Report on the diploma thesis "Steady compressible Navier-Stokes-Fourier equations in two space dimensions"
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In her diploma thesis Petra Pecharová investigates the steady complete Navier-Stokes-Fourier system in a sufficiently smooth two dimensional domain with the constitutive laws for pressure p and internal energy e,

$$p(\varrho, \vartheta) = a_1\varrho^\gamma + a_2\varrho\vartheta, \quad e(\varrho, \vartheta) = a_3\vartheta + \frac{a_1}{\gamma - 1}\varrho^{\gamma-1},$$

where ϱ denotes density, ϑ denotes temperature, with the Navier boundary conditions for the velocity (1.10) and with Neumann type conditions for the temperature, see (1.11). The problem to be solved is described in Section 1.

Section 2 is devoted to the list of mathematical preliminaries.

The main goal of the thesis is to prove existence of weak solutions to the above problem. This result is formulated in Theorem 3.2. The remaining part of the thesis is devoted to its proof.

The original problem is approached by an approximate problem introducing a special truncation function $K(t)$ containing a large parameter k - see (4.1), and a small positive parameter $\epsilon > 0$ introducing an artificial dissipation into the continuity equation, see (4.2)-(4.9).

Section 4.1 is devoted to the investigation of a priori estimates for the approximate system which are independent of the parameter ϵ. The main result is formulated in Lemma 4.1. The most involved argumentation is the proof of the positivity of temperature which is achieved by rewriting the energy equation originally with temperature in terms of a new variable $s = \log\vartheta$. This manipulation which is not justifiable at the level of original system may be perfectly justified on the level of approximations. The energy estimates are at first implicit and then completed after establishing a refined pressure estimate which is obtained by testing the momentum equation with the Bogovskii operator. The estimates require existence and a minimal regularity of solutions of the approximate system which are proved in Section 4.2. The proof relies on the Leary-Schauder fixed point theorem and is based on a linearization involving several elliptic problems.

In the whole section 4, the part "estimates" communicates and interacts with the part "designing approximations" leading to an approximate system which possesses a minimal set of energy type estimates. During this process, the author shows to be handy in deriving an manipulating estimates, to have good

Some remarks:

p.6 - (1.7) and (1.8) are conditions due to the second law of thermodynamics

p.6 - misprint in formula (1.9)

p.13 - condition $\varrho \in L^\infty(\Omega)$ is a part of the definition of a weak solution. Useless to repeat it in Theorem 3.2

p.15 - (4.1) - what is the smoothness of K?

p.16 - the real entropy has an contribution dependent of ϱ, but this contribution does note come from the elastic part

p.17 - why $\frac{\partial \varrho}{\partial n}$ does disappear on $\partial\Omega_-$?

p.18 (4.15) - integral $\int_{\partial\Omega} L(\theta)\theta_0$ is missing?

p.24 What is q in Theorem 4.2?

p.33 passage from (5.14), (5.15) to (5.16), (5.17) should be described more clearly

p.36 - l-5 - something is missing?

Where is the proof of convergence (5.35)?

(5.34), (5.35) imply (5.36) under an additional condition? e.g. Ω simply connected (in 2D)?