In the present work we study properties of orthogonality in Hilbert spaces and possibilities of extending definition to more general type of spaces, Banach spaces. We concentrate mostly on Birkhoff-James orthogonality and investigate, which properties of Hilbert space orthogonality are still valid for Banach spaces, otherwise we provide counter-examples. As the orthogonality is generally not symmetric, we have to distinguish between right and left properties. We use Birkhoff-James orthogonality to characterize smooth and strictly convex Banach spaces. Then we study properties of Hilbert space orthogonal projection and its generalizations for Banach spaces. We study projections of norm equal one and minimal projections.