In[•]:=

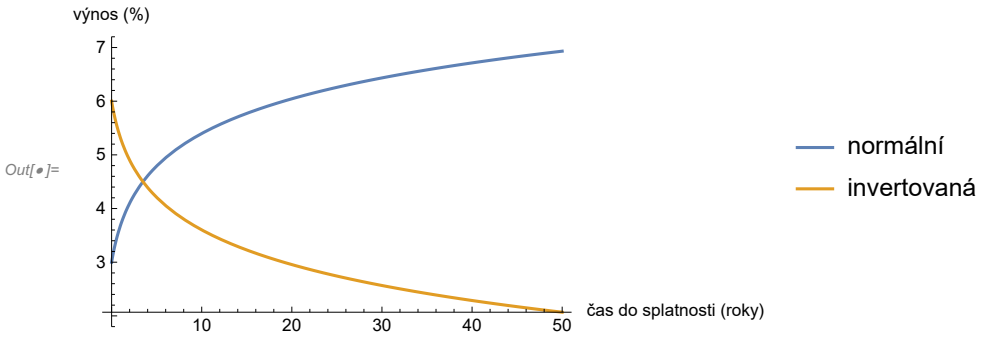
Časová struktura úrokových měr

Výnosová křivka

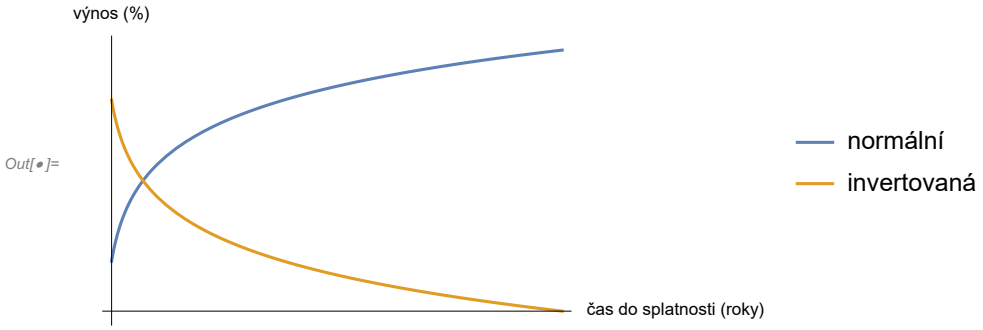
Není to funkce výnosové křivky, pouze jak by mohla výnosová křivka vypadat.

In[•]:=
```
Plot[{Log[x + 1] + 3, -Log[x + 1] + 6},
 {x, 0, 50}, PlotLegends -> {"normální", "invertovaná"},
 AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
vynos = Show[%, Ticks → None,
  Method → {"DefaultBoundaryStyle" → Automatic, "DefaultGraphicsInteraction" →
     {"Version" → 1.2`, "TrackMousePosition" → {True, False}, "Effects" →
       {"Highlight" → {"ratio" → 2}, "HighlightPoint" → {"ratio" → 2}, "Droplines" →
         {"freeformCursorMode" → True, "placement" → {"x" → "All", "y" → "None"}}}},
    "DefaultMeshStyle" → AbsolutePointSize[6], "ScalingFunctions" → None,
    "CoordinatesToolOptions" → {"DisplayFunction" →
       ({(Identity[#1] &)[#1[[1]], (Identity[#1] &)[#1[[2]]]} &), "CopiedValueFunction" →
       ({(Identity[#1] &)[#1[[1]], (Identity[#1] &)[#1[[2]]]} &)}}]
```

Out[•]=



Out[•]=




```
SetDirectory[NotebookDirectory[]]
Export["vynosovakrivka.pdf", vynos2]
```

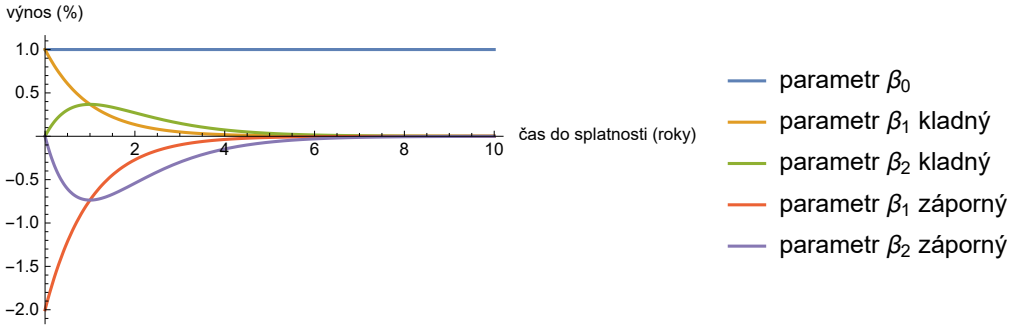
Nelson-Siegelův odhad

V grafu jsou znázorněny jednolivé části Nelson-Siegelovy funkce pro forwardovou výnosovou křivku s příslušnými parametry.

In[•]:=
```
nelsonsiegelforward[t_, beta0_, beta1_, beta2_, lambda_] :=
 beta0 + beta1 * Exp[-lambda * t] + beta2 * lambda * t * Exp[-lambda * t]
```

In[•]:=
```
nsf = Plot[{nelsonsiegelforward[t, 1, 0, 0, 1],
    nelsonsiegelforward[t, 0, 1, 0, 1], nelsonsiegelforward
[t, 0, 0, 1, 1], nelsonsiegelforward[t, 0, -2, 0, 1], nelsonsiegelforward[t, 0, 0, -2, 1]},
  {t, 0, 10}, PlotLegends → {"parametr β₀", "parametr β₁ kladný",
    "parametr β₂ kladný", "parametr β₁ záporný", "parametr β₂ záporný"},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

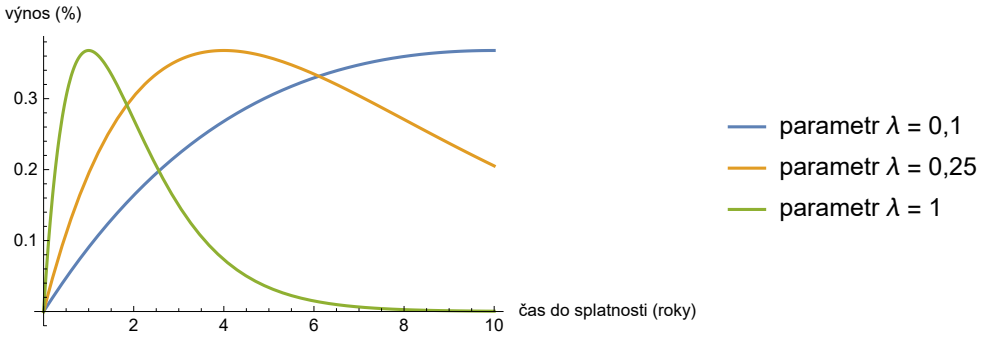
Out[•]=



Ukázáno, jak se forwardová výnosová křivka liší pro různé volby parametru λ

In[•]:=
```
nsfl = Plot[{nelsonsiegelforward[t, 0, 0, 1, 0.1],
    nelsonsiegelforward[t, 0, 0, 1, 0.25], nelsonsiegelforward[t, 0, 0, 1, 1]}, {t, 0, 10},
  PlotLegends → {"parametr λ = 0,1", "parametr λ = 0,25", "parametr λ = 1"},
  PlotRange → All, AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=

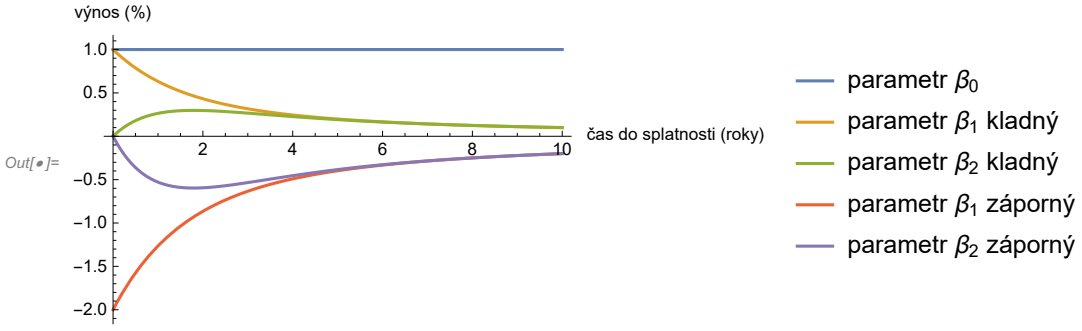


V grafu jsou znázorněny jednolivé části Nelson - Siegelovy funkce pro spotovou výnosovou křivku s příslušnými parametry

In[•]:=
```
nelsonsiegelspot[t_, beta0_, beta1_, beta2_, lambda_] :=
 beta0 + (beta1 + beta2) * ((1 - Exp[-lambda * t]) / (lambda * t)) - beta2 * Exp[-lambda * t]
```

In[•]:=
```
nss = Plot[{nelsonsiegelspot[t, 1, 0, 0, 1],
    nelsonsiegelspot[t, 0, 1, 0, 1], nelsonsiegelspot[t, 0, 0, 1, 1],
    nelsonsiegelspot[t, 0, -2, 0, 1], nelsonsiegelspot[t, 0, 0, -2, 1]},
  {t, 0, 10}, PlotLegends → {"parametr β0", "parametr β1 kladný",
    "parametr β2 kladný", "parametr β1 záporný", "parametr β2 záporný"},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

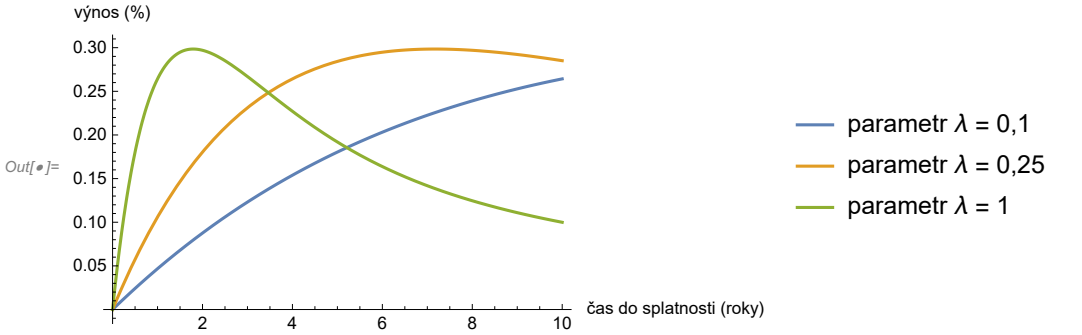
Out[•]=



Ukázáno, jak se spotová výnosová křivka liší pro různé volby parametru λ

In[•]:=
```
nssl = Plot[{nelsonsiegelspot[t, 0, 0, 1, 0.1],
    nelsonsiegelspot[t, 0, 0, 1, 0.25], nelsonsiegelspot[t, 0, 0, 1, 1]}, {t, 0, 10},
  PlotLegends → {"parametr λ = 0,1", "parametr λ = 0,25", "parametr λ = 1"},
  PlotRange → All, AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=



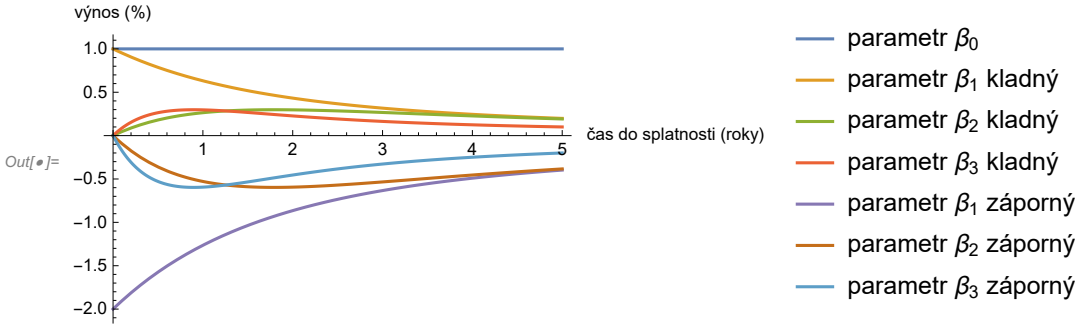
Svenssonův odhad

V grafu jsou znázorněny jednolivé části Svenssonovy funkce pro spotovou výnosovou křivku s příslušnými parametry.

In[•]:=
```
Clear[svenssonspot];
svenssonspot[t_, beta0_, beta1_, beta2_ , beta3_, lambda1_, lambda2_] :=
 beta0 + beta1 * ((1 - Exp[-lambda1 * t]) / (lambda1 * t)) +
  beta2 * ((1 - Exp[-lambda1 * t] - lambda1 * t * Exp[-lambda1 * t]) / (lambda1 * t)) +
  beta3 * ((1 - Exp[-lambda2 * t] - lambda2 * t * Exp[-lambda2 * t]) / (lambda2 * t))
```

In[•]:=
```
sven = Plot[{svenssonspot[t, 1, 0, 0, 0, 1, 2], svenssonspot[t, 0, 1, 0, 0, 1, 2],
   svenssonspot[t, 0, 0, 1, 0, 1, 2], svenssonspot[t, 0, 0, 0, 1, 1, 2],
   svenssonspot[t, 0, -2, 0, 0, 1, 2], svenssonspot[t, 0, 0, -2, 0, 1, 2],
   svenssonspot[t, 0, 0, 0, -2, 1, 2]}, {t, 0, 5}, PlotLegends →
   {"parametr β₀", "parametr β₁ kladný", "parametr β₂ kladný", "parametr β₃ kladný",
    "parametr β₁ záporný", "parametr β₂ záporný", "parametr β₃ záporný"},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

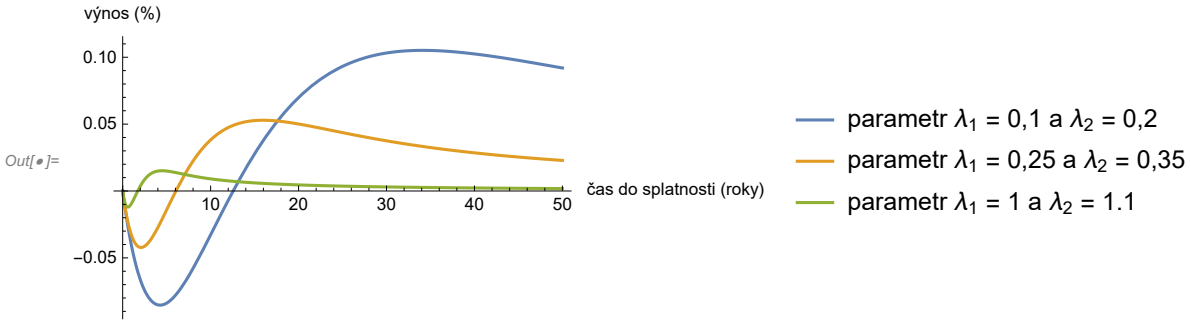
Out[•]=



Ukázáno, jak se spotová výnosová křivka liší pro různé volby parametru λ_1 a λ_2, můžeme vidět, že funkce může mít dva extrémy.

In[•]:=
```
svenl = Plot[{svenssonspot[t, 0, 0, 1, -1, 0.1, 0.2],
   svenssonspot[t, 0, 0, 1, -1, 0.25, 0.35], svenssonspot[t, 0, 0, 1, -1, 1, 1.1]},
  {t, 0, 50}, PlotLegends → {"parametr λ₁ = 0,1 a λ₂ = 0,2",
    "parametr λ₁ = 0,25 a λ₂ = 0,35", "parametr λ₁ = 1 a λ₂ = 1.1"},
  PlotRange → All, AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=



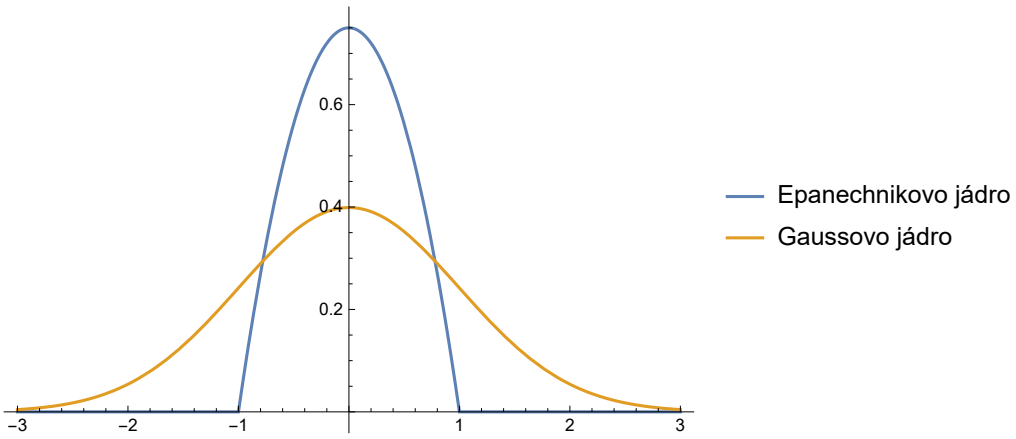
Jádrový odhad

In[2]:=
```
Clear[epanechnikov];
epanechnikov[x_] = Which[-1 < x < 1, 3/4 (1 - x^2),
                         True, 0];
```

In[4]:=
```
Clear[gauss];
gauss[x_] = PDF[NormalDistribution[], x];
```

In[•]:=
```
Plot[{epanechnikov[x], gauss[x]}, {x, -3, 3},
 PlotLegends → {"Epanechnikovo jádro", "Gaussovo jádro"}]
```

Out[•]=



In[6]:=
```
Clear[odhad];
odhad[data_, h_, jadro_, x_] := Module[{t = data[[All, 1]], y = data[[All, 2]]},
  Map[jadro, (x - t) / h].y / Total[Map[jadro, (x - t) / h]]]
```

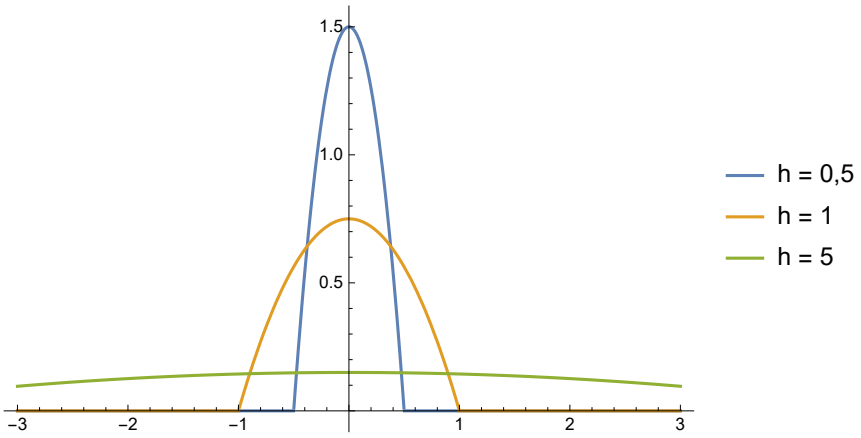
In[•]:=
```
Clear[epanechnikovh];
epanechnikovh[t_, h_] := Which[-1 < t / h < 1, 3 / (4 * h) (1 - t^2 / h^2),
                               True, 0]
```

In[•]:=
```
Clear[gaussh]
gaussh[t_, h_] := 1 / (h * Sqrt[2 * π]) * Exp[-t^2 / (2 * h^2)]
```

Rozšířená Epanechnikova jádrová funkce pro různé volby vyhlazovacího parametru h

In[•]:=
```
eh = Plot[{epanechnikovh[x, 0.5], epanechnikovh[x, 1], epanechnikovh[x, 5]},
  {x, -3, 3}, PlotLegends → {"h = 0,5", "h = 1", "h = 5"}, PlotRange → All]
```

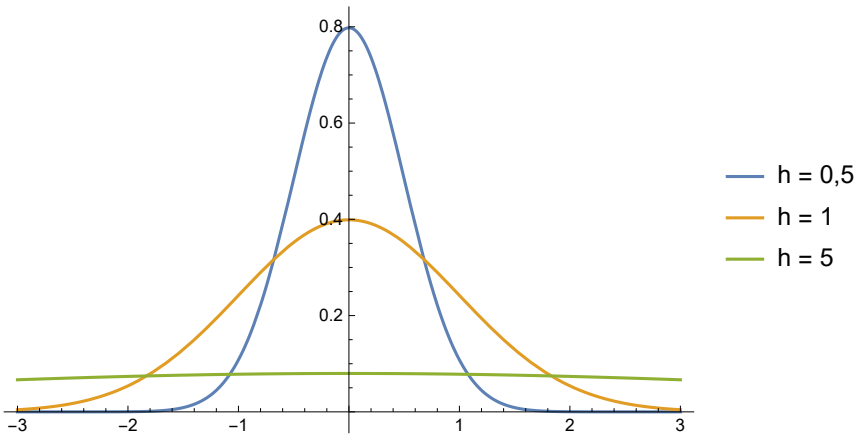
Out[•]=



Rozšířená Gaussova jádrová funkce pro různé volby vyhlazovacího parametru h

In[•]:=
```
gh = Plot[{gaussh[x, 0.5], gaussh[x, 1], gaussh[x, 5]},
  {x, -3, 3}, PlotLegends → {"h = 0,5", "h = 1", "h = 5"}, PlotRange → All]
```

Out[•]=



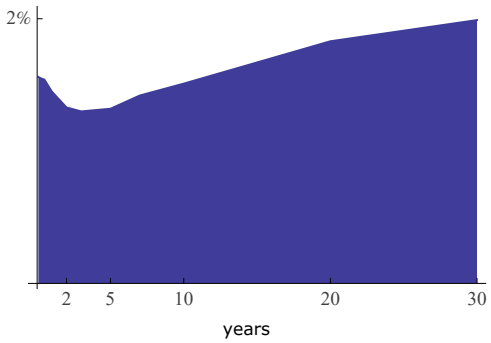
Data

In[1]:= **yield curve January 2020**

Input interpretation:

United States	treasury yield curve	January 2020

Results: More



3-month treasury bill	1.55%
1-year treasury bill	1.45%
2-year treasury note	1.33%
5-year treasury note	1.32%
10-year treasury note	1.51%
30-year treasury bond	1.99%

(January 31, 2020)

WolframAlpha

ImportExport`FileFormatQ: Cannot open C:\Users\janab\AppData\Local\Temp\m-5a6fbbfe-c80f-44b1-ae94-2508ea241e52.

In[8]:=
```
yieldcurve = {WolframAlpha["yield curve January 2020",
    {{"TreasuryYieldCurve:EconomicData", 1}, "ComputableData"},
    PodStates → {"TreasuryYieldCurve:EconomicData__More"}],
  WolframAlpha["yield curve January 2020", {{"TreasuryYieldCurve:EconomicData", 2},
    "ComputableData"}, PodStates → {"TreasuryYieldCurve:EconomicData__More"}],
  WolframAlpha["yield curve January 2020", {{"TreasuryYieldCurve:EconomicData", 3},
    "ComputableData"}, PodStates → {"TreasuryYieldCurve:EconomicData__More"}]}
```

Out[8]=
```
{Missing[NotAvailable],
 {{1-month treasury bill, 1.56%}, {3-month treasury bill, 1.55%},
  {6-month treasury bill, 1.54%}, {1-year treasury bill, 1.45%},
  {2-year treasury note, 1.33%}, {3-year treasury note, 1.3%},
  {5-year treasury note, 1.32%}, {7-year treasury note, 1.42%},
  {10-year treasury note, 1.51%}, {20-year treasury bond, 1.83%},
  {30-year treasury bond, 1.99%}}, Friday, January 31, 2020}
```

In[9]:=
```
yield2 = yieldcurve[[2]];
yield3 = Map[#[[2]] &, yield2];
rates = yield3[[All, 1]]
```

Out[11]=
```
{1.56, 1.55, 1.54, 1.45, 1.33, 1.3, 1.32, 1.42, 1.51, 1.83, 1.99}
```

In[12]:=
```
dates = {1/12, 0.25, 0.5, 1, 2, 3, 5, 7, 10, 20, 30}
```

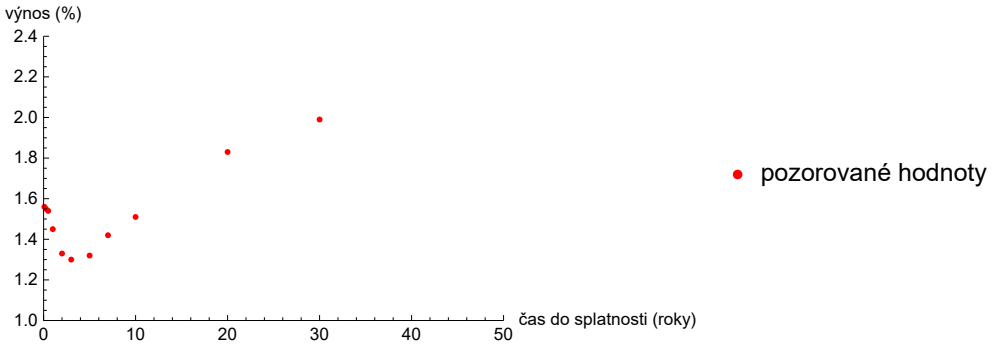
Out[12]=
```
{1/12, 0.25, 0.5, 1, 2, 3, 5, 7, 10, 20, 30}
```

In[13]:=
```
data = Thread[{dates, rates}]
```

Out[13]=
```
{{1/12, 1.56}, {0.25, 1.55}, {0.5, 1.54}, {1, 1.45}, {2, 1.33},
 {3, 1.3}, {5, 1.32}, {7, 1.42}, {10, 1.51}, {20, 1.83}, {30, 1.99}}
```

In[14]:=
```
plotdata = ListPlot[data, PlotStyle → Red,
  PlotRange → {{0, 50}, {1, 2.4}}, PlotLegends -> {"pozorované hodnoty"},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[14]=

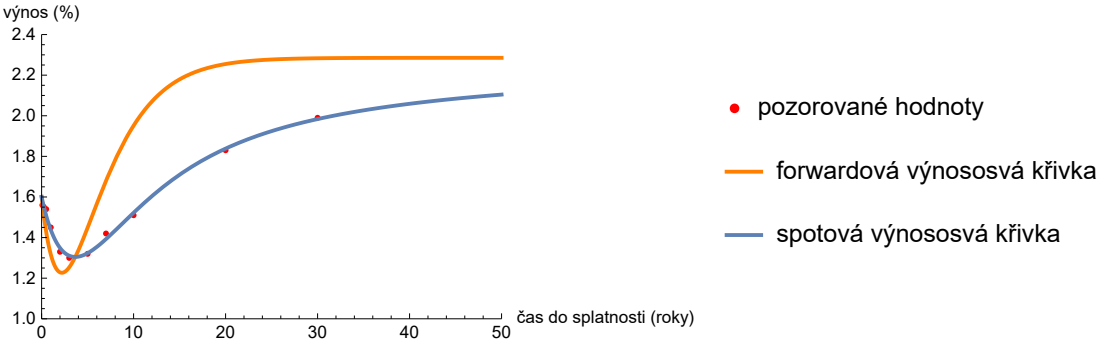


Nelson-Siegel odhad na reálných datech

```
parametryns = FindFit[data, nelsonsiegelspot[b0, b1, b2, l, t], {b0, b1, b2, l}, t]
krivkaspot = nelsonsiegelspot[b0, b1, b2, l, t] /. parametryns;
krivkaforward = nelsonsiegelforward[b0, b1, b2, l, t] /. parametryns;
nelsonsiegel =
 Show[plotdata, Plot[{krivkaforward}, {t, 0, 50}, PlotRange → {{0, 50}, {1, 2.4}},
   PlotStyle → {Orange, Thick}, PlotLegends -> {"forwardová výnosová křivka"}],
  Plot[{krivkaspot}, {t, 0, 50}, PlotRange → {{0, 50}, {1, 2.4}}, PlotStyle → {Thick},
   PlotLegends -> {"spotová výnosová křivka"}], PlotRange → {{0, 50}, {1, 2.4}},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]= {b0 → 0.3037, b1 → 2.28531, b2 → −0.684972, l → −2.06536}

Out[•]=

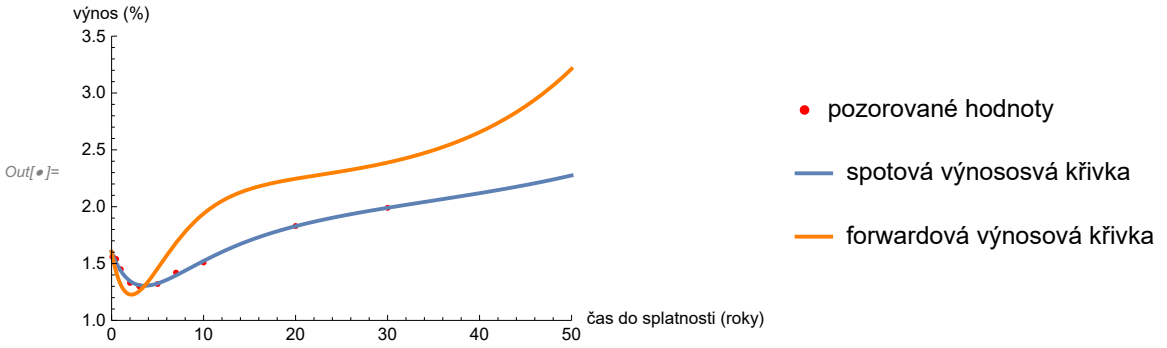


Svensson odhad na reálných datech

In[•]:=
```
Clear[svenssonforward];
svenssonforward[ beta0_, beta1_, beta2_, beta3_, lambda1_, lambda2_, t_] :=
 beta0 + beta1 * Exp[-lambda1 * t] +
  beta2 * lambda1 * t * Exp[-lambda1 * t] + beta3 * lambda2 * t * Exp[-lambda2 * t]
Clear[svenssonspot];
svenssonspot[ beta0_, beta1_, beta2_, beta3_, lambda1_, lambda2_, t_] :=
 beta0 + beta1 * ((1 - Exp[-lambda1 * t]) / (lambda1 * t)) +
  beta2 * ((1 - Exp[-lambda1 * t]) / (lambda1 * t) - Exp[-lambda1 * t]) +
  beta3 * ((1 - Exp[-lambda2 * t]) / (lambda2 * t) - Exp[-lambda2 * t])
Clear[svenssonspot2];
svenssonspot2[beta0_, beta1_, beta2_, beta3_, lambda1_, lambda2_, t_] :=
 1/t ∫_0^t svenssonforward[beta0, beta1, beta2, beta3, lambda1, lambda2, s] ⅆs
parametrys = FindFit[data, svenssonspot[b0, b1, b2, b3, l1, l2, t],
  {b0, b1, b2, b3, l1, l2}, t]
krivkaspots = sspot[b0, b1, b2, b3, l1, l2, t] /. parametrys;
krivkaforwards = sforward[b0, b1, b2, b3, l1, l2, t] /. parametrys;
svensson = Show[plotdata,
  Plot[krivkaspots, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}}, PlotStyle → {Thick},
   PlotLegends → {"spotová výnososvá křivka"}], Plot[krivkaforwards,
   {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}}, PlotStyle → {Orange, Thick},
   PlotLegends → {"forwardová výnosová křivka"}], PlotRange → {{0, 50}, {1, 3.5}},
  AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=
```
{b0 → 2.1928, b1 → -0.590027, b2 → -1.94436,
 b3 → -0.0210265, l1 → 0.321473, l2 → -0.056766}
```

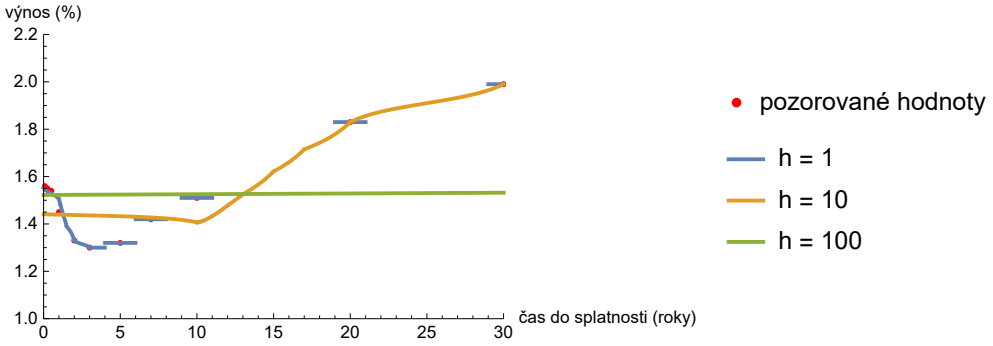
Out[•]=



Jádrový odhad na reálných datech

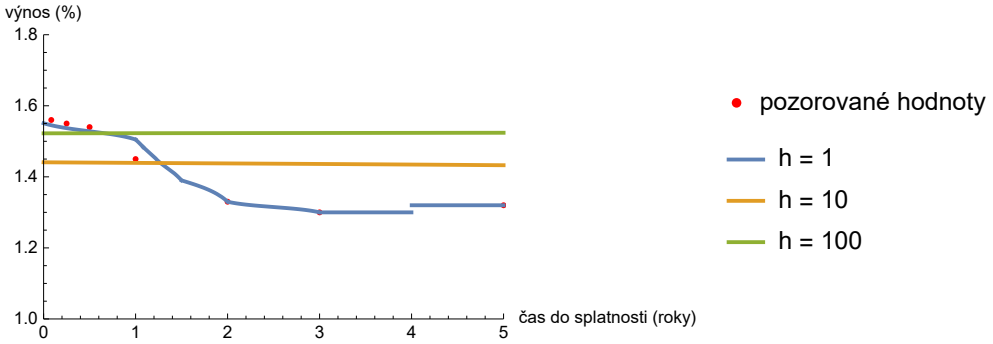
In[•]:=
```
epan = Show[ListPlot[data, PlotStyle → Red,
    PlotRange → {{0, 30}, {1, 2.2}}, PlotLegends -> {"pozorované hodnoty"},
    AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
  Plot[{odhad[data, 1, epanechnikov, x], odhad[data, 10, epanechnikov, x],
    odhad[data, 100, epanechnikov, x]}, {x, 0, 30}, PlotRange → {1, 2.2},
    PlotStyle → {Thick}, PlotLegends → {"h = 1", "h = 10", "h = 100"}]]
```

Out[•]=



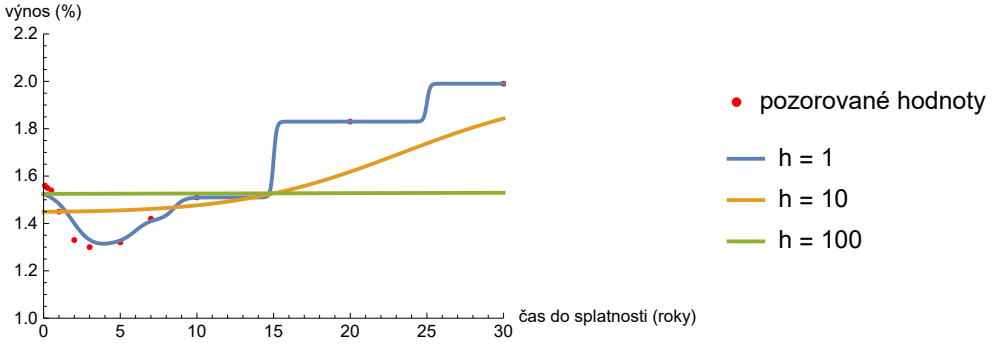
In[•]:=
```
Show[ListPlot[data, PlotStyle → Red,
    PlotRange → {{0, 5}, {1, 1.8}}, PlotLegends -> {"pozorované hodnoty"},
    AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
  Plot[{odhad[data, 1, epanechnikov, x], odhad[data, 10, epanechnikov, x],
    odhad[data, 100, epanechnikov, x]}, {x, 0, 5}, PlotRange → {1, 1.8},
    PlotStyle → {Thick}, PlotLegends → {"h = 1", "h = 10", "h = 100"}]]
```

Out[•]=



In[•]:=
```
gau = Show[ListPlot[data, PlotStyle → Red,
    PlotRange → {{0, 30}, {1, 2.2}}, PlotLegends -> {"pozorované hodnoty"},
    AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
  Plot[{odhad[data, 1, gauss, x], odhad[data, 10, gauss, x], odhad[data, 100, gauss, x]},
    {x, 0, 30}, PlotRange → {1, 2.2}, PlotStyle → {Thick},
    PlotLegends → {"h = 1", "h = 10", "h = 100"}]]
```

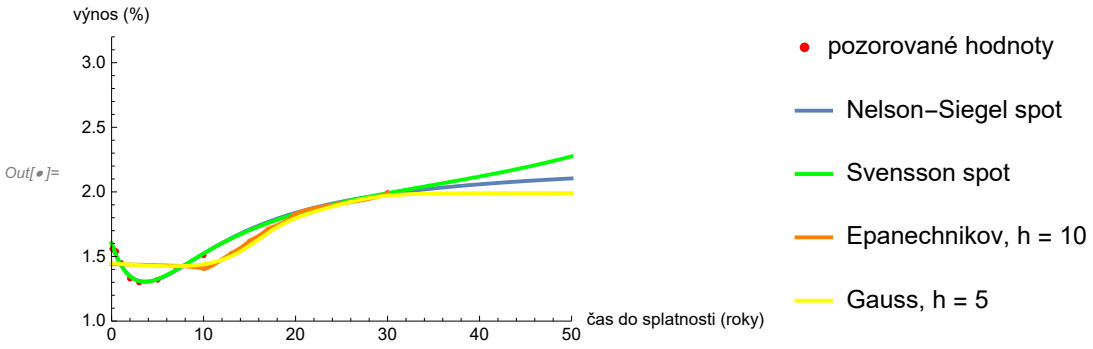
Out[•]=



Porovnání všech odhadů

```
In[•]:= porovnani1 = Show[ListPlot[data, PlotStyle → Red,
          PlotRange → {{0, 30}, {1, 3.5}}, PlotLegends -> {"pozorované hodnoty"},
          AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
        Plot[{krivkaspot}, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
          PlotStyle → {Thick}, PlotLegends -> {"Nelson-Siegel spot"}],
        Plot[krivkaspots, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
          PlotStyle → {Green, Thick}, PlotLegends → {"Svensson spot"}],
        Plot[{odhad[data, 10, epanechnikov, x]}, {x, 0, 30}, PlotRange → {1, 2.2},
          PlotStyle → {Orange, Thick}, PlotLegends → {"Epanechnikov, h = 10"}],
        Plot[{odhad[data, 5, gauss, x]}, {x, 0, 50}, PlotRange → {1, 2.4},
          PlotStyle → {Yellow, Thick}, PlotLegends → {"Gauss, h = 5"}],
        PlotRange → {{0, 50}, {1, 3.2}}, AxesLabel →
          {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=



Porovnání pro upravená data

```
In[15]:= data2 = Most[Most[data]]
```

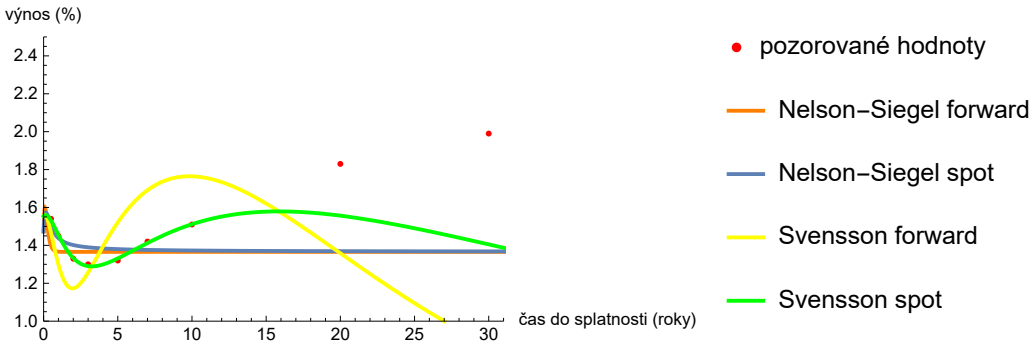
Out[15]= $\left\{\left\{\frac{1}{12}, 1.56\right\}, \{0.25, 1.55\}, \{0.5, 1.54\}, \{1, 1.45\}, \{2, 1.33\}, \{3, 1.3\}, \{5, 1.32\}, \{7, 1.42\}, \{10, 1.51\}\right\}$

In[•]:=
```
parametryns2 = FindFit[data2, nelsonsiegelspot[b0, b1, b2, l, t], {b0, b1, b2, l}, t]
krivkansspot2 = nelsonsiegelspot[b0, b1, b2, l, t] /. parametryns2;
krivkansfor2 = nelsonsiegelforward[b0, b1, b2, l, t] /. parametryns2;
parametrys2 =
 FindFit[data2, svenssonspot[b0, b1, b2, b3, l1, l2, t], {b0, b1, b2, b3, l1, l2}, t]
krivkasspot2 = svenssonspot[b0, b1, b2, b3, l1, l2, t] /. parametrys2;
krivkasfor2 = svenssonforward[b0, b1, b2, b3, l1, l2, t] /. parametrys2;
parametricke = Show[ListPlot[data, PlotStyle → Red,
    PlotRange → {{0, 30}, {1, 3.5}}, PlotLegends -> {"pozorované hodnoty"},
    AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
  Plot[krivkansfor2, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
    PlotStyle → {Orange, Thick}, PlotLegends -> {"Nelson-Siegel forward"}],
  Plot[krivkansspot2, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
    PlotStyle → {Thick}, PlotLegends -> {"Nelson-Siegel spot"}],
  Plot[krivkasfor2, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
    PlotStyle → {Yellow, Thick}, PlotLegends → {"Svensson forward"}],
  Plot[krivkasspot2, {t, 0, 50}, PlotRange → {{0, 50}, {1, 3.5}},
    PlotStyle → {Green, Thick}, PlotLegends → {"Svensson spot"}], PlotRange →
    {{0, 31}, {1, 2.5}}, AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}]
```

Out[•]=
```
{b0 → 8.58847, b1 → 1.36569, b2 → 0.107548, l → 0.514166}
```

Out[•]=
```
{b0 → 0.296952, b1 → 1.25712, b2 → 1.10744, b3 → 3.98974, l1 → 1.54087, l2 → 0.101704}
```

Out[•]=



In[16]:=
```
neparametricke = Show[ListPlot[data, PlotStyle → Red,
    PlotRange → {{0, 31}, {1, 2.4}}, PlotLegends -> {"pozorované hodnoty"},
    AxesLabel → {"čas do splatnosti (roky)", "výnos (%)"}],
  Plot[{odhad[data2, 3, epanechnikov, x], odhad[data2, 0.5, gauss, x]}, {x, 0, 31},
    PlotRange → {1, 2.4}, PlotStyle → {Thick}, PlotLegends → {"Epanechnikov", "Gauss"}]]
```

Out[16]=