Spherical harmonics and spherical monogenics are, respectively, polynomial solutions of Laplace and Dirac equations. In $\mathbb{R}^3$ these solutions form irreducible representations of Lie algebra $\mathfrak{sl}(2,\mathbb{C})$. The main aim is to construct orthogonal bases of such spaces. The well-known procedures like Gram-Schmidt orthogonalization procedure is quite clumsy and tedious. We show how to construct orthogonal bases in an easier way using representation theory. For description of rotations in $\mathbb{R}^3$ and $\mathbb{R}^4$ we use quaternions. Finally, we express constructed bases in spherical coordinates