The thesis formulates and proves a witnessing theorem for S_{PV}-provable formulas in the form $\forall x \exists y A(x, y)$ where A corresponds to a polynomial time decidable relation. By S_{PV} we understand an extension of the theory T_{PV} (the universal theory of $\mathbb{N}$ in the language representing polynomial algorithms) by additional axioms ensuring the existence of a minimum of a linear ordering defined by a polynomial time decidable relation on an initial segment. As these additional axioms are not universal sentences, the theory S_{PV} requires nontrivial use of witnessing Herbrand's and KPT theorems which have direct application only for universal theories. Based on the proven witnessing theorem, we derive a NP search problem characterizing complexity of finding y for a given x such that $A(x, y)$. A part of the thesis is dedicated to arguments supporting the conjecture that S_{PV} is strictly stronger than T_{PV}.