Report on Doctoral Thesis of Mr. Morteza Kerachian
Selected problems in relativistic cosmology.

Dr Alexander Vikman
(Institute of Physics of the Czech Academy of Sciences)

This thesis contains two main parts: one devoted to the uniformly accelerated trajectories, while another is dedicated to dynamical systems. Both parts study different aspects of the FLRW spacetimes.

I think the work of Mr Kerachian on uniformly accelerated traveller in FLRW universes is interesting and and it is very impressive that the candidate is the single author on the corresponding published paper. I believe, this paper contains some new results (in particular the study of whether uniform acceleration is enough for a return journey). This clearly demonstrates his ability for a creative scientific work.

I like the introduction to the dynamical systems in Section 3 — it is rather concise, but a comprehensive and a pleasure to read to refresh the material. Though, maybe the presentation here is a bit too dry.

I am not a big fun of Sections 5 and 6, as I think it is practically impossible to study *general* dynamics of a scalar field with a completely *unspecified* potential. For instance, one could chose a potential with infinite number of local minima, like it is the case for axion-like particles. This would introduce infinite number of critical points. Moreover, the use of normalized variables allows to draw impressive phase diagrams, but is not always very intuitive. On the other hand it is interesting to analyze the global properties of the cosmological dynamics at least for some classes of potentials. Especially, this can be useful for studies of cosmological acceleration and Dark Energy model building. Thus, these projects still have their own scientific value.

Altogether, my impression is rather positive and I have just some minor comments listed below. The author may implement optional amendments to address these issues. But even without them, I am sure, the work can be accepted and acknowledged, as a completely legitimate PhD thesis.

1. On page 6, one would better mention that (1.17) is only valid for $w = const$.

2. It is cool to see solutions (1.39)-(1.41), as I derived them at my Diploma thesis in 2001. Though, I was most probably not the first.

3. I believe, equation (1.25) could be better motivated. In particular, one could mention that this equation is just a result of the Fermi-Walker Transport prescription, see e.g. page 153 "Lecture Notes on General Relativity" by Matthias Blau http://www.blau.itp.unibe.ch/Lecturenotes.html. Indeed, we have

$$\mathcal{F}V^{\alpha} = \dot{V}^{\alpha} + \mathcal{F}^{\alpha}_{\beta}V^{\beta}\,,$$

where

$$\mathcal{F}^{\alpha}_{\beta} = a^{\alpha}u_{\beta} - u^{\alpha}a_{\beta}\,,$$

so that

$$\mathcal{F}a^{\mu} = \dot{a}^{\mu} - u^{\mu}a_{\beta}a^{\beta}\,.$$

Thus uniform acceleration corresponds to $\mathcal{F}a^{\mu} = 0$. I think the work and the candidate would profit a lot if the the literature research were more extensive.

4. In (1.53) I would be more careful with dimensions.
5. Before (3.23) I would change "To analyzing" to "To analyze".
6. On page 44 "can not" would better be changed to "cannot".
7. I would like to mention that it is OK to define a new evolution parameter τ as $d\tau = Hdt$ (as it is done between (4.5) and (4.6)) only if H cannot change the sign.
8. I think to call (4.15) and (4.16) on page 47 to be an autonomous system is a bit exaggerated. Indeed, $\ddot{\psi}$ is still present in the equations one has to use (4.19) and (4.19) to formulate this system.
9. I believe, in general here the discussion is lucking some clarity. It is obvious that the spatially-flat FLRW universe filled with barotropic hydrodynamical matter $p = w\epsilon$ and a canonical scalar field ψ has three dimensional phase space defined by $\left(\psi, \dot{\psi}, \epsilon\right)$. This is of course true for every physical system where the form of the potential $V(\psi)$ is given. I am surprised not to see this clearly written.
10. I believe, the statement below which one can read on page 48 is rather confusing.
"Note that the physical features of the universe do not dependent directly on the choice of $\Omega_{\partial V}$. In other words, different functions of the potential V does not change the phenomenological properties of the universe. This statement comes from the fact that the effective EoS parameter (4.13), the EoS parameter of the scalar field, and the relative energy density of the scalar field Ω_Ψ are independent of $\Omega_{\partial V}$."
Indeed, (4.17) does define $\Omega_{\partial V}$ and the form of the potential. Of course many physical features do depend on the form of the potential. For example, if the potential is always positive, a spatially-flat FLRW universe expands forever, while for negative potential the universe may collapse. I think the main point is that $\Omega_{\partial V}$ is not an independent variable.

Dr Alexander Vikman
(Institute of Physics of the Czech Academy of Sciences)