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Evaluating the predictability of virtual exchange rates using daily data

Bachelor's thesis

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Prague, May 3, 2021

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Abstract

Virtual worlds have garnered the attention of researchers from various disciplines and are viewed as particularly valuable to economists due to their open-ended design. In this thesis, we review a popular online multiplayer game's economy and focus on exchange rate predictability in a virtual setting as only a limited body of literature investigated this topic. The well-established unpredictability puzzle is addressed by exploiting a unique daily time series dataset using a vector autoregressive framework. Apart from a significant Granger-causal relationship between the virtual exchange rate and the player population, the system is shown to be less interconnected than expected. Furthermore, an out-of-sample exercise is conducted, and the forecasting performance of our models is examined in comparison to that of a simple no-change benchmark in the short term. Based on the evaluation methods used, the two measures of the virtual exchange rate are found to be somewhat predictable. We suggest two explanations for this inconsistency between the virtual and real-world exchange rates: data frequency and lack of complexity in the considered online economy.

JEL Classification	C22, C53, C88, F31, F37, L86
Keywords	virtual worlds, exchange rates, predictability, daily data, time series analysis
Title	Evaluating the predictability of virtual exchange rates using daily data

Abstrakt

Virtuální světy si získaly pozornost badatelů z různých oborů a jsou považovány za zvláště cenné pro ekonomy díky jejich otevřenému designu. V této práci poskytujeme shrnutí ekonomiky populární online hry pro více hráčů a zaměřujeme se na předvídatelnost směnných kurzů ve virtuálním prostředí, jelikož toto téma bylo zkoumáno pouze omezenou částí literatury. Známý problém nepředvídatelnosti směnných kurzů je řešen s pomocí unikátní datové sady denních časových řad s využitím vektorového autoregresního modelu. Kromě významného Granger-kauzálního vztahu mezi virtuálním směnným kurzem a hráčskou populací se ukázalo, že systém je méně propojený, než se očekávalo. Dále je provedeno out-of-sample cvičení a je zkoumána výkonnost předpovědí našich modelů ve srovnání s výkonem jednoduchého modelu beze změny v krátkodobém horizontu. Na základě použitých vyhodnocovacích metod lze obě míry virtuálního směnného kurzu považovat za poněkud předvídatelné. Navrhujeme dvě vysvětlení pro tuto nesrovnalost mezi virtuálními a reálnými směnnými kurzy: frekvence dat a nedostatek složitosti uvažované online ekonomiky.

Klasifikace JEL	C22, C53, C88, F31, F37, L86
Klíčová slova	virtuální světy, směnné kurzy, předvídatelnost, denní data, analýza časových řad
Název práce	Vyhodnocování předvídatelnosti virtuálních směnných kurzů pomocí denních dat

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Acronyms

ADF	Augmented Dickey-Fuller (test)
AIC	Akaike information criterion
AR	Autoregression
ARCH-LM	Autoregressive conditional heteroskedasticity Lagrange multiplier (test)
BIC	Bayes information criterion
DM	Diebold-Mariano (test)
GE	Grand Exchange
GP	Gold pieces
JB	Jarque-Bera (test)
MAE	Mean absolute error
MDA	Mean directional accuracy
MMORPG	Massively multiplayer online role-playing game
MUD	Multi-user dungeon
OLS	Ordinary least squares
OSRS	Old School RuneScape
RMSE	Root mean square error
USD	United States dollar
VAR	Vector autoregression
WOW	World of Warcraft

Chapter 1

Introduction

> *In sharp contrast to our incapacity to perform truly scientific tests in 'normal' economic settings, [...] digital economies are a marvelous test-bed for meaningful experimentation... [W]e can change the economy's underlying values, rules and settings, and then sit back to observe how the community responds, how relative prices change, the new behavioural patterns that evolve. An economist's paradise indeed.*
>
> – Ioannis Varoufakis (2012)

In an online fantasy virtual world, people may expect to find themselves facing a dragon, casting a spell, or brewing a potion. What might sound outlandish to most, however, is that some of these environments could have an economic output similar to that of a real-world country. Twenty years ago, Edward Castronova (2001) published an intriguing paper in which the economist claimed that a particular virtual realm ranked as the world's 77th wealthiest country in terms of its gross national product *per capita*. Since then, researchers from various fields have acknowledged the scientific potential of these online spaces for their ability to produce loads of data on human behavior (Bainbridge 2007). By design, nearly every aspect of these environments can be effortlessly and almost fully controlled (Castronova 2002), which may sound appealing to an economist wanting to run macroeconomic or regular experiments, especially if the in-game economy would follow real-world patterns. Indeed, Castronova *et al.* (2009a;b) and Chesney *et al.* (2009) have found that some economic behavior in a virtual space does not appear to deviate from the norm.

However, only a limited number of works have examined the topic of virtual exchange rate[1] predictability (namely Kim *et al.* (2015) and Kim *et al.* (2017)), but none seemed to have utilized standard econometric tools or recognized these environments as a potentially useful piece in the exchange rate unpredictability puzzle. Following the Meese & Rogoff (1983) rebuttal, in which the authors find that the forecasting performance of exchange rate models is as weak as a no-change prediction, researchers have yet to find a way to outperform the simple model reliably, especially in the short term (Rossi 2013).

Therefore, this thesis aims to investigate the predictability of virtual-to-real exchange rates in a particular virtual setting. Specifically, we are interested in knowing which variables influence the in-game exchange rate and whether there is enough information to surpass the simple no-change model in terms of various forecasting performance measures in the short run. Not only would this be useful to the developers for tracking the state of the in-game economy, but perhaps it may provide a clue to the exchange rate unpredictability puzzle. To answer this query, we first introduce the online environment in question and establish how the exchange rate between a real and a virtual currency is measured. Then, we analyze a unique dataset of daily time series from a popular online multiplayer game by employing the standardly used vector autoregressive framework (VAR) in the *R* statistical environment. Due to the lower number of observations than originally expected, our hypothesis is that the simple no-change model cannot be surpassed in the short term with regards to its out-of-sample performance.

The outline of this thesis is as follows. Chapter 2 surveys the existing body of literature on exchange rate predictability, virtual worlds research, and the fusion of these two topics. Next, Chapter 3 introduces the reader to the virtual economy of interest and reviews some of its issues. In the first part of Chapter 4, the considered variables are described, and the collected data is inspected. The following section then details the econometric framework used in the analysis. Subsequently, the models are constructed, estimated, and various post-estimation diagnostic tests are conducted. Chapter 5 presents and discusses the estimation results and evaluates the forecasting performance of the specified models. In Chapter 6, the findings are summarized, and the thesis is concluded. The two appendices, Appendix A & Appendix B, contain additional definitions and results, respectively.

[1] The price of the in-game currency expressed in terms of a real-world currency.

Chapter 2

Literature review

This chapter is divided into three parts. In the first section, we provide an overview of the existing literature on exchange rate predictability. The next subchapter introduces the topic of virtual worlds and surveys the current body of research, and finally, the last part briefly reviews the topic of virtual exchange rate forecasting.

2.1 Exchange rate predictability

The emergence of various methods of cashless payment dates back to 12th century Europe and can be most likely attributed to Italian traders (Denzel 2010, pp. 24-25). While a lot has changed since then, the need for liquidity has increased immensely as the world economies have become far more intertwined.

Significant movements in exchange rates have been recorded ever since the largest industrially oriented economies adopted policies that let the market forces set their value (Carbaugh 2008, p. 398). While long-term dynamics have been observed to be relatively steady, there is a high degree of volatility in the short run (Carbaugh 2008, p. 398). This should perhaps be expected due to the enormous competition on the foreign exchange market in which new information gets almost instantly processed (Kallianiotis 2013, p. 98).

Krugman & Wells (2015, p. 1016) describe exchange rates as prices at which currencies are traded on the foreign exchange market. From the perspective of policymakers, exchange rates, among other things, affect their decisions during the process of designing monetary policy; however, the experience of numerous countries around the globe differs (Devereux & Engel 2003).

There are many reasons why policymakers have been, along with economists or investors, captivated by exchange rate forecasting. For instance, Panda & Narasimhan (2007) state that they can be viewed as financial assets that might offer useful information about the state of the economy. They also suggest that central banks need to thoroughly understand the current dynamics to intervene in the foreign exchange market effectively. Last but not least, the authors allude to the possibility that companies or investors may be interested in these forecasts to make informed decisions regarding the allocation of their assets.

The highly influential paper by Meese & Rogoff (1983) marked the beginning of a new era in the research area of exchange rate predictability by highlighting the weak predictive power of numerous time series and structural models (Moosa 2013). They discovered that a naïve random walk model was able to forecast several United States dollar (USD) exchange rates with a comparable degree of out-of-sample accuracy[1] as the considered models. Moreover, the random walk almost always yielded better results over all the examined time horizons, though the improvement was often insignificant. They also attempted to apply a range of univariate time series approaches, but these were also shown to be ineffective.

The authors themselves, Meese & Rogoff (1983), assign the inability to surpass the driftless random walk to simultaneous equation bias, model misspecification, sampling error, or parameter instability. However, Taylor & Peel (2000) present a more innocuous explanation—the relationship between exchange rates and the underlying variables might not be linear. Brooks (1996), for example, has shown that this might be the case as well; on the contrary, others have found that accounting for non-linear relationships might be ineffective (Meese & Rose 1991).

Kilian & Taylor (2001) illustrate the impact of the paper with a parable of finding the holy grail as it sent economists searching for a model that would be able to outperform the random walk. While Frankel & Rose (1995) expressed an optimistic outlook for the profession, they admitted that the discovery introduced a wave of pessimism into the field of exchange rate modeling.

One might wonder whether the nearly four-decade old paper still puzzles economists to this day. Engel & West (2005) state that some have been successful in outperforming the driftless random walk in out-of-sample long-horizon

[1]Evaluated using the mean absolute error and root mean square error measures. Other methods used in the literature include: mean squared error, Diebold-Mariano test, direction of change, or Theil's U statistic (Rossi 2013).

forecasts, for instance, MacDonald & Taylor (1994) or Mark (1995). However, Rogoff & Stavrakeva (2008) call for further research on the robustness of long-run results. In contrast, the short-run dynamics of exchange rates appear to be "dominated by noise" (Mark 1995, p. 215), and there seem to be almost no positive developments thus far (Rogoff & Stavrakeva 2008).

Rossi (2013) provides an extensive overview of exchange rate predictability by examining the variables, econometric frameworks, data, and methods of evaluation used in more than 50 works concerned with the unpredictability puzzle. The author concludes that, even though certain predictors sometimes show encouraging long and even short-horizon performance improvements, no models consistently surpass the random walk in terms of its forecasting abilities. Other researchers have reached a similar conclusion; for example, Cheung *et al.* (2019) claim that even contemporary models have been unsuccessful in terms of the mean square error criterion. Moosa & Burns (2014) state that these findings should not be surprising. The authors also suggest that the task of surpassing the driftless random walk ought to be viewed as an unsolvable puzzle (going as far as calling it a *myth*). They argue that, by using root mean square error as an indicator of forecast precision, models should simply be expected to be outperformed. Instead, Moosa & Burns (2012) consider profitability and direction accuracy to be ultimately more appropriate measures of predictive power.

2.2 Virtual worlds

2.2.1 All things virtual

First of all, it is important to establish how exactly researchers define virtual worlds, and furthermore, what is considered as a 'virtual good,' a 'virtual economy,' and a 'virtual currency.'

Castronova (2001) defines a virtual world as a computer program with three distinctive characteristics: interactivity, physicality, and persistence. To elaborate, in order for a computer program to be considered a virtual world, individuals have to be able to simultaneously and remotely access the server on which the program is running, and their input should be perceived by others. The software needs to simulate a physical world with scarce resources accessible through a user interface that may resemble the real world. Finally, this program ought to keep functioning even if nobody is *online* and should retain ownership rights of items as well as the location of individuals and objects.

Furthermore, extending on Fairfield (2005), Blazer (2007) proposes a number of features that virtual property needs to possess. Analogously to real-world goods, virtual property should stay relatively unchanged whenever it is not being used (persistence). Moreover, only one person is allowed to control the property at any given time (rivalry), and others are able to interact with it (interconnectedness). The remaining two characteristics (or 'indicia' as Blazer (2007) calls them) are secondary markets—there should be a possibility for virtual property to be sold on unofficial markets, and finally, user-added value. For the specific case of virtual items, Lehdonvirta & Castronova (2014, p. 43) further subset virtual property by adding the excludability characteristic, which, along with rivalrousness, categorizes these items as 'private goods' in the traditional economic sense.

As Castronova (2008, p. 173) suggests, it is perhaps inevitable for these systems (virtual worlds with virtual goods) to naturally develop a subsystem resembling an economy, where virtual goods[2] are produced, consumed, and exchanged. On a philosophical level, the author further argues that if humans are allowed to enter such an environment, economic decision-making will unavoidably occur. This subsystem is often referred to as a virtual economy (Lehdonvirta 2009).

Be it coins, credits, gems, or gold, any form of currency arguably plays an essential role in all virtual worlds. Traditionally, money is thought to have three functions: medium of exchange, store of value, and a unit of account (Mankiw 2009, pp. 80-81). Yamaguchi (2004) indicates that virtual money often follows the same pattern. Furthermore, much like virtual currencies, the usability of real-world currencies is also constrained to particular places (though the degree of boundedness is arguably lower). On the other hand, there are some important distinctions; for instance, there are no virtual central banks issuing space credits or gems and no government assuring the value of these currencies—instead, this role is partly occupied by the developers (Lehdonvirta 2005).

2.2.2 Economics of virtual worlds

According to Lehdonvirta (2005), one of the first economics-related works on the topic of virtual worlds was that of Castronova (2001). In this paper,

[2]Some of which, as Lehdonvirta & Castronova (2014, p. 11) state, are scarce by design.

the American economist Edward Castronova materializes the seemingly unreal world of *Norrath* by estimating its various metrics such as hourly wage rate, value of its currency, or wealth distribution, and places the virtual world somewhere between Bulgaria and Russia in terms of its gross national product *per capita*.

Edward Castronova has since then published several highly-cited articles on the subject of virtual worlds and has arguably become one of the most prominent researchers in the field. For instance, in Castronova (2002), the author outlines some key characteristics of virtual economies and lists a number of situations in which traditional economics dictates otherwise. One example could be that the assumption of labor disutility is wholly reversed as work in a virtual economy generally increases utility. However, perhaps the most astute observation made in this paper is that virtual economies are, by design, almost fully and freely controllable by the developers. Moreover, Castronova *et al.* (2009a) conclude that the response of players to economic incentives is in accordance with the standard theory. Similarly, macroeconomic behavior appears to correspond to that of the physical world (Castronova *et al.* 2009b).

Another researcher, Vili Lehdonvirta, has also been active in the field. In Lehdonvirta (2005), the author investigates the suitability of micro and macroeconomic analyses of the virtual space and concludes that the real-world modeling techniques are most likely applicable. Furthermore, Lehdonvirta (2009) reviews virtual consumer behavior and discovers that individuals value in-game goods with a similar rationale as in the physical world. Finally, the two aforementioned researchers have summarized the fundamentals of virtual economics in a book (Lehdonvirta & Castronova 2014), where the discussed topics range from macroeconomic design to institutionalized crime.

Next, Chesney *et al.* (2009) have conducted several experimental games, such as the dictator game or the public goods game, within a particular virtual world to assess the potential of these environments for experimental economics. More specifically, their intention was to determine whether there would be any discrepancies between the results in real and virtual settings. Overall, they did not find any significant deviations from the normally observed behavior.

An extensive body of research is devoted to 'real-money trading,' the process of trading virtual currencies, goods, and services for real-world money, often on secondary markets (Heeks 2009). These opportunities provide the basis for an activity called 'gold farming,' which is a practice frequently occupied by people from third world countries (Lehdonvirta & Castronova 2014, p. 140) and can be described as an act of playing the game solely for financial gain either by collecting currency or by providing certain goods and services (Heeks 2009). Oftentimes, whole organizations form in the underground economy, hiring cheap labor, distributing malware, or designing artificial intelligence systems to maximize profit. Kwon *et al.* (2017) map the *modus operandi* of these groups and provide an extensive framework for detecting their activities. Another group of researchers, Keegan *et al.* (2010), report that the network structures of the gold farming organizations resemble those of drug trafficking rings.

Virtual worlds are able to produce large amounts of highly distinct data, which establishes them as a valuable tool for research (Bainbridge 2007). In particular, economics could greatly benefit from virtual economies and their design as these environments may offer novel perspectives on human behavior (Lehdonvirta & Castronova 2014, p. 270).

2.3 Forecasting virtual exchange rates

While the vast majority of massively multiplayer online games prohibit real money trading (Constantiou *et al.* 2012), secondary markets provide a way of monitoring exchange rates directly or through shadow pricing (Lehdonvirta 2009; Castronova *et al.* 2009b). Some authors have utilized these unofficial exchange rates in their works (Castronova 2001; Lehtiniemi & Lehdonvirta 2007; Heeks 2009). For instance, Heeks (2009) compares the average exchange rates of various virtual currencies in June 2005 with the values in November 2009 and discovers that most of the currencies have significantly depreciated against the dollar. The author attributes this development to an increase in competition in the gold farming *scene* over the years.

However, to the best of our knowledge, only a limited body of research exists on virtual exchange rate predictability. Furthermore, no authors seemed to have used standard linear regression analysis, utilized common economic variables as predictors, or considered the topic in the context of the unpredictability puzzle. Thus, this thesis contributes to the literature by filling these gaps. Nevertheless, we provide a review of two relevant works below.

Kim *et al.* (2015) try to predict the values of two virtual currencies using sentiment analysis, and their approach appears to be relatively accurate. They also remark that there are naturally fewer determinants of changes in the currency values in comparison with the physical world. On a related note, Wang *et al.* (2013) identify player base, structure of social hierarchy, and intensity of social networking as relevant variables that influence the price of virtual goods.

Moreover, Kim *et al.* (2017) extend on Kim *et al.* (2015) by analyzing a larger dataset of exchange rates in the game of *World of Warcraft* (WOW). To elaborate, they investigate the virtual-to-real exchange rate using 'tokens,' which are in-game items sold by the developer for real-world money that allow users to purchase *game time* (WOW is subscription-based, unlike some other games). Together with user opinion data, the authors were able to predict the next-day fluctuations with a similar degree of accuracy as Kim *et al.* (2015).

Evidently, the results of these papers seem to contradict the general consensus of short-run predictability that we have introduced earlier. One possible explanation for these findings could be that economies in the virtual space are less complex than in the physical world. However, more evidence is needed to support this claim.

Chapter 3

Background

In this chapter, we first introduce the relatively complex virtual world of *Old School RuneScape* (OSRS). We then describe the inner workings of the in-game economy by focusing on the idea of scarcity in a virtual space, production of goods, and trade. We also discuss some of its issues and the phenomenon of real-money trading.

3.1 Old School RuneScape

Old School RuneScape is the official name for the massively multiplayer online role-playing game (MMORPG) developed by the British game developer and publisher Jagex Limited and reintroduced in 2013 (OSRS Wiki 2020h). The term 'Old School' refers to the fact that the game is based on the 2007 source code of RuneScape 2, which was later updated to RuneScape 3 (OSRS Wiki 2020a). These two versions (OSRS and RuneScape 3) now run in parallel and can be essentially thought of as different games (i.e., they constitute two separate virtual worlds).

Table 3.1 summarizes the development history of RuneScape starting in 1999 with the release of its predecessor, Andrew Gower's short-lived multi-user dungeon game called *DeviousMUD*, continuing with the Gower brothers' earliest version of the project, and ending in the present day with the aforementioned duality.

It is important to keep in mind that while MMORPGs tend to set certain goals and milestones that the players can reach, there is no *winning condition.* To put it differently, unlike in other computer games, characters in MMORPGs generally continue to exist even after all the quests have been finished, items

Table 3.1: The development history of RuneScape

Game name	Developer(s)	Timeline
DeviousMUD	Andrew Gower	1999
RuneScape	Andrew and Paul Gower	2001-2004
RuneScape 2	Jagex	2004-2013
RuneScape Classic	Jagex	2004-2018
RuneScape 3	Jagex	2013-present
Old School RuneScape	Jagex	2013-present

Source: Ford (2020)

collected, and skill points obtained. Thus, it is the players who are ultimately in control of setting and achieving their goals (Yee 2004).

OSRS is no different: each player is represented by a customizable human-like character and is able to solve quests, fight monsters, duel other players, cast spells, communicate with others, trade with players and non-player characters, store their items in a bank, and various other activities (see, for instance, OSRS Wiki (2021h)). With an estimated number of unique daily players reaching well over two million (MMO-Populations 2021), OSRS undoubtedly follows the three criteria set by Castronova (2001) that are required for a program to be considered a virtual world.

The game is available for free; however, a large portion of its content is exclusive to 'members.' These are players that either pay a subscription fee or redeem a specific virtual item ('old school bond') purchasable with the in-game or real-world currency. For instance, members have access to new locations, equipment, quests, and more (OSRS Wiki 2021c). This subscriber-based approach is somewhat similar to World of Warcraft (Kim *et al.* 2017), which, as a virtual environment, arguably continues to garner the most attention in virtual worlds research.[1]

3.2 Economy of Old School RuneScape

3.2.1 Scarcity and production

One of the key issues that economics is trying to tackle is scarcity. As Lehdonvirta & Castronova (2014, p. 1) note, bits of information are far from being

[1] Relevant works to our subject area include Ducheneaut *et al.* (2006), Constantiou *et al.* (2012), Kim *et al.* (2015), or Kim *et al.* (2017).

scarce as they can be seamlessly copied and pasted. Therefore, at first glance, scarcity might not appear problematic in a virtual space. However, the authors identify three elements that, despite being in a virtual environment, are by all means scarce: attention of users, game content, and computational resources. Apart from that, Lehdonvirta & Castronova (2014, pp. 17-20) also advise developers to be aware of the issue and to introduce various forms of artificial scarcity into the game to facilitate competition and create challenging puzzles to attract players.

In OSRS, raw materials are created programmatically, and their supply is essentially unlimited. However, there is a finite number of places where players are able to gather these goods, and furthermore, the process of their collection is often rivalrous—as the resource gets depleted (a tree is cut down, an ore is extracted, a monster is slain, etc.), only one player is awarded the raw material. These resources are automatically *renewed* (the tree regrows, the ore reappears, the monster respawns, etc.) after a certain period of time (see, for example, OSRS Wiki (2021f)). Players are then able to exchange these materials with others or use them to produce consumables or final goods, which can either be utilized, stored, or sold. In almost every step of this process, players earn experience points in the respective skills such as woodcutting, firemaking, or cooking (OSRS Wiki 2021b;h). As a consequence, the prospect of 'leveling-up' is one of the reasons why raw materials, in contrast with the real world, are more costly in OSRS than the final products (Bilir 2009).

3.2.2 Trade

There are several currencies in OSRS; however, most of them only have a specific use case. The primary medium of exchange is called 'coins' but is often referred to as 'GP' (which is an abbreviation for 'gold pieces'). There are several ways players may obtain coins. For instance, small amounts of GP repeatedly appear ('spawn') in various locations around the gameworld, and thus, can be collected. Another way to acquire coins is to slay monsters—some may 'drop' gold upon perishing (OSRS Wiki 2021a). While there are other ways to obtain GP, trading is arguably the most popular approach.

Coins may be used by players to trade with other players as well as non-player characters. Moreover, each item's value during a player-to-player barter trade is also expressed in GP, which is supposed to help players make better decisions by minimizing information asymmetry. The specific amount each item is valued at corresponds to the current 'Grand Exchange' price of the particular good (OSRS Wiki 2020j).

The Grand Exchange (often abbreviated as 'GE') is a system used for trading in Old School RuneScape (OSRS Wiki 2021e). It provides a centralized way for players to exchange their virtual goods with other players for GP. The system also automatically suggests an equilibrium price for each item, which is mainly determined by the market forces of demand and supply (OSRS Wiki 2021e). Whenever a player creates an offer to buy or sell a specific quantity of an item at a certain price (which may be chosen arbitrarily, notwithstanding the suggested equilibrium price), the system tries to match their bid with another player's offer. If it successfully finds a matching or a better offer, each side of the transaction is compensated accordingly. This may happen instantaneously for commonly traded items or may take some time for less sought-after goods. In the opposite case, the system will continue searching until it finds an optimal deal, or the player may cancel the offer free of charge. Furthermore, there are no fees associated with the usage of the Grand Exchange. Finally, the whole transaction process is anonymous; that is, neither the buyer nor the seller knows who the other player is (Bilir 2009; OSRS Wiki 2021e).

With more than 5 trillion GP exchanged daily and over 3 million trades per day (Jagex 2020), the Grand Exchange generates a considerable amount of data. Thankfully, the statistics on the daily quantity and average price of nearly all items get published, archived, and are easily accessible. We provide more information on how this thesis exploits the available data in Section 4.1.

According to Bilir (2009), the Grand Exchange resembles the real-world commodity exchange in terms of the kind of goods being traded; on the other hand, it lacks, for example, the ability to trade futures. Despite the absence of various financial instruments, players have developed multiple trading strategies (collectively referred to as 'merchanting'). One of the traditional methods is, indeed, speculation. Another strategy is called 'flipping,' which is a practice of reselling items for a marginally higher price in the short run. The official OSRS Wiki (2020g) describes that this method generates profit by exploiting the differences in prices between pending buy/sell orders that arise due to the "varying degrees of patience" of some players.

The bank in OSRS (operated by the developer, not by any player) has a substantially lower number of competencies than its real-world counterparts. Players may deposit, store, and withdraw their coins free of charge, and the same procedures can also be applied to items (OSRS Wiki 2020b). Apart from that, the OSRS bank does not provide any other major services, for example, no loans or interest (Bilir 2009).

Although the space to store items and GP in a bank account is limited, it can hold a significantly larger amount of goods than a player's inventory. Furthermore, items in a bank account 'stack,' which means that, for instance, five iron pickaxes would only occupy a single slot (OSRS Wiki 2020b). The same behavior is not present in an inventory; however, players may withdraw a specific item's 'bank note'—a tradable certificate representing the ownership of a particular quantity of a certain good, allowing players to trade items on a larger scale (Bilir 2009; OSRS Wiki 2020c).

3.2.3 Issues

One of the main issues endemic to virtual economies is that, given enough time, players will simply accumulate all the available items and collect an abundance of gold. While this situation might be viewed as a positive development by growth-oriented economists, it is generally undesirable in a virtual environment (Castronova 2002; Lehdonvirta & Castronova 2014, p. 230). Castronova (2008, pp. 195-198) describes this phenomenon, which is often called 'MUDflation' (a portmanteau of multi-user dungeon and inflation), as consisting of an increase in the quantity of physical capital as well as a steady rise in the price level. The author illustrates the inflation problem using the equation of exchange:

$$M \cdot V = P \cdot T$$

In this formula, the product of the quantity of money (M) and the velocity of money (V) on the left-hand side should always be equal to the product of the price level (P) and the number of transactions during a certain period of time (T) in the economy (Mankiw 2009, pp. 86-87). Then, by isolating P, we obtain the following relation:

$$P = \frac{M \cdot V}{T}$$

This implies that the price level will rise (keeping the velocity of money constant) with the increase of the quantity of money for any fixed $T \in \mathbb{N}$. Consequently, Castronova (2008, p. 199) suggests that each inflationary transaction should later be followed by a deflationary one.

To combat the issue of MUDflation, developers utilize the 'pipes model' of the economy. The idea of the model is that virtual goods *flow* into the world using various types of 'faucets' (e.g., loot from monsters or automatically restocking shops) and are *drained* through 'sinks' (e.g., tolls or loss of items upon death). By having the ability to partially adjust the number of items that enter and exit the economy, developers are able to control the quantity of money and physical capital to a certain degree (Lehdonvirta & Castronova 2014, p. 199).

Another major difference in the functioning of a virtual and a real economy is the frequent use of price and quantity controls. Most economists seem to agree that, in general, governments should not control prices because it may lead to inefficiencies in the market (Castronova 2002, p. 4; Rockoff 2008), and a similar view is shared on quotas (Krugman & Wells 2015, p. 151). However, in Old School RuneScape, both of these restrictions are commonly imposed—according to the official OSRS Wiki (2020f), over 3500 in-game items have a fixed buying limit that resets every four hours. Moreover, the buy/sell prices of items in the in-game shops are determined by their current stock and the prespecified value set by the developers, which functions either as a price floor or a price ceiling (OSRS Wiki 2021b;d). The purpose of introducing these inefficiencies into the market is stabilization (OSRS Wiki 2021b). After all, adjusting prices and quantities is costless in a virtual economy (Castronova 2002).

3.2.4 Real-money trade

Real-money trading has been a prevailing issue in Old School RuneScape. It is prohibited as all in-game goods legally belong to Jagex (the developer). This naturally implies that players have no property rights to the items on their accounts (OSRS Wiki 2020i). That, however, does not seem to stop individuals and groups from exchanging the in-game currency for real-world money despite the possible legal ramifications (Gerhard 2011).

Unlike other means of acquiring GP in an unethical manner, such as scamming, account stealing, or glitch exploitation, gold farming has arguably been the most consistent method for money-making in OSRS. One of the strategies for *farming gold* used by individuals or organizations partaking in this activity is called 'botting' or 'macroing' (OSRS Wiki 2020d), which consists of computer-aided in-game task automation. In other words, gold farmers employ various software solutions that play the game in an efficient manner with almost no human interaction (Lehdonvirta & Castronova 2014, p. 140). To illustrate the magnitude of the issue, three years after the game's 2013 launch, there have been over 1.3 million bans issued to ‘botters,’ and just a year later, the number nearly doubled (Jagex 2017). Nevertheless, the company continues to enforce its anti-botting rules, and the number of banned accounts keeps rising (Jagex 2020).

The other strategy used by gold farmers comprises of hiring workers from low-income countries to either collect coins or provide player-for-hire services (Lehdonvirta & Castronova 2014, p. 140; Kwon *et al.* 2017). Several articles have recently been published detailing how the economic and political crisis in Venezuela encouraged people at the risk of poverty to gather virtual gold in Old School RuneScape in an attempt to earn a livable sum of money or to raise funds for their escape from the country (Good 2017; The Economist 2019; Ombler 2020). Furthermore, the 2019 nationwide power outages in Venezuela have demonstrated the extent to which the OSRS market is dependent on local gold farming as the negative supply shock led to a shortage of highly sought-after items (Jackson 2019; Ombler 2020).

While gold farming tends to be undesirable both for the players and the developers due to a myriad of reasons (Lee *et al.* 2016), these reports arguably present an interesting moral dilemma by positioning Jagex into a complicated role wherein the decision to ban an account suspected of real-money trading may hinder someone’s ability to fulfill their basic human needs.

Chapter 4

Methodology

In the first part of this chapter, we describe the variables of interest and comment on the most significant characteristics of our data. The following subchapter introduces the econometric framework that we used and details the necessary steps taken before and after estimating the models.

4.1 Description of variables and data

As we have discussed in Section 2.1, the underlying theme of exchange rate predictability seems to be the *failure to conform to the theory*. This also appears to apply to the usage of standard economic variables—Rossi (2013) reviews several predictors used in the literature in the past three decades but fails to find any variable that would help forecast exchange rates accurately in all circumstances even though most of the examined predictors were based on well established economic relationships such as the 'purchasing power parity,' 'uncovered interest rate parity,' or the 'Taylor rule.' As Carbaugh (2008, p. 417) states, the soundness of the theory does not necessarily ensure success in forecasting future adjustments.

However, that might not be an issue—Lehdonvirta (2005) argues that models of virtual economies can depart from reality and exploit these differences. This may be especially helpful for the case of our virtual economy, which only has a limited number of relevant variables that could be utilized. Let us remind ourselves that our goal is to see whether it is possible to model and forecast exchange rates *accurately* in a virtual setting.

4.1.1 Exchange rates

There is no official way to directly exchange real-world money for the in-game currency of Old School RuneScape. However, we identify an *official indirect* (estimated rate) and an *unofficial direct* (black market rate) approach to solving the issue of assigning real-money value to OSRS coins. For simplicity, we refer to the two measures of the virtual exchange rate as two separate rates. Let us begin by focusing on the latter.

Black market rate

First of all, we feel obliged to reiterate that real-world trading in OSRS is against the rules (OSRS Wiki 2020i). Therefore, we are not encouraging anyone to use the services we may mention, and we take no responsibility for anybody's incurred losses.

By means of web scraping, we were able to independently obtain publicly available data consisting of secondary market nominal spot exchange rates (daily average of United States dollars/million OSRS coins) from an online marketplace called *PlayerAuctions* owned by a Korean company *ItemMania*, where people can buy and sell various virtual currencies and goods. While there exist other websites offering similar services, we could not find any additional online marketplace that would publish their statistics.

Although the original dataset spanned from March 2017 up to late 2020, we decided to restrict our analysis to a narrower time frame upon further inspection. This decision was influenced by a multitude of factors. Firstly, the overall data quality was visibly poor in the initial years—there were unexplainable outliers (see Figure B.1 in Appendix B) and missing values (this was a problem in other variables). Subsequently, with respect to the time frame and the method we planned on using, long-run analyses were rendered unfeasible. Furthermore, Hyndman & Athanasopoulos (2018, chapter 12.7) state that it is valid to consider more recent data in a short-run forecasting exercise, especially if the model in question is relatively inflexible.

For these reasons, the final length of the time series was set to be around half a year, starting in the middle of February 2020 and ending in August 2020[1] (the black market rate series is plotted in Figure 4.1). Additionally, the observations for the following eight weeks were included for the out-of-sample exercise.

[1]Note that this time frame naturally applies to all other considered variables.

Estimated rate

In Subsection 3.2.2, we have outlined how goods are traded in Old School RuneScape and introduced the Grand Exchange, where most of the trading in OSRS occurs. Furthermore, at the end of Section 3.1, we have also briefly mentioned an in-game item that may be redeemed to elevate one's account to a 'membership' status, which allows access to all the features of the game. This virtual item, the old school bond, is purchasable either with the OSRS' virtual currency or with real-world money for \$6.99 (OSRS Wiki 2021g).

Thus, it is possible to estimate the exchange rate using the 'law of one price,' which proposes that the price of a good in a perfectly competitive market with no transportation costs and barriers to trade should be the same everywhere (Carbaugh 2008, p. 403). Following (Pugel 2015, p. 442), the law of one price may be expressed as:

$$P = e_s \cdot P^*, \tag{4.1}$$

where e_s is the nominal spot exchange rate, and P & P^* are the domestic & foreign prices of the product, respectively.

By utilizing the Grand Exchange data on old school bonds and adhering to the law of one price, we may rearrange Equation 4.1 in the following way:

$$e_s = P/P^*,$$

and obtain an estimate for the nominal spot exchange rate of the virtual currency (see Figure 4.1). In fact, Kim *et al.* (2017) have used a similar approach in their analysis by examining the 'token/Gold' exchange rate in World of Warcraft.

4.1.2 Other variables

Following the rationale outlined at the beginning of this section, we employed two other variables, one of which could be considered to mimic reality (price index) while the other seems to be rather endemic to virtual environments (player population). Unfortunately, we were unable to identify any additional predictors that could be included in the model. Some of the standardly used variables, as listed by Rossi (2013), either cannot be found in our virtual economy (e.g., interest rates or portfolio balances) or are difficult to measure (e.g., productivity or output). Finally, the data for both of these series were collected and assembled independently from public sources.

Price index

The official Old School RuneScape encyclopedia maintains a number of daily price indices tracking the state of the in-game economy. One of these indicators, the 'Common Trade Index,' includes 27 staple items such as coal, runes, or cowhide, as well as some pricier goods, which players often collect in later stages of the game (OSRS Wiki 2020e). Using the Grand Exchange data on the 27 items, we created a similar price index with the base period (t_0) set to April 1st, 2017, in the following way (see Figure 4.2 for the plot of the series):

$$\text{Price index at time } t = \frac{\text{Sum of prices at time } t}{\text{Sum of prices at time } t_0}, \text{ for } t = t_0, t_0 + 1, t_0 + 2, ...$$

While the developers publish the quantities of goods sold each day on the Grand Exchange, the dataset that we obtained was missing a large portion of these values. Thus, we were unable to construct a volume-weighted index, which could result in an imprecise representation of the economy (Scholten *et al.* 2019). Furthermore, with regards to the time period relevant to our analysis, it may seem inappropriate to choose the base date to be this far in the past. However, since we are only interested in the changes of the time series, rebasing is not necessary (i.e., the results would be the same).

The reasons for choosing this variable were relatively simple. First, price indices are used in real-world out-of-sample exercises (Rossi 2013); though, it is important to note that they are thought to influence exchange rates in the long term (Carbaugh 2008, pp. 400-403). Second, from a theoretical standpoint, it seems reasonable to believe that prices of goods would be related to exchange rates as, for example, the demand for gold pieces should increase with the rise in prices, given that players tend to behave as *typical* economic agents (as per Castronova *et al.* (2009a)).

Player population

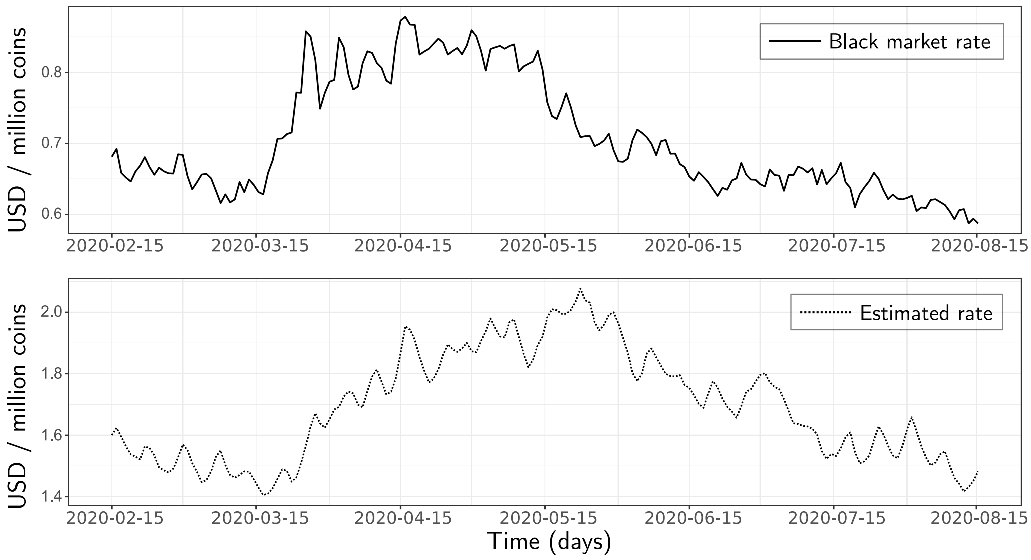
The final variable, believed to influence both the exchange rate and the price index, was player population (following Wang *et al.* (2013)). We justify the inclusion of this variable by assuming that, for example, an influx of players should increase the demand for virtual items and money, triggering a response in both the price level and the exchange rate. Conversely, fluctuations in the exchange rate may entice or discourage gold farmers from entering or leaving the market, respectively. This variable could also be thought of as a measure of popularity of the game—the more players participate in a virtual world, the more active the community is. Those who engage in real-money trading would arguably favor games with larger player populations.

We collected the average number of daily concurrent players from a community project `www.misplaceditems.com/rs_tools/graph/`[2] that periodically scrapes the number of currently online players from the official webpage of Old School RuneScape. Despite some minor inconveniences in the data-scraping process, this project offered a valuable dataset (see Figure 4.2), especially since the developers of the game do not frequently publish highly granular player-related statistics, likely due to business reasons.

4.1.3 Exploratory analysis

Figure 4.1 depicts the two considered daily exchange rates, addressing two issues. First, the high degree of similarity in the development of the rates may indicate that the black market rate seemed to have been driven by supply and demand. Without this comparison, we would only have limited evidence to validate the authenticity of the series. However, it is important to be aware of the displayed values on the two vertical axes.

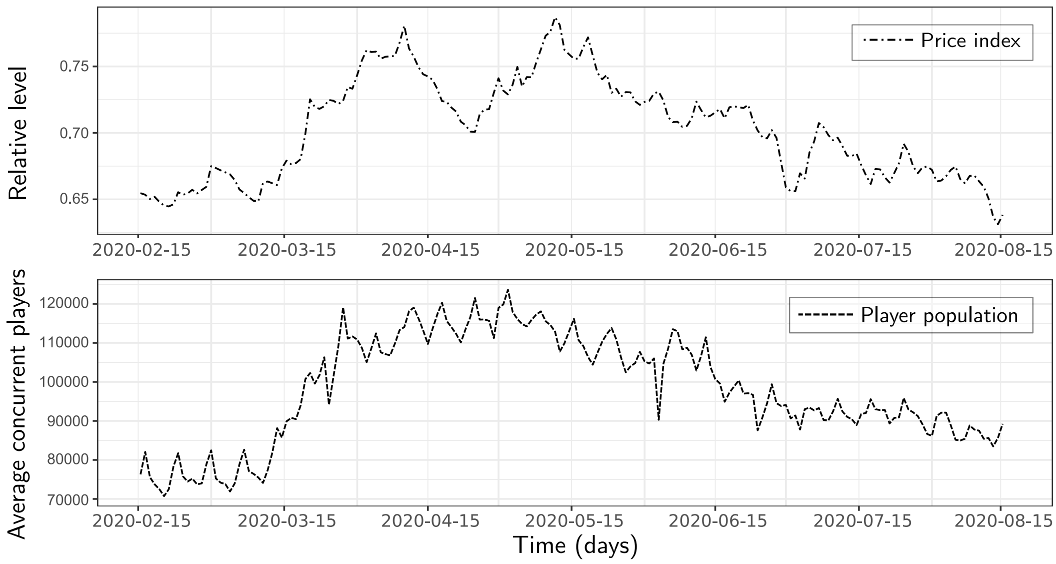
Figure 4.1: Black market rate and estimated rate graphs



[2]We tried contacting the author through the email address available on the website to give proper credit, but we received no answer. Thus, there seems to be no other way to provide attribution besides mentioning the full name of the website.

While the black market rate appeared to follow a more erratic and irregular pattern, the estimated rate's variance was larger. Furthermore, assuming that the black market rate is valid, the estimate based on the law of one price seemed to be relatively accurate in terms of the general development, though it often appeared to be slightly delayed.

Figure 4.2: Price index and player population graphs



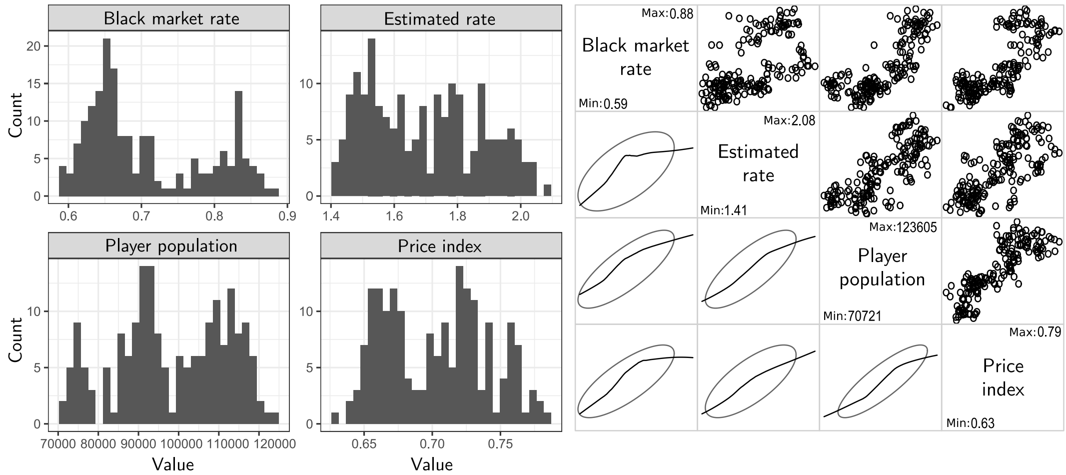
Next, from Table 4.1, it is evident that the estimated rate consistently valued the price of one million coins more than twice as higher as on the black market. While we are not entirely sure why that seems to be the case, it does not constitute a problem given the goal of our analysis.

Table 4.1: Descriptive statistics of all the variables

Sample size: 184	**Min**	**Q1**	**Median**	**Mean**	**Q3**	**Max**
Black market rate	0.587	0.646	0.673	0.706	0.785	0.878
Estimated rate	1.405	1.534	1.690	1.695	1.847	2.077
Player population	70721	88916	97040	98155	110388	123605
Price index	0.631	0.669	0.706	0.703	0.731	0.787

Lastly, the histograms in Figure 4.3 suggest that all four variables appeared to have a somewhat bimodal distribution, rendering the mean and the median in Table 4.1 unrepresentative. Moreover, the correlogram indicates that all the variables were pairwise positively correlated.

Figure 4.3: Histograms and a correlogram of all the variables



4.2 Model

4.2.1 Selected framework

Vector autoregression is an econometric framework developed in the early 1980s, which expands on the univariate 'autoregressive model' (AR) by considering a vector of variables (Stock & Watson 2001; 2019, p. 650). According to Stock & Watson (2001), vector autoregressive models have, throughout the years, established themselves as an apt method for forecasting various kinds of multidirectional dynamic relationships. Moreover, Backus (1986) believes that VARs provide a good starting point in modeling by uncovering the correlations between the variables of interest. For these reasons, VARs have been, for example, utilized to forecast wind power (Dowell & Pinson 2016) or technology introductions (Adomavicius *et al.* 2012). Finally, numerous studies have also employed vector autoregression in exchange rate forecasting (as noted by Rossi (2013) or Cuaresma *et al.* (2018)). For instance, Meese & Rogoff (1983, p. 8) use the framework in their analysis as a "representative multivariate time series model." Other works utilizing this method include Backus (1986), Dooley *et al.* (1995), Cover & Mallick (2012), or Cuaresma *et al.* (2018).[3] Therefore, we consider it reasonable to employ the VAR framework since we also believe that all the aforementioned variables influence each other.

VAR can be described as a model comprising of k time series variables, and thus, k equations, where each equation contains $p > 0$ lags of all considered

[3] We also find it important to mention the works of Průša (2010) and Mida (2013), which introduced the author of this thesis to the topic of exchange rate forecasting.

variables (Stock & Watson 2019, p. 783). The general VAR(p) model with k variables and a vector of constants can be written as:

$$x_t = A_0 + \sum_{i=1}^{p} A_i x_{t-i} + u_t, \text{ for } t = 1, 2, ..., T, \tag{4.2}$$

where x_t is a $k \times 1$ vector of variables included in the model, A_0 represents a $k \times 1$ vector of intercepts, each A_i is a $k \times k$ matrix of coefficients, and u_t represents a $k \times 1$ vector of serially uncorrelated error terms with $\mathbb{E}(u_t) = 0$ and $Cov(x_t, u_t) = \Sigma_u$ (Tsay 2005, pp. 349, 353; Enders 2015, p. 290). Under the least squares assumptions for multivariate forecasting,[4] the ordinary least squares (OLS) estimators are consistent and jointly normally distributed for T large, allowing for standard inference procedures (Stock & Watson 2019, pp. 650-651).

Furthermore, let us focus on one of the key assumptions for modeling time series data: 'stationarity.' According to Stock & Watson (2019, p. 561), a stochastic process is said to be stationary if its probability distribution remains unchanged over time. In fact, this property is especially relevant for forecasting as it might be difficult to predict something that has yet to happen (Stock & Watson 2019, p. 561). However, researchers are often unable to obtain a multitude of time series generated by a single process within the same period, and thus, a weaker assumption of 'covariance stationarity' is usually considered.[5]

A closely related term to stationarity is the 'unit root.' First, let us consider the following AR(1) process:

$$y_t = a_0 + a_1 y_{t-1} + \nu_t, \text{ for } t = 1, 2, ..., T, \tag{4.3}$$

where ν_t is a sequence of independent and identically distributed random variables with zero mean and finite variance. The AR(1) model is considered 'stable' whenever $|a_1| < 1$, rendering the process stationary (analogously for a higher-order autoregression or the VAR model). On the other hand, if $a_1 = 1$, then the root of its characteristic equation is equal to one, and $\{y_t\}$ is said to be a unit root process (Enders 2015, pp. 30-31, 287; Stock & Watson 2019, p. 605). In addition, if we also let $a_0 = 0$, then the time series involved would follow a random walk. We are particularly interested in this simple model's forecasting capabilities (or rather the lack thereof) as its best prediction for the next period's value is the current observation[6] (Stock & Watson 2019, p. 583).

[4]See Section A.1 for an overview.

[5]A precise definition can be found in Enders (2015, p. 52).

[6]This is often called the 'naïve method' (Hyndman & Athanasopoulos 2018, chapter 3.1).

4.2.2 Pre-estimation procedures

Nonstationarity

By observing the plots of all the variables (Figure 4.1 and Figure 4.2), we suspected that all of the underlying processes contained a unit root, violating the assumption of stationarity. Another apparent issue in need of addressing was that of seasonality, especially in the player population data.

Several economic variables are standardly transformed using a natural logarithm, which is done for two reasons. Firstly, their long-term growth tends to be roughly exponential, and secondly, their log-transformed standard deviations are often close to being constant (Stock & Watson 2019, pp. 556-557).

After applying the aforementioned transformation to all of our variables, we performed the augmented Dickey-Fuller tests for the existence of a unit root.[7] We failed to reject the null hypothesis at the significance level of 5% for all the variables in the driftless/trendless case as well as in the regression equation with a constant term while considering a range of lags (see Table 4.2 for the results). This non-rejection strongly suggested that a unit root was present—this was resolved by differencing the log-transformed series accordingly (see Figure 4.4), which also resulted in the removal of simple stochastic trends (Stock & Watson 2019, p. 588). Price levels are usually thought to be 'integrated of order two,' meaning that the series needs to be differenced twice to become stationary (Stock & Watson 2019, p. 660). However, we found insufficient evidence to support this claim, and thus, only the first difference was applied.

Furthermore, we observed that the raw daily player population time series exhibited a weekly seasonal pattern, which needed to be resolved. To capture the effects of seasonality, we decided to employ the standard procedure of including an appropriate number of dummy variables (Hyndman & Athanasopoulos 2018, chapter 5.4).

Lag length and cointegration

A necessary step in constructing any VAR model is to determine the lag order of all variables of interest, i.e., how many lags should be included. According to Stock & Watson (2019, p. 652), one approach of solving this problem would be to use 'information criteria' such as the 'Bayes information criterion' (BIC) or the 'Akaike information criterion' (AIC). The latter is considered to be more

[7] A brief summary of this procedure is provided in Section A.2.

Table 4.2: Augmented Dickey-Fuller test results

Variable	**Transformation**	**ADF p-value**		
		Basic	Drift	Drift & trend
	none	0.51	0.79	0.80
Black market rate	log	0.71	0.82	0.82
	difference of logs	$<$ 0.01	$<$ 0.01	$<$ 0.01
	none	0.55	0.76	0.98
Estimated rate	log	0.52	0.78	0.98
	difference of logs	$<$ 0.01	$<$ 0.01	$<$ 0.01
	none	0.59	0.57	0.77
Price index	log	0.62	0.58	0.79
	difference of logs	$<$ 0.01	$<$ 0.01	$<$ 0.01
	none	0.68	0.43	0.73
Player population	log	0.76	0.39	0.72
	difference of logs	$<$ 0.01	$<$ 0.01	$<$ 0.01

Note: The displayed values of the ADF test results were achieved using three lags; however, a range of lags was considered.

Figure 4.4: Graphs of the first differenced log-transformed series

appropriate for predictions, while BIC seems to be a more suitable measure of the overall goodness of fit (Shmueli 2010). Additionally, researchers may conduct F-tests for parameter significance (Stock & Watson 2019, p. 652).

The AIC for a particular number of lags p is computed in the following way:

$$\text{AIC}(p) = ln(det(\hat{\Sigma}_u)) + \frac{2}{T} \cdot (k^2 p + k). \tag{4.4}$$

In the above equation, $det(\hat{\Sigma}_u)$ is the determinant of the estimated covariance matrix of the vector of error terms from Equation 4.2, T is the sample size, and k represents the number of considered variables. Moreover, replacing $2/T$ with $ln(T)/T$ yields the formula for the BIC. Finally, the *ideal* lag length $\hat{p}$ is such that it minimizes the respective criterion function (Stock & Watson 2019, p. 652).

Before we determined the lag order for our VAR model, we searched for signs of 'cointegrating relationships' in both sets of our variables (price index, player population, and either of the exchange rates). For instance, given that the player population (p_t) and black market exchange rate (e_t) are integrated of order one, cointegration would mean that there exists some factor θ for which the linear combination $e_t - \theta p_t$ does not contain a unit root (Stock & Watson 2019, p. 664).

Thus, we investigated the issue by employing the Johansen test[8] with the number of lags determined by the BIC and AIC scores. We failed to reject the null hypothesis of no cointegrating relationships at the significance level of 5% for the first set of log-transformed variables. However, in the second case, the test results did not seem to be entirely robust to the lag order, resulting in the rejection of the null hypothesis at lower lag lengths (see Table 4.3). Furthermore, the Engle-Granger procedure was utilized, which first estimates θ via OLS and then tests for the presence of a unit root in the residuals using the ADF method outlined in Section A.2 (Stock & Watson 2019, p. 665). In contrast, the results of this test suggested the absence of cointegration for most evaluated models (available in Table B.1). In general, a strong suspicion of a cointegrating relationship would require utilizing an 'error-correcting mechanism' because the usual VAR model could yield inefficient forecasts (Holden 1995). However, since we were mainly interested in the short-run forecasting performance, we considered it reasonable to proceed with the standard VAR specification in first differences.

[8]See Section A.3 for a brief review of the method.

Table 4.3: Johansen test results

Model*	Johansen test			
	Lags (criterion)	Number of coint. rel.**	Trace statistic	5% Critical value
Black market rate	2 (BIC)	None	24.71	34.91
		At most 1	9.00	19.96
		At most 2	0.52	9.24
	8 (AIC)	None	23.70	34.91
		At most 1	10.99	19.96
		At most 2	1.91	9.24
Estimated rate	3 (BIC)	None	41.88	34.91
		At most 1	15.43	19.96
		At most 2	4.92	9.24
	8 (AIC)	None	34.46	34.91
		At most 1	11.00	19.96
		At most 2	3.35	9.24

Note: *Set of the three variables (exchange rate, price index, and player population). **Number of cointegrating relationships. Further, only log-transformed (not differenced) variables were used.

The final step was to determine the ideal lag length for the two sets of variables. Utilizing the information criteria described above, we calculated the AIC and BIC for a range of lags (see Table 4.4). We then estimated several models with various lag orders due to the substantial difference between the recommendations, focusing mainly on the coefficient significance, the in-sample fit, and the out-of-sample performance.

Table 4.4: Lag order selection – AIC and BIC results

Model*	Criterion	Lag order (p)					
		1	2	3	4	...	7
Black market rate	BIC(p)	$\boxed{-22.884}$	-22.767	-22.637	-22.552	...	-22.189
	AIC(p)	-23.104	-23.200	-23.188	-23.268	...	$\boxed{-23.402}$
Estimated rate	BIC(p)	-23.667	$\boxed{-23.766}$	-23.601	-23.562	...	-23.218
	AIC(p)	-23.888	-24.150	-24.153	-24.279	...	$\boxed{-24.431}$

Note: *Set of the three transformed variables.

While some state that it is possible to overfit by following the AIC's (often excessive) lag order selection, empirical evidence is not in favor of this claim (Kilian & Lütkepohl 2017, pp. 59-60). In our case, however, setting the lag length to seven yielded a number of insignificant parameters and a low F statistic in one of the equations. Thus, through several rounds of experimentation, the lag order for both models was chosen to be four, implying that the default number of parameters to be estimated in the VAR models would equate to 39 (Enders 2015, p. 290).

4.2.3 Estimation and diagnostics

Let Δle_t^{blk} and Δle_t^{est} be the first differences of the natural log of black market exchange rate and estimated rate,[9] respectively. We considered two separate models for each of the rates instead of incorporating both variables into a single system because we believed that these should be thought of as two competing measures of the official virtual-to-real exchange rate. Furthermore, let us also define Δli_t as the first difference of logs of the price index, and analogously, let Δlp_t represent the transformed player population variable.

With the goal to accurately model and forecast the changes in the two available rates, we decided to estimate the following VAR(4) models in first differences based on Equation 4.2 with $k = 3$:

$$\Delta lx_t = A_0 + \sum_{i=1}^{4} A_i \Delta lx_{t-i} + \sum_{j=1}^{6} \delta_j d_{jt} + u_t, \text{ for } t = 1, 2, ..., 183, \tag{4.5}$$

where Δlx_t is a *3 × 1* vector of variables $(\Delta le_t^{blk}, \Delta li_t, \Delta lp_t)^\intercal$ in the first model and $(\Delta le_t^{est}, \Delta li_t, \Delta lp_t)^\intercal$ in the second one. Moreover, each equation also contains six dummy variables $d_{1t}, d_{2t}, ..., d_{6t}$, which capture the weekly seasonal pattern while avoiding the 'dummy variable trap' (Hyndman & Athanasopoulos 2018, chapter 5.4).

Serial correlation

Following Eloriaga (2020), we began diagnosing the estimated models by investigating serial correlation in the residuals by employing the Portmanteau procedure detailed in Section A.4. The test was conducted for a range of lags—we started at the lowest possible order and incrementally increased the lag length, each time rerunning the procedure. The null hypothesis of no serial correlation was not rejected at the significance level of 5% in all tested cases for both models (p-values are reported in Table 4.5). If autocorrelation were to be found in the residuals, then forecasts could be rendered inefficient as there would still be some information left unexplained (Hyndman & Athanasopoulos 2018, chapter 5.3).

[9] Note that $100 \cdot \Delta le_t$ would approximately correspond to the percentage change in the raw series (Stock & Watson 2019, p. 557).

Heteroskedasticity

Next, we tested for the presence of heteroskedasticity in the VAR residuals using the 'autoregressive conditional heteroskedasticity Lagrange multiplier' test (ARCH-LM), which is briefly reviewed in Section A.5. The choice of lags followed the same strategy as in the previous procedure, and consequently, several tests were conducted. In both models and for any tested lag order, we failed to reject the null hypothesis at the 5% significance level, suggesting the absence of autoregressive conditionally heteroskedastic errors (see Table 4.5 for the results). While the presence of heteroskedasticity would not necessarily threaten estimator consistency (for a finite unconditional error variance), efficiency and inference may be contested (Kilian & Lütkepohl 2017, p. 68).

Table 4.5: Portmanteau and ARCH-LM test results

Model	**Portmanteau test**		**ARCH-LM test**	
	Lags	P-value	Lags	P-value
Black market rate VAR	5	0.400	1	0.309
	6	0.255	2	0.257
	7	0.221	3	0.090
	8	0.234	4	0.066
	9	0.329	5	0.137
	10	0.432	6	0.234
Estimated rate VAR	5	0.326	1	0.057
	6	0.654	2	0.362
	7	0.558	3	0.549
	8	0.099	4	0.099
	9	0.126	5	0.234
	10	0.199	6	0.274

Normality

We then used the multivariate Jarque-Bera (JB) test to investigate the normality of the residuals. In this procedure, the skewness and kurtosis (third and fourth moments) of the residuals are compared to those of a normal distribution, and the null hypothesis of normality is evaluated (Kilian & Lütkepohl 2017, p. 67).

In both models, the null hypothesis was rejected in the kurtosis test at the 1% significance level. Regarding skewness, however, $\mathbb{H}_0$ was rejected only in the black market rate VAR model's residuals at the 1% level of significance (see Table 4.6). Furthermore, the *R* package that we used utilized the Cholesky matrix decomposition, which is sensitive to variable ordering (Enders 2015, p. 296), and therefore, we conducted the test for all possible orderings but found no contradictory results. Kilian & Lütkepohl (2017, p. 67) state that while the presence of normally distributed residuals "facilitates predictive inference," their absence is not necessarily problematic given the goal of our analysis.

Table 4.6: Jarque-Bera test results

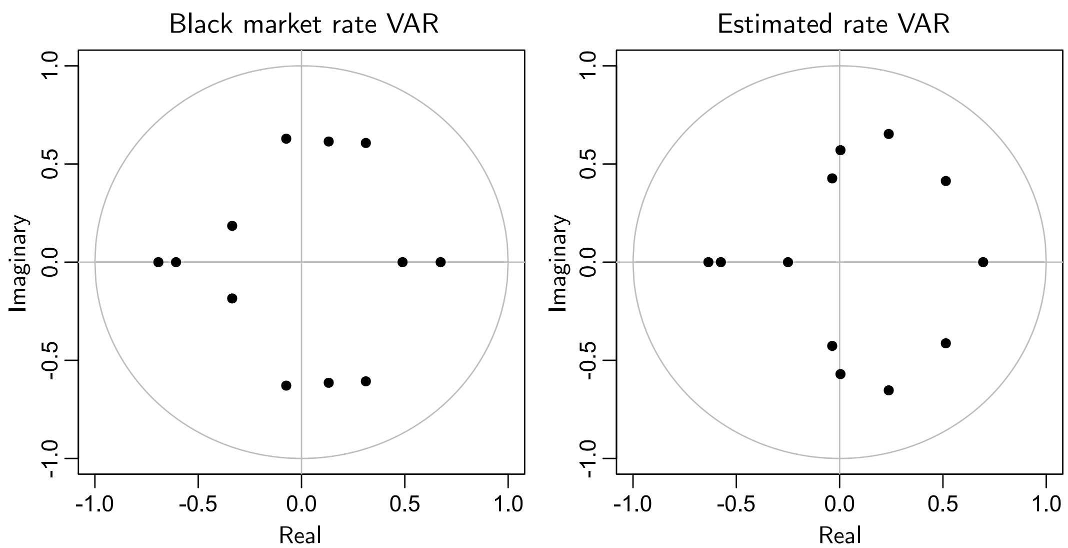
Model	**Jarque-Bera test p-values**		
	JB statistic	Skewness statistic	Kurtosis statistic
Black market rate VAR	< 0.01	< 0.01	< 0.01
Estimated rate VAR	< 0.01	0.33	< 0.01

Note: The order of the variables was $\Delta le_t, \Delta li_t, \Delta lp_t$.

Dynamic stability

Moreover, a $kp \times kp$ matrix obtained by transforming a VAR(p) model to a VAR(1) form is called the 'companion matrix' (Tsay 2005, p. 354). By examining the characteristic values of the VAR models' companion matrices, both models were found to be dynamically stable (Kilian & Lütkepohl 2017, p. 25). This is perhaps best illustrated in Figure 4.5—if the values (VAR inverse roots) are located within the complex unit circle, the condition is satisfied (Kilian & Lütkepohl 2017, p. 25; Hyndman & Athanasopoulos 2018, chapter 8.7).

Figure 4.5: VAR inverse roots inside a complex unit circle

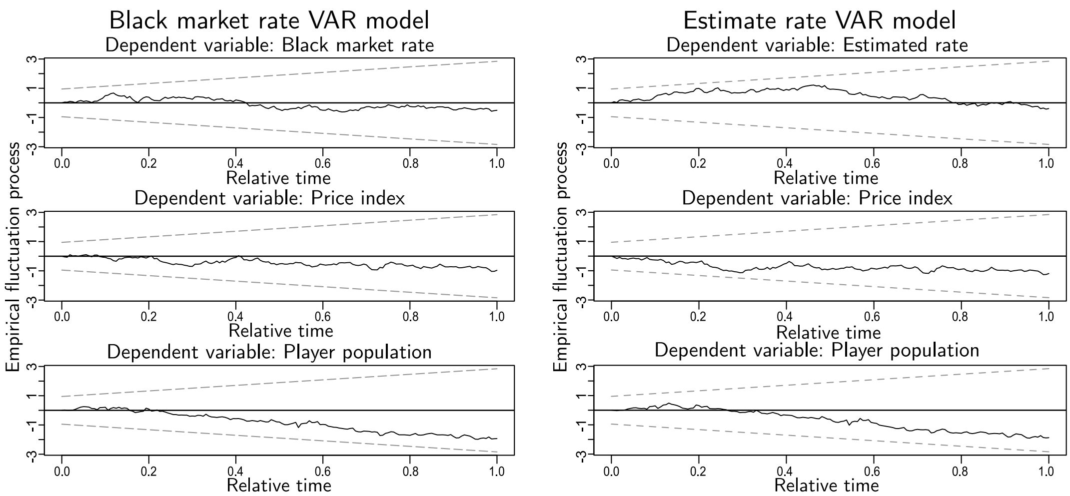


Structural stability

In terms of structural stability, we conducted a test that estimates the model with increasing sample size and forecasts the value for the next period at each step. The differences between the actual *future* value and the forecast are then summed, and statistical significance from zero is tested (Enders 2015, p. 105).

The procedure provides an intuitive assessment of coefficient stability once plotted—the dashed lines in Figure 4.6 illustrate the upper and lower confidence intervals. In other words, values above or below these lines would indicate structural change (Enders 2015, p. 106). The resulting plots, which can be observed in Figure 4.6, suggested that both models were structurally stable.

Figure 4.6: Structural stability test results



Collinearity

Lastly, we also investigated the condition index of the data matrices, following Adomavicius *et al.* (2012), and found evidence of collinearity. While this may threaten the precision of our forecasts, researchers who utilize the VAR framework often disregard the issue and accept the consequences (Adomavicius *et al.* 2012). Moreover, Hyndman & Athanasopoulos (2018, chapter 5.9) add that strong collinearity may not be as problematic for forecasting if the future observations of our predictors do not significantly deviate from the historical norm.

Chapter 5

Results and discussion

The first section of this chapter briefly introduces tools useful for interpreting estimated VARs and follows by putting forward our findings. After that, the forecasting results are presented. These are then reviewed and discussed in the final section.

5.1 Empirical results

Researchers employing vector autoregression are often not concerned with slope parameters and the usual goodness of fit criteria[1] (Stock & Watson 2001; Kilian & Lütkepohl 2017, pp. 37, 59). Instead, they focus on other measures, such as 'Granger causality' or 'innovation accounting,' which are thought to be more insightful (Stock & Watson 2001; Enders 2015, p. 302).[2]

Nevertheless, we report the standard goodness of fit measures in Table B.2 and Table B.3 for each model. First, the usual R^2 was relatively high in all equations—this was expected due to the number of lagged variables.[3] On the other hand, the adjusted R^2 seemed to penalize the price index equations in both models the most (proportionally). This could indicate that the non-volume-weighted index was, indeed, imprecise (as discussed in Subsection 4.1.2), which was further suggested by the relatively low F statistic, though still statistically significant at the 5% level. In all other cases, however, the adjusted coefficient of determination and the results of the F-tests indicated a satisfactory fit.

[1]In our case, reporting the slope parameters was also unfeasible and unnecessary due to the difficulty of typesetting and interpreting such a large number of coefficients, respectively.

[2]We follow a similar structure to Stock & Watson (2001), Adomavicius *et al.* (2012), Cover & Mallick (2012), and Pavlíček & Kristoufek (2019) in reporting the results.

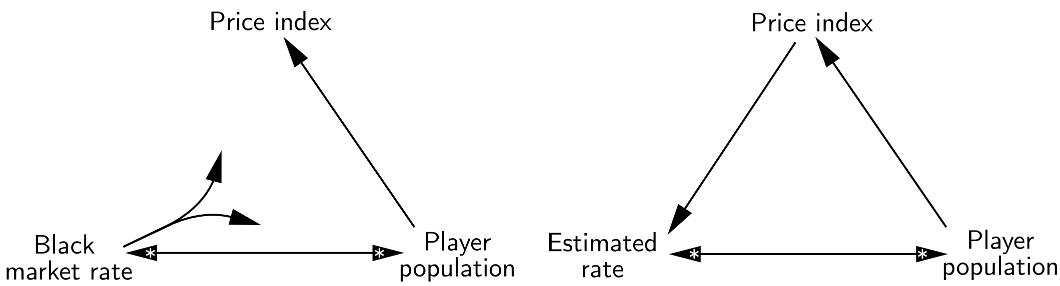
[3]Note that R^2 either increases or stays constant with the addition of a new variable (Stock & Watson 2019, p. 223).

5.1.1 Granger causality

Granger causality describes a relationship wherein the past values of a time series variable are helpful in predicting the future course of another variable (Stock & Watson 2001). Kilian & Lütkepohl (2017, p. 198) note that Granger causality merely implies precedence since establishing causation is a much more demanding task. Thus, researchers need to be careful not to commit the *post hoc* fallacy while interpreting the results. In a VAR model, it is possible to uncover *Granger-causal* relationships by, for example, testing whether the relevant coefficients are statistically different from zero (no relationship), assuming that stationarity holds (Enders 2015, p. 306).

We report the results of the pairwise as well as the multiple-relationship analysis in a visual form in Figure 5.1 by considering a range of lags. Each arrow represents the direction and the significance of Granger causality at the level of 10% (indicated by an arrow with no asterisk) or 5% and lower (a single asterisk). First of all, there seemed to be a highly significant bidirectional Granger causality between the exchange rates and the player population variables in both models. Furthermore, the transformed[4] player population series appeared to Granger-cause the price index in the black market rate and the estimated rate VAR models at the significance level of 10%. This suggested that player population seemed to be an important variable in both systems. On the other hand, there appeared to be a Granger-causal relationship coming from the price index series only in the second model. In the black market rate VAR system, the exchange rate also seemed to Granger-cause the subvector of the other two series. However, no such feedback was found in the latter model.

Figure 5.1: Granger causality between the variables of interest



[4]First difference of natural logs, as described in Subsection 4.2.3. We omit this specification for the sake of brevity. Therefore, unless otherwise noted, the transformed series ought to be assumed.

5.1.2 Innovation accounting

Innovation accounting is a set of tools comprising of 'forecast error variance decomposition' and 'impulse response function' analysis used to investigate the effect of sudden shocks on the variables of interest (Enders 2015, p. 302). Similarly to the JB test examined in Subsection 4.2.3, the Cholesky matrix decomposition is utilized in these procedures. However, as Enders (2015, p. 302) states, low correlations among error terms may render variable ordering irrelevant, which seemed to be the case in our analysis.

For our intents and purposes, it is helpful to rewrite the general VAR(p) model from Equation 4.2 as a function of its past errors:

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i u_{t-i}, \text{ for } t = 1, 2, ..., T, \tag{5.1}$$

where μ is the unconditional mean of x_t and ϕ_i are matrices of coefficients representing the effects of past errors u_{t-i} on x_t (Tsay 2005, p. 362; Enders 2015, pp. 294-295).

Forecast error variance decomposition

A shock in a specific sequence of errors may trigger a system-wide response. The decomposition attempts to assess the extent to which this shock explains the forecast error variance of a particular variable for a specific time horizon (Enders 2015, p. 302).

Letting $\sigma_e(n)^2$, $\sigma_i(n)^2$, and $\sigma_p(n)^2$ to be the n-days-ahead forecast error variances of the respective variables in the black market rate model (see Equation 4.5), we may, for example, express the proportion of $\sigma_e(n)^2$ due to the shock in its *own* error as:

$$\frac{\sigma_e^2 \left[\sum_{j=0}^{n-1} \phi_{11}(j)^2\right]}{\sigma_e^2 \left[\sum_{j=0}^{n-1} \phi_{11}(j)^2\right] + \sigma_i^2 \left[\sum_{j=0}^{n-1} \phi_{12}(j)^2\right] + \sigma_p^2 \left[\sum_{j=0}^{n-1} \phi_{13}(j)^2\right]}$$

where ϕ_{1k} are the parameter matrices from Equation 5.1 (Enders 2015, pp. 301-302).

We provide an overview of the results in Table 5.1 for the time horizon of up to 56 days, though the values were more or less stabilized after two weeks. Overall, each variable was, by far, best at explaining its forecast error variance, especially in the short run, which should generally be expected (Enders 2015, p. 302). In case of the two rates, the proportion of the effects of own shocks lowered to 92.6% for the black market rate and roughly 90% for the latter rate after two weeks. Furthermore, we see that a sudden change in the price index explained about 6% of the estimated rate's error variance after seven days and onward. An even weaker pattern was present in the black market rate model, indicating that the future values of both rates were not strongly affected by a sudden change in the price index. Similarly, shocks in the player population variable were only able to explain a small fraction of the error variance in both exchange rates. Moreover, the exchange rates also accounted for a relatively low proportion of the variance in other variables. However, it is worth noting that the black market rate explained approximately 8.5% of the forecast error variance in the player population variable.

Table 5.1: Forecast error variance decomposition results

Dependent variable		**Shock**		
	Days ahead	Exchange rate*	Price index	Player population
Black market rate	1	100.0000%	0.0000%	0.0000%
	7	92.6962%	2.9384%	4.3654%
	14	92.6437%	2.9689%	4.3874%
	56	92.6431%	2.9690%	4.3879%
Price index	1	0.5543%	99.4457%	0.0000%
	7	2.9641%	95.1993%	1.8366%
	14	3.0279%	95.1102%	1.8619%
	56	3.0283%	95.1097%	1.8620%
Player population	1	3.6846%	0.0482%	96.2672%
	7	8.3261%	2.3438%	89.3301%
	14	8.4875%	2.3998%	89.1126%
	56	8.4883%	2.3999%	89.1118%
Estimated rate	1	100.0000%	0.0000%	0.0000%
	7	90.1089%	5.9115%	3.9796%
	14	89.9847%	5.9376%	4.0777%
	56	89.9841%	5.9379%	4.0779%
Price index	1	0.1082%	99.8918%	0.0000%
	7	2.2779%	95.6939%	2.0282%
	14	2.3699%	95.5186%	2.1115%
	56	2.3703%	95.5172%	2.1124%
Player population	1	0.5135%	0.1560%	99.3306%
	7	0.7044%	2.5989%	96.6967%
	14	0.7729%	2.7471%	96.4800%
	56	0.7730%	2.7473%	96.4798%

Note: *This variable represents either the black market rate or the estimated rate depending on the considered model. Moreover, the order of the variables was $\Delta le_t, \Delta li_t, \Delta lp_t$.

Impulse response function analysis

Returning to Equation 5.1, ϕ_i also represents the impact of the current errors u_t on x_{t+i} (i.e., the future observations), from which the name 'impulse response function' is derived. To circumvent the problem of correlation in u_t, the Cholesky matrix decomposition is used. The resulting parameter matrices ϕ_i^* are considered for interpretation purposes (Tsay 2005, p. 362).

According to Stock & Watson (2001), impulse response functions, in their visual form, depict the reaction of each variables' present and future values to a positive shock in a specific error at time t.[5] Furthermore, it is important to include confidence intervals in the analysis due to the fact that estimated parameters are used in the process of obtaining the results, which introduces uncertainty[6] (Enders 2015, p. 299).

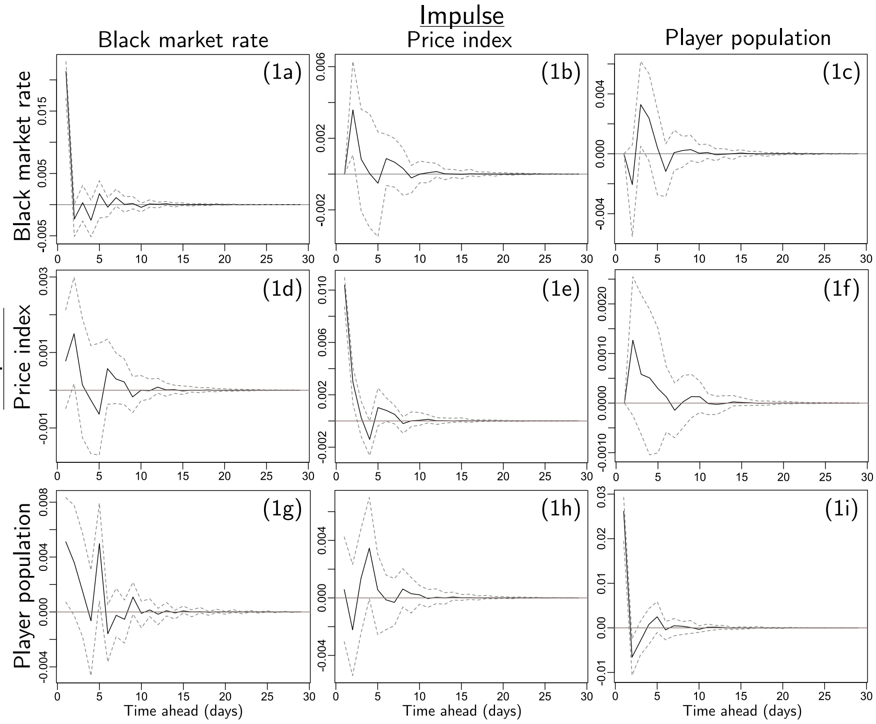
In Figure 5.2 and Figure 5.3, we illustrate the responses of variables to shocks in the fitted black market rate VAR and the estimated rate VAR, respectively. First, in Figure 5.2, we see that a positive shock in the black market rate (Panel (1a)) naturally caused a jump in its value, but there were no other lasting or significant effects with 95% confidence intervals. The estimated rate in Panel (1a) from Figure 5.3 followed the same pattern.

Moreover, while the short-term response of the two exchange rates to a shock in the price index was significantly different from zero (Panels (1b) & (2b) in Figure 5.2 & Figure 5.3), the direction of the change was, oddly enough, opposite in each. This could perhaps be explained by the delayed reactions of the estimated rate (i.e., it often appears to appreciate or depreciate later than the black market rate), which are best visible in Figure 4.1. On the other hand, the response of both rates to an unexpected increase in player population was somewhat similar as there was a narrowly significant rise three and five days after the shock occurred, respectively (Panels (1c) & (2c)). In addition, the reaction of the price index variable to a shock in player population appeared to be almost identical in both models (Panels (1f) & (2f)), which also seemed to be the case in the opposite direction (Panels (1h) & (2h)). All in all, we see that all the effects gradually converged to zero, which follows from the notion that shocks have no permanent impact on stationary time series (Enders 2015, p. 295).

[5] Given that other error terms amount to zero and the error of interest reverts to zero in the following periods.

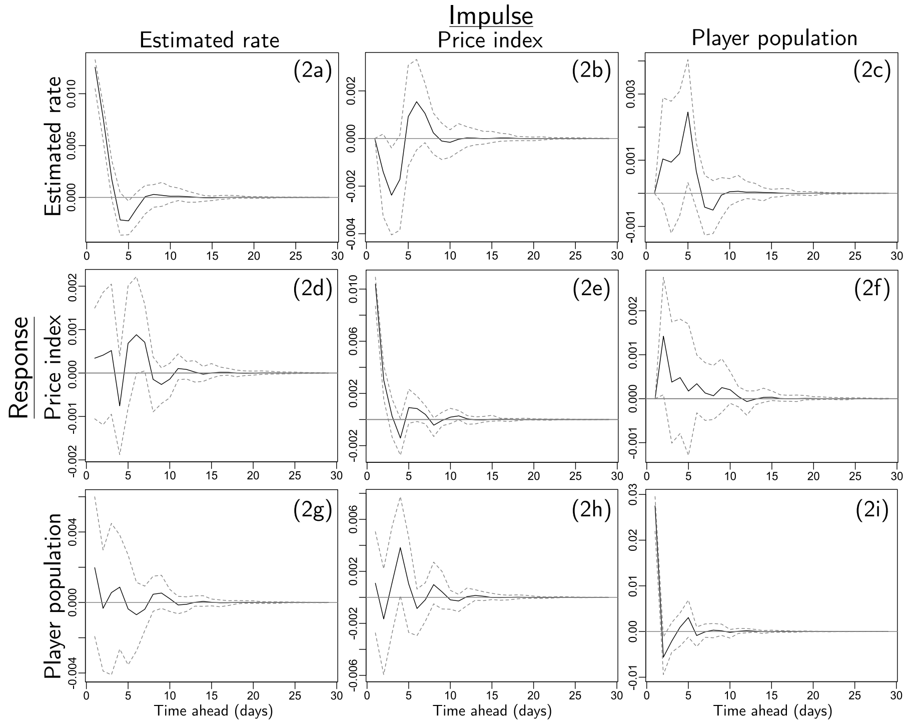
[6] Bootstrap confidence intervals were utilized as per Enders (2015, pp. 299-300).

Figure 5.2: Impulse response functions – black market rate VAR model



Note: 95% bootstrap confidence intervals with 100 runs were used. Further, the order of the variables was $\Delta le_t^{blk}, \Delta li_t, \Delta lp_t$.

Figure 5.3: Impulse response functions – estimated rate VAR model



Note: 95% bootstrap confidence intervals with 100 runs were used. Further, the order of the variables was $\Delta le_t^{est}, \Delta li_t, \Delta lp_t$.

5.1.3 Evaluation of out-of-sample performance

In a VAR system, a one-period-ahead forecast is computed for every equation in the model. For instance, the next-day forecast of x_t from Equation 4.2 would simply be calculated as:

$$\hat{x}_{T+1|T} = \hat{A}_0 + \hat{A}_1 x_T + ... + \hat{A}_p x_{T+1-p},$$

where $\hat{A}_i$ are the estimated parameter matrices (Hyndman & Athanasopoulos 2018, chapter 11.2). We utilized this procedure in a recursive forecasting scheme, which is standardly used in the literature (Ferraro *et al.* 2011; Rossi 2013). After computing the first one-period-ahead forecast, the sample was extended by one observation due to its relatively small size, and the models were re-estimated.[7] Then, another one-period-ahead forecast was generated, and the process was repeated until the final data points were reached, generating a sequence of next-day forecasts (Zivot 2013).

For the out-of-sample exercise, we employed two benchmark models, against which our VAR forecasts were compared. The first one was a driftless random walk (naïve / simple no-change model), briefly described in Subsection 4.2.1, which is the industry standard. We also included a simple rolling average[8] to gauge the quality of the main models. All the predictions can be seen in Figure 5.4 and Figure 5.5, together with the actual values of the two exchange rates.

Following Rossi (2013), we assessed the forecasting performance of all the models using commonly used evaluation methods at time horizons of 1 to 56 days. Moreover, Hyndman & Athanasopoulos (2018, chapter 3.4) define the error of the h-steps-ahead forecast (ε_{T+h}) as the difference between the actual future value and the predicted value of a time series variable. Then, the first two measures of h-steps-ahead forecasting precision, mean absolute error (MAE) and root mean square error (RMSE), can be expressed as (Gujarati 2014, p. 323):

$$\text{MAE}_n = \frac{1}{n} \cdot \sum_{h=1}^{n} |\varepsilon_{T+h}|, \ \text{RMSE}_n = \sqrt{\frac{1}{n} \cdot \sum_{h=1}^{n} \varepsilon_{T+h}^2}.$$

We also evaluate the directional accuracy of our forecasts by utilizing the mean

[7]These generally yielded similar diagnostic results. We omit to report them for brevity.

[8]The mean value of the dataset is re-calculated after each addition of a new observation (mean forecast).

directional accuracy (MDA) measure, which outputs the proportion of *directionally* correct predictions, and is given as (Pavlíček & Krištoufek 2019):

$$\mathrm{MDA}_n = \frac{1}{n} \cdot \sum_{h=1}^{n} a_h,$$

where a_h is equal to 1 if the sign of the actual and predicted change is the same, i.e., $sign(\Delta le_{T+h}) = sign(\Delta \widehat{le}_{T+h|T})$; otherwise, a_h is zero.

Figure 5.4: Transformed black market rate forecasts

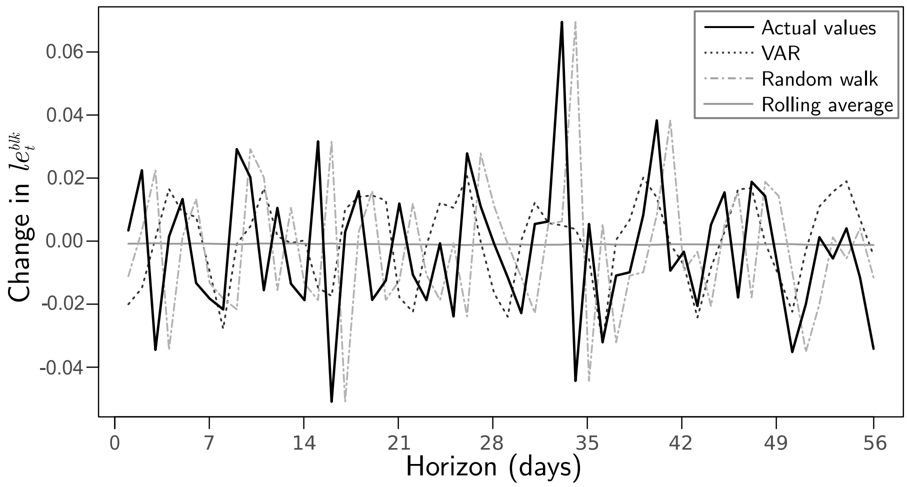


Figure 5.5: Transformed estimated rate forecasts

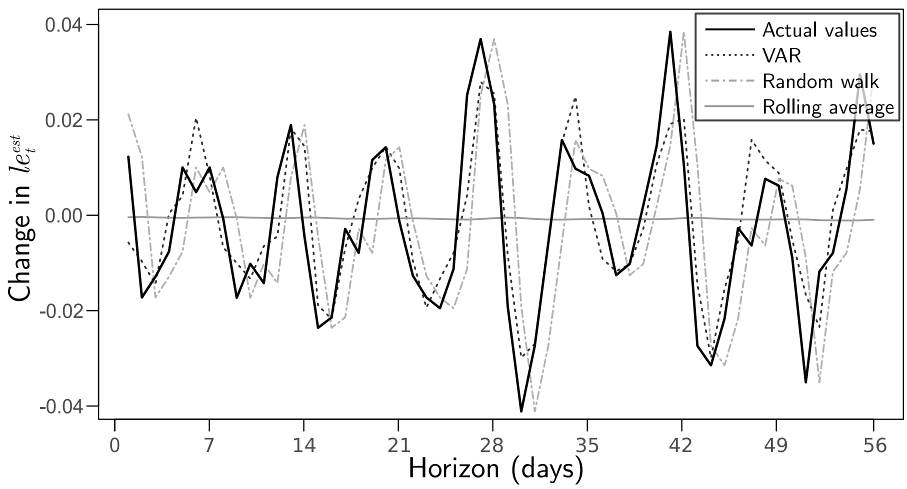
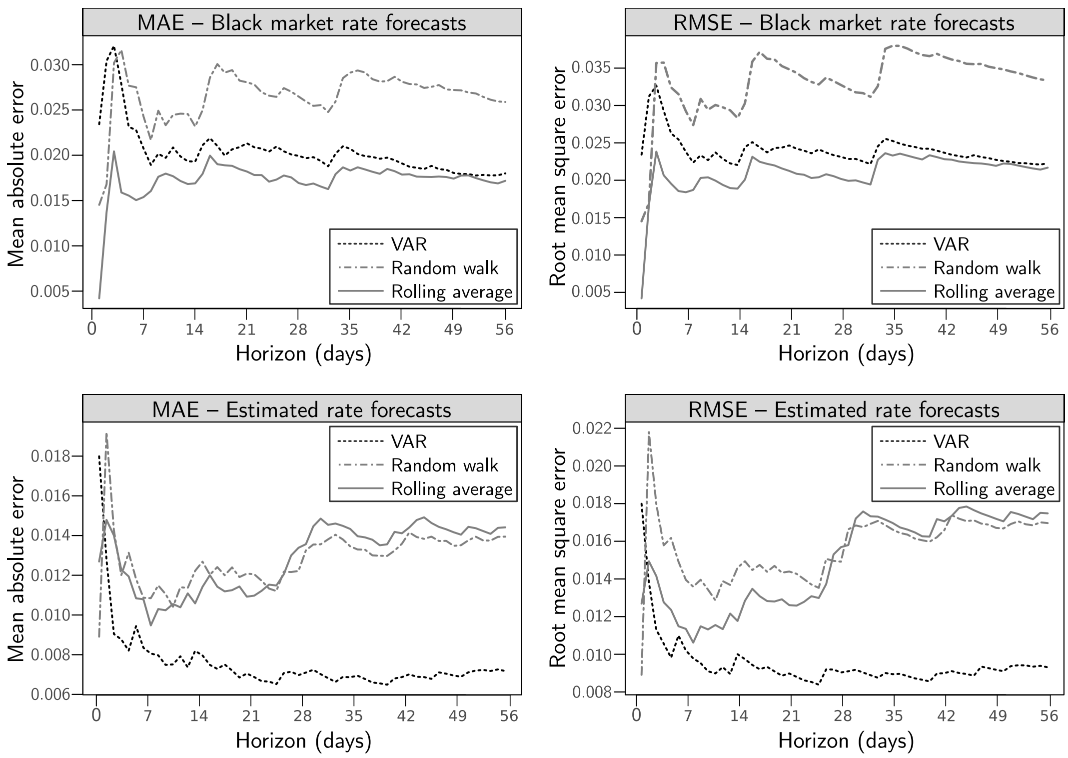


Figure 5.6 shows the graphed forecasting performance results for the considered VAR models in first differences and the corresponding benchmarks over the time period of eight weeks starting from August 17th, 2020.[9] Firstly, let us focus on the results of the black market rate models (top panels in Figure 5.6).

[9]See Table B.4 in Appendix B for the tabular results.

We see that the black market rate VAR failed to surpass the naïve model in terms of both accuracy measures only at the very beginning. However, after less than a week, the VAR model achieved higher precision in both statistics. It continued to perform better than the driftless random walk since, especially with regards to the RMSE—the VAR model was roughly one and a half times more accurate than the random walk at the time horizon of four weeks and later. Nonetheless, it is important to note that by no means were the black market rate VAR's predictions precise as the rolling average appeared to outperform the first model at all time horizons. Though, its accuracy seemed to decrease in time, suggesting that the VAR might eventually surpass the simple model.

Figure 5.6: MAE and RMSE – graphical results



Furthermore, focusing on the bottom two panels in Figure 5.6, the estimated rate VAR forecasts were also initially bested by the random walk. However, we see that this changed relatively quickly. At the time horizon of about three days, the VAR model was able to forecast the estimated rate more accurately than the benchmarks in terms of the two considered evaluation methods and remained unsurpassed since. For instance, after 56 days, the MAE of the VAR model was equal to 0.007175, while the same statistic for the random walk was almost twice as large (see Table B.4). Moreover, we may observe that the rolling average generated worse forecasts than the VAR, unlike in the previous instance.

Next, following Pavlíček & Krištoufek (2019), we report the MDA results in Table 5.2. We see that the predictions generated by the VAR models were the most directionally accurate for both variables in question, though narrowly in the first case. The black market rate VAR correctly identified more than 57% of the changes in the exchange rate, while the estimated rate VAR scored even better (82.14%). In contrast, the black market rate forecasts generated by the driftless random walk were the least precise, direction-wise. However, in terms of the estimated rate, the simple no-change model correctly identified the direction of change in more than two-thirds of the data points.

Table 5.2: Mean directional accuracy results

Variable	**Model**	**MDA**
Black market rate	VAR	57.14%
	Random walk	46.43%
	Rolling average	55.36%
Estimated rate	VAR	82.14%
	Random walk	71.43%
	Rolling average	57.14%

Note: The full sequences of forecasted values were used.

Finally, we conducted the Diebold-Mariano test (DM), which is used to compare the predictive accuracy of two competing models (Diebold & Mariano 2002; Rossi 2013; Zivot 2013). In this procedure, the precision of each method is assessed through a loss function (typically absolute or squared error), and the null hypothesis of equal forecast precision is tested against the alternative of non-equal accuracy using the Diebold-Mariano statistic. More specifically, the DM test considers the difference of the two loss functions and evaluates the following hypothesis (Zivot 2013):

$$H_0 : \mathbb{E}(L(\varepsilon^1_{T+h}) - L(\varepsilon^2_{T+h})) = 0$$

where L represents the chosen loss function, and ε^i_{T+h} are the forecast errors of the first ($i = 1$) or the second model ($i = 2$).

In our case, we compared the forecast precision of the VARs against the random walk models using a one-sided hypothesis test with the alternative of lower accuracy of the naïve method. From the results in Table 5.3, the rejection of the null hypothesis at the 1% significance level suggested that the VAR models' forecasts were more precise in both cases and for both loss functions.

Table 5.3: Diebold-Mariano test results

Models	Diebold-Mariano test		
	Loss function	Statistic	P-value*
Black market rate VAR vs Random walk	Absolute	-3.1675	< 0.01
	Squared	-2.6421	< 0.01
Estimated rate VAR vs Random walk	Absolute	-5.3502	< 0.01
	Squared	-4.3199	< 0.01

Note: *The alternative hypothesis is that the random walk yielded less accurate predictions. Furthermore, the full sequences of forecast errors were used for the comparison.

Overall, we see that both VAR models were better than the driftless random walk at forecasting the respective exchange rates based on the DM test results, MDA, and the two error measures after a relatively short *burn-in* period, failing to support our original hypothesis. However, it is important to interpret these results with regards to the forecasting strategy used; that is, we have to remember that all of the predictions were, essentially, one-period-ahead.

Although the two models are not necessarily comparable as they consider two different measures of the official exchange rate, it is evident that the VAR framework performed significantly better than the benchmarks in the latter case. This could perhaps be explained by the smoother, less erratic, and more seasonal pattern of the estimated rate, which is best illustrated in Figure 5.5. On the other hand, the black market rate might be closer to behaving like a real-world rate in terms of its fluctuations. Moreover, in Figure 5.1, we may observe that the price index variable was only seen to Granger-cause the estimated rate, further helping in forecasting the variable. Nevertheless, it is important to remark that price levels are thought to influence exchange rates in the long term, as noted in Subsection 4.1.2.

5.2 Discussion

We view these findings as interesting, especially in the context of the somewhat poor results of forecast error variance decomposition and impulse response functions—while there were some narrowly significant movements in the impulse response functions, shocks in the price index and player population variables accounted only for a small amount of variance of the forecast error. All in all, this suggested that the system was less interconnected than initially thought. In contrast, the Granger causality tests uncovered a few significant relationships, namely the bidirectional feedback between player population and exchange rates.

Proposed explanations for finding predictive ability

Clearly, our results differ from the usual exchange rate forecasting literature in that we find some predictive ability in the short term, similarly to Kim *et al.* (2015) and Kim *et al.* (2017) in the case of virtual exchange rates. There are a few reasons that we can think of to justify this relative success. In the real world, Ferraro *et al.* (2011) have found a robust relationship between exchange rates and oil prices in the short run using daily data. They attributed this predictive ability partly to the frequency of the series by comparing the forecasting accuracy results with monthly and quarterly data. While this could also be the case in our analysis, at this point in time, it would be difficult to investigate this assertion within the virtual economy in question because there exists only a limited amount of high-quality data, rendering long-run analyses unfeasible.

Another explanation could be that despite its level of complexity, the virtual economy of Old School RuneScape may not be as complicated *as it would need to be* in order for exchange rates to be unpredictable. In other words, it is possible that the more diverse and interconnected an online economy is with the physical world, the less predictable virtual-to-real exchange rates might be.

Limitations

Despite our relative success in the out-of-sample exercise, we believe that the performance and the in-sample fit of our models could be improved in certain ways. Firstly, a price index weighted by the quantities of the individual items sold on the GE may more precisely represent the price level of the in-game economy, possibly yielding better results. Secondly, other variables could be added to the model—for instance, the proportion of gold farmers in the player population as these individuals appear to have an influence on the market (see Subsection 3.2.4). Another way to possibly enhance the accuracy and fit would be to implement more flexible econometric frameworks.

Contribution

This thesis demonstrates that it is possible to approach virtual economies in a similar manner as their real-world counterparts. Although there are some key differences in these two environments, applying regular economic analysis is shown to be feasible. While our results may not be helpful in solving the exchange rate unpredictability puzzle, they may at the very least provide game developers with a framework for monitoring the developments in the exchange rate of their virtual currency since selling the in-game items is prohibited in OSRS (see Subsection 3.2.4). However, as Papagiannidis *et al.* (2008) report, real-money trading is allowed in some virtual worlds, and thus, perhaps there may be potential to achieve *real* value elsewhere.

Recommendations for future research

These findings generate additional challenges. For instance, how complex should a virtual economy be to reliably mimic reality? Another question worthwhile examining could be whether the efficient market hypothesis holds in in-game markets, i.e., are the markets for virtual goods informationally efficient? We believe that answering these queries may provide meaningful insights into the nature of virtual and real economies.

Chapter 6

Conclusion

Virtual worlds have been acknowledged as environments with high research potential by generating massive amounts of data on human behavior, garnering the attention of academics from various fields (Bainbridge 2007). These spaces tend to feature complex in-game economies that are open to effortless manipulation by design (Castronova 2002). In addition, some have shown that virtual economic behavior is consistent with the physical world (Castronova *et al.* 2009a;b; Chesney *et al.* 2009), which could be particularly useful to experimental economists or macroeconomic researchers.

Our goal was to examine the predictability of virtual-to-real exchange rates as only a limited amount of research has investigated this topic. Unlike any other works, we were interested in testing whether the findings of Meese & Rogoff (1983), who discovered that forecasts generated by various exchange rate models were as inaccurate as those of a driftless random walk, were applicable to a virtual setting. Therefore, we believe that the added value of this thesis is twofold. First, it approaches the topic of virtual exchange rate forecasting in the context of the unpredictability puzzle, bridging the gap between two widely different bodies of research. Second, past works concerned with virtual currencies have not examined the influence of standard economic variables on exchange rate fluctuations.

To address this query, we focused on a popular online fantasy game world of Old School RuneScape and compiled a unique time series dataset with a daily frequency from various online sources. The data consisted of two competing measures of the official virtual-to-real exchange rate, the in-game price index, and the average daily player population. After conducting an exploratory analysis, the standardly used vector autoregressive framework was utilized, and models were constructed. The diagnostic tests and the goodness of fit measures of the two fitted VARs in first differences generally yielded satisfactory results. Apart from a highly significant bidirectional Granger-causal relationship between the two measures of the exchange rate and the player population, the interconnectedness of the system in question was shown to be weaker than initially expected.

We hypothesized that the driftless random walk would remain unsurpassed in terms of the considered evaluation methods in the short run. Thus, one-period-ahead recursive forecasts were generated and evaluated using four commonly employed measures of accuracy (MAE, RMSE, MDA, and the DM test). However, despite the subpar estimation results, the predictions generated by the two VARs bested the naïve forecasts in both cases, failing to support our hypothesis. These outcomes suggested that the virtual economy in question did not appear to be consistent with the real world in terms of exchange rate predictability. We suggested two explanations for this successful yet disappointing outcome—daily data frequency or the economy's lack of complexity.

Further research could focus on extending the timeframe and evaluating the exchange rate dynamics in the long run, perhaps with aggregated monthly or quarterly data. We would also recommend analyzing other virtual economies, finding additional variables, implementing different econometric frameworks, constructing more precise predictors (namely, a volume-weighted price index), and testing the efficient market hypothesis in the in-game markets to provide new evidence. Overall, we believe that virtual economies should be given more attention—after all, masses of people from all around the world daily participate in these rich environments, each with different goals and motivations. In Edward Castronova's (2011) words, "people in the game industry are sitting on experiments that are going on for years with millions of people in them," and yet, they hardly ever publish any research.

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Table 6.1: Used R packages

Packagename	**Version**	**Maintainer**
aTSA	3.1.2	Debin Qiu
car	3.0.10	John Fox
carData	3.0.4	John Fox
corrgram	1.13	Kevin Wright
dplyr	1.0.5	Hadley Wickham
forecast	8.13	Rob Hyndman
ggplot2	3.3.2	Thomas Lin Pedersen
lmtest	0.9.38	Achim Zeileis
MASS	7.3.53	Brian Ripley
Metrics	0.1.4	Michael Frasco
pacman	0.5.1	Tyler Rinker
papeR	1.0.4	Benjamin Hofner
rstudioapi	0.13	Kevin Ushey
sandwich	3.0.0	Achim Zeileis
stargazer	5.2.2	Marek Hlavac
strucchange	1.5.2	Achim Zeileis
tidyr	1.1.2	Hadley Wickham
tseries	0.10.48	Kurt Hornik
urca	1.3.0	Bernhard Pfaff
vars	1.5.3	Bernhard Pfaff
xtable	1.8.4	David Scott
zoo	1.8.8	Achim Zeileis

Note: R is an open-source language and environment for statistical computing developed by the R Foundation.

Appendix A

Additional definitions

In this section, we summarize the more complicated definitions and procedures used in this thesis.

A.1 Least squares assumptions for forecasting

As specified by Stock & Watson (2019, pp. 159, 571-573), the least squares assumptions for time series forecasting consider the following general model with k regressors, p representing the number of lags of the dependent variable, and q_k depicting the lag order of z_k:

$$\begin{aligned} y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \ldots + \beta_p y_{t-p} + \delta_{11} z_{1t-1} + \delta_{12} z_{1t-2} + \\ + \ldots + \delta_{1q_1} z_{1t-q_1} + \ldots + \delta_{k1} z_{kt-1} + \delta_{k2} z_{kt-2} + \ldots + \\ + \delta_{kq_k} z_{kt-q_k} + \nu_t \quad \text{for } t = 1, 2, \ldots, T, \end{aligned}$$

where the following is assumed to hold:

1. The expected value of ν_t given $y_{t-1}, y_{t-2}, \ldots, z_{1t-1}, z_{1t-2}, \ldots, z_{kt-1}, z_{kt-2}, \ldots$ is zero.

2. The distributions of all considered variables are stationary. Moreover, for a large *time distance* j, $(y_t, z_{1t}, \ldots, z_{kt})$ and $(y_{t-j}, z_{1t-j}, \ldots, z_{kt-j})$ are independent (stationarity and weak dependence).

3. The variables $y_t, z_{1t}, \ldots, z_{kt}$ have finite and nonzero kurtoses (no significant outliers).

4. No predictor is a linear function of another predictor (no perfect multicollinearity).

A.2 Augmented Dickey–Fuller test

Following Enders (2015, p. 206), we consider a simple model $y_t = a_1 y_{t-1} + \nu_t$ and continue by subtracting y_{t-1} from both sides. The Dickey-Fuller procedure assesses whether a unit root is present with the use of the following equations:

$$\Delta y_t = \gamma y_{t-1} + \nu_t,$$
$$\Delta y_t = c_0 + \gamma y_{t-1} + \nu_t,$$
$$\Delta y_t = c_0 + \gamma y_{t-1} + d_0 t + \nu_t,$$

where $\Delta y_t = y_t - y_{t-1}$, $\gamma = a_1 - 1$, c_0 is a constant term, t represents time, and ν_t is a sequence of independent & identically distributed random variables with zero mean and finite variance σ^2. Thus, the null hypothesis of $\gamma = 0$ is tested against the alternative of $\gamma \neq 0$. Finally, the augmented test includes lags of Δy_t, and an analogous hypothesis is tested (Enders 2015, p. 207).

A.3 Johansen test for cointegration

In this section, we briefly summarize Enders's (2015, pp. 383, 389-393) practical description of the Johansen procedure by applying it to our data. The reader is encouraged to consult the source material for a more precise explanation.

The first step in testing for cointegration using this method is to determine the ideal lag length. One could use the lag selection method for a VAR model on the original data (not differenced). In our case, for instance, the AIC and BIC suggested using eight and two lags for the first set of variables, respectively.

Next, let x_t be the *3 × 1* vector of variables. Then, the following model with a vector of intercepts B_0 is estimated:

$$\Delta x_t = B_0 + \pi x_{t-1} + \sum_{i=1}^{p-1} \pi_i \Delta x_{t-i} + u_t$$

with the left-hand side equal to $\Delta x_t = x_t - x_{t-1}$, p being the lag order, and u_t representing a vector of serially uncorrelated error terms with zero mean and covariance matrix Σ_u. Finally, the characteristic roots (estimates) of the π matrix are of particular interest, and the null hypothesis of $rank(\pi) = 0$ (i.e., no cointegrating relationships) is tested.[1]

[1] In our case, a 'trace' statistic was calculated. If its value is greater than the corresponding critical value, the null hypothesis is rejected.

A.4 Portmanteau test for serial correlation

As described by Kilian & Lütkepohl (2017, pp. 52-53), the Portmanteau procedure tests for the presence of serial correlation in the residuals by considering the following statistic:[2]

$$Q_h = T \cdot \sum_{j=1}^{h} trace(\hat{C}_j^{\intercal} \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}).$$

In the above equation, $\hat{C}_j = 1/T \cdot \sum_{t=j+1}^{T} \hat{u}_t \hat{u}_{t-j}^{\intercal}$, where $\hat{u}_t$ are the residuals from estimating Equation 4.2. The test statistic is approximately χ^2 distributed for $h/T \xrightarrow{T \to \infty} 0$, and rejecting the null hypothesis at the, for example, 5% threshold would suggest the presence of autocorrelation.

A.5 Autoregressive conditional heteroskedasticity Lagrange multiplier test

Tsay (2005, pp. 102-103) outlines the autoregressive conditional heteroskedasticity framework as a way of modeling volatility. Following Stock & Watson (2019, pp. 669-670), let ν_t be a normally distributed error term of a single-equation time series regression with zero mean and variance σ_t^2 dependent on lagged squares of ν_t. Then the ARCH model of order p can be written as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \nu_{t-1}^2 + \alpha_2 \nu_{t-2}^2 + ... + \alpha_p \nu_{t-p}^2.$$

Further, the multivariate case of the ARCH model with p lags of VAR errors u_t, as described by Kilian & Lütkepohl (2017, pp. 68-69), is expressed as:

$$vech(\Sigma_{u_{t|t-1}}) = \delta_0 + D_1 vech(u_{t-1} u_{t-1}^{\intercal}) + ... + D_p vech(u_{t-p} u_{t-p}^{\intercal}),$$

where $vech$ expresses vectorization by stacking columns of a square matrix, $\Sigma_{u_{t|t-1}}$ represents the covariance matrix of u_t conditional on u_{t-1}, u_{t-2}, and so forth, δ_0 is a vector of coefficients, and D_p is a matrix of parameters.

The ARCH-LM test is thus concerned with testing the null hypothesis of $D_1 = D_2 = ... = D_p = 0$ against the alternative of at least one nonzero term using a particular Lagrange multiplier test statistic. Rejecting the null hypothesis at a specified significance level would suggest the presence of heteroskedasticity in the residuals (Kilian & Lütkepohl 2017, p. 69).

[2]Note the difference between T (time periods) and $^{\intercal}$ (transpose).

Appendix B

Additional results

This appendix contains figures and tables with results to which we refer in the later chapters of the thesis.

Figure B.1: Full black market rate time series

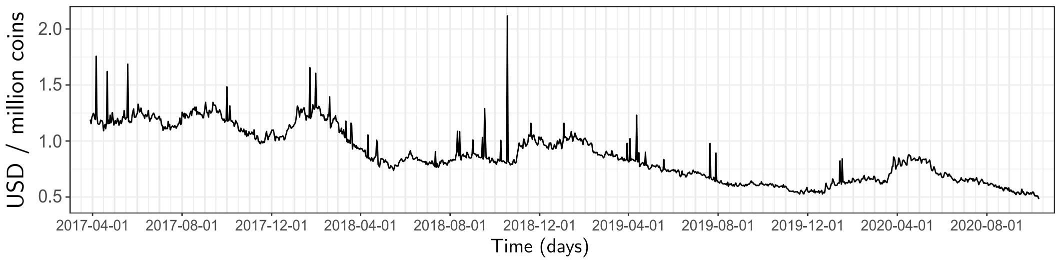

Table B.1: Engle-Granger test results

Response	**Engle-Granger test**			
	Lags (criterion)	No trend p-value	Linear trend p-value	Quadratic trend p-value
Black market rate	2 (BIC)	0.06	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10
Price index	2 (BIC)	< 0.01	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10
Player population	2 (BIC)	< 0.01	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10
Estimated rate	3 (BIC)	> 0.10	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10
Price index	3 (BIC)	< 0.01	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10
Player population	3 (BIC)	0.06	> 0.10	> 0.10
	8 (AIC)	> 0.10	> 0.10	> 0.10

Note: The response is regressed on the other two variables with respect to the two considered sets of variables. Furthermore, only log transformations were applied.

Table B.2: Goodness of fit of the black market rate VAR model

	Dependent variable		
	Exchange rate	Price index	Player population
R^2	0.3507	0.1583	0.5825
Adjusted R^2	0.2777	0.0636	0.5355
Residual Std. Error (df = 160)	0.0214	0.0104	0.0266
F Statistic (df = 18; 160)	4.8017***	1.6716**	12.3993***

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Table B.3: Goodness of fit of the estimated rate VAR model

	Dependent variable		
	Exchange rate	Price index	Player population
R^2	0.6217	0.1624	0.5520
Adjusted R^2	0.5791	0.0681	0.5015
Residual Std. Error (df = 160)	0.0126	0.0104	0.0276
F Statistic (df = 18; 160)	14.6055***	1.7229**	10.9503***

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

Table B.4: MAE and RMSE – tabular results

Horizon	**Model**	**Black market rate**		**Estimated rate**	
		MAE	RMSE	MAE	RMSE
1 Day	VAR	0.023428	0.023428	0.017991	0.017991
	Random walk	0.014520	0.014520	0.008908	0.008908
	Rolling average	0.004203	0.004203	0.012692	0.012692
3 Days	VAR	0.032083	0.032672	0.009041	0.011304
	Random walk	0.030189	0.035683	0.014258	0.017973
	Rolling average	0.020430	0.023826	0.014004	0.014152
7 Days	VAR	0.020755	0.023795	0.008347	0.010203
	Random walk	0.024282	0.029240	0.010867	0.013951
	Rolling average	0.015381	0.018397	0.010790	0.011351
14 Days	VAR	0.019306	0.022039	0.008193	0.010004
	Random walk	0.023169	0.028325	0.012224	0.014594
	Rolling average	0.016898	0.018878	0.010583	0.011768
28 Days	VAR	0.019931	0.023128	0.006971	0.009035
	Random walk	0.026482	0.032742	0.012217	0.014916
	Rolling average	0.016935	0.020178	0.013385	0.015688
56 Days	VAR	0.018000	0.022262	0.007175	0.009309
	Random walk	0.025879	0.033337	0.013945	0.016965
	Rolling average	0.017180	0.021684	0.014414	0.017483