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Dear Colleagues,

Please find bellow my report on the doctoral thesis of Martin MAREŠ, entitled:

"GRAPH ALGORITHMS".

The aim of this thesis is to propose an unifying approach to the analysis of two well-known problems of algorithmic graph theory, namely: finding a minimum spanning tree and computing the rank of a permutation (and conversely a permutation from its rank). It mainly considers two computational models: the *Word-RAM* and the *Pointer Machine* models.

This thesis surveys the important results related to Minimum Spanning Tree (MST) and permutation ranking problems and proposes a set of analysis tools for the considered models of computation allowing a coherent and simplified analysis of graph algorithms. The analysis of the classical and more advanced algorithms are simplified and important missing details of the original versions are filled.

Chapter 1 presents the classical approaches to MST problem and contractive versions of these. In particular, a lower bound for Contractive Borůvka's algorithm is given.

Chapter 2 introduces the considered models of computation and provides a handy set of basic data structures tuned according to the particularities of these computation models, and it is shown how bucket sorting and unification can be used to perform flattening and tree isomorphism test efficiently.

Chapter 3 starts with the observation that the sparseness of proper minor closed classes is sufficient to guarantee the linearity of Contractive Borůvka's and Local Borůvka's algorithm for graphs restricted to such a class. Then follow the analysis of advanced MST algorithms and, in particular, of a simplification of King's verification algorithm. At this occasion, omissions of the original author are corrected.

Chapter 4 is devoted to Chazelle's soft heaps and its use by MST algorithms. This chapter is the occasion for a simplification of both the data structure and its analysis.

Chapter 5 deals with dynamic graph algorithms. Using the fully dynamic connectivity algorithm of Holm et al., a fully dynamic MSF algorithm for graphs with limited edge weight is proposed, which works in time $O(k \log^2 n)$ amortized per operations for graphs on n vertices with only k distinct edge weights allowed. This has to be compared with the original fully dynamic MSF algorithm of Holm et al. for the unconstrained case, which works in time $O(\log^4 n)$ amortized per operations for graphs on n vertices.

Chapter 6 then reviews some applications of the MST problem.

Chapter 7 is devoted to the application of the techniques introduced in Chapter 2 to a really different problem, namely the ranking of combinatorial structures. The observation that the ranks of the considered structures have to be large numbers then allows to design within the Word-RAM model efficient algorithms to compute ranking and unranking functions. For instance, the lexicographic ranking and unranking of permutations of $\{1, \ldots, n\}$ are shown to be computable in time $O(n)$. The case of restricted permutations is then considered and both interesting results are obtained, either positive (test in time $O(\sqrt{n}m)$ of the existence of a permutation satisfying a given restriction graph of order n and size m) or negative (non-existence of a polynomial-time algorithm for lexicographic ranking of permutations with a set of restrictions which is part of the input under the assumption $P \neq \#P$). Finally, a linear time algorithm for lexicographic ranking and unranking is proved to exist for the special case of derangements, modulo an initialization phase performed in time $O(n^2)$.

Martin Mareš demonstrated convincingly his ability of thorough and original scientific work and in the addition of things mentioned above the thesis contains many small improvements of classical algorithms. The very good English in which this thesis has been carefully written makes it a pleasure to read it.

In my opinion, and according to international academic standards, this thesis satisfies all requirements for a doctoral thesis in the field of Computer Science.

Best Regards,

Patrice Ossona de Mendez