



June 20th, 2021

To: To Whom It May Concern

Subject: Evaluation of Master's Thesis

"Interpolation of logarithmically convex combinations of operators" by Jakub Takac

General characteristic:

Submitted work is divided into 5 chapters with the total length 68 pages. The length of the work is more than appropriate for Master Thesis in mathematics. The main focus of the thesis is on a study of logarithmically convex combination of operators and their application in interpolation theory. The main underlying theme of this work is focusing on the development of new techniques for interpolation of operators which could help recover existing results by a novel approach and improve the existing results. The author's approach is based on studying an abstract interpolation of Calderón spaces, using Calderón-Lozanovskii construction and utilizing pointwise estimates for Calderón operators.

The main results are in chapters 3 and 4. The main statements in chapter 3. "Abstract factorization of couples of interpolation functions" are **Theorem 3.2** (which shows that operator T , constructed as a product of powers of two linear maps (see 3.1), is bounded on the smallest possible Banach function space generated by a corresponding interpolation functor) and **Theorem 3.10** (which describes simple conditions on Orlicz spaces under which interpolation, among four spaces, behaves nicely). In chapter 4. "A method based on Calderon estimates" the main results are **Theorem 4.3** (is claimed that if Young function $A(t)$ satisfies some rather weak condition (4.7) (there is a typo in (4.7)) and a quite weak condition on level sets of the map T (4.10) than the map T is a bounded map from Orlicz space generated by Young function A into Orlicz space generated by Young function derived from $A(t)$) and **Theorem 4.8** (which combines the previous theorem with logarithmically convex combination of linear maps). All these results are complemented by a long list of examples.



Evaluation of Thesis:

The thesis contains a huge quantity of material and is obvious that author spends plenty of time and effort to collect pieces of information that were scattered over literature. Presented results seem to be original and novel and should be considered for possible publishing in a research journal. Since there isn't an Index it is quite hard for a reader to follow the author's passage through the work and also in some parts it is quite hard to follow the author's ideas (this could be by the complexity of the problems and techniques discussed and amounts of materials). I think that the submitted work satisfies the requirements for a Master's Thesis.

Comments/questions:

page 4, line 10 – Which well know principle? (add quote)

page 6, line 2 – There should be added a quote for the inequality.

page 8, line 2 – It will be nice, to the reader, mention why only resonant measures are considered.

page 20, line 9 – Why there is constant 2 in Riesz-Thorin interpolation Theorem? Perhaps add a note that there could be 1 (complex vs real numbers versions).

page 20, formula (2.2) – Here is the construction of Calderon-Lozanovskii which is important, in this work, and I think that this construction should deserve to be labeled as a definition. Also, it should be put in the Index.

page 21, Proposition 2.3 – That is a really nice statement!

page 23, line -5 – Should there be the absolute value on the left side of the inequality? (i.e. $|f| \leq \dots$)

page 24, line -1 – I can not see where (2.9) was proved.

page 28, line 17-20 – I do not understand this definition. What is a relation between T and S? I am sorry but this does not make sense.

page 30, formula (3.1) – It will be better to use T with index θ .

page 31, line 16 – Where was "Int" introduced? I did not find it in the previous text.

page 31, Theorem 3.2 – It will be good for the reader to know what is expected from X.

page 34, line 7 – I do not think that "that is that" is well-fitting here.

page 34, line -1 – "already shown" – please consider add where.

page 35, line 1 – Why modified Theorem 3.2 for the situation when X is Banach space? Theorem 3.2 does not mention any requirements on X.

page 43, line 7 – Please what is the definition of T? It will be nice for the reader to recall how T is built from S_1 and S_2 . Same in example 3.15.

page 44, line 5-6 – Perhaps too many "however".

page 46, line 11 – Where is the definition of classical Calderon operator?

page 46, Lemma 4.1 – Please consider to recall definitions of S_1 and S_2 .

page 47, Proposition 4.2 – Please consider to recall, for readers, definitions of Calderon operators $\{\tilde{S}_1\}$ and $\{\tilde{S}_2\}$ associate with a given segment.

page 47, formula (4.6) – What is the upper bound of integration?

page 47, formula (4.7) – What is the lower bound of integration? (in the present form, these conditions do not make sense)

page 48, line 4 – Move ")" one line higher.



page 52, line 1-2 – Too many “This means”.

page 61, line 13 - An interesting question! Any idea about the answer?

Conclusion:

I think that this work can be considered acceptable for the Master's thesis. But the author should address, to the committee, my comments/questions which were mentioned above.

Sincerely,

Jan Lang

PS. Based on recommendation for Master thesis from the web page of MFF UK:

(see: https://www.mff.cuni.cz/data/web/obsah-puvodni/studium/bcmqr/prace/dp_uprava.pdf and <https://www.mff.cuni.cz/cs/vnitri-zalezitosti/predpisy/prikazy-a-smernice-dekana/smernice-dekana-c-1-2015>)

The Thesis (Diplomová práce) should contain Epilog/Conclusion (Závěr), List of Tables & Pictures, and List of Abbreviations/Index which I did not find in this work. But the committee would know if these requirements are essential.