This thesis is focused on the Minimal $SO(10)$ Grand Unified Theory and, in particular, on the $\mathbf{45} \oplus \mathbf{126}$ Higgs sector. It has been shown that in the scalar spectrum of the model there are potentially tachyonic instabilities. However, these issues might be resolved by considering quantum effects at the 1-loop level. This entails the study of the second derivatives of the effective potential. In order to establish the stability of the results of this perurbative approach, it is necessary to evaluate the beta functions of the scalar couplings in the Higgs sector. This way one also addresses the questions regarding the Landau poles of the scalar couplings which tend to be around the unification scale. In this work, the running of the scalar couplings in $\mathbf{45}$ in the $SO(10)$ Higgs model is evaluated using two approaches: direct summation of Feynman graphs and the effective potential method.