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DOCTORAL THESIS REVIEW

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Selected exact spacetimes in Einstein's gravity

In this doctoral thesis, five new "crystal-like" exact solutions to the four-dimensional Einstein-Maxwell and Einstein-Maxwell-dilaton equations are found employing two methods, linearity the field equations and dimensional reduction. In all cases, the geometry, horizons, the presence of singularities and electrogeodesics are studied and accompanied by diagrams.

The first four chapters are devoted to finding four-dimensional axially symmetric Majumdar-Papapetrou (MP) solutions to the Einstein-Maxwell equations. The linearity of one of the field equations allows for infinite superposition of MP extremely charged black holes having thus "crystal-like" structure with discrete translational symmetry along a symmetry axis with field sources.

After a review given in Chapter 1, in Chapter 2 a solution representing an "alternating" crystal with an anti-period potential with alternating charges is found and studied. The resulting spacetime has several convenient properties, such as the uniform convergence of the potential and asymptotic flatness at the cylindrical infinity. However, naked singularities are present that can be eliminated, at the cost of introducing a non-zero stress-energy tensor.

In Chapter 3, a solution representing a uniform crystal with only positive charges is constructed. Here, the potential has to be modified to achieve the uniform convergence. However, naked singularities are still present.

In Chapter 4, a smooth crystal is found as a sum of screened, Yukawa-like potentials. The potential converges absolutely uniformly and so singularities are present at the expense of an additional matter content - charged dust. The uniform crystal can be obtained in an appropriate limit.

All the above solutions contain sources aligned along the symmetry axis. Thus these spacetimes can be also expressed using the Weyl metric, reviewed in Chapter 5 in the vacuum and electrovacuum case.

The above four-dimensional solutions are not presented in a closed form. However, higher-dimensional solutions can admit a closed form. Then four-dimensional closed-form solutions can be constructed from such higher-dimensional solutions with a compact spacelike Killing coordinate using the dimensional reduction (reviewed in Chapter 6) to generate a lower-dimensional solution with an additional scalar field.

In Chapter 7, five-dimensional uniform and alternating crystals are found and studied. In Chapter 8, the dimensional-reduction technique is then used for a more suitable five-dimensional uniform crystal, resulting in an asymptotically flat four-dimensional reduced crystal with a scalar field, a closed-form metric and no naked singularities.

This thesis contains a number of interesting and timely original results. It clearly demonstrates the author's ability to perform original research independently. It is very well written. Apart from a very few minor typos, I have not noticed any mistakes.

The thesis is of high quality and fully satisfies the required conditions for a doctoral thesis and thus I am happy to recommend its acceptance as a doctoral thesis at the Faculty of Mathematics and Physics of Charles University.

I would like to ask the following question:
Would it be possible to construct solutions belonging to MP family while relaxing the assumption of equal masses or equal spacing between black holes and still preserving certain periodicity, e.g. by alternating two masses or two spacing distances?

August 13, 2020

Mgr. Alena Pravdová, Ph.D.

I have noticed only some minor typos, e.g.

Page 5: eq. (10) (which implies $\mathrm{Ric}^{l}{}_{i} = - R$) is in a contradiction with eq. (9) ($\mathrm{Ric}^{l}{}_{i} = R$)

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