A report on the doctoral thesis by

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Additive combinatorics and number theory

The title of the thesis covers rather broad part of combinatorics. The real content of the thesis restricts to the investigation of the behaviour of the growth functions of ideals of selected combinatorial structures. The thesis consists of three essential chapters. In the first one, the author deals with ideals in the poset of integer partitions, in the second one, the author investigates the ideals of ordered hypergraphs, the third one presents an exposition of the known results on the Wilf equivalence in general posets suplemented by a few own results. In all the three parts the author presents his own results in the broader context of known results. Each of the three chapters is finished by research problems opening a door for further research. Let me now discuss the contributions of the author in detail.

The integer partitions is a classical object of study in combinatorics. The seminal result by Hardy ans Ramanujan (1918) establishes their assymptotics. After that, Erdos et.all. (1989) derived the assymtotics of the partitions with excluded (finite) set of parts. The first result presents an alternative (elegant) proof of the above theorem (called the complementary Schur theorem here). The main original result in Chapter 1 is Theorem 1.12, where the author proves that there is a partition ideal X such that the growth function $p(n, X)$ is divergent taking for infinitely many integers 0, while for another sequence of integers its value is almost the number of all partitions $p(n)$ of n. Both main results are valuable contributions to the theory of integer partitions.

The starting point for the second chapter is a seminal paper by Balogh, Bollobas and Morris (2007) characterising possible asymptotics of growth functions of the ideals of the ordered graphs. Note that the ideal here means a hereditary property of graphs. A generalisation to hypergraphs seems to be difficult. Therefore the author restricts to uniform hypergraphs. Two main results are presented. The first is Theorem 2.11 which can be considered as a first attempt to a generalisation of the results from graphs to hypergraphs. The second strengthtens the result for the 3-uniform hypergraphs. Both results are original and interesting. In the proof(s) the author first finds a reformulation of the problem in terms of certain graph colourings. In the constructive part(s) of the proofs the problem reduces to counting particular (binary) matrices and even tensors. Chapter 2 is therefore interesting from the methodological point of view as well. I think this is the hardest part of the thesis containing a lot of new original ideas.

Summing together, the thesis is well written, contains enough new, original and interesting results. I did not find a gap in the arguments. Although, the main results of the thesis were not published, I expect their publication in top specialised journals (such as JCT A, for instance). I **recommend** the thesis for the defence. Based on the thesis **I recommend to award the author by the degree PhD.**

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prof. Roman Nedela