One of the problems in Euclidean Ramsey theory is to determine the chromatic number of the Euclidean space. The chromatic number of a space is the minimum number of colors with which the whole space can be colored so that no two points of the same color are at unit distance. We prove that the chromatic number of the six-dimensional real space is at least 11
and that the chromatic number of the seven-dimensional rational space is at least 15. In addition we give a new proof of the lower bound 9 for the chromatic number of the five-dimensional real space. We also simplify the proof of the lower bound 7 for the four-dimensional real space. It is known that the chromatic number of the n-dimensional real space grows exponentially in n. We show some of its subspaces, in which the growth is slower than exponential. We also summarize previous results for normed spaces in general and for some interesting non-Euclidean spaces.