Posudek školitele na doktorskou disertační práci

L. Křižka: Moduli spaces of Lie algebroid connections

Lie groupoids and Lie algebroids were introduced in 60's. At the beginning, attention was concentrated to transitive Lie algebroids, non-transitive ones became popular later. Many notions in the theory of Lie algebroids are parallel to notions coming from differential geometry, due to the fact that the Lie algebroids form a generalization of tangent bundles of manifolds. In particular, a theory of Lie algebroid connections was developed in 80's.

A main part of the presented thesis is devoted to moduli problem of Lie algebroid connections, resp. their flat versions. Various moduli problems form a very important part of traditional problems in differential and algebraic geometry, they were studied from various point of views (in particular results, on moduli spaces of holomorphic structure and moduli spaces of holomorphic vector bundles form famous examples of such problems).

The thesis has three parts. The first one introduces Lie and Courant algebroids and describes their differential geometry. It also contains a description of generalized complex structures. This part is pedagogical, it introduces in a smooth and understandable way main notions needed later. It contains a lot of examples of Lie algebroids.

The second and the third part contain new and interesting results. The second part is devoted to linear Lie algebroid connections (L-connections) on a given vector bundle E. The author discusses here the action of gauge transformations on L-connections, introduces notion of irreducible L-connections. He also introduces Sobolev completions of the spaces of smooth L-connections. The first important theorem proved in this part states that the moduli space of irreducible L-connections modulo gauge transformation form a locally Hausdorff Hilbert manifold. The second one gives a local description of moduli space of irreducible flat L-connections on E modulo action of the gauge group, it shows that the moduli space near a given point has a structure of a finite dimensional manifold.

In the third part, the author studies principla Lie algebroid connections. Some analogues of *standard* notions are developed here (principal L-connections, induced linear connections on associated vector bundles, parallel transport along a Lie algebroid path and a notion of a holonomy. The main new result here shows that the holonomy group and the corresponding isotropy group are isomorphic.

During the work on the dissertation, L. Krizka showed a very good ability to learn a lot of new things, to study systematically and from various sources and to work independently. The dissertation is written carefully and precisely, results contained in the thesis are interesting and important. I am sure that the dissertation has a very good quality.

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