost, která je použita v naivním modelování dat experimentu Daya Bay. Z modelu plyne, že experiment je schopen při určitých hodnotách parametrů nestandardních interakcí změřit jejich efekt. Pokud však nebudou nestandardní interakce pozorovány, experiment Daya Bay nedokáže stanovit silnější limity na jejich parametry.

Za druhé jsou zkoumány oscilace a rozpady neutrin. Je ukázáno, že experiment Daya Bay může naměřit jejich efekt pouze při velké hodnotě směšovacího úhlu θ_{13} a při době života hmotového stavu ν_3 $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$.

Klíčová slova: Oscilace neutrin, nestandarní interakce, rozpady neutrin, experiment Daya Bay.

Title: Neutrino oscillations

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Abstract: In the master thesis there is investigated the possibility of the measurement nonstandard phenomenons in the Daya Bay experiment.

At first we investigate the nonstandard interactions in the neutrino source and the detector. We calculated the oscillation probability and used it in the naive model of the Daya Bay experiment data. The experiment can measure the effect of nonstandard interactions for a particular set of nonstandard parameters. If there will not be any evidence of their effect Daya Bay experiment can not set stronger limits on nonstandard interactions parameters.

We investigated the neutrino oscillations and decays scenario. The Daya Bay experiment is able to measure its effect only for large value of mixing angle θ_{13} and for lifetime of mass eigenstate ν_3 $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$.
Keywords: Neutrino oscillations, nonstandard interactions, neutrino decays, Daya Bay experiment.

Chapter 1

Introduction

There is now compelling evidence of neutrino flavor changing. The most plausible theory how to treat these transitions is neutrino oscillations. The phenomenon of neutrino oscillation was first proposed by Bruno Pontecorvo in 1969 [1] as an analogy with K^0 $\bar{K}^0$ oscillation. Since that time various neutrino experiments were performed. Finally in 2004 the Super-Kamiokande experiment brought the first compelling evidence of neutrino oscillations [2].

The neutrino oscillations parameters were measured in several experiments. Currently we know the values of mass splittings Δm^2_{21} and Δm^2_{32}. We also have the values for two mixing angle θ_{12} and θ_{23}. What remains unknown is mixing angle θ_{13} and CP-violating phase δ. Therefore there are future experiments with goal to measure these parameters.

One of them is the Daya Bay experiment [3]. The aim of the experiment is the precision measurement of the mixing angle θ_{13}. The present limit is given by the CHOOZ experiment as $\sin^2 2\theta_{13} < 0.17$ [4]. For a large value of θ_{13} the Daya Bay experiment can provide valuable data for further studies.

The aim of this thesis is to investigate the effect of the nonstandard phenomenons in neutrino physics in the Daya Bay experiment and the possibility of its measurement. We focus on the two issues. First is the nonstandard interactions in the matter in the source and the detector and the other is neutrino oscillations and decays.

In the first part of this thesis we show the formalism for neutrino oscillation, present knowledge of the oscillation parameters and we introduce the Daya Bay experiment. In the second part we present the nonstandard interactions formalism and we perform probability calculations to investigate the possible effect of the nonstandard interactions in the Daya Bay experiment. Finally in the third part we introduce the formalism for neutrino oscillations and decays and we calculate the effect for the particular reactor and neutrino experiment.

Chapter 2

Phenomenology of neutrino oscillations

2.1 Neutrino mixing

There are three neutrino flavor eigenstates ν_e,ν_μ and ν_τ in the Standard model. These states are not coincident with neutrino mass eigenstates but they are their linear combination. We can express the transformation as

$$|\nu_\alpha\rangle = \sum_{i=1,2,3} M^*_{\alpha i} |\nu_i\rangle \qquad (\alpha = e, \mu, \tau) \tag{2.1}$$

where $|\nu_\alpha\rangle$ stands for neutrino eigenstate with flavor α and $|\nu_i\rangle$ stands for mass eigenstate with mass m_i.

In the three neutrino case, the unitary mixing matrix M is called the Pontecorvo-Maki-Nakgawa-Sakata matrix. It generally depends on three mixing angles θ_{12}, θ_{23}, θ_{13}, CP-violating phase δ and two Majorana phases α_1 and α_2. The standard parametrization of mixing matrix M is

$$M = U \ diag\left\{e^{ia_1/2}, e^{ia}, 1\right\}, \tag{2.2}$$

where U is given by

$$\begin{aligned} U &= \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \tag{2.3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. As we will see later Majorana phases do not have any effect on the oscillation probability.

2.2 Neutrino time evolution

From elementary quantum mechanics we know that time propagation amplitude of neutrino is in the rest frame given by

$$|\nu_i(\tau)\rangle = \exp\left(-\frac{i}{\hbar} m_i \tau\right) |\nu_i(0)\rangle\,, \tag{2.4}$$

where m_i is the mass of the i-th neutrino mass eigenstate. In our calculations we take m_i in the units of energy.

Using Lorentz invariance we can come to the laboratory frame and the time evolution is

$$|\nu_i(t)\rangle = \exp\left(-\frac{i}{\hbar c}\left(E_i ct - p_i L\right)\right) |\nu_i(0)\rangle\,, \tag{2.5}$$

where t is the laboratory time and L is propagation distance in the laboratory frame. Now we can use relativistic approximation

$$p_i = \sqrt{E_i^2 - m_i^2} \simeq E_i - \frac{m_i^2}{2E_i} \tag{2.6}$$

and $ct \simeq L$. Moreover, we are not able to measure the energy of different mass states, because the differences are small. Therefore we can set $E_i = E$ for all mass eigenstates. The propagation amplitude is after these assumptions given by

$$|\nu_i(L)\rangle = \exp\left(-\frac{i}{\hbar c}\frac{m_i^2}{2E}L\right)|\nu_i(0)\rangle . \tag{2.7}$$

We can calculate the propagation amplitude for neutrino flavor eigenstate. Using transformation (2.1) in (2.7) we get

$$|\nu_\alpha(L)\rangle = \sum_{i=1,2,3} M_{\alpha i}^* \exp\left(-\frac{i}{\hbar c}\frac{m_i^2}{2E}L\right)|\nu_i(0)\rangle . \tag{2.8}$$

2.3 Neutrino oscillation probability

Let us calculate the amplitude of probability that neutrino born in flavor state $|\nu_\alpha\rangle$ is detected as a neutrino in flavor eigenstate $|\nu_\beta\rangle$. Using (2.1) and (2.8), we get

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_{i=1,2,3} M_{\beta i} M_{\alpha i}^* \exp\left(-\frac{i}{\hbar c}\frac{m_i^2}{2E}L\right). \tag{2.9}$$

Now we prove, that the oscillation amplitude does not depend on Majorana phases. From (2.2), we have

$$\begin{aligned} \sum_i M_{\beta i} M_{\alpha i}^* &= \left(MM^\dagger\right)_{\beta\alpha} = \left(U \; diag\left\{e^{ia_1}, e^{ia_2}, 1\right\} \; diag\left\{e^{-ia_1}, e^{-i\alpha_2}, 1\right\} U^\dagger\right)_{\beta\alpha} \\ &= \left(UU^\dagger\right)_{\beta\alpha} = \sum_i U_{\beta i} U_{\alpha i}^* . \end{aligned} \tag{2.10}$$

Therefore the amplitude of probability is

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_{i=1,2,3} U_{\beta i} U_{\alpha i}^* \exp\left(-\frac{i}{\hbar c}\frac{m_i^2}{2E}L\right) \tag{2.11}$$

and does not depend on Majorana phases.

According to (2.11) the oscillation probability is given by

$$\begin{aligned} P(\nu_\alpha \quad \rightarrow \quad & \nu_\beta) = |A(\nu_\alpha \rightarrow \nu_\beta)|^2 = \\ & \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\right)\sin^2\left[\frac{\Delta m^2_{ij}L}{4\hbar cE}\right] \\ & +2\sum_{i>j} \mathrm{Im}\left(U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}\right)\sin\left[\frac{\Delta m^2_{ij}L}{2\hbar cE}\right], \end{aligned} \tag{2.12}$$

where we have introduced notation $\Delta m^2_{ij} \equiv m^2_i - m^2_j$.

Assuming CPT invariance holds we can write the relation between neutrino and antineutrino oscillations as

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\overline{\nu}_\beta \rightarrow \overline{\nu}_\alpha). \tag{2.13}$$

From equation (2.12) we can see that using complex conjugated elements of the matrix U, we get the oscillation probability inverted in time. We can express this result as

$$P(\nu_\alpha \rightarrow \nu_\beta, U) = P(\nu_\beta \rightarrow \nu_\alpha, U^*). \tag{2.14}$$

Using (2.14) in (2.13) we get

$$P(\nu_\alpha \rightarrow \nu_\beta, U) = P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta, U^*). \tag{2.15}$$

If the PMNS matrix is real the oscillation probability is same for neutrinos and antineutrinos. If the CP-violating phase δ is not zero the probabilities are different. Anyway if $\alpha = \beta$ it holds

$$P(\nu_\alpha \rightarrow \nu_\alpha) = P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\alpha). \tag{2.16}$$

The probability that neutrino with flavor α is detected with the same flavor α is equal for neutrinos and antineutrinos.

In several cases we can use instead if three neutrino mixing the two neutrino approximation. We consider only two flavor eigenstates and two mass eigenstates. Then the unitary mixing matrix is

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \tag{2.17}$$

Using the same procedure as we used above we get the oscillation probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 L}{4\hbar c E}\right] \qquad (\alpha \neq \beta) \tag{2.18}$$

$$P(\nu_\alpha \rightarrow \nu_\alpha) = 1 - \sin^2 2\theta \sin^2 \left[\frac{\Delta m^2 L}{4\hbar c E}\right]. \tag{2.19}$$

The oscillation probability is a function of two parameters, mixing angle θ and mass splitting Δm.

2.4 Oscillation parameters

The oscillation probability (2.12) depends in general on six parameters. Three mixing angles θ_{12}, θ_{23}, θ_{13}, CP-violating phase δ and two independent squared-mass splittings Δm^2_{21} and Δm^2_{32}. The present knowledge is summarized in figure 2-1.

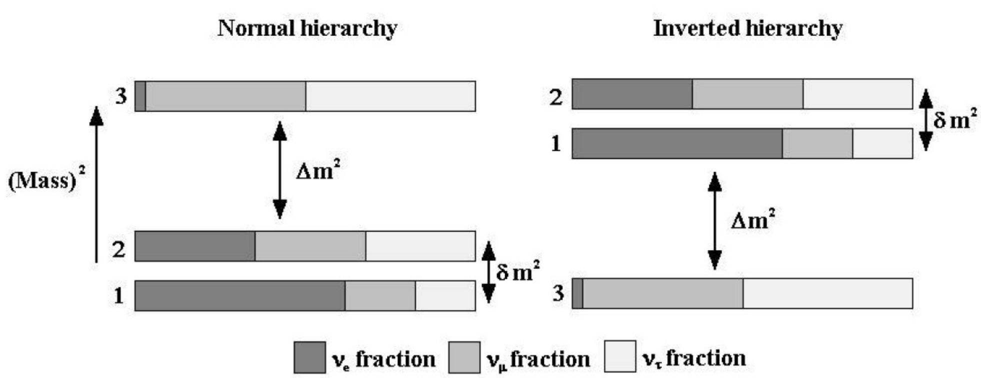

Figure 2-1: Normal and inverted neutrino mass hierarchy.

Mixing angle θ_{12} and mass splitting Δm^2_{21} are measured in solar neutrino and reactor neutrino experiments such as Homestake experiment, SNO, KamLAND, SAGE, Gallex. Solar experiments use electron neutrinos, reactor experiments use electron antineutrinos. Due to the small value of mixing angle θ_{13} the oscillation probability can be treated as a two neutrino oscillations between mass eigenstates ν_1 and ν_2. The best fit combining solar and reactor neutrino data is [5]

$$\sin^2\theta_{12} = 0.31^{+0.016}_{-0.023} \tag{2.20}$$

$$\Delta m^2_{21} = (7.59 \pm 0.21) \times 10^{-5}\ eV^2. \tag{2.21}$$

Due to the Mikheyev-Smirnov-Wolfenstein (MSW) effect presented in the neutrino propagation through matter we know that $m_2 > m_1$.

Accelerator and atmospheric neutrino experiments are sensitive to the parameters θ_{23} and Δm^2_{32} where muon neutrinos are used. Because $\Delta m^2_{21} << \Delta m^2_{32} \simeq \Delta m^2_{31}$ most of the experiments are not sensitive to the small mass splitting. Therefore we can use the two neutrino approximation with mass eigenstate ν_3 and effective combination of ν_1 and ν_2 for the data interpretation.

The Super-Kamiokande experiment have brought first compelling evidence of neutrino oscillations [2]. Present value given by the Super-Kamiokande for θ_{23} is [6]

$$\sin^2\theta_{23} = 0.50 \pm 0.063. \tag{2.22}$$

The most precise value of Δm^2_{32} is given by the MINOS experiment [7]

$$\left|\Delta m^2_{32}\right| = (2.43 \pm 0.13) \times 10^{-3}\ eV^2. \tag{2.23}$$

We are able to measure only the absolute values of Δm^2_{32} so far. Therefore we can not distinguish if m_2 is lighter or heavier than m_3. This leads us to two possible neutrino mass hierarchies, normal hierarchy with $m_2 < m_3$ the inverted with $m_2 > m_3$, figure 2-1.

At present we know only the upper bound on θ_{13}. The limit is given by the CHOOZ experiment [4] as

$$\sin^2 2\theta_{13} < 0.17. \tag{2.24}$$

Recent experiments such as Daya Bay experiment [3] plan to measure θ_{13} more precisely.

Last parameter is CP-violating phase δ. We do not have any restrictions on this parameter.

Chapter 3

Daya Bay experiment

The Daya Bay experiment [3] is antineutrino reactor experiment. It will perform the precision measurement of survival probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ to find the value of mixing angle θ_{13}. It is known only the upper bound of this oscillation parameter given by the CHOOZ experiment as $\sin^2 2\theta_{13} < 0.17$ [4]. The goal of the Daya Bay experiment to reach the sensitivity of 0.01 or better in $\sin^2 2\theta_{13}$ [3].

The experiment is located near the Daya Bay nuclear power plant complex in China. The complex consists of two pairs of nuclear reactors named Daya Bay and Ling Ao. One more pair of reactors Ling Ao II is currently under construction and it will be finished in 2011. These reactors provide the source of electron antineutrinos.

3.1 Detection method

The electron antineutrino detection method is so called inverse beta-dacay

$$\overline{\nu_e} + p \rightarrow n + e^+. \tag{3.1}$$

This reaction has the threshold

$$E_\nu^{thr} = \frac{(m_n + m_e)^2 - m_p^2}{2m_p} = 1.806\ MeV. \tag{3.2}$$

Comparing the reactor antineutrino energy spectrum and the cross-section of inverse beta-dacay we get the count rate with maximum about 4MeV see figure 3-1.

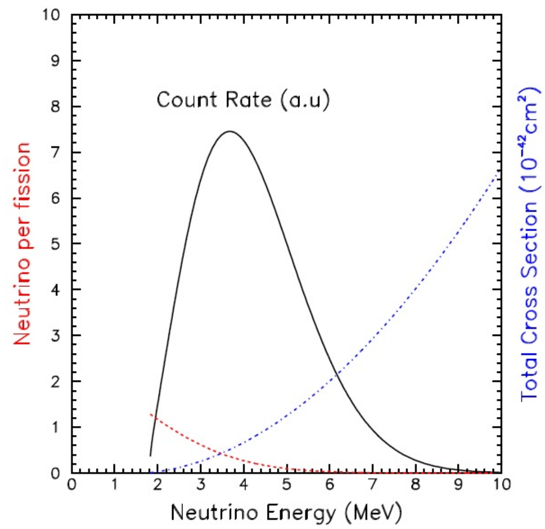

Figure 3-1: Detected antineutrino energy spectrum in the Daya Bay experiment [3].

3.2 Antineutrino detector

The detector module is designed as a cylindrical vessel consist of three nested cylindrical zones. In the inner is 20 tons of liquid scintillator doped with the atoms of Gadolinium. This element has large cross-section of neutron capture.

Positron from inverse beta-decay is stopped emitting γs along its trajectory and annihilates with electron and produced two γs. Neutrons is captured on the Gadolinium and new nucleus deexcite emitting also γs. The liquid scintillator shifts the wavelength to the visible spectrum and it is detected by PMTs on the surface of the outer vessel. There is pure liquid scintillator in the middle zone for catching γs which escape form the inner part. The outer zone where PMTs are installed is filled with the mineral oil. Its basic purpose is shielding from the γ background. The antineutrino detector module is shown on the figure 3-2.

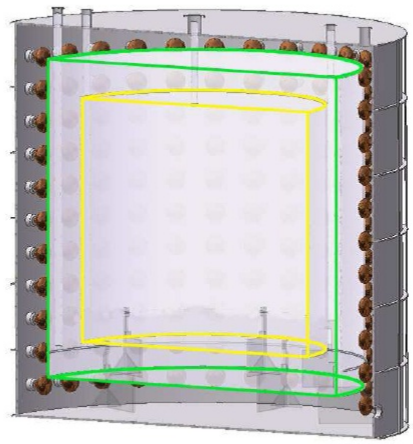

Figure 3-2: Cross sectional slice of a 3-zone antineutrino detector module showing vessel holding the Gd-doped liquid scintilator at the center (20 tons), liquid scintillator between the acrylic vessels (20 tons) and mineral oil (40 tons) in the outer region. The PMTs are mounted on the inside walls of the stainless steel tank [3].

3.3 Experimental layout

The Daya Bay experiment consists of one far detector with four antineutrino detector modules and two near detectors both with two antineutrino detector modules. They are placed in experimental halls under the surface due to background shielding. Halls are connected with entrance tunnels. The experimental layout is shown on the figure 3-3.

The oscillation probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ is using (2.12) given by

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 4s_{12}^2c_{12}^2\left(\frac{\Delta m_{21}^2 L}{4\hbar cE}\right)^2 - 4s_{13}^2\sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar cE}\right). \quad (3.3)$$

The first oscillation minimum is according to (3.3) in $\frac{L(km)}{E(MeV)} \simeq 0.5$. The antineutrino count rate has the maximum about 4 MeV therefore the far detector is in the distance about 2 km. The near detectors are placed next to the reactor cores to measure precisely the antineutrino flux to reach better sensitivity.

In the table1 there are distances between the detectors and the center of the reactor pairs.

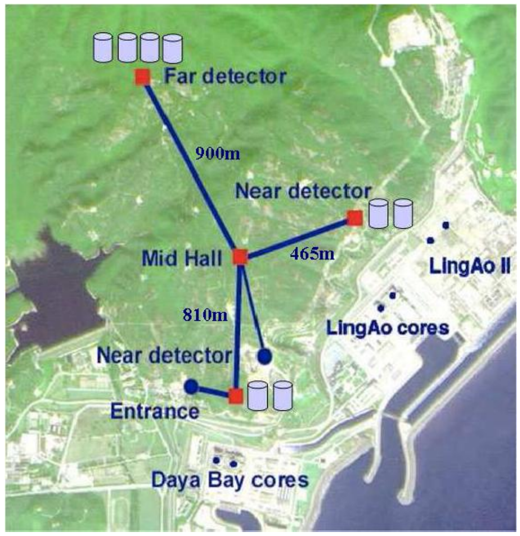

Figure 3-3: Experimental layoutof the Daya Bay experiment [3].

Sites/Detectors	DYB	LA	Far
DYB cores	363	1347	1985
LA cores	857	481	1618
LA II cores	1307	526	1613

Sites/Detectors	DYB	LA	Far
DYB cores	L_{DYBDYB}	L_{DYBLA}	L_{DYBFAR}
LA cores	L_{LADYB}	L_{LALA}	L_{LAFAR}
LA II cores	$L_{LAIIDYB}$	L_{LAIILA}	$L_{LAIIFAR}$

Table 1: Distances in meters between detectors and center of reactor pairs [3] and their notation.

Chapter 4

Nonstandard interactions in the neutrino source and detector

Let us consider some possible physics beyond the Standard model in neutrino source and detector. We assume that neutrino with flavor α is not always coupled to the lepton with the same flavour in the production and the detection process. This does not conserve the family lepton number. But neutrino oscillations themselves in general do not conserve family lepton number. Therefore we want to investigate the effect of these nonstandard interactions and the possibility of their measurement. Now we introduce the formalism proposed in [8].

Neutrinos born in the source can be treated as a superposition of flavor eigenstates

$$|\nu_\alpha^s\rangle = \frac{1}{N_\alpha^s}\left(|\nu_\alpha\rangle + \sum_{\beta=e,\mu,\tau} \varepsilon_{\alpha\beta}^s |\nu_\beta\rangle\right). \tag{4.1}$$

Similarly for detector, we get

$$\left\langle\nu_\alpha^d\right| = \frac{1}{N_\alpha^d}\left(\left\langle\nu_\alpha^d\right| + \sum_{\beta=e,\mu,\tau} \varepsilon_{\beta\alpha}^d \left\langle\nu_\beta^d\right|\right). \tag{4.2}$$

N_α^s and N_α^d are normalization factors. Matrices $\varepsilon_{\alpha\beta}^s$, $\varepsilon_{\beta\alpha}^d$ parametrize size of the non-standard interactions in the source, respectively detector. Since we are not specifying the interaction they are in general arbitrary.

Let us calculate the oscillation amplitude that neutrino produced in the source with flavor α is measured in the detector with flavor β. The amplitude is using (4.2),(4.1) given by

$$\begin{aligned} A(\nu_\alpha^s \to \nu_\beta^d) = \left\langle \nu_\beta^d | \nu_\alpha^s (L) \right\rangle &= \frac{1}{N_\alpha^s N_\alpha^d} \left(\langle \nu_\beta | \nu_\alpha (L) \rangle + \sum_{\gamma=e,\mu,\tau} \varepsilon_{\alpha\gamma}^s \langle \nu_\beta | \nu_\gamma (L) \rangle \right. \\ &\left. + \sum_{\gamma=e,\mu,\tau} \varepsilon_{\beta\gamma}^d \langle \nu_\gamma | \nu_\alpha (L) \rangle + \sum_{\gamma,\lambda=e,\mu,\tau} \varepsilon_{\alpha\gamma}^s \varepsilon_{\beta\lambda}^d \langle \nu_\lambda | \nu_\gamma (L) \rangle \right). \end{aligned} \tag{4.3}$$

The amplitudes $\langle \nu_\rho | \nu_\sigma (L) \rangle$ appearing in (4.3) are standard amplitudes given by equation (2.11). Therefore we can write

$$A(\nu_\alpha^s \to \nu_\beta^d) = \sum_{i=1,2,3} T_{\alpha\beta}^i \exp\left(-\frac{i}{\hbar c} \frac{m_i^2}{2E} L \right), \tag{4.4}$$

where $T_{\alpha\beta}^i$ is given by

$$T_{\alpha\beta}^i = \frac{1}{N_\alpha^s N_\alpha^d} \left(U_{\alpha i}^* U_{\beta i} + \sum_{\gamma=e,\mu,\tau} \varepsilon_{\alpha\gamma}^s U_{\gamma i}^* U_{\beta i} + \sum_{\gamma=e,\mu,\tau} \varepsilon_{\beta\gamma}^d U_{\alpha i}^* U_{\gamma i} + \sum_{\gamma=e,\mu,\tau} \sum_{\lambda=e,\mu,\tau} \varepsilon_{\alpha\gamma}^s \varepsilon_{\beta\lambda}^d U_{\gamma i}^* U_{\lambda i} \right). \tag{4.5}$$

Since we have the amplitude, we are able to calculate the oscillation probability as

$$\begin{aligned} P(\nu_\alpha^s \to \nu_\beta^d) &= \left| A(\nu_\alpha^s \to \nu_\beta^d) \right|^2 \\ &= \sum_{i,j} T_{\alpha\beta}^i T_{\alpha\beta}^{j*} - 4 \sum_{i>j} \mathrm{Re}\left(T_{\alpha\beta}^i T_{\alpha\beta}^{j*} \right) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4\hbar c E} \right) \\ &+2 \sum_{i>j} \mathrm{Im}\left(T_{\alpha\beta}^i T_{\alpha\beta}^{j*} \right) \sin\left(\frac{\Delta m_{ij}^2 L}{2\hbar c E} \right). \end{aligned} \tag{4.6}$$

The oscillation probability (4.6) is valid for neutrinos. Let us discuss antineutrino oscillation probability. If the identity $\varepsilon^s = \varepsilon^{d\dagger}$ is satisfied the oscillation probability for antineutrinos is

$$\begin{aligned} P(\bar{\nu}_\alpha^s \to \bar{\nu}_\beta^d) &= \sum_{i,j} T_{\alpha\beta}^{i*} T_{\alpha\beta}^{j} - 4 \sum_{i>j} \mathrm{Re}\left(T_{\alpha\beta}^{i*} T_{\alpha\beta}^{j} \right) \sin^2\left(\frac{\Delta m_{ij}^2 L}{4\hbar c E} \right) \\ &+2 \sum_{i>j} \mathrm{Im}\left(T_{\alpha\beta}^{i*} T_{\alpha\beta}^{j} \right) \sin\left(\frac{\Delta m_{ij}^2 L}{2\hbar c E} \right). \end{aligned} \tag{4.7}$$

We just change term $T^i_{\alpha\beta}$ to the complex conjugated term $T^{i*}_{\alpha\beta}$ in the probability formula (4.6).

But if the process of neutrino detection is not exactly inverse to the neutrino production process some features which violate the condition $\varepsilon^s = \varepsilon^{d\dagger}$ could be present. In this case the oscillation probability is

$$\begin{aligned} P(\bar{\nu}^s_\alpha \to \bar{\nu}^d_\beta) &= \sum_{i,j} T^i_{\alpha\beta} T^{j*}_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left(T^i_{\alpha\beta} T^{j*}_{\alpha\beta}\right) \sin^2\left(\frac{\Delta m^2_{ij} L}{4\hbar c E}\right) \\ &\quad +2\sum_{i>j} \mathrm{Im}\left(T^i_{\alpha\beta} T^{j*}_{\alpha\beta}\right) \sin\left(\frac{\Delta m^2_{ij} L}{2\hbar c E}\right), \end{aligned}$$

where $T^i_{\alpha\beta}$ is given by

$$T^i_{\alpha\beta} = \frac{1}{N^s_\alpha N^d_\alpha}\left(U_{\alpha i}U^*_{\beta i} + \sum_{\gamma=e,\mu,\tau} \varepsilon^s_{\alpha\gamma} U_{\gamma i} U^*_{\beta i} + \sum_{\gamma=e,\mu,\tau} \varepsilon^d_{\beta\gamma} U_{\alpha i} U^*_{\gamma i} + \sum_{\gamma=e,\mu,\tau}\sum_{\lambda=e,\mu,\tau} \varepsilon^s_{\alpha\gamma}\varepsilon^d_{\beta\lambda} U_{\gamma i} U^*_{\lambda i}\right)$$

and $\varepsilon^s_{\alpha\beta}$, $\varepsilon^d_{\alpha\beta}$ are the parameters for antineutrinos. They can be in general different from $\varepsilon^s_{\alpha\beta}$, $\varepsilon^d_{\alpha\beta}$ for neutrinos.

Chapter 5

Possibility of the measurement of the nonstandard interactions in the Daya Bay experiment

5.1 Modeling of the electron antineutrino spectra in the Daya Bay experiment

Let us perform naive calculation of the antineutrino spectra. The Daya Bay experiment will use three pairs of nuclear reactors as a antineutrino source. The thermal power of each core is 2.9 GW [3]. Fission of uranium and plutonium isotopes produces daughter nuclei. They are neutron rich therefore most of them beta decays. There is released in average 200 MeV of energy and approximately six antineutrinos [3]. Therefore we can estimate the total number of fissions in the reactor site as

$$NF = \frac{Number\ of\ reactors \times Thermal\ power}{Energy\ per\ fission} = \frac{2 \times 2.9\ GW}{200\ MeV} = 1.8 \times 10^{20}\ s^{-1}. \qquad (5.1)$$

Most of the neutrinos produces in the reactor are below the detection threshold. The neutrino energy spectrum per fission can be parametrized as

$$\Phi(E) = \exp\left(\gamma - \alpha E - \beta E^2\right) \qquad (5.2)$$

where $\alpha = 0.16$, $\beta = 0.091$ and $\gamma = 0.87$ [9].

As it is mentioned above the detection method is the inverse beta decay $\overline{\nu_e} + p \rightarrow n + e^+$. The cross-section of this reaction can be calculated as [10]

$$\sigma(E) = 0.0952 \times 10^{-46} m^2 \left(E - (m_n - m_p)\right) \sqrt{\left(E - (m_n - m_p)\right)^2 - m_e^2} \frac{1}{MeV^2} \tag{5.3}$$

with the threshold $E = 1.806\ MeV$. This reaction will be proceeded on free protons in the liquid scintillator. There are 6.29×10^{22} free protons per cm^3 in the scintillator and density is $\rho = 0.86\ g/cm^3$ [11]. In each detection module there is 20 tons of liquid scintillator. Therefore there is $NT = 1.46 \times 10^{30}$ targets in one detection module.

The number of detected neutrinos depends also on the oscillation probability. It is a function of neutrino energy, distance and the oscillation parameters. We use denotation $P(E, L, ...)$.

Let us calculate the number of detected neutrinos in each detector. There are two detection modules in the DYB near detector. The distances between the detector and reactors are in table 1. For the three years run we get

$$\begin{aligned} N_{DYB}(E) &= 3 \times 365 \times 24 \times 3600 \times NT \times 2 \times eff \times NF \times \frac{\Phi(E)}{4\pi} \times \sigma(E) \\ &\times \left(\frac{P(E, L_{DYBDYB}, ...)}{L^2_{DYBDYB}} + \frac{P(E, L_{LADYB}, ...)}{L^2_{LADYB}} + \frac{P(E, L_{LAIIDYB}, ...)}{L^2_{LAIIDYB}} \right). \end{aligned} \tag{5.4}$$

Similarly for the near LA detector and the FAR detector we get

$$\begin{aligned} N_{LA}(E) &= 3 \times 365 \times 24 \times 3600 \times NT \times 2 \times eff \times NF \times \frac{\Phi(E)}{4\pi} \times \sigma(E) \\ &\times \left(\frac{P(E, L_{DYBLA}, ...)}{L^2_{DYBLA}} + \frac{P(E, L_{LALA}, ...)}{L^2_{LALA}} + \frac{P(E, L_{LAIILA}, ...)}{L^2_{LAIILA}} \right), \end{aligned} \tag{5.5}$$

$$\begin{aligned} N_{FAR}(E) &= 3 \times 365 \times 24 \times 3600 \times NT \times 4 \times eff \times NF \times \frac{\Phi(E)}{4\pi} \times \sigma(E) \\ &\times \left(\frac{P(E, L_{DYBFAR}, ...)}{L^2_{DYBFAR}} + \frac{P(E, L_{LAFAR}, ...)}{L^2_{LAFAR}} + \frac{P(E, L_{LAIIFAR}, ...)}{L^2_{LAIIFAR}} \right). \end{aligned} \tag{5.6}$$

We apply detection efficiency $eff = 0.78$ [3].

We integrate this energy spectrum to the several bins to obtain data like from the real measurement. Then we are able to investigate the effect of the nonstandard interactions in the experiment.

5.2 Oscillation probability

We investigate the possible effect of the nonstandard interactions presented in the source and in the detector in the Daya Bay experiment. The oscillation probability (4.6) depends on terms $T^i_{\alpha\beta}$. The Daya Bay experiment will measure the probability $P(\bar{\nu}_e \to \bar{\nu}_e)$. Let us calculate appropriate terms $T^3_{ee}T^{1*}_{ee}, T^3_{ee}T^{2*}_{ee}, T^2_{ee}T^{1*}_{ee}$. Using (4.5) we get

$$\begin{aligned}
T^3_{ee}T^{1*}_{ee} &= \Big(s^2_{13}c^2_{12}c^2_{13} + \varepsilon^s_{e\mu}c^2_{12}s_{23}c_{13}s_{13}e^{-i\delta} + \varepsilon^s_{e\tau}c^2_{12}c_{23}c_{13}s_{13}e^{-i\delta} \\
&\quad + \varepsilon^d_{\mu e}c^2_{12}s_{23}c_{13}s_{13}e^{i\delta} + \varepsilon^d_{\tau e}c^2_{12}c_{23}c_{13}s_{13}e^{i\delta} + \varepsilon^s_{e\mu}\varepsilon^d_{\mu e}c^2_{12}s^2_{23} \\
&\quad + \varepsilon^s_{e\tau}\varepsilon^d_{\tau e}c^2_{12}c^2_{23} + \varepsilon^s_{e\mu}\varepsilon^d_{\tau e}c^2_{12}s_{23}c_{23} + \varepsilon^s_{e\tau}\varepsilon^d_{\mu e}c^2_{12}s_{23}c_{23}\Big) \\
&\quad + \mathcal{O}\left(\varepsilon s^2_{13}, \varepsilon^2 s_{13}\right) \qquad (5.7) \\
T^3_{ee}T^{2*}_{ee} &= \Big(s^2_{13}s^2_{12}c^2_{13} + \varepsilon^s_{e\mu}s^2_{12}s_{23}c_{13}s_{13}e^{-i\delta} + \varepsilon^s_{e\tau}s^2_{12}c_{23}c_{13}s_{13}e^{-i\delta} \\
&\quad + \varepsilon^d_{\mu e}s^2_{12}s_{23}c_{13}s_{13}e^{i\delta} + \varepsilon^d_{\tau e}s^2_{12}c_{23}c_{13}s_{13}e^{i\delta} + \varepsilon^s_{e\mu}\varepsilon^d_{\mu e}s^2_{12}s^2_{23} \\
&\quad + \varepsilon^s_{e\tau}\varepsilon^d_{\tau e}s^2_{12}c^2_{23} + \varepsilon^s_{e\mu}\varepsilon^d_{\tau e}s^2_{12}s_{23}c_{23} + \varepsilon^s_{e\tau}\varepsilon^d_{\mu e}s^2_{12}s_{23}c_{23}\Big) \\
&\quad + \mathcal{O}\left(\varepsilon s^2_{13}, \varepsilon^2 s_{13}\right) \qquad (5.8) \\
T^2_{ee}T^{1*}_{ee} &= \big(c^2_{12}s^2_{12}c^4_{13} + 2\,\mathrm{Re}\left(\varepsilon^s_{ee}\right)c^2_{12}s^2_{12}c^4_{13} - \varepsilon^{s*}_{e\mu}s^3_{12}c_{12}c_{23}c_{13} \\
&\quad + \varepsilon^s_{e\mu}s_{12}c^3_{12}c_{23}c_{13} + \varepsilon^{s*}_{e\tau}s^3_{12}c_{12}s_{23}c_{13} - \varepsilon^s_{e\tau}s_{12}c^3_{12}s_{23}c_{13} \\
&\quad + 2\,\mathrm{Re}\left(\varepsilon^d_{ee}\right)c^2_{12}s^2_{12}c^4_{13} - \varepsilon^{d*}_{\mu e}s^3_{12}c_{12}c_{23}c_{13} + \varepsilon^d_{\mu e}s_{12}c^3_{12}c_{23}c_{13} \\
&\quad + \varepsilon^{d*}_{\tau e}s^3_{12}c_{12}s_{23}c_{13} - \varepsilon^d_{\tau e}s_{12}c^3_{12}s_{23}c_{13}\Big) + \mathcal{O}\left(\varepsilon^2, \varepsilon s_{13}\right). \qquad (5.9)
\end{aligned}$$

We are omitting the normalization factor $\frac{1}{N^s_\alpha N^d_\alpha}$ because it is compensated with the change of the production and detection cross-sections in the real experiment. Using (5.7), (5.8), (5.9) in

(4.6) the oscillation probability is $P(\bar{\nu}_e^s \to \bar{\nu}_e^d)$ is

$$
\begin{aligned}
&P(\bar{\nu}_e^s \to \bar{\nu}_e^d) = 1 - 4c_{12}^2 s_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 \\
&-4\left[s_{13}^2 + \mathrm{Re}\left(\varepsilon_{e\mu}^s e^{-i\delta} + \varepsilon_{\mu e}^d e^{i\delta}\right) s_{23} s_{13}\right. \\
&+\mathrm{Re}\left(\varepsilon_{e\tau}^s e^{-i\delta} + \varepsilon_{\tau e}^d e^{i\delta}\right) c_{23} s_{13} + \mathrm{Re}\left(\varepsilon_{e\mu}^s \varepsilon_{\mu e}^d\right) s_{23}^2 \\
&\left.+\mathrm{Re}\left(\varepsilon_{e\tau}^s \varepsilon_{\tau e}^d\right) c_{23}^2 + \mathrm{Re}\left(\varepsilon_{e\mu}^s \varepsilon_{\tau e}^d + \varepsilon_{e\tau}^s \varepsilon_{\mu e}^d\right) s_{23} c_{23}\right] \sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar c E}\right) \\
&+2\left[\mathrm{Im}\left(\varepsilon_{e\mu}^s e^{-i\delta} + \varepsilon_{\mu e}^d e^{i\delta}\right) s_{23} s_{13} + \mathrm{Im}\left(\varepsilon_{e\tau}^s e^{-i\delta} + \varepsilon_{\tau e}^d e^{i\delta}\right) c_{23} s_{13}\right. \\
&\left.+\mathrm{Im}\left(\varepsilon_{e\mu}^s \varepsilon_{\mu e}^d\right) s_{23}^2 + \mathrm{Im}\left(\varepsilon_{e\tau}^s \varepsilon_{\tau e}^d\right) c_{23}^2 + \mathrm{Im}\left(\varepsilon_{e\mu}^s \varepsilon_{\tau e}^d + \varepsilon_{e\tau}^s \varepsilon_{\mu e}^d\right) s_{23} c_{23}\right] \sin\left(\frac{\Delta m_{32}^2 L}{2\hbar c E}\right) \\
&+2\left[\mathrm{Im}\left(\varepsilon_{e\mu}^s + \varepsilon_{\mu e}^d\right) c_{12} s_{12} c_{23} - \mathrm{Im}\left(\varepsilon_{e\tau}^s + \varepsilon_{\tau e}^d\right) c_{12} s_{12} s_{23}\right] \left(\frac{\Delta m_{21}^2 L}{2\hbar c E}\right) \\
&+\mathcal{O}\left(s_{13}^3, \varepsilon^3, \varepsilon s_{13}^2, \varepsilon^2 s_{13}, \varepsilon^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right), \varepsilon s_{13} \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right), \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^3\right). \qquad (5.10)
\end{aligned}
$$

The oscillation probability for standard neutrino oscillations i.e. without nonstandard interactions is using (2.12) given by

$$
\begin{aligned}
P(\bar{\nu}_e \to \bar{\nu}_e) &= 1 - 4s_{12}^2 c_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 - 4s_{13}^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar c E}\right) \qquad (5.11) \\
&+\mathcal{O}\left(s_{13}^3, \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)\right)
\end{aligned}
$$

Due to the Daya Bay experiment probability resolution which will be 0.01 [3] we have neglected terms $s_{13}^3, \varepsilon^3, \varepsilon s_{13}^2, \varepsilon^2 s_{13}, \varepsilon^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right), \varepsilon s_{13} \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right), \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^3$ which are in order of 0.001. If we take the experiment parameters which are approximately $L = 1800m$ and mean value of neutrino energy $E \simeq 4MeV$ it holds that $\frac{\Delta m_{21}^2 L}{4\hbar c E} \simeq 0.04$. The limit on θ_{13} is $\sin\theta_{13} \lesssim 0.2$ [4] and the limit on the NSI parameters is $|\varepsilon| < 0.1$ [12]. In the following calculations we do not explicitly mention these neglected terms.

If the process of neutrino detection is exactly inverse to the neutrino production CPT invariance implies $\varepsilon^s = \varepsilon^{d\dagger}$. If this condition is satisfied the nonstandard interactions mimic the neutrino oscillations signal and we measure the effective mixing angle which is different from the angle appearing in the PMNS matrix [8], [13]. Adopting $\varepsilon^s = \varepsilon^{d\dagger}$ in (5.10) we get

$$
\begin{aligned}
P(\bar{\nu}_e^s \to \bar{\nu}_e^d) &= 1 - 4c_{12}^2 s_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 \\
&-4\left[s_{13}^2 + 2\,\mathrm{Re}\left(\varepsilon_{e\mu}^s e^{-i\delta}\right) s_{23}s_{13} + 2\,\mathrm{Re}\left(\varepsilon_{e\tau}^s e^{-i\delta}\right) c_{23}s_{13}\right. \\
&\left.+\left|\varepsilon_{e\mu}^s\right|^2 s_{23}^2 + \left|\varepsilon_{e\tau}^s\right|^2 c_{23}^2 + 2\,\mathrm{Re}\left(\varepsilon_{e\mu}^s \varepsilon_{e\tau}^{s*}\right) s_{23}c_{23}\right] \sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar c E}\right).
\end{aligned}
\tag{5.12}
$$

If we introduce notation

$$
\begin{aligned}
s_{13eff}^2 &= s_{13}^2 + 2\,\mathrm{Re}\left(\varepsilon_{e\mu}^s e^{-i\delta}\right) s_{23}s_{13} + 2\,\mathrm{Re}\left(\varepsilon_{e\tau}^s e^{-i\delta}\right) c_{23}s_{13} \\
&+\left|\varepsilon_{e\mu}^s\right|^2 s_{23}^2 + \left|\varepsilon_{e\tau}^s\right|^2 c_{23}^2 + 2\,\mathrm{Re}\left(\varepsilon_{e\mu}^s \varepsilon_{e\tau}^{s*}\right) s_{23}c_{23}
\end{aligned}
\tag{5.13}
$$

the oscillation probability (5.12) get the same form as the pure oscillation probability (5.11) only with effective mixing angle θ_{13}^{eff}. Therefore we are not able to distinguish the effect of pure oscillations and the effect of nonstandard interactions.

But if the processes of production and detection are different identity $\varepsilon^s = \varepsilon^{d\dagger}$ does not have to hold. Violation of this condition lead not only to the effective mixing angle θ_{13}^{eff} but also to the change of the oscillation curve what is shown later. Let us investigate this case and the results for the Daya Bay experiment.

We assume that condition $\varepsilon^s = \varepsilon^{d\dagger}$ is violated. We can parametrize relevant elements of ε^s by

$$
\begin{aligned}
\varepsilon_{e\mu}^s &= \left|\varepsilon_{e\mu}^s\right| e^{i\phi_{e\mu}^s} \\
\varepsilon_{e\tau}^s &= \left|\varepsilon_{e\tau}^s\right| e^{i\phi_{e\tau}^s}.
\end{aligned}
\tag{5.14}
$$

Then we assume that violation is only in the phase factor and not in the absolute value. Using this assumption, we can write

$$
\begin{aligned}
\varepsilon_{\mu e}^d &= \left|\varepsilon_{e\mu}^s\right| e^{-i\phi_{e\mu}^s + i\psi_{e\mu}} \\
\varepsilon_{\tau e}^d &= \left|\varepsilon_{e\tau}^s\right| e^{-i\phi_{e\tau}^s + i\psi_{e\tau}}.
\end{aligned}
\tag{5.15}
$$

Phases $\psi_{e\mu}$ and $\psi_{e\tau}$ are violating phases. If they are both equal to zero, the condition $\varepsilon^s = \varepsilon^{d\dagger}$

e.g. $\varepsilon^s_{\alpha\beta} = \varepsilon^{d*}_{\beta\alpha}$ holds. Using (5.15),(5.14) in (5.10) we get

$$
\begin{aligned}
&P(\bar{\nu}^s_e \to \bar{\nu}^d_e) = 1 - 4c^2_{12}s^2_{12}\left(\frac{\Delta m^2_{21}L}{4\hbar cE}\right)^2 \qquad (5.16)\\
&-4\left\{s^2_{13} + s_{23}s_{13}\left|\varepsilon_{e\mu}\right|\left[\cos\left(\phi_{e\mu} - \delta - \psi_{e\mu}\right) + \cos\left(\phi_{e\mu} - \delta\right)\right]\right.\\
&+c_{23}s_{13}\left|\varepsilon_{e\tau}\right|\left[\cos\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right) + \cos\left(\phi_{e\tau} - \delta\right)\right]\\
&+s^2_{23}\left|\varepsilon_{e\mu}\right|^2\cos\left(\psi_{e\mu}\right) + c^2_{23}\left|\varepsilon_{e\tau}\right|^2\cos\left(\psi_{e\tau}\right)\\
&+s_{23}c_{23}\left|\varepsilon_{e\mu}\right|\left|\varepsilon_{e\tau}\right|\left[\cos\left(\phi_{e\mu} - \phi_{e\tau} - \psi_{e\mu}\right)\right.\\
&\left.\left. + \cos\left(\phi_{e\mu} - \phi_{e\tau} + \psi_{e\tau}\right)\right]\right\}\sin^2\left[\frac{\Delta m^2_{32}L}{4E}\right]\\
&+2\left\{s_{23}s_{13}\left|\varepsilon_{e\mu}\right|\left[\sin\left(\phi_{e\mu} - \delta\right) - \sin\left(\phi_{e\mu} - \delta - \psi_{e\mu}\right)\right]\right.\\
&+c_{23}s_{13}\left|\varepsilon_{e\tau}\right|\left[\sin\left(\phi_{e\tau} - \delta\right) - \sin\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right)\right]\\
&+s^2_{23}\left|\varepsilon_{e\mu}\right|^2\sin\left(\psi_{e\mu}\right) + c^2_{23}\left|\varepsilon_{e\tau}\right|^2\sin\left(\psi_{e\tau}\right)\\
&+s_{23}c_{23}\left|\varepsilon_{e\mu}\right|\left|\varepsilon_{e\tau}\right|\left[\sin\left(\phi_{e\mu} - \phi_{e\tau} + \psi_{e\tau}\right)\right.\\
&\left.\left. - \sin\left(\phi_{e\mu} - \phi_{e\tau} - \psi_{e\mu}\right)\right]\right\}\sin\left[\frac{\Delta m^2_{32}L}{2E}\right]\\
&+2\left\{\left|\varepsilon_{e\mu}\right|c_{23}\left[\sin\left(\phi_{e\mu}\right) - \sin\left(\phi_{e\mu} - \psi_{e\mu}\right)\right]\right.\\
&\left. - \left|\varepsilon_{e\tau}\right|s_{23}\left[\sin\left(\phi_{e\tau}\right) - \sin\left(\phi_{e\tau} - \psi_{e\tau}\right)\right]\right\}s_{12}c_{12}\left[\frac{\Delta m^2_{21}L}{2E}\right].
\end{aligned}
$$

We have omitted upper script s in $|\varepsilon_{\alpha\beta}|$ and $\phi^s_{\alpha\beta}$. In following calculations we set $\psi_{e\mu} = \psi_{e\tau}$ and $\phi_{e\mu} = \phi_{e\tau}$. Therefore the oscillation probability (5.16) is given by

$$
\begin{aligned}
&P(\bar{\nu}^s_e \to \bar{\nu}^d_e) = 1 - 4c^2_{12}s^2_{12}\left(\frac{\Delta m^2_{21}L}{4\hbar cE}\right)^2\\
&-4\left\{s^2_{13} + s_{13}\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\cos\left(\phi_{e\tau} - \delta\right) + \cos\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right)\right]\right.\\
&\left. + \cos\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin^2\left[\frac{\Delta m^2_{32}L}{4E}\right]\\
&+2\left\{s_{13}\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi_{e\tau} - \delta\right) - \sin\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right)\right]\right.\\
&\left. + \sin\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin\left[\frac{\Delta m^2_{32}L}{2E}\right]\\
&+2\left[c_{23}\left|\varepsilon_{e\mu}\right| - s_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi_{e\tau}\right) - \sin\left(\phi_{e\tau} - \psi_{e\tau}\right)\right]s_{12}c_{12}\left[\frac{\Delta m^2_{21}L}{2E}\right]. \qquad (5.17)
\end{aligned}
$$

We can simply calculate the probability difference to investigate if the effect of nonstandard interactions is measurable. Using (5.11) and (5.17) we get

$$
\begin{aligned}
&P(\bar{\nu}_e^s \to \bar{\nu}_e^d) - P(\bar{\nu}_e \to \bar{\nu}_e) = \qquad (5.18)\\
&-4\left\{s_{13}\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\cos\left(\phi_{e\tau} - \delta\right) + \cos\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right)\right]\right.\\
&\left. + \cos\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin^2\left[\frac{\Delta m_{32}^2 L}{4E}\right]\\
&+2\left\{s_{13}\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi_{e\tau} - \delta\right) - \sin\left(\phi_{e\tau} - \delta - \psi_{e\tau}\right)\right]\right.\\
&\left. + \sin\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\mu}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin\left[\frac{\Delta m_{32}^2 L}{2E}\right]\\
&+2\left[c_{23}\left|\varepsilon_{e\mu}\right| - s_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi_{e\tau}\right) - \sin\left(\phi_{e\tau} - \psi_{e\tau}\right)\right] s_{12}c_{12}\left[\frac{\Delta m_{21}^2 L}{2E}\right].
\end{aligned}
$$

In following figures 5-1, 5-2 and 5-3 we have plotted the probability difference (5.18) for three energies. We set $s_{13}^2 = 0.04$ the limit given by the CHOOZ experiment [4], $\delta = 0$ and $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$[12]

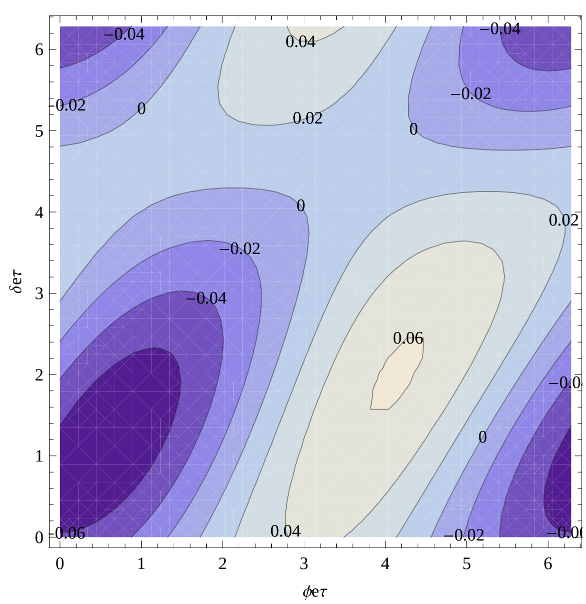

Figure 5-1: The probability difference as a function of $\phi_{e\tau}$ and $\psi_{e\tau}$ for E=2.5 MeV, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$, $s_{13} = 0.04$.

The previous plots show that the probability difference is large enough to be measured in the Daya Bay experiment. In principle we are able to reveal the NSI for a particular set of oscillation and nonstandard interaction parameters. On the other hand there are regions, where

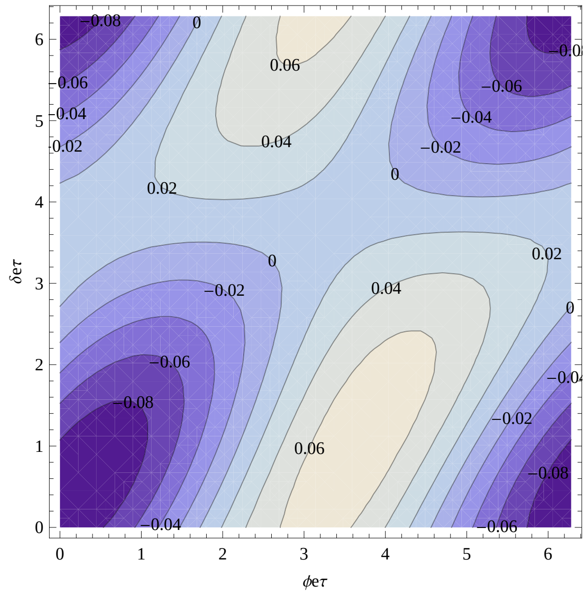

Figure 5-2: The probability difference as a function of $\phi_{e\tau}$ and $\psi_{e\tau}$ for E=3 MeV, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$, $s_{13} = 0.04$.

the oscillation probability is very close to the case with no NSI, even if the NSI parameters $\phi_{e\tau}$ and $\psi_{e\tau}$ are not zero.

If the probability difference is large it does not mean that we are able to distinguish the effect of the nonstandard interactions. For parameter $\psi_{e\tau} = 0$, e.g. $\varepsilon^s = \varepsilon^{d\dagger}$ holds, we effectively measure different mixing angle θ_{13} and we can not distinguish the effect of the pure oscillations and the NSI.

Therefore let us investigate the oscillation probability in few interesting points $[\phi_{e\tau}; \psi_{e\tau}]$. For revealing the nonstandard interactions it is important the term proportional to $\sin\left(\frac{\Delta m^2 L}{2E}\right)$ in (5.18). If it is different from zero results in the shift of the first minimum of the oscillation probability as we show later. The term proportional to $\sin\left(\frac{\Delta m^2 L}{2E}\right)$ is from (5.18) given by

$$F = 2\left\{s_{13}\left[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|\right]\left[\sin(\phi_{e\tau}) - \sin(\phi_{e\tau} - \psi_{e\tau})\right] + \sin(\psi_{e\tau})\left[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|\right]^2\right\}. \quad (5.19)$$

We can find extremes for $F = F(\phi_{e\tau}, \psi_{e\tau})$. They have to satisfy equations:

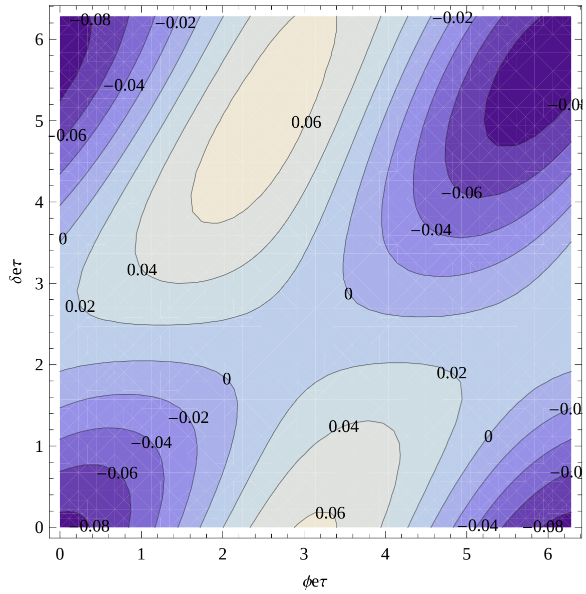

Figure 5-3: The probability difference as a function of $\phi_{e\tau}$ and $\psi_{e\tau}$ for E=5 MeV, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$, $s_{13} = 0.04$.

$$\cos(\phi_{e\tau}) = -\frac{[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|]}{s_{13}} \cos(\psi_{e\tau}) \tag{5.20}$$

$$\cos(\phi_{e\tau}) = \cos(\phi_{e\tau} - \psi_{e\tau}) \,. \tag{5.21}$$

Solving them, we get:

$$\cos(\psi_{e\tau}) = \frac{1}{4}\left(\frac{s_{13}}{[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|]}\right)^2 \left(1 - \sqrt{1 + 8\left(\frac{[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|]}{s_{13}}\right)^2}\right)$$

$$\cos(\phi_{e\tau}) = -\frac{1}{4}\left(\frac{s_{13}}{[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|]}\right) \left(1 - \sqrt{1 + 8\left(\frac{[s_{23}|\varepsilon_{e\mu}| + c_{23}|\varepsilon_{e\tau}|]}{s_{13}}\right)^2}\right)$$

For $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$, $\theta_{23} = 45^\circ$ and $\theta_{13} = 0.2$ we get the minimum of factor F in point $[\phi_{e\tau}; \psi_{e\tau}] \simeq [4.97; 3, 65]$ and the maximum $[\phi_{e\tau}; \psi_{e\tau}] \simeq [1.32; 2, 63]$. These results are in agreement with figure 5-4.

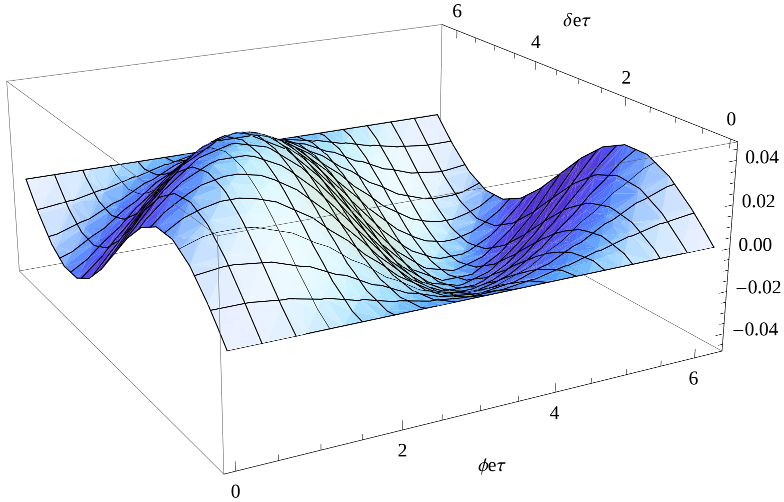

Figure 5-4: Factor F for $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$, $s_{13} = 0.04$ as a function of $\phi_{e\tau}$ and $\psi_{e\tau}$,

Now we are able to show the consequences. If $\psi_{e\tau} = 0$, what is equivalent to the condition $\varepsilon^s = \varepsilon^{d\dagger}$, ΔP is proportional to $\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$. This fact leads to conclusion, that we measure effectively different mixing angle θ_{13} as it is shown above. We are not able to reveal the NSI. On the contrary if the factor F is different from zero the oscillation probability curve is changed and the first disappearance minimum is shifted. These results are shown in the following plots 5-5, 5-6, 5-7.

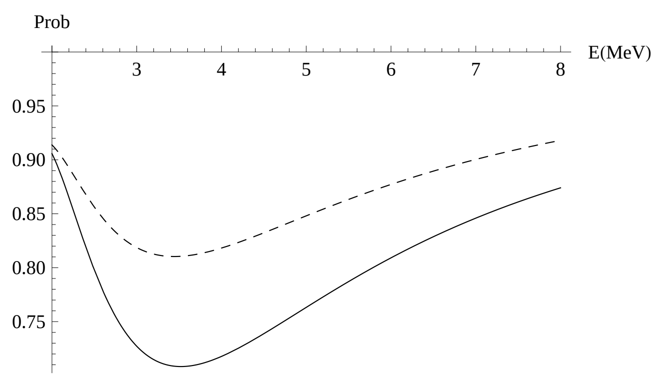

Figure 5-5: The oscillation probability with no NSI (dashed) and with NSI (full) for $[\phi_{e\tau}; \psi_{e\tau}] = [0; 0]$, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$. There is only negligible shift of the first disappearance minimum.

From this simple calculation of the oscillation probability results that in case of large value

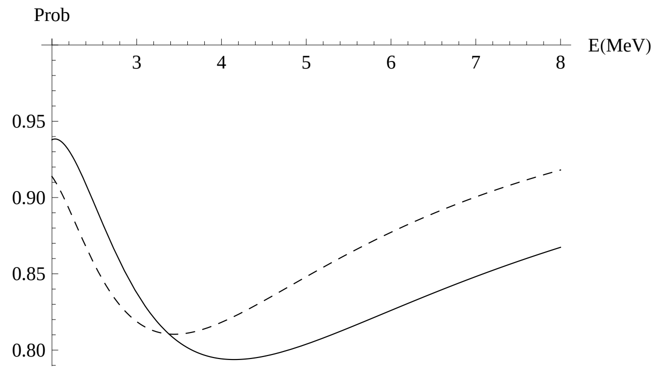

Figure 5-6: The oscillation probability with no NSI (dashed) and with NSI (full) for $[\phi_{e\tau};\psi_{e\tau}] = [4.97; 3.65]$, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$. The first oscillation minimum is shifted by 0.68 MeV to the higher energies.

of mixing angle θ_{13} the Daya Bay experiment is able to reveal the nonstandard interactions for particular set of the nonstandard interaction parameters.

5.3 Constrains given by the Daya Bay experiment

Let us find the regions of nonstandard interactions parameters for which we are able to reveal the nonstandard interactions e.g. the measured data are not consistent with pure neutrino oscillations only. Using our naive estimation of the Daya Bay experiment data we can get the bounds on the nonstandard interactions parameters.

For our analysis we make a simplification and assume $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}|$. Using this condition in (5.17) and assuming $\theta_{23} = 45°$ we get the oscillation probability as

$$\begin{aligned} P(\bar{\nu}_e^s \to \bar{\nu}_e^d) &= 1 - 4c_{12}^2 s_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 \\ &-4\left\{s_{13}^2 + s_{13}\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\cos\left(\phi_{e\tau}-\delta\right) + \cos\left(\phi_{e\tau}-\delta-\psi_{e\tau}\right)\right]\right. \\ &\left.+\cos\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin^2\left[\frac{\Delta m_{32}^2 L}{4E}\right] \\ &+2\left\{s_{13}\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi_{e\tau}-\delta\right) - \sin\left(\phi_{e\tau}-\delta-\psi_{e\tau}\right)\right]\right. \\ &\left.+\sin\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin\left[\frac{\Delta m_{32}^2 L}{2E}\right]. \end{aligned} \tag{5.22}$$

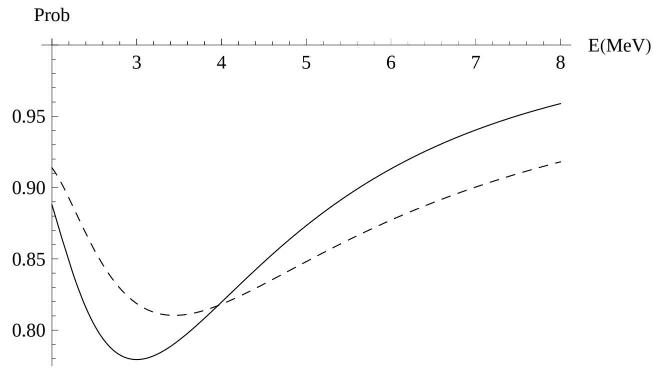

Figure 5-7: The oscillation probability with no NSI (dashed) and with NSI (full) for $[\phi_{e\tau}; \psi_{e\tau}] = [0; 0]$, $|\varepsilon_{e\mu}| = |\varepsilon_{e\tau}| = 0.041$. The first oscillation minimum is shifted by 0.44 MeV to the lower energies.

Now we can combine CP-violating phases $\phi_{e\tau}$ and δ using notation $\phi\prime = \phi_{e\tau} - \delta$. Final oscillation probability is given by

$$
\begin{aligned}
&P(\bar{\nu}_e^s \to \bar{\nu}_e^d) = 1 - 4c_{12}^2 s_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 \qquad (5.23)\\
&-4\left\{s_{13}^2 + s_{13}\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\cos\left(\phi\prime\right) + \cos\left(\phi\prime - \psi_{e\tau}\right)\right]\right.\\
&\left.+\cos\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin^2\left[\frac{\Delta m_{32}^2 L}{4E}\right]\\
&+2\left\{s_{13}\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]\left[\sin\left(\phi\prime\right) - \sin\left(\phi\prime - \psi_{e\tau}\right)\right]\right.\\
&\left.+\sin\left(\psi_{e\tau}\right)\left[s_{23}\left|\varepsilon_{e\tau}\right| + c_{23}\left|\varepsilon_{e\tau}\right|\right]^2\right\}\sin\left[\frac{\Delta m_{32}^2 L}{2E}\right].
\end{aligned}
$$

It is a function of four unknown parameters $|\varepsilon_{e\tau}|$, $\phi\prime$, ψ, θ_{13} and several oscillation parameters for which we use current best fits.

General idea how to search for the regions where we can reveal the nonstandard interactions is to generate the data with the nonstandard interactions and try to fit them with data obtained by the pure neutrino oscillations with an effective mixing angle θ_{13}^{eff}. If we are not able to find any θ_{13}^{eff} consistent with the nonstandard interactions data we can measure the effect of them. On contrary if we are able to find θ_{13}^{eff} the data can be explained by effective mixing angle and the nonstandard interactions stay hidden.

As it was described above the data are obtained from the antineutrino energy spectra (5.4),(5.5), (5.6) with using appropriate probability (5.23) for the nonstandard interactions and

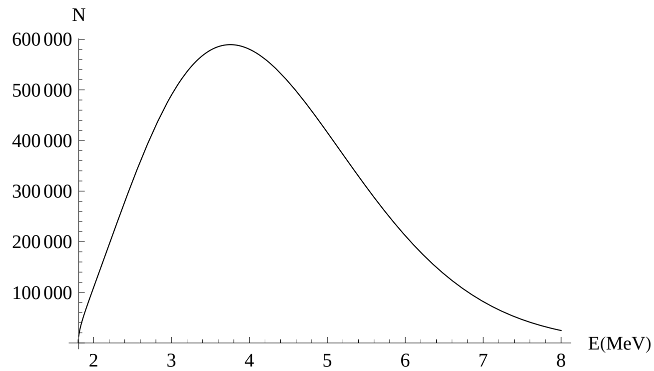

Figure 5-8: Detected antineutrino spectrum in the near DYB detector for the pure oscillations with $\theta_{13} = 0.2$ for three years run.

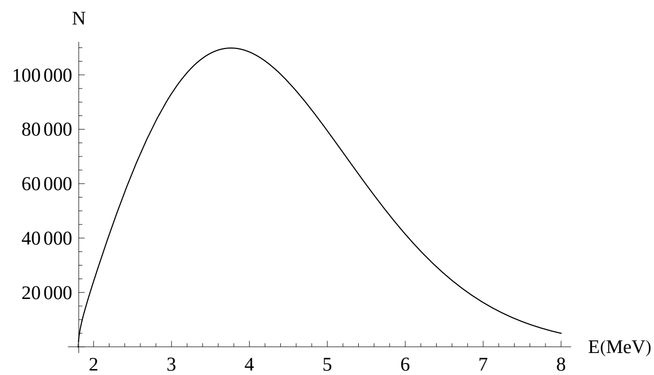

Figure 5-9: Detected antineutrino spectrum in the FAR detector for the pure oscillations with $\theta_{13} = 0.2$ for three years run.

(5.11) for pure oscillations. The spectra are put into the 15 energy bins.

There is an example of antineutrino spectra. On the figures 5-8 and 5-9 there are antineutrino spectra from the DYB near and FAR detector for pure oscillations with $\theta_{13} = 0.2$. Three years run was considered.

On the figure 5-10 there is a ratio between DYB near detector and FAR detector with statistical error for pure oscillations with $\theta_{13} = 0.2$ and for three years run. This form of data is used in following χ^2 analysis.

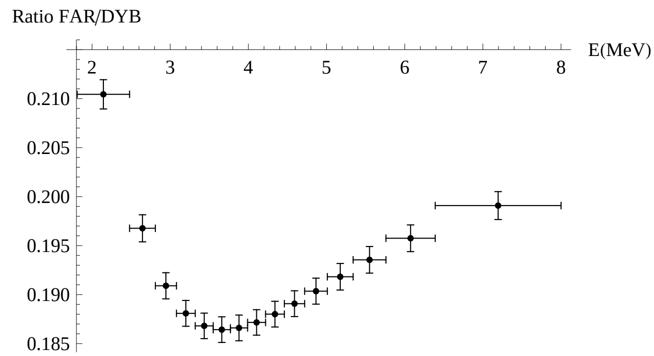

Figure 5-10: Ratio between DYB and FAR detector for the pure oscillations with $\theta_{13} = 0.2$ with statistical error for three years run.

χ^2 functional is defined as

$$\chi^2 = \sum_i \left(\frac{X_i - \mu_i}{\sigma_i} \right)^2 \tag{5.24}$$

where for our purpose X_i is the value of pure oscillations, μ_i is the value of nonstandard interactions data and σ_i is the error of the pure oscillation data for bin i. We are using only statistical error in our calculations. If the value of χ^2 is lower than bound given by the P-value and degrees of freedom the tested data can be explained by the expected hypothesis. If the value of χ^2 is higher the measured data does not fit to the expected hypothesis. Because we have one degree of freedom θ_{13}^{eff} and we set the P-value on 90% the critical value of χ^2 is 2.71. We minimize the value of χ^2 in the parameter θ_{13}^{eff} for a particular set of parameters $|\varepsilon_{e\tau}|$, $\phi\prime$, ψ, θ_{13}. If the minimal value is higher then 2.71 the nonstandard interactions described by $|\varepsilon_{e\tau}|$, $\phi\prime$, ψ, θ_{13} are revealed with at least 90% certainty. If the value of χ^2 is lower then 2.71 the data can be treated on 90% as pure oscillations.

We investigated the possibility of revealing the nonstandard interactions for particular values of $\phi\prime$. On the figures 5-11,5-12 there are plotted regions for $\phi\prime = 1.32$ respectively $\phi\prime = 4.97$ where the Daya Bay experiment is able distinguish the effect of nonstandard interactions for any mixing angle θ_{13}. Because the present bound on $|\varepsilon_{e\tau}|$ is $|\varepsilon_{e\tau}| < 0.041$ [12] experiment can measure the nonstandard interactions effect for particular values $\phi\prime$.

If there will be any evidence of nonstandard interactions in the Daya Bay data we do not know anything about $\phi\prime$. Therefore in the figure 5-13 is plotted the region for different values

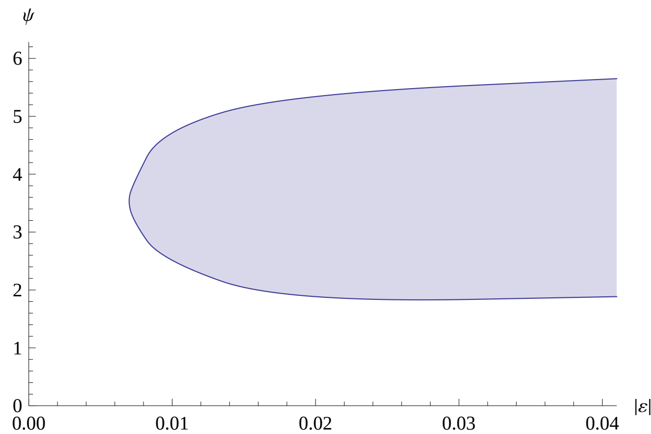

Figure 5-11: Shadow region shows where theDaya Bay experiment can reveal the nonstandard interactions for $\phi\prime = 4.97$ and for any mixing angle θ_{13}.

of $|\varepsilon_{e\tau}|$ and violating phase ψ for any value of $\phi\prime$ and θ_{13} where the nonstandard interactions can be revealed.

According to the limit $|\varepsilon_{e\tau}| < 0.041$ [12] we can result from figure 5-13 that the Daya Bay experiment can not in general set the better constrains on the nonstandard parameter $|\varepsilon_{e\tau}|$.

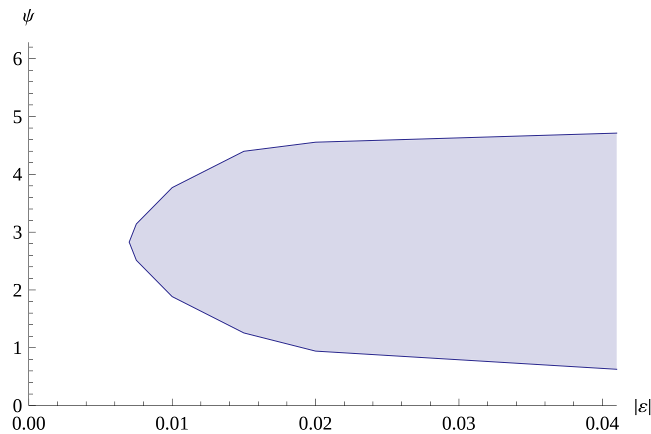

Figure 5-12: Shadow region shows where theDaya Bay experiment can reveal the nonstandard interactions for $\phi\prime = 4.97$ and for any mixing angle θ_{13}.

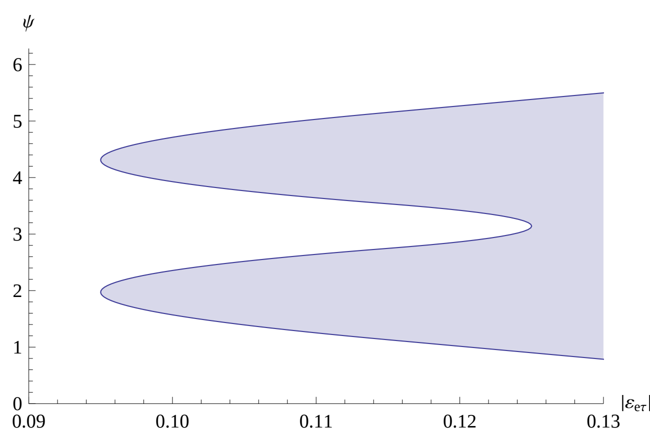

Figure 5-13: Shadow region shows the excluded parameters $|\varepsilon_{e\tau}|$ and ψ if we will not observe any evidence for the nonstandard interactions in the Daya Bay experiment.

Chapter 6

Neutrino decays and oscillations formalism

The Super-Kamiokande experiment have brought first compelling evidence of neutrino oscillation [2]. The ratio between data and predicted muon neutrino flux is shown in the figure 6-1. There is significant minimum in the $L/E \approx 600km/GeV$ which fits to the neutrino oscillation scenario. Neutrino decay hypothesis is disfavored by this minimum.

Nevertheless, we can suppose both neutrino oscillations and decays. Let us introduce the formalism for this scenario.

We focus on three neutrino oscillations and decays. Oscillations are treated in the standard way. In the neutrino decays we are not interested in the decay process and the products. We describe it phenomenologically by neutrino mass eigenstate mean lifetime. Also if the neutrino mass eigenstate is decaying to the lighter mass state, we do not consider any effect in the measured data.

Let us start with time development. For vector $|\psi(t)\rangle$ we can write the Schrőedinger's equation

$$i\hbar\frac{\partial \mid \psi(t)\rangle}{\partial t} = \widehat{H}_{eff}\,|\psi(t)\rangle\,. \tag{6.1}$$

with formal solution

$$|\psi(t)\rangle = \exp\left(-\frac{i}{\hbar}\widehat{H}_{eff}t\right)|\psi(0)\rangle\,. \tag{6.2}$$

Term $\exp\left(-\frac{i}{\hbar}\widehat{H}_{eff}t\right)$ is the time evolution operator.

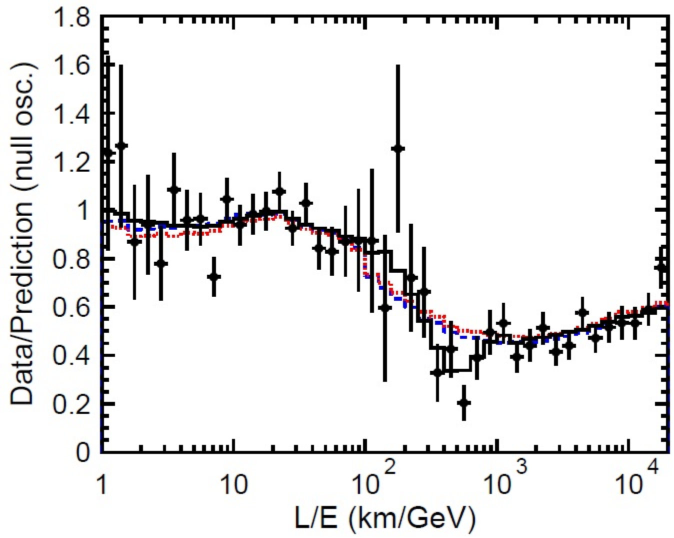

Figure 6-1: The ration between data and predicted atmospheric muon neutrino flux in the Super-Kamiokande experiment [2]. Best fit for neutrino oscillation (solid line), best fit for neutrino decays (dashed line).

In general all three mass eigenstates can be instable. Therefore the H_{eff} is in the neutrino rest frame diagonal and can be expressed by

$$H_{eff}^{diag} = \begin{pmatrix} m_1 - \frac{i}{2}\Gamma_1 & 0 & 0 \\ 0 & m_2 - \frac{i}{2}\Gamma_2 & 0 \\ 0 & 0 & m_3 - \frac{i}{2}\Gamma_3 \end{pmatrix}, \tag{6.3}$$

where Γ_i is appropriate total decay rate of mass state ν_i. Coming to the laboratory frame we write

$$\sqrt{p_i^2 + \left(m_i - \frac{i}{2}\Gamma_i\right)^2} \simeq \sqrt{p_i^2 + m_i^2 - i m_i \Gamma_i} \simeq E_i - \frac{i}{2}\Gamma_i \frac{m_i}{E_i}. \tag{6.4}$$

Using (6.4) we get the H_{eff} in the laboratory frame as

$$H_{eff}^{diag} = \begin{pmatrix} E_1 - \frac{i}{2}\Gamma_1 \frac{m_1}{E_1} & 0 & 0 \\ 0 & E_2 - \frac{i}{2}\Gamma_2 \frac{m_2}{E_2} & 0 \\ 0 & 0 & E_3 - \frac{i}{2}\Gamma_3 \frac{m_3}{E_3} \end{pmatrix}. \tag{6.5}$$

Instead of total decay rate Γ_i we can introduce the mean lifetime τ_i. It holds that $\Gamma_i = \frac{\hbar}{\tau_i}$ therefore H_{eff}^{diag} is given by

$$H_{eff}^{diag} = \begin{pmatrix} E_1 - \frac{i}{2}\frac{\hbar}{\tau_1}\frac{m_1}{E_1} & 0 & 0 \\ 0 & E_2 - \frac{i}{2}\frac{\hbar}{\tau_2}\frac{m_2}{E_2} & 0 \\ 0 & 0 & E_3 - \frac{i}{2}\frac{\hbar}{\tau_3}\frac{m_3}{E_3} \end{pmatrix}. \tag{6.6}$$

Using the approximation

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p} \tag{6.7}$$

and $p \simeq E$, we get

$$H_{eff}^{diag} = E \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} \frac{m_1^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_1}\frac{m_1}{E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_2}\frac{m_2}{E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_3}\frac{m_3}{E} \end{pmatrix}. \tag{6.8}$$

If we transform the Hamiltonian to the flavor eigenstate base it is no more diagonal and we can write

$$H_{eff}^{flavor} = E \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + U \begin{pmatrix} \frac{m_1^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_1}\frac{m_1}{E} & 0 & 0 \\ 0 & \frac{m_2^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_2}\frac{m_2}{E} & 0 \\ 0 & 0 & \frac{m_3^2}{2E} - \frac{i}{2}\frac{\hbar}{\tau_3}\frac{m_3}{E} \end{pmatrix} U^\dagger. \tag{6.9}$$

Introducing H_{eff}^{flavor} to the time evolution operator $\exp\left(-\frac{i}{\hbar}\widehat{H}_{eff}t\right)$ and we get due to mixing matrix unitarity

$$\exp\left(-\frac{i}{\hbar}\widehat{H}_{eff}t\right) = \exp\left(-\frac{i}{\hbar}Et\right) UTU^\dagger \tag{6.10}$$

where

$$T = \begin{pmatrix} \exp\left(-i\frac{m_1^2}{2\hbar E}t - \frac{m_1}{2\tau_1 E}t\right) & 0 & 0 \\ 0 & \exp\left(-i\frac{m_2^2}{2\hbar E}t - \frac{m_2}{2\tau_2 E}t\right) & 0 \\ 0 & 0 & \exp\left(-i\frac{m_3^2}{2\hbar E}t - \frac{m_3}{2\tau_3 E}t\right) \end{pmatrix}. \tag{6.11}$$

The amplitude $A(\nu_\alpha \to \nu_\beta)$ is

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \exp\left(-\frac{i}{\hbar}Et\right) UTU^\dagger |\nu_\alpha\rangle = \exp\left(-\frac{i}{\hbar}Et\right)\left(UTU^\dagger\right)_{\beta\alpha}. \tag{6.12}$$

Omitting the phase factor, which does not influence the probability, and using approximation $L \approx ct$ the amplitude is given by

$$A(\nu_\alpha \to \nu_\beta) = \langle \nu_\beta | \nu_\alpha(L)\rangle = \sum_{i=1,2,3} U_{\beta i} U^*_{\alpha i} \exp\left(-\frac{i}{\hbar c}\frac{m_i^2 L}{2E} - \frac{m_i L}{2c\tau_i E}\right). \tag{6.13}$$

Now we are able to calculate the survival probability as

$$\begin{aligned} P(\nu_\alpha \quad \to \quad & \nu_\beta) = |A(\nu_\alpha \to \nu_\beta)|^2 = \\ & \sum_{i,j} U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j} \exp\left(-\frac{m_i L}{2c\tau_i E} - \frac{m_j L}{2c\tau_j E}\right) \\ & -4\sum_{i>j} \mathrm{Re}\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \exp\left(-\frac{m_i L}{2c\tau_i E} - \frac{m_j L}{2c\tau_j E}\right) \sin^2\left[\frac{\Delta m^2_{ij} L}{4\hbar c E}\right] \\ & +2\sum_{i>j} \mathrm{Im}\left(U_{\beta i} U^*_{\alpha i} U^*_{\beta j} U_{\alpha j}\right) \exp\left(-\frac{m_i L}{2c\tau_i E} - \frac{m_j L}{2c\tau_j E}\right) \sin\left[\frac{\Delta m^2_{ij} L}{2\hbar c E}\right]. \end{aligned} \tag{6.14}$$

From equation (6.14) we can see that in the case with neutrino decays the probability is in general function of nine parameters. Six of them were discussed above. In addition we have mean lifetimes of three neutrino mass eigenstates τ_1, τ_2 and τ_3.

We can see in equation (6.14) that effect of the neutrino decays is contained in the term $\exp\left(-\frac{m_i L}{2c\tau_i E} - \frac{m_j L}{2c\tau_j E}\right)$. While parameters L and E are determined by the experiment as long as we do not know absolute neutrino mass scales we are able to measure only τ_i/m_i as a neutrino mean lifetime. The best direct constrain on ν_1 lifetime is from supernova SN1987A neutrino detection. The limit is about $\tau_1/m_1 > 10^5\ s/eV$ [14]. The limit on ν_2 lifetime is given by the measurement of solar neutrinos and it is $\tau_2/m_2 > 10^{-5}\ s/eV$ [15]. Finally the bound on ν_3 lifetime is imposed by atmospheric and accelerator neutrino measurement. Reasonable fit for atmospheric neutrinos is $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$ for $\Delta m^2_{32} = 2.6 \times 10^{-3}\ eV^2$ and $\theta_{23} = 34^\circ$ [16]. But including accelerator data this solution is disfavored dropping to the 99%CL and on this confidence level the ν_3 lifetime limit is $\tau_3/m_3 \gtrsim 10^{-10}\ s/eV$ [16].

In the normal hierarchy the mass eigenstate ν_3 is the heaviest and therefore we are expecting the shortest lifetime. On the contrary, in the inverse hierarchy is ν_3 the lightest therefore it is reasonable to assume that the lifetime of ν_3 is longer than lifetime of two other states. As long as the limit on ν_1 lifetime is $\tau_1/m_1 > 10\ ^5 s/eV$ the lifer time of ν_3 has to satisfy the same limit and we get $\tau_3/m_3 > 10^5\ s/eV$. As we show later this limit is so strong to see any effect in the reactor and accelerator neutrino experiments. Therefore we are assuming normal hierarchy in the neutrino decays and oscillations calculations.

Chapter 7

Possibility of the measurement of neutrino oscillations and decays the Daya Bay experiment

7.1 Possibility of the measurement of neutrino decays

Let us consider the neutrino oscillation experiment with baseline L, neutrino energy E and with given sensitivity. The effect of neutrino decays is expressed by the term $\exp\left(-\frac{m_i L}{2c\tau_i E} - \frac{m_j L}{2c\tau_j E}\right) = \exp\left(-\frac{m_i L}{2c\tau_i E}\right)\exp\left(-\frac{m_j L}{2c\tau_j E}\right)$ in the equation (6.14). Therefore if term $\frac{m_i L}{2c\tau_i E}$ for given i is small compared to the experiment sensitivity we are not able to measure any effect of ν_i decays.

7.2 Neutrino decays in the Daya Bay experiment

The Daya Bay experiment is described in the chapter above. Let us sum important properties. The experiment is designed to measure oscillation probability $P\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right)$. The neutrino energy spectrum is about $E \in \langle 2\ MeV, 8\ MeV\rangle$. The mean value is $E = 4\ MeV$. The baseline is approximately $L = 1800\ m$ and the expected sensitivity is 1% [3].

Let us investigate the effect of neutrino decays in the Daya Bay experiment. We are assuming normal hierarchy and only one instable neutrino mass eigenstate ν_3. Other mass eigenstates are treated as stable because the effect of their decays is again negligible. Using (**??**) we get the survival probability as

$$P\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right) = 1 - \left(\frac{m_3 L}{c\tau_3 E}\right) s_{13}^2 - 4 s_{12}^2 c_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 - 4 s_{13}^2 \left(1 - \frac{m_3 L}{2c\tau_3 E}\right) \sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar c E}\right) - 4 c_{12}^2 s_{13}^2 \sin\left(\frac{\Delta m_{32}^2 L}{2\hbar c E}\right)\left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right) + \mathcal{O}\left(\left(\frac{m_3 L}{2c\tau_3 E}\right)^2 s_{13}^2, s_{13}^4, \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^4, \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 s_{13}^2\right). \tag{7.1}$$

The oscillation probability without neutrino decays is using (2.12) given by

$$P_{no\ decays}\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right) = 1 - 4 s_{12}^2 c_{12}^2 \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 - 4 s_{13}^2 \sin^2\left(\frac{\Delta m_{32}^2 L}{4\hbar c E}\right) - 4 c_{12}^2 s_{13}^2 \sin\left(\frac{\Delta m_{32}^2 L}{2\hbar c E}\right)\left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right) + \mathcal{O}\left(s_{13}^4, \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^4, \left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 s_{13}^2\right). \tag{7.2}$$

It holds $\left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right) = 0.04$ for the baseline $L = 1800m$ and energy $E = 4MeV$, $s_{13} < 0.2$ and we assume $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$ [16]. Therefore we neglected terms $\left(\frac{m_3 L}{2c\tau_3 E}\right)^2 s_{13}^2$, s_{13}^4, $\left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^4$, $\left(\frac{\Delta m_{21}^2 L}{4\hbar c E}\right)^2 s_{13}^2$ which are in order of 10^{-4}.

Using (7.1),(7.2) we get the probability difference

$$P\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right) - P_{no\ decays}\left(\overline{\nu}_e \rightarrow \overline{\nu}_e\right) = -s_{13}^2 \left(\frac{m_3 L}{c\tau_3 E}\right) \cos\left(\frac{\Delta m_{32}^2 L}{2\hbar c E}\right). \tag{7.3}$$

In figure 7-1 we plotted both oscillation probabilities (7.1),(7.2). We set $\sin\theta_{13}^2 = 0.04$, $\Delta m_{21}^2 = 7.5 \times 10^{-5} eV^2$. We assume fit from [16] $\Delta m_{32}^2 = 2.6 \times 10^{-3}\ eV^2$, $\theta_{23} = 34^\circ$ and $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$.

In the figure 7-2 we set $\sin\theta_{13}^2 = 0.04$ what is the upper bound given by CHOOZ experiment [4]. Let us investigate the effect of different values of θ_{13}. In figure 7-2 we plot the probability difference (7.3) for three values of $\sin\theta_{13}^2$. For $\sin\theta_{13} < 0.11$ is the probability difference lower than 1% which is the sensitivity of the experiment. Therefore we are able to measure any effect of neutrino decays in the Daya Bay experiment only for large mixing angle θ_{13}.

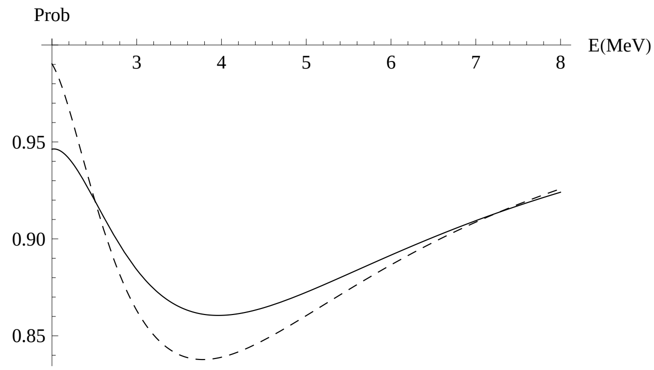

Figure 7-1: The survival probability in the Daya Bay experiment as a function of neutrino energy with decays (full) and without decays (dashed) for fit [16] and $s_{13} = 0.2$.

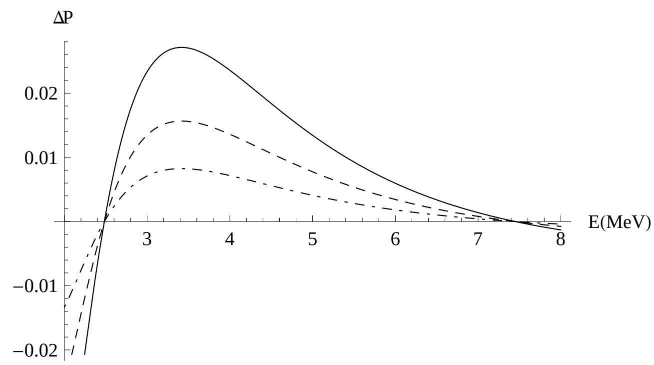

Figure 7-2: The probability difference in the Daya Bay experiment as a function of neutrino energy for fit [16] and for $s_{13} = 0.2$ (full), $s_{13} = 0.16$ (dashed), $s_{13} = 0.11$ (dash-dotted) .

Moreover previous plots are referring to the fit $\Delta m^2_{32} = 2.6 \times 10^{-3}\ eV^2$, $\theta_{23} = 34°$ and $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$ [16]. This fit is disfavored on the 99%CL and the bound is $\tau_3/m_3 \gtrsim 10^{-10}\ s/eV$ $(99\%CL)$. In this case the probability difference (7.3) is fifty times lower. Therefore we will not be able to measure any effect in the Daya Bay experiment even for large value of θ_{13}.

7.3 Constrains given by the Daya Bay experiment

Let us focus on fit $\Delta m^2_{32} = 2.6 \times 10^{-3}\ eV^2$, $\theta_{23} = 34°$ and $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$ [16]. We want to investigate for which values of mixing angle θ_{13} is Daya Bay experiment able to see the

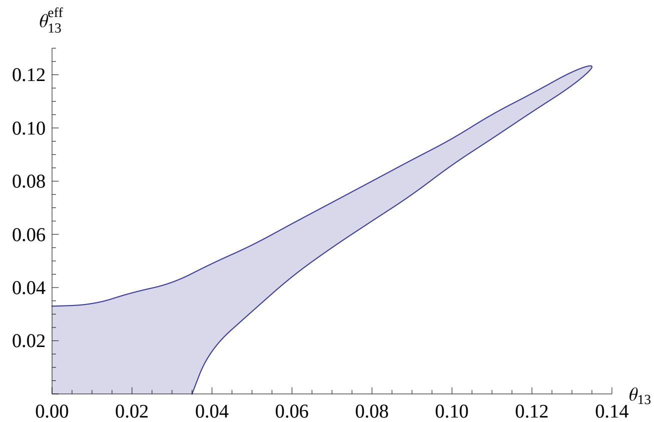

Figure 7-3: Shadow region shows where oscillations and decays are indistinguishable from pure oscillations for the Daya Bay experiment for fit [16].

neutrino oscillations and decays effect or which values we can this fit disfavored. The basis idea is to obtain the oscillations and decays data and try to find pure oscillations effective mixing angle $\theta_{13}^{eff,decay}$ which can mimic decays. Using our naive model described in the previous chapter we can perform the χ^2 analysis.

χ^2 distribution is given by

$$\chi^2 = \sum_i \left(\frac{X_i - \mu_i}{\sigma_i} \right)^2 . \tag{7.4}$$

μ_i is the value for oscillations and decays scenario obtained by using (7.1) in (5.4),(5.5), (5.6). X_i is the pure oscillation value with effective angle $\theta_{13}^{eff,decay}$ calculated using (7.2) in (5.4),(5.5), (5.6) and σ_i is its statistical error. We set the confidence level on 90%. Therefore if χ^2 value is lower than 2.71 we are not able to distinguish between pure oscillations and oscillations with decays.

On figure 7-3 there is shown the region where the neutrino oscillations and decays can not be distinguished from pure oscillations for $\Delta m^2_{32} = 2.6 \times 10^{-3}\ eV^2$, $\theta_{23} = 34^\circ$ and $\tau_3/m_3 = 2.6 \times 10^{-12}\ s/eV$.

For $\theta_{13} > 0.135$ we are able to measure the effect of neutrino decays or disfavor fit form [16]. For the lower values of mixing angle θ_{13} we are not able to distinguish between pure oscillations and oscillations and decay scenario.

Chapter 8

Conclusions

We performed calculations of the oscillation probability with nonstandard interactions in the source and the detector for the Daya Bay experiment.

- If $\varepsilon^s = \varepsilon^{d\dagger}$ holds the effect can be described by effective mixing angle θ_{13}^{eff}, ine general different from real θ_{13}, and the experiment is not able to distinguish between pure oscillations and nonstandard interactions.

- If condition $\varepsilon^s = \varepsilon^{d\dagger}$ is not satisfied we are able to measure the effect for a particular set of nonstandard interaction parameters. There can be set stronger constrains on $|\varepsilon_{e\alpha}|$ then present limits.

- The Daya Bay experiment can not improve present bounds on the parameter $|\varepsilon_{e\alpha}|$ if we will not observe any evidence for nonstandard interactions.

We investigated the possible effect of neutrino oscillations and decays in the Daya Bay experiment.

- We are not able to observe the effect of this scenario for inverted hierarchy due to the present limits on the neutrino mass eigenstate lifetimes.

- For the normal hierarchy and fit from [16] we are able to measure neutrino oscillations and decays only for large mixing angle $\theta_{13} > 0.135$. If this fit is disfavored the Daya Bay experiment can not see any effect.

Bibliography

[1] V. Gribov and B. Pontecorvo, Phys. Lett. B28 (1969) 493.

[2] The Super-Kamiokande Collaboration; Evidence for an oscillatory signature in atmospheric neutrino oscillation; arXiv:hep-ex/0404034v1

[3] The Daya Bay collaboration; A Precision Measurement of the Neutrino Mixing Angle theta_13 using Reactor Antineutrinos at Daya Bay; arXiv:hep-ex/0701029v1

[4] M. Apollonio at al.; Search for neutrino oscillations on a long base-line at the CHOOZ nuclear power station; arXiv:hep-ex/0301017v1

[5] The KamLAND Collaboration; Precision Measurement of Neutrino Oscillation Parameters with KamLAND; arXiv:0801.4589v3 [hep-ex]

[6] The Super-Kamiokande Collaboration; Atmospheric neutrino oscillation analysis with subleading effects in Super-Kamiokande I, II, and III; arXiv:1002.3471v2 [hep-ex]

[7] The MINOS Collaboration; Measurement of Neutrino Oscillations with the MINOS Detectors in the NuMI Beam; arXiv:0806.2237v1 [hep-ex]

[8] Tommy Ohlsson, He Zhang;Non-Standard Interaction Effects at Reactor Neutrino Experiments; 0809.4835v2 [hep-ph]

[9] P. Vogel , J. Engel; Phys. Rev. D39, 3378 (1989)

[10] P. Vogel, J. F. Beacom; The angular distribution of the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$; arXiv:hep-ph/9903554v1

[11] I.Nemchenok; Liquid scintillator on the base of the linear alkybenzene; Daya Bay internal report

[12] Carla Biggio, Mattias Blennow, Enrique Fernandez-Martinez; 0907.0097v3 [hep-ph]

[13] P.Huber, T.Schwetz, J.W.F.Valle; Confusing non-standard neutrino interactions with oscillations at a neutrino factory; arXiv:hep-ph/0202048v2

[14] K. Hirata et al.; Observation of a neutrino burst from the supernova SN1987A; Phys. Rev. Lett. 58, 1490–1493 (1987)

[15] Anjan S. Joshipura, Eduard Masso, Subhendra Mohanty; Constraints on decay plus oscillation solutions of the solar neutrino problem; arXiv:hep-ph/0203181v1

[16] M.C. Gonzalez-Garcia, Michele Maltoni; Status of Oscillation plus Decay of Atmospheric and Long-Baseline Neutrinos; arXiv:0802.3699v2 [hep-ph]