

In this thesis, we study the discrete logarithm problem in the context of **TFNP** – the complexity class of search problems with a syntactically guaranteed existence of a solution for all instances. Our main results show that suitable variants of the discrete logarithm problem, which we call **INDEX** and **DLOG**, are complete for the classes **PPP** and **PWPP**, respectively. Additionally, our reductions provide new structural insights into **PWPP** by establishing two new **PWPP**-complete problems. First, the problem **DOVE**, a relaxation of the **PPP**-complete problem **PIGEON**. **DOVE** is the first **PWPP**-complete problem not defined in terms of an explicitly shrinking function. Second, the problem **CLAW**, a total search problem capturing the computational complexity of breaking claw-free permutations. In the context of **TFNP**, the **PWPP**-completeness of **CLAW** matches the known intrinsic relationship between collision-resistant hash functions and claw-free permutations established in the cryptographic literature.