

The limit ultrapower is generalized to complete distributive lattices equipped with a ultrafilter and a partition system. This construction provides a complete characterization of the internal universe in models of nonstandard set theory: we prove that bounded part of an elementary extension of a set universe is given by suitable partition ultrapower. Our special interest is in models where a weak form of standardization holds. The Rudin–Keisler preorder on ultrafilters is defined on partition systems on ultrafilters such that it corresponds to embeddings of related partition ultrapowers, whereas Rudin–Frolík ordering characterizes those embeddings which are standardizable. Finally, the problem whether set-many elements are always enough to generate the internal universe from its standard part is considered. It's shown that the existence of a narrow elementary extension which doesn't rise from adjunction of set-many elements implies an existence of highly nonregular ultrafilters, and thus is equivalent to a large-cardinal hypothesis.