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Sector ETF Portfolio Optimization Using Differential Evolution

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Abstract

This thesis examines the use of differential evolution in a real-world portfolio optimization task based on US stock data. We empirically test the capability of the algorithm to find an inter-sector allocation that outperforms a broad-market stock index. Two constrained sector ETF portfolios are constructed to simulate realistic agent-based settings and performance of the competing portfolios is analyzed in terms of both return and risk. The results are further extended to include Markowitz' global minimum variance portfolio and a naive 1/N portfolio. We show that the constructed portfolios are indeed capable of outperforming the market whilst simultaneously maintaining lower tail risk, however, the performance significantly deteriorates if the portfolios are rebalanced based on rolling data windows. Overall the algorithm delivers satisfying results while providing the user with a relative freedom when choosing portfolio constraints.

JEL Classification: C61, G11, G17, G19

Keywords: portfolio optimization, exchange-traded funds, differential evolution, empirical analysis

Title: Sector ETF Portfolio Optimization Using Differential Evolution

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Abstrakt

Tato práce se zabývá realistickým využitím diferenciální evoluce pro optimalizaci akciového portfolia za užití amerických burzovních dat. Schopnost algoritmu nalézt mezisektorovou alokaci, která by výkonnostně předčila akciový index, je empiricky testována. Jsou zkonstruována dvě portfolia sektorových ETF s omezujícími podmínkami, které simulují nastavení, jimž čelí reální agenti a výkonnost těchto soupeřících portfolií je analyzována jak z pohledu výnosnosti, tak z pohledu rizika. Výsledky jsou dále rozšířeny o Markowitzovo portfolio s minimálním rozptylem a portfolio naivní diversifikace. Ukážeme, že zkonstruovaná portfolia jsou schopna porazit trh a současně vykazovat nižší míru chvostového rizika, avšak výkonnost významně klesá, jsou-li portfolia rebalancována na základě nejnovějších historických dat. Celkově lze říci, že algoritmu se daří dosáhnout uspokojivých výsledků a současně poskytuje uživateli relativní svobodu při určování omezujících podmínek.

Klasifikace JEL:	C61, G11, G17, G19
Klíčová slova:	optimalizace portfolia, exchange-traded funds, diferenciální evoluce, empirická analýza

Název práce:	Optimalizace Portfolia Sektorových ETF Pomocí Diferenciální Evoluce

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Declaration of Authorship

I hereby proclaim that I wrote my bachelor thesis on my own under the leadership of my supervisor and that the references include all resources and literature I have used.

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Prague, July 29, 2020

Signature

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Bachelor's thesis proposal

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Research question and motivation

Passive investing, especially in the form of broad market index-tracking, has grown in popularity over the past few decades (see, e.g., Anadu et al., 2019). In my thesis, I would like to focus on the following question: Would dynamic allocation of assets, where each of them represents one of nine U.S. stock market sectors, beat the strategy of simply investing in a broad market index? The answer to this question largely depends on the approach chosen to find the optimal allocation of assets. Conventional mean-variance approach to asset portfolio optimization established by Harry Markowitz more then sixty years ago has its limitations. One of such limitations is its inefficiency in finding solutions to problems with non-smooth objective functions and often non-linear real world constraints, namely cardinality constraints, e.g., an upper bound for the number of assets in the portfolio, upper bound on the number of short positions to limit leverage etc. Since I would like to use these real world constraints to simulate settings that real world agents such as mutual funds or hedge funds must face, the task of finding the solution to our optimization problem will be given to a metaheuristic algorithm called differential evolution (DE) (Storn & Price, 1997). Numerous researchers have already found success using the DE to solve complex optimization tasks. A research paper with a topic nearest to the one I propose here was composed by Krink & Paterlini (2009), who focus on realistic portfolio optimization, although they use altered version of the original DE algorithm to solve their optimization tasks. They have shown that traditional quadratic programming solutions obtained from mean-variance optimization are substantially worse compared to solutions obtained from their DE-based algorithm when interpreted in the objective function space of a realistic portfolio optimization problem.

Contribution

Constrained index-tracking using differential evolution (DE) has already been studied by Krink et al. (2009) or Andriosopoulos et al. (2013). Answering a question, whether "picking" among sectors would bring superior results in terms of risk and return, could be valuable for portfolio managers or individual investors as well as for the academia. Moreover, the investigation will bring further insight into using nature-inspired algorithms for portfolio choice.

Methodology

The empirical section of the thesis will be dedicated to out-of-sample testing of the sector portfolio and comparing the results to benchmarks: The S&P 500 index, naive (equal weights) portfolio and Markowitz' global minimum variance portfolio. The testing will be carried out under two different scenarios. In the first scenario, the portfolio is annually rebalanced and thus, additional transaction costs arise and must be taken into account. The second scenario represents a passive buy and hold approach. Two set of portfolio constraints will be constructed to represent two diversely regulated agents. I will be using SPDR sector exchange-traded funds to represent sectors within the S&P 500 index. Publicly available data will be used to carry out the analysis and downside risk measures such as conditional value at risk along with conventional mean-variance performance measures will be used to evaluate the results.

Outline

1 Introduction

2 Literature Review

3 Theoretical Section

3.1 Portfolio optimization: mean-variance framework

3.2 Risk measures: VaR and CVaR

3.3 Differential Evolution

3.4 Exchange-traded funds

4 Empirical Section

Core bibliography

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Contents

List of Tables xi

1 Introduction 1

2 Literature Review 3

3 Theoretical Background 6

3.1 Global Minimum Variance Portfolio 6

3.2 Differential Evolution 8

3.3 Returns and Risk Measures 10

3.4 Exchange-Traded Funds 15

4 Empirical Investigation 18

4.1 Data 18

4.2 Methodology 20

4.3 Performance Analysis 23

5 Conclusion 32

Bibliography 33

Appendix 37

List of Tables

1 Summary Statistics 19

2 B&H – performance summary 25

3 Annual rebalancing – performance summary 27

4 Qaurterly rebalancing (3-year ISP) – performance summary 29

5 Qaurterly rebalancing (1-year ISP) – performance summary 30

A1 B&H – asset weights 37

A2 Annual rebalancing – asset weights; monthly data 38

A3 Annual rebalancing – asset weights; weekly data 39

A4 Annual rebalancing – asset weights; daily data 40

1 Introduction

Modern methods of portfolio management rarely incorporate classical mean-variance framework for portfolio optimization, despite its theoretical appeal. This is mainly due to the following reasons. Firstly, errors in estimates of the mean returns have a severe impact on performance of the model. Michaud (1989) states in his paper, that mean-variance optimizers magnify the input errors so that the optimization technique can, in many cases, be outperformed by an equally weighted portfolio. Secondly, the model restricts the user from employing realistic risk measures and real world constraints that managers have to consider. And thirdly, the model relies on the assumption that asset returns are normally distributed. As presented in Cont (2001), this assumption is flawed since returns exhibit certain stylized facts, e.g. fat tails and excess kurtosis. In this thesis, a heuristic search algorithm called differential evolution (DE) is used as an alternative quantitative approach to portfolio selection. Due to a completely different nature, the algorithm bypasses the above mentioned issues of the mean-variance framework which are often subject to criticism. In fact, it does not even require the optimization problem to be specified as a mathematical formula. Instead, it evolves a randomly generated population of solutions from the feasible space until it converges to a proposed solution. The evolutionary mechanism applied by the algorithm is inspired by the evolutionary processes from nature – mutation, crossover and selection, and is motivated by the user-specified objectives (in our case, to maximize mean return and minimize tail risk). The algorithm is employed to provide an optimal allocation to a portfolio composed of US sector exchange-traded funds (ETFs) which mimic the performance of the individual industry sectors. To motivate the use of the algorithm, realistic agent-based constraints are introduced along with a risk measure reflecting the investors' asymmetric perception of risk – the Conditional Value-at-Risk. The Conditional Value-at-Risk is chosen as it meets the requirements to be recognized as a coherent risk measure (Artzner et al., 1999). Apart from employing different rebalancing scenarios, we also test different data-collection frequencies to increase robustness of the out-of-sample analysis. We further add the equal weighted portfolio (1/N) and Markowitz' global minimum variance portfolio to serve as benchmark portfolio construction models. The results for the time period under investigation are generally quite positive. The algorithm is capable of beating the market in terms of returns if shorting is allowed (even though the short sales are limited by a set of constraints defined by the author), while providing lower tail risk. The best result is achieved under the buy and hold scenario and although there are a few exceptions, it can be said that increasing the rebalancing frequency or data-collection frequency both lead to a significant deterioration of performance.

The thesis is structured in a following manner: first, the theoretical background is provided and subsequently, an empirical investigation is presented together with the results. Then next Section 2 gives a summary of the existing literature dedicated to the topic. Specifically, we review the literature on the classical mean-variance optimization and the shift to more modern and robust optimization techniques. Several specific applications of differential evolution in finance and portfolio management are also presented. Section 3 presents a description of the key theoretical concepts used in this work – the competing optimization techniques (global minimum variance and DE), measures of economic performance and risk and lastly, the exchange-traded funds. The following part, Section 4, presents characteristics of the dataset, methodology and most importantly, analysis of performance. The attention is given mainly to a description of the portfolio construction process under the different constraints and portfolio management strategies and the corresponding out-of-sample results. Section 5 concludes the paper, summarising the main findings.

2 Literature Review

It has been more than sixty years since Harry Markowitz laid the foundations of the Modern Portfolio Theory (MPT) with his *Portfolio Selection* (1952) – a ground-breaking work following up on the already established concept of security portfolio diversification. He later extended his work through the publication of his book, *Portfolio Selection: Efficient Diversification* (1959). Another major contribution to the MPT framework was the capital asset pricing model (CAPM), independently introduced by Jack Treynor (1961), William F. Sharpe (1964), John Lintner (1965) and Jan Mossin (1966). Sharpe, Markowitz and Merton Miller jointly received the 1990 Nobel Memorial Prize in Economics for their pioneering work in the theory of financial economics[1]. The original Markowitz mean-variance model for portfolio optimization is a linearly constrained quadratic programming (QP) problem and although it remains a frequently discussed and revisited topic in the academia, the available solution methods employing QP algorithms encounter limitations when being used to solve optimization problems with non-smooth objective functions and often non-linear real world constraints. Attempts have been made to linearize the portfolio optimization problem (see Mansini et al. (2014) and references therein). The solution techniques employing linear programming (LP), however, often require problem simplification or alteration in order to be applicable (Krink et al. (2009); Mansini et al. (2007)). The focus has thus shifted to alternative optimization methods such as metaheuristic algorithms (see, e.g., Yang (2010)). One of the metaheuristics falling into the category of nature-inspired evolutionary algorithms is the differential evolution (DE), extensively described in the theoretical section of this paper. There is a limited amount of literature concerning portfolio optimization using the DE algorithm, however, the results presented in the research generally show that the algorithm is capable of effectively finding solutions to optimization problems with real world settings.

Hagströmer and Binner (2009) apply DE to solve full-scale optimization[2] (FSO) asset selection problems of 97 assets under complex utility functions. The authors show that the problem is computationally feasible and that DE yields approximations that appear to converge to the FSO optimum. Furthermore, their results demonstrate that

[1] Markowitz for the theory of portfolio choice, Sharpe for CAPM and Miller for his contributions to the theory of corporate finance.

[2] Utility maximization approach to portfolio choice problems.

when investors are loss averse, FSO improves stock portfolio performance compared to mean-variance portfolios.

Ma et al. (2012) examine a portfolio optimal model with cardinality constraints[3], in which the minimum of Value-at-Risk is taken as the objective function. Using sixteen stocks from Shanghai and Shenzhen stock market and a hybrid[4] DE algorithm to solve the model, they conclude that the given model is reasonable and the given algorithm is effective.

Another study relevant to the topic was carried out by Krink and Paterlini (2011). The researchers construct an algorithm for portfolio optimization based on DE which they call DEMPO (Differential Evolution for Multiobjective Portfolio Optimization) and use it to solve three different multiobjective optimization problems, comparing it with the traditional quadratic programming (QP) and another well-known evolutionary algorithm for multiobjective optimization called NSGA-II. The most important conclusion drawn by the researchers is that DEMPO can tackle a realistically defined portfolio optimization problem and in reasonable runtime, whereas QP can only solve simplified problem instances, such as mean-variance optimization.

In this study we use ETFs to mimic the performance of equity indexes. ETFs are investment vehicles that pool various assets such as stocks, bonds or commodities and issue shares that are openly traded on exchanges. The characteristics which differentiate ETFs from mutual funds are thoroughly described in Section 3.4. Apart from the use of ETFs, an alternative way to track a market index is to replicate it by selecting a group of assets – a sample from all index constituents, and calibrating the asset weights with the aim to keep the tracking-error as low as possible over time. Since this approach imposes cardinality constraints, it is plausible to use a search heuristic such as DE to tackle the optimization task. Krink et al. (2009) have done so, using a method which is partly based on DE and on combinatorial search. They analyze the in-sample and out-of-sample performance of the tracking portfolios for the Dow Jones 65 and the Nikkei 225 indices, rebalancing them periodically under a rolling window-scheme. They show that their extension of DE works remarkably well for this quantitative approach to index-tracking, succeeding in reasonable runtime.

[3]Constraints on the number of stocks to hold in a portfolio, due to the costs of monitoring and portfolio re-weighting (Fieldsend et al., 2004).

[4]Produced by combining penalty function method (described in the paper) and differential evolution.

Another interesting finding is that their algorithm obtains results as good as QP for simplified formulations of the original index-tracking problem, although the computation time is, as expected, significantly shorter for QP. Furthermore, they show that increasing the maximum number of assets in the tracking portfolio leads to a lower tracking-error volatility up to a certain limit, but not necessarily to a higher out-of-sample excess return. A research paper touching a very similar subject was written by Andriosopoulos et al. (2013). The group of authors focus on more specific class of indexes, namely shipping stock indexes. They construct an international market-capitalisation-weighted shipping index and its performance is replicated in a similar manner as described above, employing the DE algorithm and a genetic algorithm to pick and weight the stocks. Out-of-sample testing under annual, quarterly and monthly rebalancing frequencies reveals that in terms of RMSE, the best tracker is the genetic algorithm basket with maximum number of assets set to ten, when weights are rebalanced on a monthly basis. As one would expect, it is shown that the higher the rebalancing frequency, the lower the RMSE. However, frequent rebalancing overall pares down the excess return due to transaction costs.

3 Theoretical Background

3.1 Global Minimum Variance Portfolio

Probably the most crucial concept of the MPT that originated from Markowitz' early work is the efficient frontier. It is a set of portfolios offering the highest expected return for a given level of risk (expressed as a standard deviation of returns), represented by a curve in the mean-variance space. Analytical derivation of the frontier can be found in Merton (1972). According to the MPT, the optimal portfolio for a mean-variance investor is a combination of a tangency portfolio[5] and a riskless asset. The construction of such portfolio, however, requires knowledge of both mean and a variance-covariance matrix of the expected returns which, in reality, are not known to the investor. Parameter uncertainty arises and thus the main challenge behind constructing the optimal portfolio in practice, lies in the estimation of the parameters from the data. As shown in Kan and Zhou (2007), the standard approach of simply plugging the sample parameter estimates into the model and treating them as the true parameters can lead to very poor out-of-sample results. Furthermore, the authors show that a portfolio that optimally combines a riskless asset, a sample tangency portfolio, and a sample global minimum variance portfolio dominates a portfolio with just the riskless asset and the sample tangency portfolio. The global minimum variance (GMV) portfolio is found on the left edge of the efficient frontier (it is the vertex point of the mean-variance parabola) giving the investor the lowest possible volatility for his selection of assets and as such, it solves the mean-variance risk minimization task.

Despite its limitations, the mean-variance framework is still frequently used as a benchmark in portfolio optimization research (see, e.g., Meucci, 2009). We have decided to use the GMV portfolio as a portfolio construction benchmark since it can be found analytically, using the matrix algebra and it does not require estimation of means of expected returns, as the only input is the variance-covariance matrix of asset returns. Additionally, numerous empirical studies suggest that the GMV might produce better out-of-sample results than the tangency portfolio, which the MPT describes as being superior to the GMV portfolio for its Sharpe efficiency

[5] A portfolio lying on the efficient frontier, maximizing the Sharpe ratio – a commonly used return-to-risk performance measure.

(Sharpe, 1964). This is due to the fact that the GMV portfolio optimization does not suffer from the mean estimation errors problem. Jorion (1985) shows that the errors in estimating expected returns have a critical impact on the portfolio analysis, both in terms of out-of-sample performance and instability of the optimum weights. Importantly, he concludes that both of these issues can be explained by a wide variation in sample means and that the uncertainty in variances and covariances is not as critical because they are more precisely estimated. And quite surprisingly, it is shown that the GMV portfolio significantly outperforms the tangency portfolio even in terms of profitability. Chopra and Ziemba (2013) also discovered that using inaccurate forecasts of the expected returns can considerably degrade the mean-variance optimization performance[6] and that errors in means are approximately ten times as important as errors in variances and covariances. These findings strengthen the argument for using the GMV portfolio as a benchmark.

The following part of this section presents the analytic solution to finding the GMV portfolio optimal weights as proposed in Merton (1972). The GMV optimization task with a quadratic objective function for a portfolio consisting of N assets can be written as:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^\top \mathbf{\Sigma} \mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^\top \epsilon = 1 \end{aligned} \tag{1}$$

where $\mathbf{w} = (w_1, \ldots, w_N)^\top \in \mathbb{R}^N$ is a vector of asset weights, $\mathbf{\Sigma} \in \mathbb{R}^{N \times N}$ is a positive-definite variance-covariance matrix of the expected returns and $\epsilon = (1, \ldots, 1)^\top \in \mathbb{R}^N$ is a vector of ones, with the appropriate dimensions. Thus, our goal is to find the vector of optimal weights $\mathbf{w}^* = (w_1^*, \ldots, w_N^*)^\top$ under the constraint that the weights add up to one, while short selling and leverage is allowed. The optimal weights, which are found using the Lagrange multipliers, are given as:

$$\mathbf{w}^* = \frac{\mathbf{\Sigma}^{-1} \epsilon}{\epsilon^\top \mathbf{\Sigma}^{-1} \epsilon}. \tag{2}$$

Hence we arrive to a relatively straightforward solution, which is, as shown further in the thesis, implemented using the R software. Specifically, we will be using R functions developed by Zivot (2008) to simulate holding the GMV portfolio under three different scenarios – buy and hold, annual and quarterly rebalancing.

[6] They measure the impact of the errors by a cash equivalent loss from holding the suboptimal portfolio (with parameter estimates) instead of the true optimal portfolio.

3.2 Differential Evolution

As mentioned earlier in the text, one of the alternatives to the classical mean-variance approach are the evolutionary algorithms. These search heuristics utilize trial-and-error selection to find the optimal solution (in our case, a vector of optimal asset weights) in a parameter space given by the optimization constraints. Differential evolution (Storn and Price, 1997) is a well suited choice for the purpose of our work since it is a relatively simple algorithm and being based on genetic processes from nature, its use in finance is indeed a fascinating subject of research. The DE can be used to tackle optimization problems with real-world constraints since it does not impose any assumptions on their mathematical properties. Moreover, it does not require that the objective functions have certain restricting properties, such as smoothness. It is therefore robust enough to allow for the use of coherent risk measures (described further in the text) which reflect the investors' actual perception of risk, contrary to the classical variance. Nevertheless, the framework has certain drawbacks. Firstly, the results might not be satisfyingly accurate and secondly, the computation time can potentially be very long for complex tasks. The latter can be, to some extent, lessened by parallelization[7] for which the algorithm is suitable. In fact, parallelizability is one of the four user requirements the DE was designed to fulfill. They are (Storn and Price, 1997, p. 342):

1) Ability to handle non-differentiable, nonlinear and multimodal cost functions[8].
2) Parallelizability to cope with computation intensive cost functions.
3) Ease of use, i.e. few control variables to steer the minimization. These variables should also be robust and easy to choose.
4) Good convergence properties, i.e. consistent convergence to the global minimum in consecutive independent trials.

As explained below in greater detail, the DE is a stochastic search heuristic which evolves a randomly chosen population of solution candidates by iteratively replacing inferior candidates by new ones created by the processes of mutation and crossover,

[7] A computation method in which calculations are carried out simultaneously, using multi-core processors (see Asanovic et al. (2006)).

[8] The authors refer to their objective function as cost function since it is minimized.

until a convergence criterion is met (e.g., after a given number of population generations is explored). The following description, coming from the original paper, explains the search process to minimize a given cost function.

For each generation G, there is a population of solution candidates consisting of NP D-dimensional parameter vectors $\{\mathbf{x}_{i,G} | \, i = 1, \ldots, NP\}$. The initial population is chosen randomly from the parameter space and NP remains constant throughout the minimization process. We assume a uniform probability distribution for all random decisions.

Mutation

To each candidate vector $\mathbf{x}_{i,G}$, $i = 1, \ldots, NP$ from generation G a mutation operator is applied according to the following formula:

$$\mathbf{v}_{i,G+1} = \mathbf{x}_{r_1,G} + F \cdot (\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G}), \tag{3}$$

where $r_1, r_2, r_3 \in \{1, 2, \ldots, NP\}$ are randomly chosen integer indexes, such that $r_1 \neq r_2 \neq r_3 \neq i$. This imposes a condition that the parameter NP must be at least greater or equal to four. $F \in \mathbb{R}$ is a predefined factor such that $F \in [0, 2]$. It controls the amplification of the differential variation $(\mathbf{x}_{r_2,G} - \mathbf{x}_{r_3,G})$ from which the algorithm got its name.

Crossover

The original vector $\mathbf{x}_{i,G}$ is then recombined with the mutant vector $\mathbf{v}_{i,G+1}$ from Equation 3 to increase the diversity of the evolving parameter vectors. This process is driven by the crossover operator, according to which a trial vector $\mathbf{u}_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, \ldots, u_{Di,G+1})$ is generated, where

$$\mathbf{u}_{ji,G+1} = \begin{cases} \mathbf{v}_{ji,G+1} & \text{if } \mathbf{\Psi}(j) \leq CR \text{ or } j = \mathbf{\Omega}(i), \\ \mathbf{x}_{ji,G} & \text{if } \mathbf{\Psi}(j) > CR \text{ or } j \neq \mathbf{\Omega}(i) \end{cases} \tag{4}$$
$$j = 1, 2, \ldots, D.$$

The $\mathbf{\Psi}$ operator in Equation 4 draws a uniform random number $\in [0, 1]$ for each $j = 1, 2, \ldots, D$. CR is the crossover constant $\in [0, 1]$ preselected by the user. It regulates the impact of $\mathbf{v}_{i,G+1}$ on the original vector. $\mathbf{\Omega}$ draws a random number such that $\mathbf{\Omega}(i) \in j = 1, 2, \ldots, D$. Therefore, the trial vector $\mathbf{u}_{i,G+1}$ must get at least one parameter from the mutant.

The newly created trial vector, however, only replaces the original vector in the next generation if it passes through the selection filter.

Selection

The selection mechanism is simple – a comparison of the trial vector $\mathbf{u}_{i,G+1}$ against the original vector $\mathbf{x}_{i,G}$ is made on the basis of the greedy criterion. If $\mathbf{u}_{i,G+1}$ yields a lower cost function (e.g., lower risk for the selected asset weights) than $\mathbf{x}_{i,G}$, $\mathbf{x}_{i,G+1}$ is set equal to $\mathbf{u}_{i,G+1}$ so that the inferior original vector is replaced. Otherwise, it is retained so that $\mathbf{x}_{i,G+1} = \mathbf{x}_{i,G}$.

These processes are repeated until the algorithm converges. The choice of DE's control variables is discussed in the empirical part of the study.

3.3 Returns and Risk Measures

This section describes investment performance measures that we use for the optimal portfolio construction and the evaluation of results.

Returns

Returns are a tool used to measure economic performance of an investment. As such, they measure a profit or loss on an investment over a period of time as a relative change in an asset's price[9], expressed as a proportion of the amount invested. The two most frequently used types of returns used to measure a single period performance are linear (simple) returns

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \tag{5}$$

and logarithmic returns

$$L_t = log\left(\frac{P_t}{P_{t-1}}\right), \tag{6}$$

where P_t is the asset's price for period t (usually last or closing price). For smaller values, simple and logarithmic returns are virtually equal. However, for larger values,

[9] For simplicity, we consider a plain vanilla stock investment that does not pay dividends.

using logarithmic returns as an approximation of simple returns can lead to significant estimation errors (Pini, 2009). Linear returns will be used in this thesis since they possess a property of additivity across assets and have a more intuitive interpretation. The former can be conveniently used to calculate a return of a whole asset portfolio over period t:

$$R_{P,t} = \sum_{i=1}^{N} w_{i,t} R_{i,t} \tag{7}$$

where $w_{i,t}$ is a weight of asset i at the beginning of period t. When analysing portfolio performance, however, we hardly ever look at individual periodic returns. We compare investment returns over longer holding periods, typically longer than one year and thus it is required to use measures of central tendency over standardized periods to conveniently compare returns of two different investments. One of such measures, very frequently used, is the geometric average or annualised return. As described in Bacon (2008), the average annual return over a number of years can be calculated as follows:

$$R_G = \left[\prod_{t=1}^{T} (1 + R_t) \right]^{\frac{f}{T}} - 1 \tag{8}$$

where T is the number of periods and f is the number of periods within a year (for instance, $f = 12$ if we work with monthly periodic returns). A useful property of the geometric average is that when compounded with itself for a cumulative (holding) period, it results in a cumulative return. In the empirical part of the thesis, geometric chaining is always used to compute the average or cumulative return, unless stated otherwise.

Risk Measures

There are numerous ways to quantify financial risk. In this thesis, we are concerned with one particular type of financial risk – the market risk, arising from the ever-changing market prices of securities. The Markowitz' era is characterized by measuring risk associated with an asset investment as a deviation from the mean return – variance (or standard deviation) and in the case of a portfolio of assets, as a covariance between all pairs of assets – i.e., we want to capture the extent to which each asset contributes to the overall portfolio risk. Despite still being widely used as a universal risk-quantification method, variance is not consistent with the investor's actual perception of risk and the theoretical assumptions that support its use as a risk measure are rather restrictive (Harlow, 1991). Namely, either returns have to be normally distributed or investors' behaviour has to be describable by quadratic utility function. There is a large volume of literature documenting that the former is a flawed assumption as normal distribution cannot adequately describe features that the asset returns exhibit, such as excess kurtosis and heavy tails (see Baillie and Bollerslev (2002) or Cont (2001)). Concerning the latter assumption, investors with this utility function require higher risk premia as their wealth increases – inconsistent with both intuition and observed investor behaviour (see Sarnat (1974)).

In the context of asset management, the investors' risk perception is shifted towards asymmetric risk measures, which do not take into account the upside potential of the investment and rather focus on a tail of the appropriate return distribution, below a predefined threshold level. Such quantile-based risk measures include the ubiquitous Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR, also known as Expected Shortfall). Artzner et al. (1999) developed a framework for a construction of such risk measures that ensures that these measures are coherent. In order for a risk measure ρ to be coherent, it must satisfy a following set of properties:

For investments[10] x and y:

- Positive homogeneity: $\rho(\lambda x) = \lambda\rho(x)$ for $\lambda \in \mathbb{R}$.
- Subadditivity: $\rho(x + y) \leq \rho(x) + \rho(y)$
- Monotonicity: $x \leq y$ implies $\rho(x) \leq \rho(y)$
- Transitional invariance: $\rho(x + rC) = \rho(x) - C$ for all riskless rates r and $C \in \mathbb{R}$

Let me make a few comments about the economic implications of these properties. The homogeneity condition simply states that an investment position risk is proportional to its size. Subadditivity signifies that diversification reduces overall risk – a risk of two different investments made together can never exceed adding the two risks separately. The monotonicity requirement rules out any semi-variance type of risk measure (Szegö, 2002). And finally, the transitional invariance implies that adding a riskless investment C decreases overall risk for any riskless rate r. In other words, adding cash to a portfolio only decreases risk. In what follows, the formerly men-

[10] Investments (portfolios) x and y have unknown outcomes and are random variables.

tioned risk measures VaR and CVaR are described in detail, together with remarks about their coherence.

Value-at-Risk

As described in Dowd (2007), the development of VaR as a risk measurement system started in the late 1980s, as financial firms became more complex and lacked the means to aggregate their risks, seeking for a reliable method that would allow them to do so. In 1994, JP Morgan published a simplified version of the firm's own internal risk model – the so called RiskMetrics, which contained information about a methodology for building and operating VaR-based risk management systems. A rapid adoption of VaR systems by financial institutions, such as securities houses and investment banks, followed.

The VaR is a quantile-based risk measure trying to answer a question: "For a given level of confidence, what proportion of our portfolio value is at risk over a given period of time?" As such, it focuses on extreme losses and contrary to variance, it allows for a use of more general distributions that capture the non-normality of returns. A mathematical formulation that follows comes from Pflug (2000) since I found it elegant and rather easily comprehensible.

Let Y be a random loss variable[11] and let F_Y be its cumulative distribution function, i.e. $F_Y(u) = \mathbb{P}\{Y \leq u\}$. Let $F_Y^{-1}(p)$ be its right continuous inverse – a quantile function for the loss r.v., i.e. $F_Y^{-1} : [0,1] \to \mathbb{R}$, $F_Y^{-1}(p) = inf\{u : F_Y(u) > p\}$. For a given confidence level α, the VaR_α is defined as

$$\mathrm{VaR}_\alpha(Y) = F_Y^{-1}(\alpha). \tag{9}$$

And so, the VaR_α is simply the α-quantile of the loss distribution. Calculating VaR in practice involves choosing the two arbitrary parameters – the confidence level and the holding period – a period over which the potential loss is calculated. As presented in Dowd (2007), using confidence levels in a region of 95% to 99% is the industry standard. The choice of the holding period depends on the context of use, yet for backtesting it is essentially given by the frequency of returns which are used

[11] A loss distribution is a return distribution with a "flipped" x-axis – losses are positive and profits are negative. It is used here for a more intuitive definition of VaR and CVaR. For the purpose of calculations, a return distribution is always used in its stead.

as the model input. VaR, however, has serious limitations. Firstly, it only tells us the maximum value of a loss if the tail event does *not* occur – it gives no indication about a magnitude of a loss we could suffer if the tail event occured. Secondly, VaR is not a coherent risk measure since it does not satisfy the subadditivity condition unless we impose a strict assumption on the return distribution. Specifically, the subadditivity condition only holds for elliptical distributions (Artzner et al., 1999). An important practical implication of this is that VaR does not reward diversification and also, it means that at least from a standpoint of coherence, VaR does not have to be regarded as a proper risk measure at all. Fortunately, the introduction of CVaR tackles both of these issues.

Conditional Value-at-Risk

Rockafellar and Uryasev (2000) defined Conditional Value-at-Risk as an expected loss exceeding the VaR. Again, using mathematical notation from Pflug (2000), it is given as

$$\mathrm{CVaR}_\alpha(Y) = \mathbb{E}[Y \,|\, Y \geq \mathrm{VaR}_\alpha(Y)]. \tag{10}$$

Alternatively, i.e.

$$\begin{aligned} \mathrm{CVaR}_\alpha(Y) &= \mathbb{E}[Y \,|\, Y \geq F_Y^{-1}(\alpha)] & (11) \\ &= \frac{1}{1-\alpha} \int_\alpha^1 F_Y^{-1}(p)\, \mathrm{d}p. & (12) \end{aligned}$$

CVaR is therefore more sensitive to the shape of the tails than VaR as it measures the conditional expectation of a loss in case of tail event occurrence, while VaR is simply a quantile. It holds that $\mathrm{CVaR}_\alpha(Y) \geq \mathrm{VaR}_\alpha(Y)$. CVaR satisfies the subadditivity condition along with all other properties of coherence and is therefore coherent (the proof can be found in Acerbi (2004)). In terms of its properties, CVaR is preferred to VaR and is used here as one of the optimization objectives (is minimized).

Both measures are, however, subject to estimation risk. In this work, a function that is used to estimate the return distributions uses sample estimates to calculate the first moments. Additionally, it implements statistical factor model based on the work of Boudt et al. (2015) to calculate the second, third and fourth moments which are used to model the distributions (in a case when CVaR is chosen as the objective function).

3.4 Exchange-Traded Funds

The last section of the theoretical part of this thesis is dedicated to a description of financial instruments that we use to represent the individual sectors of the American economy. In order to grasp how a single financial asset's performance can reflect the performance of a whole economic sector, it is important for the reader to understand how ETFs work.

ETFs are pooled investment vehicles that issue shares that are publicly traded on exchanges. Since their introduction in the 1990s, ETFs have substantially grown in popularity, with total assets under management breaching the $4 trillion mark in July 2019, according to a report by ETF.com (Rosenbluth, 2019a). This would mean approximately 26-fold increase of the ETF market size compared to the data from year-end 2003 (Antoniewicz and Heinrichs, 2014). There are numerous factors explaining the rising popularity of ETFs. Primarily, it is the shift from active to passive investing over the course of the past couple of decades (see Anadu et al. (2019)). This shift could be explained by the inability of actively managed funds to beat the benchmarks, especially over long-term horizons – according to Morningstar, Inc., only 23 % of all active funds managed to outperform the average of passive funds over the 10 year period ended June 2019 (Johnson, 2019). Intraday tradability, transparency and extensive variety of covered markets are other factors contributing to the surging demand for ETFs.

Just like a mutual fund[12], an ETF enables the investor to buy a portion (a share) of a pool of stocks, bonds and other instruments. The major difference, however, lies in the pricing mechanism. Shares of mutual funds are bought and sold at net asset value (NAV) per share (typically computed at the end of each business day), either directly from the fund company or an intermediary, e.g. a financial advisor. ETFs are bought and sold throughout a day via broker-dealers at market-determined prices (a secondary market activity relevant for most investors, dual structure of the ETF market is explained below). ETF can either be actively managed or have a passive investment objective – typically a market index replication. Yet, active funds amounted for approximately $100 billion in assets at the end of 2019 (Rosenbluth, 2019b) – a mere fraction of the ETF market.

[12]Mutual fund is a term used mainly in the U.S., a European alternative being the SICAV (investment company with variable capital).

The primary market

The following explanation of an ETF dual market structure comes from Antoniewicz and Heinrichs (2014). An ETF originates from a sponsor – an entity with investing expertise (a fund manager) that sets the investment objective for the ETF. Another agent that comes into play after the origination is an authorized participant (AP), also called participating dealer – a large institutional investor such as a brokerage house that provides liquidity to finance the creation/redemption mechanism. This mechanism regulates the amount of shares outstanding in the ETF based on demand. The shares are created when the AP applies for a creation unit – a specified number of shares, usually ranging from 25 to 200 thousand. A creation basket, settled to equal the fund's NAV per creation unit, is transferred to the sponsor and simultaneously, the shares are transferred to the AP. The creation basket contains either the physical securities that constitute the ETF portfolio[13] (an in-kind transfer) or it is limited to a subset of the portfolio's securities and the remainder is covered by a corresponding amount of cash. The latter is permitted or even required by the sponsor particularly when certain instruments in the basket are traded on less liquid markets as it is often the case with, for instance, high-yield bonds, or when the securities are odd lots that do not fit into the basket. In the case of stock ETFs, accumulated dividends can be another reason for a cash component to be a part of the basket. The redemption mechanism simply works in the opposite way. The AP redeems a creation unit by acquiring the corresponding number of shares through private purchases or from exchanges (the secondary market, discussed below) and transferring them to the sponsor and receives the redemption basket of securities and/or cash in return. At the end of each business day, the sponsor is required to publish a portfolio composition file – a list of securities and their quantities (plus the cash component) that equals the ETF's NAV per creation unit. This file informs the AP about the composition of the creation and redemption baskets for the next business day and thus serves as a pricing tool for the primary market.

The secondary market

As mentioned earlier in the text, mutual funds and ETFs differ in the way they are priced for individual investors, since ETFs can be traded on exchanges on an

[13] In the case of an index-based ETF, the securities in the basket are weighted to match their weighting in the index.

intraday basis. Therefore, once APs receive the created ETF shares, they can sell them in large blocks to big institutional investors or to individual investors in the secondary market – the exchanges. The imbalances in supply and demand can cause the floating price of an ETF share to deviate from its NAV per share. There are, however, counter-forces that eliminate any significant deviations and thus prevent from any long-lasting arbitrage opportunities for investors. The ability of APs to create or redeem shares keeps the floating price closely aligned with the share's underlying value. If, for example, the shares trade at a premium to their intraday value, the AP can use the creation mechanism to sell the freshly created shares at a higher price than the value of the securities comprising the corresponding basket. This creates a risk-free profit opportunity for the APs and it should, eventually, drive the price down in reaction to the newly-emerged excess supply.

4 Empirical Investigation

4.1 Data

The focus of this study is to examine whether our algorithm is able to generate alpha[14] through the allocation among the US sectors. For this purpose we use historical price data covering the period from January 3, 2012 to December 31, 2019 for the SPDR Sector Select ETFs to represent the individual US sectors and for the SPDR S&P 500 Trust (SPY) ETF, tracking the S&P 500 index[15], to represent the benchmark. Even though the data collection period starts from 2012, the investment period starts on January 2, 2015 since a 3 year-long estimation window is used for the portfolio construction. Each sector ETF is a capitalization-weighted selection of stocks from the S&P 500 index that belong to the corresponding sector[16]. The main reasons for choosing ETFs are their favorable price data availability and the fact that the ETF market is well accessible even to retail investors. The entire dataset was obtained from the publicly available database provided by Yahoo! Finance.

A few adjustments to our data were made to make them usable as an input into our model. Firstly, the raw dataset was cleaned to only contain daily closing prices. And secondly, simple (linear) daily returns as described in the theoretical section were calculated from the closing prices together with weekly and monthly returns to test different sampling frequencies. The list of instruments as well as summary statistics of returns are presented in Table 1. The nature of our data differs with the sampling frequency. Daily returns are economically insignificant, exhibit slightly negative skewness and have heavy tails. This is consistent with Cont (2001). Running the Jarque-Bera (JB) test on the daily returns results in rejecting the null hypothesis of normality at the 1% significance level for all instruments. The same conclusions are made when analyzing the weekly returns. However, we fail to reject the null hypothesis for the monthly returns and the magnitude of excess kurtosis is also

[14] Not to be confused with the α-quantile previously used in the VaR and CVaR formulas, here the term alpha refers to a return in excess of the broad market (represented by the SPY ETF in this paper). It is a commonly used term in asset management.

[15] The S&P 500 index is capitalization-weighted US stock market index that tracks the performance of 500 large companies listed in the US.

[16] The sector division is consistent with the Global Industry Classification Standard (GICS) taxonomy as it applied at the beginning of the period covered by our data.

lessened when compared to the lower sampling frequencies. Nevertheless, it has been shown (see, e.g., Thadewald and Büning (2007)) that the JB test lacks power when small-sized samples are used and there are less than 100 observations of the monthly returns in out dataset. The results thus suggest using higher moments to estimate return distributions in our optimization model.

Table 1: Summary Statistics

				Daily Returns				
Ticker	Sector	Mean	St. dev.	Maximum	Minimum	Skewness	Kurtosis	Jarque-Berra*
XLU	Utilities	0.03%	0.84%	2.98%	-4.18%	-0.50	4.75	0.00
XLP	Consumer Staples	0.04%	0.72%	2.95%	-3.63%	-0.40	4.92	0.00
XLK	Technology	0.07%	1.01%	6.04%	-5.05%	-0.32	6.33	0.00
XLY	Consumer Discretionary	0.06%	0.90%	5.93%	-3.99%	-0.35	5.60	0.00
XLB	Basic Materials	0.04%	1.02%	4.48%	-4.36%	-0.21	4.27	0.00
XLF	Financial Services	0.06%	1.03%	4.53%	-5.36%	-0.30	5.20	0.00
XLI	Industrial	0.05%	0.94%	4.66%	-4.54%	-0.36	4.96	0.00
XLV	Healthcare	0.06%	0.89%	4.36%	-4.43%	-0.32	5.13	0.00
XLE	Energy	0.00%	1.25%	6.22%	-6.42%	-0.15	4.85	0.00
SPY		0.05%	0.81%	5.05%	-4.21%	-0.40	6.31	0.00

				Weekly Returns				
Ticker	Sector	Mean	St. dev.	Maximum	Minimum	Skewness	Kurtosis	Jarque-Berra*
XLU	Utilities	0.15%	1.74%	4.72%	-5.30%	-0.38	3.24	0.01
XLP	Consumer Staples	0.17%	1.55%	5.43%	-7.93%	-0.55	5.25	0.00
XLK	Technology	0.33%	2.03%	6.02%	-8.32%	-0.58	4.58	0.00
XLY	Consumer Discretionary	0.30%	1.90%	5.85%	-8.37%	-0.51	4.29	0.00
XLB	Basic Materials	0.17%	2.14%	9.06%	-7.74%	-0.23	4.41	0.00
XLF	Financial Services	0.28%	2.25%	11.19%	-7.26%	-0.20	4.81	0.00
XLI	Industrial	0.23%	2.02%	8.11%	-7.16%	-0.34	4.15	0.00
XLV	Healthcare	0.28%	1.91%	6.95%	-7.06%	-0.45	4.56	0.00
XLE	Energy	0.01%	2.55%	8.85%	-9.81%	-0.38	4.44	0.00
SPY		0.24%	1.68%	4.71%	-7.59%	-0.72	5.14	0.00

				Monthly Returns				
Ticker	Sector	Mean	St. dev.	Maximum	Minimum	Skewness	Kurtosis	Jarque-Berra*
XLU	Utilities	0.67%	3.51%	8.03%	-9.05%	-0.52	3.01	0.13
XLP	Consumer Staples	0.74%	3.09%	6.41%	-9.74%	-0.60	3.76	0.03
XLK	Technology	1.42%	3.99%	10.51%	-8.77%	-0.37	3.11	0.35
XLY	Consumer Discretionary	1.29%	3.78%	9.87%	-10.10%	-0.40	3.70	0.18
XLB	Basic Materials	0.73%	4.32%	13.43%	-10.71%	-0.05	3.85	0.42
XLF	Financial Services	1.21%	4.35%	14.03%	-11.68%	-0.37	3.72	0.21
XLI	Industrial	1.00%	3.97%	11.43%	-11.21%	-0.36	4.03	0.10
XLV	Healthcare	1.19%	3.59%	8.08%	-9.76%	-0.58	3.74	0.04
XLE	Energy	-0.01%	5.27%	11.21%	-13.25%	-0.26	3.16	0.60
SPY		1.04%	3.19%	8.51%	-9.33%	-0.63	4.04	0.01

** Note: P-values for the Jarque-Bera test of normality.*

4.2 Methodology

In this section, we employ the concepts described in the theoretical part of the thesis to give a detailed explanation of the portfolio construction process.

We construct four different sector ETF portfolios and examine each portfolio's capability to generate alpha whilst keeping the risk as low as possible, over a five year-long investment horizon starting in January 2012. Two of these portfolios serve as benchmarks for optimization techniques: a 1/N portfolio and a GMV portfolio. The other two are DE-optimized portfolios, each with a different set of constraints to simulate a realistic agent-based setting. The 1/N portfolio, also called a naive portfolio, is simply a portfolio giving equal weighting to each asset. The GMV portfolio seeks to minimize the portfolio volatility over the investment period. The DE-optimized portfolios are both assigned two objectives – to maximize the mean return and minimize the Conditional Value-at-Risk (CVaR) calculated at a 95% confidence level. The first DE-optimized portfolio, to which we refer to as a Long Only portfolio (LO), is designed to simulate a strictly regulated market participant, e.g. a pension fund. It is therefore constrained to only contain long position investments and to keep leverage below 2%.[17] The other portfolio simulates a more loosely regulated participant – e.g. a hedge fund and as such its set of constraints allows for shorting, although each single short position's size is limited to a 20% of the capital invested and the overall leverage exposure, where leverage is defined as the sum of the absolute values of asset weights, is capped at 2. Just as with the LO portfolio, the weight sum constraint is relaxed to the 98% - 102% range. This portfolio is referred to as a With Shorting (WS) portfolio further in the text.

There are three strategies under which the portfolios are managed: a passive buy and hold approach, annual rebalancing and quarterly rebalancing. The rebalancing strategies employ a 3 year-long rolling window of returns to construct the portfolios[18]. The portfolios are thus re-optimized on each rebalancing date with the most recent observations (this of course does not apply to the 1/N portfolio, which is just rebalanced to keep the assets equally weighted). Regarding the buy and hold

[17] This is just a 98% - 102% relaxation of the 100% weights sum constraint to enable for situations where it is impossible to keep the capital invested at exactly 100%. These situations emerge, for instance, due to round lots. Relaxing the weight sum constraint also helps the algorithm to find more feasible candidate solutions.

[18] Additionally, a 1 year-long window is also tested when rebalancing quarterly.

strategy, a single 3 year-long estimation window is used for the portfolio construction.

In what follows, there is an explanation of the optimization techniques utilized in this study. The GMV portfolio allowing for short sales is found using the analytic approach. For this purpose, we use R functions developed by Zivot (2008) with a covariance matrix as input. The covariance matrix is estimated from the data windows using a sample covariance method. The DE portfolios are constructed in accordance with the set of objectives and constraints described above, using an R function package called *PortfolioAnalytics* (Peterson and Carl, 2018). The package provides a common interface to specify optimization constraints and objectives that can be solved by a supported solver (DE, particle swarm, QP methods etc.). The original version of the DE algorithm (Storn and Price, 1997) is extensively described in the theoretical section of the thesis. We use a few extensions to the original scheme, coming from Zhang and Sanderson (2009), to make the algorithm more convenient and less time consuming to use.

Primarily, we use *adaptive parameter control* so the algorithm does not require the user to prespecify the control parameters CR and F. Instead, feedback from the evolutionary search is used to dynamically change the parameters throughout the search process. The crossover probability CR from Equation 4 and the mutation factor F from Equation 3 become random variables CR_i and F_i, respectively. Both parameters are independently generated at each population generation and are associated with each individual candidate vector $\mathbf{x}_i$. The CR_i parameter is randomly drawn from a normal distribution $N_i(\mu_{CR}, 0.1)$ of mean μ_{CR} and standard deviation of 0.1., according to a following scheme:

$$CR_i = N_i(\mu_{CR}, 0.1). \tag{13}$$

Let S_{CR} be a set of all *successful* crossover probabilities CR_i's at generation G. Additionally, let $c \in [0, 1]$ be a constant preselected by the user − in our case chosen to be 0.4 and $\text{mean}_A(\cdot)$ be the usual arithmetic mean operator. The mean μ_{CR} is initialized to be 0.5 and then updated at the end of each generation as follows:

$$\mu_{CR} = (1 - c) \cdot \mu_{CR} + c \cdot \text{mean}_A(S_{CR}). \tag{14}$$

The adaptation of the F_i parameter follows a similar arrangement. At each generation G, it is drawn from a Cauchy distribution $C_i(\mu_F, 0.1)$ with location parameter

μ_F and scale parameter 0.1[19], i.e.,

$$F_i = C_i(\mu_F, 0.1). \tag{15}$$

Let S_F be a set of all *successful* mutation factors F_i's at generation G and $\text{mean}_L(\cdot)$ be the Lehmer mean:

$$\text{mean}_L(S_F) = \frac{\sum_{F \in S_F} F^2}{\sum_{F \in S_F} F}. \tag{16}$$

The location parameter μ_F is initialized to be 0.5 and then updated at the end of each generation as follows:

$$\mu_F = (1 - c) \cdot \mu_F + c \cdot \text{mean}_L(S_F). \tag{17}$$

The logic behind choosing the Cauchy distribution over the normal distribution in Equation 16, is to achieve better diversification in the mutation factors and thus avoid premature convergence. Lehmer mean is used in order to place more weight on larger successful mutation factors, and in consequence, to speed up the convergence of the algorithm.

Another extension we use alters the mutation process so that it incorporates the information about the best solutions. The mutant vector is therefore generated as:

$$\mathbf{v}_{i,G+1} = \mathbf{x}_{i,G} + F_i \cdot (\mathbf{x}^p_{best,G} - \mathbf{x}_{i,G}) + F_i \cdot (\mathbf{x}_{r_1,G} - \mathbf{x}_{r_2,G}), \tag{18}$$

where $\mathbf{x}^p_{best,G}$ is a $p$best solution randomly chosen from the top $100p\%$ candidate vectors from the current population. We set $p \in (0, 1]$ to be equal to 0.2 as it is recommended in the literature. F_i is the mutation factor generated by the above described adaptive mechanism and $r_1, r_2 \in \{1, 2, \ldots, NP\} \backslash \{i\}$ are distinct random integers.

Finally, since we specify two optimization objectives, an extension that ensures comparability between candidate solutions in a multi-dimensional space needs to be employed. This extension is based on a Pareto dominance and crowding density[20] and its explanation is beyond the scope of this thesis (see Zhang and Sanderson (2009) for a detailed overview).

[19]Location and scale parameters are used to specify the Cauchy distribution since both its mean and variance are undefined.

[20]To compare two solutions where neither dominates the other, crowding density is used. The solution with lower density is preferred.

4.3 Performance Analysis

This section presents empirical findings on the out-of-sample performance of the sector ETF portfolios under three different portfolio management strategies: buy and hold, annual rebalancing and quarterly rebalancing. To analyse economic performance, annualised (geometric average) returns are used, together with cumulative returns to capture the effect of not realizing any capital gains/losses throughout the investment period. Conditional Value-at-Risk is used as a measure of risk and we also examine standard deviation of portfolio returns since the aim of the GMV portfolio is its minimization. Our focus is to analyse whether the DE-optimized portfolios generate higher returns than the benchmark index (SPY), while also keeping the tail risk below the benchmark's values. In all of the above mentioned strategies, the investment period is 5 years, starting in January 2015 and ending in December 2019. This is in accordance with the GIPS performance presentation standards that require at least 5 years of performance history[21] to avoid "cherry picking" of time periods and ensure transparency and comparability.

Buy and Hold

In the buy and hold strategy, the weights are estimated using a 3 year-long in-sample window of asset returns covering the period from January 2012 till December 2015. Having obtained the portfolio weights, we use the portfolio returns formula from Equation 7 to calculate the portfolio returns with the same periodicity as the asset returns time series. The portfolio returns are then used to calculate the performance measures. These calculations are realized using monthly, weekly and daily returns as inputs. Table 2 presents a performance summary for the constructed portfolios and the benchmark. The weight allocations can be found in Appendix in the Table A1.

When using monthly data, the DE-optimized portfolios outperform the benchmark in terms of both return and tail risk, although the WS portfolio is slightly more volatile and the alpha of the two portfolios is not very significant. While the LO portfolio generates nearly identical return as the WS portfolio, it has lower tail risk with the CVaR reduced by almost 1% compared to its counterpart. The best-performing investment is the GMV portfolio, generating a 2.4% alpha and a cumulative return of almost 75% over the 5-year holding period with a monthly tail risk down approxi-

[21] Global Investment Performance Standards (GIPS), as cited in Bacon (2008).

mately 1.3% against the benchmark. It does not, however, beat the LO portfolio in terms of standard deviation, even though it is still less volatile than the benchmark. The outperformance of the GMV portfolio can be linked to its heavy allocation in the technology sector ETF (44% weight[22] at the initiation of the investment), which generated an alpha of almost 8% throughout the holding period. Due to the allowed short sales, the leverage exposure at the beginning of the investment period is 1.36 and 1.47 for the GMV and the WS portfolio, respectively. The 1/N portfolio performs the worst of the four constructed portfolios, lagging behind the benchmark in terms of returns while being almost as risky. This finding is not surprising since only two of the individual sector ETFs outperform the broad market over the holding period, namely the technology ETF and the consumer discretionary ETF.

The weekly data exhibit substantially different results. Only the WS and the GMV portfolios now outperform the market with the WS portfolio being a clear winner of the buy and hold scenario from the viewpoint of the risk-return tradeoff. It delivers a 12% annualized return, beating the benchmark by almost 3% whilst simultaneously having a 0.5% lower weekly CVaR. The WS portfolio is exposed to a leverage of 1.55 under the initial weight allocation, noticeably less than the GMV's leverage of 1.71, making it not just a better performing contender but also less costly in terms of borrowed capital. Yet it should be mentioned that the GMV is the least risky of the sector portfolios, both in terms of tail risk and volatility. The outperformance of the WS portfolio is a result of its 34.6% allocation in the technology ETF which performs best of all the sector ETFs and a 20% allocation in a short sale of the energy ETF which, on the other hand, is the only ETF that generates a negative annualized return of -0.5% over the period.

The use of daily data as input into the models leads to uniformly worse results. None of the sector portfolios outperform the market, even though both return volatility and CVaR are slightly reduced compared to the benchmark. This finding is quite surprising as data of higher frequency carry more information. Nevertheless, it is precarious to suggest any reasoning for the DE portfolios outperformance when lower data frequency is used, since the algorithm is practically a black box. The fact that

[22]Due to the presence of short sales in this portfolio, this number should be interpreted as a fraction of own capital invested rather than as a percentage of the total amount invested. If the number is summed with all other assets' absolute values of weights, it gives a value of 136% indicating the portfolio's exposure to leverage.

monthly data work best for the GMV portfolio might, however, be the result of monthly returns being closest to normality – an assumption of the mean-variance framework.

Table 2: B&H – performance summary

Monthly data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	**9.9%**	**10.9%**	**6.5%**	**60.5%**
With Shorting	**10.0%**	12.5%	**7.4%**	**61.3%**
1/N	7.8%	**11.6%**	**7.4%**	45.6%
GMV	**11.8%**	**11.3%**	**6.2%**	**74.6%**
SPY	9.4%	12.1%	7.5%	56.6%

Weekly data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	8.2%	**11.6%**	**4.6%**	48.5%
With Shorting	**12.0%**	**12.1%**	**4.4%**	**76.4%**
1/N	7.5%	**12.2%**	**4.7%**	44.1%
GMV	**9.7%**	**11.6%**	**4.3%**	**59.2%**
SPY	9.1%	12.8%	4.9%	54.9%

Daily data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	6.0%	**11.0%**	**1.9%**	33.5%
With Shorting	8.5%	**11.7%**	**1.9%**	50.4%
1/N	7.8%	**12.8%**	**2.3%**	45.6%
GMV	8.8%	**11.9%**	**2.0%**	52.6%
SPY	9.4%	13.5%	2.5%	56.6%

Note: This table presents the performance of the sector portfolios under the buy and hold scenario based on different data collection frequencies. 1/N and GMV are the benchmark portfolio construction models and SPY is the performance benchmark, representing the broad market. The annualized return is calculated according to the formula in Equation 8 and the cumulative return is calculated as the annualized return compounded over the investment period. The annualized st. deviation is scaled by the approximate number of observations in one year. These calculation methods come from Bacon (2008). Calculation of the Conditional Value-at-Risk is described in Section 3. The in-sample period (ISP) is set as 3 years and the holding period as 5 years, covering January 2015 to December 2019. All values in **bold** font indicate outperformance of the broad market.

Annual rebalancing

Subsequently, we select a more dynamic strategy where portfolios are rebalanced at the end of each calendar year[23]. First, it is important to illustrate the difference in the approaches to rebalancing the individual sector portfolios. The portfolios which use historical data to estimate optimal weights (the DE portfolios and GMV) are initially constructed using a 3 year-long data window and then re-optimized on a 3 year-long rolling window at each rebalancing date. When rebalancing takes place, the weights obtained from the models are used to reallocate the assets to reflect the most recent history and thus account for the development on the equity markets. Therefore, for these portfolios the term rebalancing does not refer to the process of readjusting the asset weights for the sole purpose of maintaining the initial weightings, as it is often defined in the literature. The classical definition, however, applies to the naive portfolio, where assets are bought and sold to prevent the asset weights from drifting away from the desired 1/N allocation. This simple approach ensures that the risk profile of the investment remains generally constant over the investment period. Its disadvantage is that it is not responsive to current market conditions and price movements. The investment period is again 5 years and so the portfolios are rebalanced 4 times. The results of this strategy can be found in Table 3.

When using monthly data, only the WS portfolio now beats the benchmark both in terms of return and risk. It generates a 1.6% alpha and a cumulative return of 68.6% while its monthly tail risk is 1.1% lower compared to the benchmark and, in fact, the lowest of all the portfolios. It is also less volatile than the benchmark, even though its long only counterpart exhibits a lower standard deviation value of 10.3%. Both shorting portfolios reach maximum leverage exposure during 2016 – 1.85 for WS and 2.06 for GMV. This is most likely due to the fact that 2015 was not very prosperous year for equities and the inflow of the most recent data into the models suggested more shorting. A complete overview of asset allocations throughout the investment period can be found in Appendix (tables A2, A3 and A4).

A few interesting remarks can be made when analyzing weekly and daily data results.

[23]This approach is often referred to as calendar rebalancing. Another popular approach is the so-called percentage of portfolio rebalancing where the portfolio proportions are maintained through buying or selling assets if their weights breach predefined tolerance bands, to adjust them to the original weighting. Thus in the case of the percentage strategy, the rebalancing transactions are not prearranged but rather triggered by rising volatility.

Firstly, the performance of the DE portfolios deteriorates with higher data frequency. And secondly, the sector portfolios reduce risk in practically all cases even though the reduction is not very significant. Overall, it cannot be said that annual rebalancing improves performance of the portfolios and neither considerably reduces risk.

Table 3: Annual rebalancing – performance summary

	Monthly data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	7.2%	**10.3%**	**6.9%**	41.5%
With Shorting	**11.0%**	**11.3%**	**6.4%**	**68.6%**
1/N	7.5%	**11.6%**	7.5%	43.5%
GMV	7.7%	**10.8%**	**6.8%**	45.2%
SPY	9.4%	12.1%	7.5%	56.6%
	Weekly data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	6.7%	**11.0%**	**4.3%**	38.6%
With Shorting	6.8%	**11.9%**	**4.0%**	39.1%
1/N	7.3%	**12.1%**	**4.7%**	42.1%
GMV	4.7%	**10.9%**	**3.9%**	25.7%
SPY	9.1%	12.8%	4.9%	54.9%
	Daily data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	3.8%	**11.5%**	**2.0%**	20.7%
With Shorting	3.6%	**12.1%**	**2.1%**	19.6%
1/N	7.5%	**12.7%**	**2.3%**	43.5%
GMV	6.1%	**11.2%**	**2.0%**	34.2%
SPY	9.4%	13.5%	2.5%	56.6%

Note: This table presents the performance of the sector portfolios under the annual rebalancing scenario based on different data collection frequencies. The in-sample period (ISP) is set as 3 years for each rebalancing and the investment period as 5 years, covering January 2015 to December 2019. All values in **bold** font indicate outperformance of the broad market. See Table 2 for additional notes regarding the performance measures.

Quarterly rebalancing

Quarterly rebalancing is implemented in a very similar manner to annual rebalancing. The portfolios are rebalanced at the end of each quarter, although this time we employ not only 3 year-long rolling windows, as in the previous case, but also 1 year-long rolling windows to test whether shorter in-sample period (ISP) works better if we rebalance weights more frequently[24]. With the 1 year-long ISP we thus create a more dynamic system with the allocations reflecting only the more recent stock market development. The quarterly rebalancing strategy would be preferred to the ones described above only if it suggested superior out-of-sample performance since it generates higher transaction costs paring down the returns. The results of quarterly rebalancing are presented in Tables 4 and 5 for 3-year ISP and 1-year ISP, respectively. Asset allocations under the quarterly rebalancing strategies can be found in an auxiliary Excel file that can be provided upon request.

Overall, the results are disappointing for all data frequencies and both ISP lengths. In the case of 1-year ISP, the monthly data now feed each model with only 12 past observations, rendering the models ineffective and making the resulting performance very poor – the WS portfolio has the highest monthly CVaR of 8.8% while generating even lower return than the average risk-free rate[25] of approximately 2%. The GMV shows rather confusing results, generating an impressive 20.8% annualized return but failing to limit the risks. An annualized standard deviation of 20% suggests that the data window is indeed too short for monthly inputs. Moreover, the model proposes unacceptably high leverage exposure – for instance, for the third quarter of 2016, the allocation generates a leverage of 22.82. The 3-year ISP brings no significant improvement.

With weekly and daily inputs, the 1-year ISP really does improve performance of the DE portfolios compared to the 3-year ISP, even though the figures are still unsatisfactory. The GMV produces very similar results for both estimation window lengths. Ultimately, it can be said that neither annual nor quarterly rebalancing consistently enhances the performance of any of the sector portfolios in comparison with the buy and hold approach.

[24] The window lengths apply to both initial portfolio construction and regular rebalancing.

[25] 10-year US treasury yield is used here to represent the risk-free rate. The average yield over the investment period is approximately 2%.

Table 4: Qaurterly rebalancing (3-year ISP) – performance summary

Monthly data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	-0.2%	**10.5%**	**7.1%**	-0.9%
With Shorting	4.3%	12.6%	7.7%	23.3%
1/N	7.5%	**11.5%**	**7.4%**	43.7%
GMV	5.6%	**11.0%**	**6.6%**	31.4%
SPY	9.4%	12.1%	7.5%	56.6%

Weekly data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	0.2%	**10.8%**	**4.0%**	1.1%
With Shorting	1.3%	**11.7%**	**4.1%**	6.8%
1/N	7.3%	**12.1%**	**4.6%**	42.3%
GMV	3.8%	**10.8%**	**4.0%**	20.5%
SPY	9.1%	12.8%	4.9%	54.9%

Daily data				
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	-0.9%	**11.8%**	**2.2%**	-4.6%
With Shorting	-0.6%	**12.2%**	**2.2%**	-2.7%
1/N	7.5%	**12.7%**	**2.3%**	43.7%
GMV	5.9%	**11.1%**	**1.9%**	33.1%
SPY	9.4%	13.5%	2.5%	56.6%

Note: This table presents the performance of the sector portfolios under the quarterly rebalancing scenario based on different data collection frequencies. The in-sample period (ISP) is set as 3 years for each rebalancing and the investment period as 5 years, covering January 2015 to December 2019. All values in **bold** font indicate outperformance of the broad market. See Table 2 for additional notes regarding the performance measures.

Table 5: Qaurterly rebalancing (1-year ISP) – performance summary

	Monthly data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	2.6%	**10.7%**	**6.8%**	13.7%
With Shorting	1.7%	13.1%	8.8%	8.8%
1/N	7.5%	**11.5%**	**7.4%**	43.7%
GMV	**20.8%**	20.0%	8.4%	**157.0%**
SPY	9.4%	12.1%	7.5%	56.6%

	Weekly data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	1.5%	**10.9%**	**3.9%**	7.7%
With Shorting	4.6%	**12.0%**	**4.4%**	25.1%
1/N	7.3%	**12.1%**	**4.6%**	42.3%
GMV	3.1%	**11.1%**	**4.4%**	16.7%
SPY	9.1%	12.8%	4.9%	54.9%

	Daily data			
	Annualized Return	Annualized St. Dev.	Conditional Value-at-Risk	Cumulative Return
Long Only	1.0%	**11.8%**	**2.2%**	5.0%
With Shorting	4.5%	**12.2%**	**2.1%**	24.5%
1/N	7.5%	**12.7%**	**2.3%**	43.7%
GMV	6.2%	**11.1%**	**2.0%**	34.8%
SPY	9.4%	13.5%	2.5%	56.6%

Note: This table presents the performance of the sector portfolios under the quarterly rebalancing scenario based on different data collection frequencies. The in-sample period (ISP) is set as 1 year for each rebalancing and the investment period as 5 years, covering January 2015 to December 2019. All values in **bold** font indicate outperformance of the broad market. See Table 2 for additional notes regarding the performance measures.

Note on the transaction costs

Transaction costs associated with investing in ETFs are largely dependent on market liquidity as well as on specific conditions determined by financial intermediaries. Market liquidity affects transaction costs through bid-ask spreads, making the costs time-varying and their estimation rather challenging. For these reasons, we have decided to disregard transaction costs in all performance calculations. However, it is not inadequate to make a supposition regarding the costs. As already stated above, the higher the rebalancing frequency, the higher the overall costs, simply due to the rising number of transactions. The additional reduction in returns generated by the rebalancing strategies would make the use of rebalancing even less desirable, most likely leading to a rejection of quarterly and potentially even annual rebalancing of the portfolios.

5 Conclusion

The aim of this thesis is to examine whether a nature-inspired heuristic search algorithm is capable of finding an inter-sector allocation that outperforms a broad market index whilst simultaneously being less risky. For this purpose we construct two sector ETF portfolios, each with a different set of constraints to simulate a realistic agent-based setting: a long only portfolio and a shorting portfolio. The results are further extended to include Markowitz' global minimum variance portfolio (GMV) and a naive 1/N portfolio. We test three different data input frequencies – monthly, weekly and daily and additionally, we employ three different strategies under which the portfolios are managed – a passive buy and hold approach, annual rebalancing and quarterly rebalancing. Although the long only portfolio provides similar results in terms of tail risk, the shorting portfolio delivers higher annualized returns in nearly all cases. The best performance is reached under the buy and hold approach, with the shorting portfolio outperforming the benchmark by almost 3% while providing lower tail-risk. Even though this result is achieved when using weekly data as an input, the monthly periodicity provides the best results overall. Rebalancing impairs the performance of the tested portfolios while simultaneously adding transaction costs. This suggests that the historical data covering the period from 2012 to 2014, used for the initial portfolio construction, provide the model with the best guidance for the future equity market development. Ultimately it can be said that picking among sectors to outperform the broad market is a challenging task in case the portfolios are heavily constrained. On the other hand the algorithm is able to deliver satisfying results while providing the user with a relative freedom when choosing portfolio constraints. The study could further be extended by testing different lengths of in-sample periods used for estimating the optimal allocations or by adding different asset classes to construct the portfolios.

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Appendix

Table A1: B&H – asset weights

	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
Monthly data										
Long Only	23%	0%	22%	11%	4%	0%	3%	34%	1%	0,98
With Shorting	7%	8%	8%	-6%	3%	2%	23%	73%	-18%	1,47
GMV	38%	-2%	44%	-7%	13%	-4%	18%	5%	-5%	1,36
Weekly data										
Long Only	11%	55%	22,6%	3%	1%	1%	3%	1%	0%	0,98
With Shorting	23%	31%	34,6%	5%	-8%	0%	18%	15%	-20%	1,55
GMV	26%	45%	33%	-12%	17%	-8%	-4%	15%	-12%	1,71
Daily data										
Long Only	34%	43%	0,2%	1%	2%	7%	0%	10%	1%	0,98
With Shorting	30%	46%	21,2%	-3%	5%	-14%	3%	16%	-4%	1,42
GMV	30%	51%	25%	3%	-4%	-15%	-2%	10%	3%	1,43

Note: This table presents the beginning-of-period asset weights under the buy and hold scenario. The beginning-of-period leverage exposure where leverage is defined as the sum of absolute values of asset weights is also presented.

Table A2: Annual rebalancing – asset weights; monthly data

					Long Only					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	17%	5%	11%	3%	5%	0%	5%	54%	0%	0,99
12/31/2015	38%	0%	33%	21%	1%	3%	2%	0%	0%	0,99
12/30/2016	29%	8%	24%	26%	2%	1%	0%	5%	3%	0,98
12/29/2017	32%	16%	1%	12%	1%	1%	16%	1%	19%	0,98
12/31/2018	58%	0%	2%	18%	19%	0%	2%	0%	0%	0,99
					With Shorting					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	17%	1%	15%	22%	5%	2%	-8%	58%	-16%	1,45
12/31/2015	27%	-19%	40%	29%	-18%	0%	16%	29%	-6%	1,85
12/30/2016	43%	-5%	39%	27%	-3%	22%	-12%	-1%	-9%	1,62
12/29/2017	33%	31%	-10%	34%	-5%	11%	13%	-5%	-3%	1,47
12/31/2018	42%	-1%	33%	22%	25%	11%	-18%	-1%	-14%	1,66
					GMV					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	38%	-2%	44%	-7%	13%	-4%	18%	5%	-5%	1,36
12/31/2015	37%	-10%	71%	-13%	-30%	17%	5%	10%	13%	2,06
12/30/2016	23%	34%	7%	-15%	-15%	21%	7%	25%	14%	1,62
12/29/2017	36%	21%	-15%	19%	-8%	10%	8%	13%	16%	1,46
12/31/2018	41%	21%	4%	15%	22%	19%	-19%	-6%	4%	1,50

Note: This table presents the beginning-of-period asset weights under the annual rebalancing scenario, using monthly data. The beginning-of-period leverage exposure where leverage is defined as the sum of absolute values of asset weights is also presented.

Table A3: Annual rebalancing – asset weights; weekly data

					Long Only					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/26/2014	12%	40%	40%	1%	0%	3%	0%	1%	0%	0,98
12/31/2015	41%	6%	2%	35%	4%	0%	10%	0%	1%	0,98
12/30/2016	36%	30%	2%	23%	1%	0%	2%	3%	0%	0,98
12/29/2017	47%	3%	0%	39%	0%	2%	2%	1%	4%	0,99
12/28/2018	52%	35%	0%	2%	2%	6%	0%	1%	0%	0,99
					With Shorting					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/26/2014	7%	32%	27%	14%	-13%	-19%	18%	51%	-19%	2,00
12/31/2015	28%	7%	44%	33%	-16%	-8%	16%	13%	-18%	1,83
12/30/2016	36%	29%	-3%	33%	-20%	1%	10%	19%	-8%	1,58
12/29/2017	29%	31%	-19%	23%	0%	-10%	40%	12%	-7%	1,71
12/28/2018	65%	25%	-6%	7%	2%	17%	-15%	-3%	4%	1,45
					GMV					
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/26/2014	26%	45%	33%	-12%	17%	-8%	-4%	15%	-12%	1,71
12/31/2015	27%	43%	2%	18%	22%	17%	-29%	6%	-6%	1,69
12/30/2016	25%	55%	-23%	12%	14%	19%	-20%	19%	0%	1,86
12/29/2017	27%	42%	-19%	10%	12%	14%	-14%	21%	7%	1,65
12/28/2018	47%	18%	-9%	11%	18%	2%	-9%	8%	15%	1,36

Note: This table presents the beginning-of-period asset weights under the annual rebalancing scenario, using weekly data. The beginning-of-period leverage exposure where leverage is defined as the sum of absolute values of asset weights is also presented.

Table A4: Annual rebalancing – asset weights; daily data

	Long Only									
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	46%	43%	7%	0%	1%	0%	0%	0%	1%	0,98
12/31/2015	34%	54%	4%	0%	1%	2%	0%	1%	2%	0,98
12/30/2016	14%	70%	0%	3%	4%	0%	0%	6%	0%	0,98
12/29/2017	22%	59%	0%	3%	2%	2%	1%	8%	1%	0,98
12/31/2018	47%	10%	1%	0%	2%	1%	1%	37%	0%	0,99
	With Shorting									
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	47%	34%	14%	5%	4%	6%	-19%	17%	-8%	1,53
12/31/2015	28%	53%	14%	17%	32%	-7%	-20%	0%	-19%	1,90
12/30/2016	23%	72%	-18%	32%	7%	16%	-18%	-5%	-9%	2,00
12/29/2017	23%	45%	-19%	38%	9%	-7%	31%	-7%	-14%	1,93
12/31/2018	54%	10%	-15%	16%	2%	26%	-18%	36%	-13%	1,89
	GMV									
	XLU	XLP	XLK	XLY	XLB	XLF	XLI	XLV	XLE	Leverage Exposure
12/31/2014	30%	51%	25%	3%	-4%	-15%	-2%	10%	3%	1,43
12/31/2015	23%	59%	17%	9%	4%	-12%	-1%	-1%	1%	1,28
12/30/2016	20%	58%	-10%	10%	6%	2%	7%	9%	-2%	1,25
12/29/2017	22%	43%	-10%	15%	2%	2%	13%	14%	-2%	1,25
12/31/2018	36%	26%	-17%	21%	2%	11%	3%	19%	0%	1,34

Note: This table presents the beginning-of-period asset weights under the annual rebalancing scenario, using daily data. The beginning-of-period leverage exposure where leverage is defined as the sum of absolute values of asset weights is also presented.