The report on the bachelor thesis by Lenka Slavikova

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In the present work the author presents a systematic study of the so-called almost-compact embeddings between Banach function spaces. I want to mention that the almost-compact embeddings are the same as the absolutely-continuous embeddings. Such embeddings was introduced by Luxemburg and Zaanen (Compactness of integral operators in Banach function spaces, *Math. Ann.* **149** (1963), 150--180). This type of embeddings plays a very crucial role especially in the case when we study embeddings into Banach function spaces. In general, to show that an embedding is compact, is very hard. Compactness of embeddings between r.i. spaces is closely related to the absolute continuity of these embeddings. In the paper mentioned above it was shown that the week compactness or the compactness in measure, together with an absolute continuity gives compactness of the embeddings. To show compactness in measure or weak compactness is also hard but in the case of Sobolev or Besov spaces embeddings into rearrangement invariant (r.i.) Banach function spaces, the former follows from a continuous embedding. Therefore, in this in order to case to show compactness it is enough to show almost-compactness. Motivated by these examples it is very important to study absolutely continuous embeddings between r.i. spaces and obtained a good characterization of it.

The main results of the work are presented in sections 3-7. In Section 3 she gives several equivalent characterizations of almost-compact embeddings between various Banach function spaces. Using these characterizations she gives sufficient conditions for compact embeddings of the Sobolev space modeled on r.i. into r.i. spaces. In Section 4 she studies the question when an r.i. space does not coincide with either L^1 or L^∞. These characterizations are given in terms of fundamental function. At the end of the section a characterization of almost-compactness of certain product operator is presented. The characterization of almost compactness in terms of fundamental function are given in Section 5. In Section 6, all possible pairs of Lorents and Marcinkiewicz endpoint spaces for which almost-compact embeddings holds are characterized in a very elegant way. I want to point out the recent paper P. Fernández-Martínez, A Manzano, E Pustylnik. Absolutely continuous Embeddings of Rearrangement-Invariant Spaces, *Mediterr. J. Math.* (2010), where also the absolute continuity of embeddings between r.i. spaces is studied

The quality of work is excellent. The results presented in the work are very interesting and presented in a very elegant way. The work is written in English and its level is very good. It demonstrates that the author's knowledge in both language and mathematics is excellent.

Without any doubt I recommended the presented work to be recognized as an excellent bachelor thesis and I recommend to be qualified by the mark "vyborne" (in Czech) .

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