The goal of this thesis is to study real quadratic number fields $\mathbb{Q}(\sqrt{D})$ such that, for a given rational integer $m$, all $m$-multiples of totally positive integers are sums of squares. We prove quite sharp necessary and sufficient conditions for this to happen. Further, we give a fast algorithm that verifies this property for specific $m, D$ and that for a fixed $m$ finds all such fields in polynomial time.

