Review of "Sums of squares in number fields"

The thesis of Martin Raška is about the representation of integers as the sums of squares. This is a rather classical area of number theory, yet, it is an active field of research, especially, when one considers such representations over the rings of integers of number fields.

In his thesis, the student considers the question above for the ring of integers of real quadratic number fields. Specifically, he tackles the problem of finding a natural number m such that all integers in a given ring of integers that are divisible by m can be written as a sum of squares. He finds lower bounds for the discriminant of the corresponding number fields (Theorem 2) and thus, generalises a recent work of Kala & Yatsyna (2020). Furthermore, he classifies all the real quadratic number fields where all algebraic integers that are divisible by four are represented as the sum of squares (Theorem 9). In Chapter 4, by studying indecomposable integers and continued fractions, the author shows necessary and sufficient conditions for totally positive integers divisible by a defined integer m to be represented by the sums of squares (Theorems 10 and 11). The thesis concludes with an algorithm to decide whether in a fixed real quadratic number field all totally positive integers divisible by a given integer are represented as the sum of squares.

The thesis is self-contained and includes some known results that never appeared in English (like Peters' theorem in Chapter 3). Furthermore, it is concise and well written. Much of the work (Chapters 2, 4 and 5) is original and publishable. Proofs use a variety of tools and interesting novel concepts. The author managed to observe, and subsequently prove, a beautiful extension of the recent result (in Chapter 2). In Chapter 4, these ideas advanced further to prove "positive" results (that are generally more difficult) in an infinite family of cases. Hence, I will recommend the top mark (výborně) for this thesis.

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Date