With the increasing amount of digital data in the form of unstructured text, the importance of natural language processing (NLP) increases. The most successful technologies of recent years are deep neural networks. This work applies the state-of-the-art methods, namely transfer learning of Bidirectional Encoders Representations from Transformers (BERT), on three Czech NLP tasks: part-of-speech tagging, lemmatization and sentiment analysis. We applied BERT model with a simple classification head on three Czech sentiment datasets: mall, facebook, and csfd, and we achieved state-of-the-art results. We also explored several possible architectures for tagging and lemmatization and obtained new state-of-the-art results in both tagging and lemmatization with fine-tunning approach on data from Prague Dependency Treebank. Specifically, we achieved accuracy 98.57% for tagging, 99.00% for lemmatization, and 98.19% for joint accuracy of both tasks. Best models for all tasks are publicly available.