In this thesis, the single-level correlated quantum dot attached to two BCS superconducting leads is analyzed. A difference in the superconducting phases of the leads induces the DC Josephson supercurrent in the junction. In this setup, the influence of asymmetrical dot-lead couplings on transport properties is clarified analytically. The coupling asymmetry and the phase difference can be combined into one function, which allows us to calculate physical properties of a system with coupling asymmetry from the properties of its effective symmetric counterpart. The coupling asymmetry turns out to be an important parameter which influences the position of the $0-\pi$ quantum phase transition even in the strongly correlated Kondo regime.

Further, this thesis contributes to the interpretation of an AC Josephson current measurement, in which a surprising drop in the amplitude was observed in the Kondo regime. The experimental setup is characterized using numerical renormalization group calculations of the equilibrium many-body spectra. Possible quantum-point-contact-based interpretations are discussed. Although a drop in the AC Josephson current at the experimental bias voltage is also expected in a quantum point contact, we conclude that the physical mechanisms causing it in the quantum dot system are likely not the same.