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The supervisor's report on the Ph.D. thesis by

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Universal solutions in gravity, electrodynamics and nonabelian gauge theories

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The theme of this thesis is (almost) universality of fields, such as gravitational, electromagnetic, and non-abelian gauge fields. Such fields then solve (almost) all corresponding field equations derived from a (polynomial) Lagrangian containing arbitrary higher-order corrections. The connection between universality and geometrical properties of these fields, curvature algebraic type, and scalar invariants is studied using higher-dimensional generalizations of the algebraic tensor classification based on null alignment, of the NP and GHP formalisms, and the balance-tensor approach. This thesis is based on four papers (two published in Physics Review D, one in Classical and Quantum Gravity, and one submitted to a journal).

Chapter 1 concentrates on the universality and almost universality of the gravitational field. Universal spacetimes are spacetimes for which all conserved symmetric second-rank tensors constructed as polynomials in the metric, the Riemann tensor, and its covariant derivatives are constant multiples of the metric. This implies that vacuum field equations of any theory with the Lagrangian being a polynomial in curvature (including arbitrary derivatives of the Riemann tensor) reduce to just one algebraic equation. For almost universal spacetimes, the universality condition is relaxed to allow also one nontrivial traceless type N component. Field equations then reduce to one algebraic and one differential equation. Necessary and sufficient conditions for (almost) universal spacetimes are determined. For example, it follows that non-Einstein almost universal spacetimes are necessarily CSI (constant scalar invariant) Kundt spacetimes of Weyl type III or more special. As a practical application of these results, explicit solutions of specific modified theories of gravity, such as quadratic gravity and 6-dimensional conformal gravity, are constructed.

In Chapter 2, electromagnetic field is also included. A class of plane-fronted gravitational and electromagnetic waves in the Weyl type III Kundt class propagating in a flat spacetime along a common recurrent null vector is identified as universal. These coupled Einstein-Maxwell fields solve all generalizations of the Einstein-Maxwell field equations where higher-order gravitational, electromagnetic, and interaction corrections to the Einstein-Maxwell Lagrangian are considered. The explicit form of the solution is also given.

In the final two chapters, the previous techniques and results are extended to gauge covariant fields. In Chapter 3, these techniques are applied to find VSI nonabelian gauge fields and universal nonabelian (test) gauge fields in

Yang-Mills theories (on a fixed background) in arbitrary dimension solving any Yang-Mills field equations with higher-order corrections. It has been proven there that a gauge field is VSI iff it is null and aligned with a degenerate Kundt spatime and furthermore, that an aligned null Yang-Mills field in a degenerate Kundt spacetime of Weyl and traceless Ricci type III is universal. Explicit examples are also given.

Finally, in Chapter 4, universal solutions to the full Einstein-Yang-Mills system which are immune to higher-order corrections are identified as a combination of a VSI spacetime with a VSI gauge field subject to an additional tensorial condition. They represent gravitational and YM plane-fronted waves propagating in a flat spacetime along the common recurrent null vector. To illustrate that this approach can be applied to more involved theories including a spacetime metric, gauge fields, scalars, and p-forms, universal solutions to the bosonic part of ten-dimensional heterotic supergravity is studied.

M. Kuchynka turned out to be an excellent student, bright, creative, and hard-working. During his studies, he published six papers. Two papers have been published in Physical Review D, two in Classical and Quantum Gravity, one in General Relativity and Gravitation, and one paper is now submitted to a journal. After we published our first three papers, M. Kuchynka developed an interest to generalize and extend our results on universal and almost universal spacetimes to the case with generalized Maxwell fields (p-forms) and gauge fields. After he obtained his preliminary results in the Einstein-Maxwell case, we found that M. Ortaggio was working on a very similar problem independently. Both of them thus joined forces in their paper on Einstein-Maxwell fields with vanishing higher-order corrections. Two subsequent papers, which also incorporate gauge and Yang-Mills fields, were written by M. Kuchynka as a single author.

This thesis contains a considerable amount of original research results. It clearly demonstrates the author's ability to perform an original research independently. The thesis is of high quality and fully satisfies the required conditions for a doctoral thesis. Thus I am happy to recommend its acceptance as a doctoral thesis at the Faculty of Mathematics and Physics of Charles University.

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