Reader report on PhD thesis by Morteza Kerachian

Selected problems in relativistic cosmology

In contemporary cosmology one starts with a coarse-grained description of the universe as a spatially homogeneous and isotropic spacetime filled with matter, called FLRW (Friedmann–Lemaitre–Robertson–Walker) model, which provides us with the knowledge of the evolution of background metric and distribution of matter in the early stage, and smoothed-out metric and distribution of matter in later stages. The FLRW model of the actual universe is known, since it is uniquely defined by a handful of parameters that have been determined with high precision in the WMAP and Planck experiment. Still, for the sake of better understanding of gravity – one of the four fundamental forces in Nature, described in a highly nonintuitive manner in general relativity –, it makes sense to scrutinize how physical systems behave in FRWL models depending on the values of their parameters, and revisit the evolution of the models themselves. In the thesis two problems of such kind are considered: the author analyzes motion of a uniformly accelerated observer ("rocketeer") in FLRW models, and classifies FLRW models from the point of view of the theory of dynamical systems.

In a FLRW universe, equations for the worldline of a uniformly accelerated observer are just a slight generalization of the equations that hold in Minkowski space; however, they are solved, in a sense, in complementary way. While in Minkowski space we obtain for the observer's worldline parametric equations with the proper time as parameter, in a FLRW universe we assign proper time to the points of the worldline, if we are interested in it at all, *after* establishing the form of the worldline. This is easily seen from the form of the equation for rapidity (hyperbolic angle between the worldline and the time axis, defined in terms of the velocity of the observer v as $\zeta = \operatorname{arctanh} v$). The equation reads $d(R \sinh\zeta)/dt = aR$, where R is scale parameter, a is acceleration and t is cosmological time; and since $d(\sinh\zeta)/dt = \cosh\zeta d\zeta/dt = d\zeta/d\tau$, where τ is proper time, for Minkowski space, which has $R = const$, we immediately obtain $\zeta = a\tau$. On the other hand, in a FLRW universe – even in the Milne universe, which is just the interior of a future light cone in Minkowski space covered by hyperbolic spherical coordinates – we need to solve for $\zeta(t)$, and only then we can find $\zeta(\tau)$ from $\tau = \int \frac{dt}{\cosh\zeta}$. All in all, the problem would make a nice exercise in a problem book in general relativity. (If it is not there yet. But I surely did not find it in my Lightman-Press-Price-Teukolsky.)

The dynamics of FLRW models is given by a system of first order differential equations for the scale parameter R and the density/densities of matter ρ, and if the theory contains a scalar field ϕ, second order differential equation for ϕ. In addition, we have algebraic equations $p = p(\rho)$ and $V = V(\phi)$, where p is pressure and V is potential. This is all we need for model building; however, if we want to apply the methods of qualitative theory of differential equations to a particular model, we rather pass to new variables. To get rid of the square root in the equation for R, R is to be replaced by $H = \dot{R}/R$; to suppress infinities when approaching the initial and final singularity, ρ is to be replaced by $\Omega = 8\pi G\rho/(3D^2)$, where $D^2 = H^2$ for flat universe and $H^2 + R^{-2}$ for curved universe (the additional term ensuring that Ω's stay finite at the moment of maximal expansion of the universe); and to shift the singularities to the times $\mp\infty$, t is to be replaced by $\tau = \int D^{-1}dt$. The full set of Ω's has to include Ω_p and Ω_V, since p and V appear in the equations on equal footing with ρ. But these Ω's bring with themselves $\Omega_{\partial p}$, $\Omega_{\partial V}$, these bring with themselves $\Omega_{\partial^2 p}$, $\Omega_{\partial^2 V}$ etc., and to cut short the ever-increasing series, we need to restrict ourselves to special classes of functions $p(\rho)$ and $V(\phi)$. Such classes are considered also in the thesis, where the general idea of the analysis, known from the literature, is applied to the models with barytropic fluids and non-minimally coupled scalar fields. In particular, one class of V's to be found in the thesis are exponentials introduced in Copeland, Liddle and Wands (1998).

The main body of the thesis are three papers, one single-person and two written in collaboration. The thesis also contains explanatory notes that do fine work to help the reader to understand the content of the papers, however, in my opinion the author should pay more attention to the language – both grammar and logic of sentences. The exposition is well organized, and even if it rises a modest objection here and there (how the condition $\epsilon \geq 0$ became two conditions $\epsilon \geq 0$ and $\epsilon = 0$; where the condition $1 \leq w \leq 2$ for coinciding Penrose diagrams came from; how the fact that V' is negative for some ϕ implies that V is not suitable for deciding about stability; etc.), none of them is principal.

The author clearly displayed ability to work independently, so I recommend to accept the thesis and grant the author the title PhD.

Bratislava, 11/27/2020

Vladimír Balek
Dept. of Theoretical Physics
Comenius University