The presented study gives a comprehensive overview of the theory and the evidence for a systematically varying stellar initial mass function (IMF). Then we focus on the impact of this paradigm change, that is, from the universal invariant IMF to a variable IMF, on galaxy chemical evolution (GCE) studies. For this aim, we developed the first GCE code, GalIMF, that is able to incorporate the empirically calibrated environment-dependent IMF variation theory, the integrated galactic initial mass function (IGIMF) theory. In this theory, the galaxy-wide IMF is calculated by summing all the IMFs in all embedded star clusters which formed throughout the galaxy in 10 Myr time epochs. The GalIMF code recalculates the galaxy-wide IMF at each time step because the integrated galaxy-wide IMF depends on the galactic star formation rate and metallicity. The resulting galaxy-wide IMF and metal abundance evolve with time. Using this code, we examine the chemical evolution of early-type galaxies (ETGs) from dwarf to the most massive. We find that the introduction of the non-canonical IMF affects the best estimation of the galaxy properties such as their mass, star formation history, and star formation efficiency. Moreover, we are able to provide an independent estimation on the stellar formation timescale of galaxies, the type Ia supernova production efficiency, and constrain the IMF variation law of the low-mass stars. This work provides to the community the publicly available GalIMF code with improved constraints on the IMF variation and has, for the first time, resolved the discrepancy between the galaxy formation timescales obtained from stellar population synthesis and chemical enrichment studies.