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How much do we pay for a real estate ownership? A simulation approach

Bachelor's thesis

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Declaration of Authorship

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Prague, July 31, 2020

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Abstract

This thesis compares rent vs home ownership based on the net present value, within the periods of the Great Recession and current year. The analysis is focused on the Czech Republic real estate market as a whole. Rent and real estate price are forecasted, and factors determining the price of rent and real estate are identified. The ARIMA model used for forecasting performs accurate short-term predictions. The results expect 3,2 percent annual growth of rent in the following year and 7,2 percent increase for the real estate prices. The results of net present value analysis indicate, that for years 2008 and 2009 renting was superior choice, while for years 2011, 2013 and 2019 home ownership was to be preferred from financial aspect.

JEL Classification	F12, C32, C53, R21, R31
Keywords	real estate price, rent, net present value, forecasting
Title	How much do we pay for a real estate ownership? A simulation approach

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Abstrakt

Táto práca porovnáva kúpu bytov verzus prenájom na základe čistej súčasnej hodnoty počas obdobia dopadov Veľkej Recesie v Českej republike. Analýza je zamerená na Českú Republiku ako celok. V praci sú prognózované ceny prenájmov a ceny bytov a faktory ovplyvňujúce tieto ceny sú taktiež sledované. Na prognostiku boli použité ARIMA modely, ktoré vytvárajú presné krátkodobé prognózy. Výsledky naznačujú rast cien aj v nasledujúcom roku, presnejšie rast cien nájmov o 3,2 percent a cien nehnuteľností o 7,2 percent. Výsledky analýzy čistej súčasnej hodnoty ukazujú, že zatiaľ čo počas rokov 2008 a 2009 bolo výhodnejšie bývať v nájme, v rokoch 2011, 2013 a 2019 to bola kúpa nehnuteľností.

Klasifikácia JEL	F12, C32, C53, R21, R31
Kľúčové slová	cena nehnuteľností, nájom, čistá súčastná hodnota, prognostika
Názov práce	Koľko skutočne platíme za vlastníctvo nehnuteľností?

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Contents

List of Tables

List of Figures

Acronyms

ADF Augmented Dickey-Fuller test

AIC Akaike Criterion

ARIMA Autoregressive Integrated Moving Average

ARIMAX ARIMA with exogenous variables

ARMA Autoregressive Moving Average

CNB Czech National Bank

CPI Consumer Price Index

CZK Czech Crown

CZSO Czech Statistical Office

EUR Euro

GDP Gross National Income

MAFE Mean Absolute Forecasting Error

MSFE Mean Squared Forecasting Error

Bachelor's Thesis Proposal

Author	Ivana Gallova
Supervisor	Petr Pleticha, M.Sc.
Proposed topic	How much do we pay for a real estate ownership? A simulation approach

Research question and motivation One of the biggest financial decision many people must make is whether to take a mortgage and buy a real estate or rent one. The aim of my thesis is to find out what is the premium people are willing to pay for real estate ownership. I will focus on real estate market in Prague, examine current prices of real estates, rent and interest rate of mortgages, and design models simulating value development of different portfolios over a thirty years' horizon.

Contribution There is extensive difference between rent/own ratio across Europe - from 38% in Switzerland to 95% in Romania. Home ownership is proportionally related to experienced macroeconomic factors such as inflation (Malmendier, Steiny, 2017). The low home ownership of Switzerland is caused mainly by high real estate prices compared to income (Bourassa, Hoesli, 2010). So, for them, it is more convenient to rent a house. The aim of my thesis is to demonstrate where the Czech Republic stands, mostly Prague and to show how big is the difference between taking a mortgage and paying interests rate and renting a house/apartment.

Methodology Using time series data, I will make econometrics models to compare under what conditions it will be more efficient to take a mortgage and buy a real estate. In the first part, I will estimate regression model with independent variables as the current price of properties, average rent, interest rate of mortgages, inflation, wage, and age to find out which decision is more efficient at present. In the second part, I will make simulations with forecasting variables from the first model in 30 years' horizon and I will estimate the new model with the new variables to determine whether the conclusion from the first part still holds. I will use data provided by Eurostat and the Czech Statistical Office.

Outline

1. Introduction
2. Literature review
3. Data
4. Methodology
5. Results and discussion
6. Conclusion

Core bibliography

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Author

Supervisor

Chapter 1

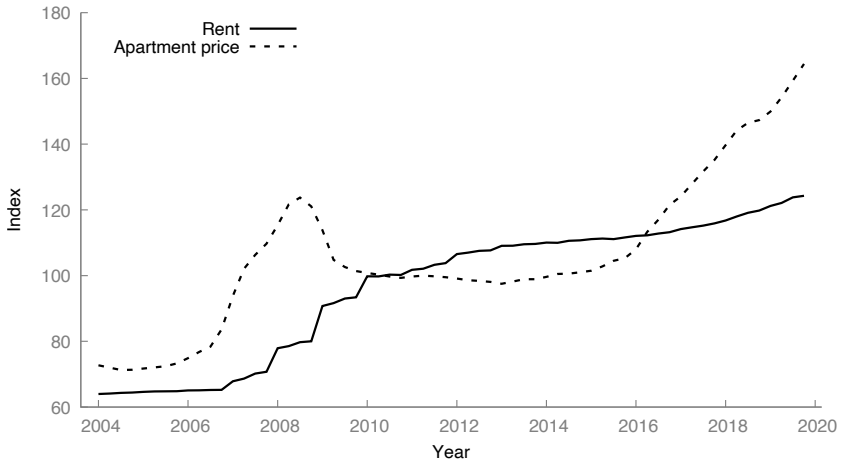
Introduction

Should I buy a real estate, or live in a rented property? Nowadays it is one of the most important questions people face at least once during a lifetime (Tabner 2016). Both tenure choices have positive and negative effects. Renting is more affordable because to buy a real estate people need to have funds for the down payment. Tenants are relatively free to move out from a rented property, while homeowners are less flexible in this aspect. Home ownership means, that after individuals repay the mortgage it becomes their own property, and from that moment, their costs for living will be decreased. Home ownership provides equity, and it supports senior citizens' budget. The question is whether it is just a safer alternative, or it is also a smarter choice from financial perspective. In the Czech Republic 21.3 percent of people lived and rented dwelling during the 2018 (EUROSTAT).

The prices of both real estate and rent have increased sharply since the analyzed period. The prices of rent have been increasing continuously, while property prices dropped during the Great Recession in 2008 (see Figure 1.1). The rent price growth rate between 2007 and the first quarter of 2020 was 82.2 percent in the Czech Republic, which is the second highest growth rate in the European Union. Growth rate in house prices ranks at the fourth place with 63 percent among European states (EUROSTAT). This increase is caused by massive demand growth for real estate which results in the shortage of available dwellings and low mortgage interest rates (Somogyi 2019).

The objective of the thesis is to compare the costs and benefits of home-ownership with rent prices between 2008 and 2019 as this era is characterized by various economic turnovers. We will evaluate the tenure choice using net present value (NPV). Specially we are interested in the years 2008, 2009, 2011,

Figure 1.1: Real estate price and rent development



2013 as the Great Recession affected the housing market during these years, and 2019 to have current comparison results. For the assumed growth rate of apartment prices and rent we need to forecast the values for each period separately. We will find the best model, make the predictions and identify the price determining factors of rent and property purchase.

Main questions we want to answer are how much the prices will change in following years, which factors determine the price of both tenure choices and whether it is more beneficial to rent or buy.

This bachelor thesis is structured as follows. After the introduction part in Chapter 2 there is a short literature review focused on the comparison of homeownership versus renting and real estate market in the Czech Republic. Data used in the thesis are precisely described in the Chapter 3. Chapter 4 consists of the used methodology and models description. In Chapter 5 empirical results of the econometric analysis are presented and discussed. In Chapter 6 there is a detailed analysis of net present value with its results and the last chapter, the last chapter provides Chapter 7.

Chapter 2

Literature review

Extensive numbers of studies have been made about real estates, but the comparison between rent and homeownership is still scarce (Tabner 2016, Wainer & Zabel 2020). It shows that there is a great number of factors determining advantages and disadvantages of both buying and renting (Beracha & Johnson 2012). These factors are not only economic but also socioeconomic and even psychological (Diaz-Serrano 2009). The first part covers all these factors in comparison between rent and homeownership internationally, the second covers the most important and similar studies for this thesis focused on the real estate market in the Czech Republic. At the end there is evaluation of the real estate prices and rent evolution in the Czech Republic.

2.1 Renting vs homeownership worldwide

The most common ways to calculate economical difference and advantages of rent vs. homeownership is rent to price ratio and price to income ratio. Despite of fluctuation in the short run, these ratios are reliable indicators in the long run as they return near to average (André *et al.* 2014).

Most people think about homeownership as an "American dream" (Cauley *et al.* 2007). On the other hand, when comparing financial aspect of renting vs homeownership, many studies show the advantages of renting. (Beracha & Johnson 2012). In despite of people's urge to own, renting and reinvesting the saved money had been the more suitable option during the studied timeframe 1978-2009. To gain financial advantage of renting is possible only by reinvesting put aside money to safe bonds or stocks. By spending this money on goods

individuals would have been satisfied in the present but would not gain any wealth in the long run (Beracha & Johnson 2012).

Timing in the decision process whether to buy or rent during the end of the 20th century and in the early 2000s was crucial. Studies based on the panel data in the USA during this period show that gaining wealth by becoming a homeowner was depending on timing (Di *et al.* 2007). In the short run, gaining wealth by becoming a homeowner is certainly sensitive to the time period of the purchase. In the long run it is more favorable to purchase but it still depends on the cyclical period (Di *et al.* 2007).

By putting individuals renting and purchasing homes side to side in different time periods during the first decade of the 2000s using intent-to-treat framework Newman & Holupka (2016) found that renting was more convenient in the long-run. "The intent-to-treat principle refers to a set of criteria for the evaluation of the benefits and risks of a new therapy that essentially calls for the complete inclusion of all data from all patients randomized in the final analyses" (Lachin 2000). In the short run there was found a significant difference in the outcome by ethnic groups. While white individuals acquired small gain in wealth, African Americans lost some of their wealth. Authors believe that demographic factors, such as different location preference were the main reasons in this distinction.

A study by Wainer & Zabel (2020) was aiming to prove that for low-income households it is easier to gain wealth by buying a real estate. Data used for the study were available every five years during the examined period from 1984 to 2000 and since 2001 till 2013 it was available biennially. Gain in wealth was observed for individuals who became homeowners in the years between 1989 and 1999. For persons who purchased property in the early 2000s would have been more beneficial to stay as renters.

Regardless of the survey saying that almost 86 percent of Swiss prefer owning a real estate to renting, Switzerland has the lowest rate of homeowners in Europe (Eurostat; Bourassa & Hoesli 2010). Deciding whether to buy or rent in Switzerland is much more based on economic factors then in any other state. The biggest issue is high price to income ratio. This is caused mainly by the lack of new real estate due to geographic restrictions and strict laws (Borowiecki 2009). The likelihood of owning is different across age groups. Bourassa & Hoesli (2010) detected that "while for young people under the age of thirty it is only 13% for the older generation it is 57%".

Financial aspect is not everything. Finally becoming a homeowner also

brings happiness. Subjective well-being has been studied by psychologists for a long time; in recent years also, economists has started to examine this issue (Diaz-Serrano 2009). Based on the panel data obtained in Germany it was proven that buying a home raises subjective well-being both in the long run and in the short run. However, short-term effect was higher than the result in the long-run. Compared to renters, new homeowners scored one point higher in average on subjective well-being scale from 0 to10. After five years this difference decreased to 0.7 points. The analysis also shows that age is also a determining factor. Younger new homeowners had in average higher short-term increase in subjective well-being then older homeowners (Stotz 2019).

Homeowners are usually less flexible and are staying further at one place without moving. This is mainly due to the higher transaction costs compared to moving from a rented apartment (Dietz & Haurin 2003). The fact that renters move more frequently than homeowners is related to employment (Oswald 1999). According to the author, unemployment is directly proportional to homeownership. This was confirmed by Green & Hendershott (2001). They detected that this relationship is strongest in the middle-aged population since they are usually already settled and still in labor force. This is opposed by the findings of Borg & Branden (2018) who researched Swedish housing and labor market in the recent years. They had come to conclusion that despite of the fact that in the areas with higher rate of homeowners the unemployment rate is bigger, it cannot be justified by the tenure choice of people, even though it is connected.

2.2 Rent vs ownership evidence from the Czech Republic

Applying Net Present Value as main decisive factor, living in rented apartment in Prague is more convenient from monetary perspective than buying (Ulrich 2010, Tláskalová 2013). Diploma thesis by Ulrich (2010) examines advantages of homeownership versus renting using various methods of financing. Using forecasting methods for 15 years, the author concludes that the biggest difference in rent vs homeownership is buying using only own capital. In this case the gap is enormous 6.7 million CZK. The "best" option of financing is mortgage, where the difference in NPV is 4.3 million. The difference in rent vs buy is caused by impossibility of investing into bonds or stocks. The anal-

ysis is extremely sensitive to changes in the rent amount and inflation. Just a small change, for example raise in the rent amount by 238 CZK would balance the difference and erase the advantage of renting (the original rent was 17 318 CZK).

In contrast to Ulrich (2010), Tláskalová (2013) compares various different investing strategies in her work, for example buying a real estate, selling it and buying a new one. The only option in favor of homeownership is renting followed by buying in tower blocks. As stated before, author uses rent to price ratio. This ratio was also applied to variables influencing the price. The study shows that ground floored apartments are more lucrative for buying. On the other hand, renting is better for bigger apartments with more rooms is more effective.

Another way to compare profitability of buying vs renting is monthly mortgage payment to monthly rent payment ratio. Suchánek (2008)) in his thesis applies this method. The lower the ratio the more it pays off to buy a dwelling. Study is focused on the Czech Republic and shows that the best cities for homeownership are Teplice and Usti nad Labem with this ratio below 1. On the other side of scale are Prague - west and Prague - east with the ratio being equal to 1.98. Suchanek also shows yearly increase in prices in Prague by 8.2 percent and in Czech Republic by 9.1 percent between the years 1998 and 2006.

2.3 Price market in the Czech Republic

In the beginning of 2008 when already the Great Recession was going on in the Czech Republic, real estate prices were still increasing despite price decreasing trends in the world around. That lead Hlaváček & Komárek (2009) to look for the price bubbles examining the real estate market between the years 1997 - 2007. Two price bubbles were determined; the first one, and surprisingly bigger one was in 2002/2003 and the second one in 2007/2008.

The low mortgage interest rate allows real estate price increase in the Czech Republic. The growth is higher compared to the other countries, mostly because the slow number of new dwellings (Fait 2019). Shortage of the available real estates is caused mostly by the building restrictions (Somogyi 2019). In 2018 the prices were overstated by 10 to 15 percent. Compared to the real estate prices in 2008 still being relatively low (Plašil & Andrle 2019). According to the CNB in the following year, it was overstated by 15 to 20 percent (Fait 2019).

The most significant economic factors determining the price of real estate in the Czech Republic are the unemployment rate, wage and mortgage volume. But not only economic factors determine the property price; divorce rate was also significant variable, as the separations of partners increase demand for the dwellings (Hlaváček & Komárek 2009).

Homeowners in the Czech Republic have a lower consumption and higher net savings comparing to the tenants. This was most notable during the sharp increase of the property prices however the gap stayed even after the price decline in 2009 (Bruha *et al.* 2013).

Chapter 3

Data

We collected time series data from various sources (CZSO, CNB, EUROSTAT). Monetary and macroeconomic data were carefully chosen based on previous, mostly international, real estate's studies and forecasting papers (e.g. Wilson *et al.* 2000, Rapach 2007). We tried to look only for the data collected quarterly, but for some variables only monthly data were available. For the model we needed to have all data collected in the same intervals therefore if this was a case we made and average of each quarter. Also, some of the variables had to be seasonal adjusted (e.g. wage). Some data were available in the nominal values, therefore we had to clean them to real values using CPI deflator with base year 2010. In our analysis we use quarterly panel data from 2004 to 2019.

Table 3.1: Data Summary

variable	Unit	N	Mean	Median
Apartment price	CZK/m^2	64	19634	18662
Rent	CZK/m^2	64	104.4	113.7
CZK/EUR currency rate	CZK	64	26.81	26.28
Goverm. Bonds rate_10	%	64	2.930	3.214
GDP	$bilCZK$	64	1031	1015
Mortgage interest rate	%	64	3.931	3.818
Mortgage volume	$milCZK$	64	12425	12848
Price to income	$ratio$	64	4.598	4.523
Unemployment	%	64	5.621	6.182
Wage	CZK	64	24169	23587

Real estate price The prices of real estate were collected from CZSO. CSZO has been doing statistics on prices of the real estate since 2000. Every year

CZSO publishes paper that evaluates the prices of the previous three years, the oldest one being the most accurate. For example, in the 2019 paper there are prices for the years 2016 to 2018. The 2016 year was also evaluated in the publication comes from cooperation of CZSO with the Ministry of Finance. It is derived from completed transfer prices, hence the new apartments are not included. From 2014 the expert testimony is not mandatory hence the data comes only from the transaction with it. Two types of the data are published every year. Annual price for square meter and quarterly house price index. To find the real value of meter squared for every quartal we combined these values. For our analysis we chose apartments because they are the most homogenous type of real estate, cause the fact, that the price changes mostly due time and not due to wear rate and location. Prices are in real terms with the base year 2010.

Rent Rent time series was build by combining data from Eurostat and Ministry of Regional Development of the Czech Republic. The minsitry did a Unfortunately, the data comes only for the Czech Republic as a whole, therefore the analysis will be based on the figures representing the whole republic. It is an average price of one square meter.

Unemployment Unemployment takes and important role in the homeownership rate (Oswald, 1999) and on that account it also affects the prices of both apartments and rent. Data are procured from CZSO.

Average Wage Average real gross wage is one of the most important factors determining the price of real estate. Data are collected by CSZO and including bonuses, that are usually paid once a year, therefore in our analysis wage had to be seasonally adjusted. Prices are in real terms with the base year 2010.

Price-to-income ratio Price to income ratio is the ratio between the property price and annual income. It is often used as an indicator of the purchase availability (Zhang *et al.* 2016). Simply put, this ratio shows us how many years people have to work to be able to buy a real estate. The lower the ratio, the better the availability. This variable was included in many models as macroeconomic indicator for example in the study of Rapach (2007).

Currency rate CZK/EUR Currency exchange rate can be also used as one of the macroeconomic variables affecting real estate prices (Vishwakarma 2013). It is expected that the most important currency for the Czech Republic is the Euro currency, due to its location next to states of Eurozone. The rate was obtained from CNB.

Government bonds Investing into the government bonds is one of the safes investments (Codogno *et al.* 2003). If people think about buying a real estate as an investment, other investment choices need to be reflected. We will use the bonds with 10 years holding period. Data were collected from the CNB.

Mortgage interest rate For the model we chose mortgage interest rate before the general loan interest rate with the purpose of buying a real estate, because it is the most popular way to finance the buying property in the Czech Republic (CNB, ARAD statistics). The dataset has been available since 2004, when there was a change in methodology. For the studied period of time the mortgage rate is lower by 0.16 percentage points comparing to classic loan. (CNB, ARAD statistics).

Volume of mortgages Similar to mortgage interest rate, this variable is also obtained from CNB. The volume means the millions of Czech crowns, that people borrowed from banks in order to buy a property. The high volume of mortgages in the recent years causes liquidity decline (Hlaváček & Komárek 2009). Therefore, we will use also this variable in our model.

Gross national income GDP is important macroeconomic variable as it is the main indicator of the economy. Price of real estates and rent are highly correlated with GDP (Quan & Titman 1999) and we assumed that it should be also included in our analysis.

Chapter 4

Theory and Methodology

In the empirical analysis we are going to estimate and forecast prices of real estates and rents in the Czech Republic using time series data and autoregressive integrated moving average (ARIMA) model.

4.1 ARMA, ARIMA, ARIMAX and SARIMA models

ARIMA and SARIMA models are popular time series forecasting methods, often used in economic models, due to properties that are well understood (Zhang 2003). ARIMA model is accurate in the short and medium term forecasting, however it does not show the turning points precisely. (Chen & Yu 2010).Crawford & Fratantoni (2003) compares ARIMA forecasting to other methods on the sample of various states in the US real estate prices. It shows that neither method is the best one all the time, it always depends on the sample, although ARIMA was superior in most of the cases. Also, it shows ARIMA as significantly more suitable model for out of sample forecasting. We chose this model based on previous studies about estimation and forecasting real estate market and prices (Tse 1997, Wilson *et al.* 2000, Chen & Yu 2010).

Autoregressive process AR of order p is defined as

$$x_t = f(t) + \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + x_t \phi_p x_{t-p} + \epsilon_t \tag{4.1}$$

Where $f(t)$ is the deterministic component and can be constant or constant with a trend, $\phi_1, ..\phi_p$ are finite set of weight parameters and ϵ_t are independent and identically random variables normally distributed (Box *et al.* 2008).

Moving average (MA) of order q is defines as

$$x_t = f(t) + \epsilon_t - \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + ... + \theta_q\epsilon_{t-q} \tag{4.2}$$

Where $f(t)$ is the deterministic component and can be constant or constant with trend, $\theta_1, ..., \theta_q$ are finite set of weight parameters and $\epsilon_t \sim iidN(0, \sigma_\epsilon^2)$ (Box *et al.* 2008).

By putting these two parts together we get Autoregressive moving average (ARMA). This model is the fundamental model of ARIMA family. It consists of AR(p) and MA(q) and is given as (Box *et al.* 2008)

$$x_t = f(t) + \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + \epsilon_t - \theta_1\epsilon_{t-1} + \theta_2\epsilon_{t-2} + ... + \theta_q\epsilon_{t-q} \tag{4.3}$$

The problem with this model is that it works only if all its parts are stationary. As in reality most variables are not static, we need to adjust them. This is possible by transformation. The most popular transformation method is differentiation. By adding differentiation to ARMA(p,q) model we get ARIMA (p,d,q) where d is the order of differentiations and indices how many times we have to differentiate model to be stationary. ARIMA model consists of three parts: autoregressive (AR), integrated (I) and moving average (MA) with the following equation (Box *et al.* 2008)

$$\begin{aligned} \bigtriangledown^d x_t = f(t) + \phi_1 x_{t-1} + \phi_2 x_{t-2} + ... + \phi_p x_{t-p} + \\ + \epsilon_t - \theta_1\epsilon_{t-1} - \theta_2\epsilon_{t-2} - ... - \theta_q\epsilon_{t-q} \end{aligned} \tag{4.4}$$

where $\epsilon_t \sim iidN(0, \sigma_\epsilon^2)$ (Box *et al.* 2008). Time series data are usually collected in regular intervals, e.g. yearly, monthly or in our case, quarterly. This may occasionally result in seasonality phenomenon. Checking the seasonal stationarity is as important as the classic stationarity. Adding seasonal character to the ARIMA(p,d,q) model we get SARIMA(P,D,Q)s where s indicates the number of seasons (4 represents quarters, 12 months, etc.) and P,D,Q represent the order of AR, differentiations and MA respectively (Box *et al.* 2008).

ARIMAX is ARIMA model with exogenous input. It means that we can add other explanatory variables to the model.

4.2 Identification

For the time series data analysis we need stationary data. Strict stationarity is defined as a joint distribution function of the stochastic process that is independent of time. It implies that that the characteristics of the stochastic process do not change when different time periods are considered. (Wooldridge, 2012).

$$F(x_1, x_2, ..., x_T) = F(x_{1+h}, x_{2+h}, ..., x_{T+h}) = F(x_{1-h}, x_{2-h}, ..., x_{T-h}) \tag{4.5}$$

For every $h = 1, 2, ...$ Weak stationarity is defined as the second order stationarity or covariance stationarity. It requires that the first and second order moments of the stochastic process' distribution do not depend on time (Wooldridge 2016).

$$\begin{aligned} E(x_t) = E(x_{t+h}) = \mu < \infty \\ Var(x_t) = Var(x_{t+h}) = \sigma^2 < \infty \\ Cov(x_t, x_s) = Cov(x_{t+h}, x_{s+h}) = \gamma < \infty \end{aligned} \tag{4.6}$$

For checking whether our model is stationary or not, we will use Augmented Dickey-Fuller (ADF) test. ADF is a classical unit root test described by Dickey and Fuller (Box *et al.* 2008). Comparing ADF test to τ statistics test ADF is more powerful for testing stationarity.

Now we can test the hypothesis

$$\begin{aligned} H_o = |\rho| = 1 \\ H_1 = |\rho| < 1 \end{aligned} \tag{4.7}$$

The null hypothesis states that the model comprises of one unit root, i.e. is non-stationary, opposite to alternative hypothesis, which says that the model is stationary. Testing to find the correct model and differentiate the difference stationarity from trend stationarity is important for the following estimation and forecasting of the series (Cochrane 1991)

4.3 Estimation

To find the most suitable model we use various tests, as mentioned in the subsection before. Firstly, we made sure that our time series is stationary. The

next step is to find the best model that fits our time series data. For this purpose, we will use Akaike information criterion (AIC) introduced by Akaike (1974)

$$AIC_{p,q} = -2ln(ML) + 2rln(\hat{\sigma}_a^2) + 2r + constant \tag{4.8}$$

$$AIC_{p,q} = -2ln(ML) + 2r \times \frac{1}{n} ln(\hat{\sigma}^{2_a}) + 2r \times \frac{1}{n} + constant \tag{4.9}$$

Where $\hat{\sigma}_a^2$ is the maximum likelihood estimate of σ_a^2 and $r = p + q + 1$ is the number of estimated parameters, including a constant. The Equation 4.8 is based on Akaike (1974) and the Equation 4.8 is the original equation normalized by sample n as suggested by Box *et al.* (2008). We will use the second equation and as the best model will be chosen the one with the lowest AIC.

4.4 Validation

We have to also validate the estimated model, using three points (Box *et al.* 2008)

- the significance of the estimated coefficients - the parameters of autoregressive and moving average components have to be statistically significant
- stationarity fulfillment - the stationarity of ARIMA model is satisfied if the roots are greater than 1
- behaviour of estimation's residuals - we have to chech the residuals' correlation using Q-test of Ljung-Box. If we can not reject the null hypothesis, that correlation coefficient are equal to zero, we can say that the residuals behave as white noise.

4.5 Forecasting

Forecasted ARIMA model at the time $t + n$ can be written as

$$\begin{aligned} \bigtriangledown^d x_{t+n} = f(t) + \phi_1 x_{t+n-1} + \phi_2 x_{t+n-2} + ... + \phi_p x_{t+n-p} + \\ + \epsilon_t - \theta_1 \epsilon_{t+n-1} - \theta_2 \epsilon_{t+n-2} - ... - \theta_q \epsilon_{t+n-q} \end{aligned} \tag{4.10}$$

Where x_{t+n}, is the forecasted variable, $n \geq 1$ when we are currently on time t and n is the forecasting time (Box *et al.* 2008). After the forecast is done it is

important to test the accuracy, to check whether our model is well specified or has accurate inputs. We will use the frequently used evaluation tests, based on forecast error. Forecast error $\hat{e}_1$ is defined as the actual value minus the forecast value Brooks & Tsolacos (2010) We will use the Mean absolute forecast error (MAFE) and Mean squared forecasting error (MSFE) in our analysis.

Mean absolute forecast error can be derived as the

$$MAFE = \frac{1}{n} \sum |\hat{e}_i| \tag{4.11}$$

Where $\hat{e}_i$ is the forecast error and n is the number of forecast period. We expect that the $\hat{e}_i$i is zero or close to zero, and the lower is the MAFE the better forecast.

Mean squared forecasting error takes into account variance of the forecast error

$$MSFE = \frac{1}{n} \sum \hat{e}_i^2 \tag{4.12}$$

Chapter 5

Empirical results

Firstly, stationarity of the original series is checked using ADF test and modified if it is non-stationary. Then the ARIMAX model is built and the actual casual effects on dependent variables, rent and apartment prices, are discussed and tracked.

5.1 Stationarity

To build our models we need to find the order of integration and secure stationarity. To check the stationarity, we use the ADF test, which was explained in the methodology section. In the original time series, we cannot reject null hypothesis, that the model has one unit root, i.e. all variables are non-stationary. After the first differentiation most variables were stationary; however, for wage and GDP we also have to use seasonal differentiation to reject the null hypothesis. As mentioned in the data section, we had to apply seasonal differentiation due to the mechanism wage data were collected, as they also include bonuses. Our detection of seasonal component in GDP confirms the findings of Hylleberg *et al.* (1993). After these modifications are made all series are stationary. Variables defined in percentage were not log-transformed (unemployment rate, mortgage interest rate and government bond rate), but all the others were. The results of the AD test are presented in Table 5.1.

5.2 ARIMAX

Having all variables stationary, we can construct our models. We want to check the significance of independent variables on the apartment prices and

Table 5.1: Results of ADF test

note: the p-values are in the parentheses

	original	$1^{st}diff$	seasonal diff
l_apartment	-0.045	-0.209	
	(0.356)	(0.337)	
l_rent	-0.050	-0.663	
	(0.789)	(0.043)	
l_CZK/EUR	-0.139	-1.041	
	(0.197)	(<0.001)	
l_GDP	-0.112	-0.597	-1.05999
	(0.601)	(0.626)	(0.003)
l_gbond	-0.119	-1.043	
	(0.669)	(<0.001)	
mrt_int_rate	-0.003	-0.679841	
	(0.933)	(0.015)	
l_mrtg_vol	-0.021	-0.205	
	(0.237)	(0.0138)	
l_price-to-income	-0.137	-0.839	
	(0.621)	(0.017)	
unemployment	-0.068	-0.610	
	(0.765)	(0.048)	
l_wage	-0.244	-0.843	-1.701
	(0.343)	(0.580)	(0.001)

the rent prices. To check this, we use ARIMAX models based on the Box-Jenkins methodology stated in the previous chapter. As the same model was selected for both predicted variables, it is further described once. Now we can write our models for apartment price Equation 5.1 and for rent Equation 5.2

$$\begin{aligned} ln(apprice) = \beta_0 + \beta_1 ln(eur) + \beta_2 ln(rent) + \beta_3 ln(pricetoincome) + \\ + \beta_4 ln(mrtgvolume) + \beta_5 ln(sd\_wage) + beta_6 ln(sd\_gdp) + \\ + \beta_7 unempl + \beta_8 mrtginterest + \beta_9 gbond \end{aligned} \tag{5.1}$$

$$\begin{aligned} ln(rent) = \beta_0 + \beta_1 ln(eur) + \beta_2 ln(apprice) + \beta_3 ln(pricetoincome) + \\ + \beta_4 ln(mrtgvolume) + \beta_5 ln(sd\_wage) + beta_6 ln(sd\_gdp) + \\ + \beta_7 unempl + \beta_8 mrtginterest + \beta_9 gbond \end{aligned} \tag{5.2}$$

First, we have to do the model differentiation. As we already know that our original series is non-stationary; hence, we have to do the first difference. Followed by model identification ARIMAX (1,1,0) model is selected as the best one based on the Akaike criterion. Next we have to validate the estimated

model. All estimated coefficients of AR (1) are significant, root of the autoregressive part is greater than one in absolute value, that means that they are stationary as the weak stationarity condition is met. As the last requirement to validate this model we have to check the residuals using Ljung-Box Q-test. Failing to reject the null hypothesis indicates that the residuals behave as white noise and our model is specified accurately.

We may conclude that both R squared, and adjusted R squared are high for both models. As we can see in the Table 5.2 in the first model where apartment price is an explained variable only exchange currency rate CZK/EUR and mortgage interest rate are not significant. In the second model (rent is dependent variable) exchange currency rate CZK/EUR and GDP are not significant.

We will compare the actual casual effects of both dependent variables. People have two tenure choices. They either live in their own property or rent one. Therefore, we expect, that if the variable has a positive effect on the real estate model than it has negative effect on the rent model and vice versa. This means that if the price of real estate increased, the price of rent would decrease. As shown in Figure 1.1 this is not the case. Yet, the higher price of real estate can raise the demand for rent; hence, the prices of rent will increase as well (Hlaváček & Komárek 2009). Our results confirm both of these assumptions. Some of the independent variables have the same correlation sign, some have the opposite one.

Mortgage volume Mortgage volume is significant at 1 percent in both models. In the model in which apartment price is a dependent variable the effect is positive, while in the model with rent being dependent variable the effect is negative. This confirms our assumption that the model contrasts opposite effects. The higher is the mortgage volume the higher is the demand for real estate and as a result the prices are increasing (Clayton *et al.* 2010).

Unemployment rate Another significant variable at 1 percent for both models is the unemployment rate. The negative effect on apartment prices was expected. People usually buy real estate using mortgage and to get a mortgage people have to have permanent income (Česká spořitelna 2019). The same effect was also found by Hlaváček & Komárek (2009). We believe that the impossibility of getting mortgage increases the prices of rent, which was also confirmed by our model.

Table 5.2: Results of the ARIMAX models

Variable	Apartments	Rent
const	0.112*	0.014*
	(1.109)	(0.008)
phi_1	0.984***	0.786*
	(0.020)	(0.084)
l_CZK/EUR	0.109	0.068
	(0.091)	(0.070)
l_apartmp	-	0.393**
	-	(0.001)
l	0.291***	-
	(0.103)	-
l_price_to_income	0.080***	0.082**
	(0.039)	(0.027)
l_mrtg_vol	-0.135***	0.124**
	(0.024)	(0.041)
sd_l_Wage	0.172***	-0.042*
	(0.127)	(0.085)
sd_l_GDP	â'0.229**	- 0.112
	(0.121)	(0.086)
unemployment	-0.021***	0.016***
	(0.006)	(0.006)
mrtg_interest	0.042	â'0.015*
	(0.014)	(0.016)
gbond_10	-0.006**	0.004*
	(0.005)	(0.004)
$R\hat{2}$	0.995	0.994
Adjusted $R\hat{2}$	0.994	0.994
Akaike	â'306.479	â'312.749

Wage Positive effect of the average wage on apartment prices means that people on high income will prefer buying a property since there is better affordability and they will not be looking for renting a place; therefore, in the model where rent is dependent variable the coefficient is negative.

Rent/Apartment prices When rent is the explanatory variable it is significant at 1 percent. It has positive effect on the model, which means that the higher prices of rent will increase the demand for apartments thus the apartment prices will increase as well (Hlaváček & Komárek 2009). Also, the apartment price as independent variable is significant at 5 percent with positive correlation towards rents. The same principle applies here. This indicates that increase in apartment prices leads to rent price growth.

5.3 Forecasting

Our NPV analysis requires prediction of the annual growth rate of both apartment and rent prices. Using ARIMA model we forecast the prices for all five periods. As we need annual growth, we make forecasts for four observations, since our time series data was collected quarterly. We are interested in the years 2008, 2009, 2011, 2013 and 2019. For the periods before 2019 we can compare the accuracy of the forecasted values to the real values.

Firstly, we want to forecast the apartment prices using Box-Jenkins methodology same as in the previous subsection. Stationarity of the variable is satisfied after making the first differentiation as the null hypothesis of the ADF is rejected. Based on Akaike criterion ARIMA (1,1,1) model was selected as the best one. All estimated coefficients of both AR (1) and MA (1) are significant and both roots are greater than one in absolute value that means that they are stationary as the weak stationarity condition is satisfied. The last part of validation requires to check the residuals using Ljung-Box Q-test. The null hypothesis could not be rejected meaning that the residuals behave as white noise and our model is specified accurately. Now we can forecast the future values.

We forecast the rent the same way as the apartment prices were forecasted. The variable is non-stationary, so we have to do the first difference. However, the seasonal component was also identified. Again, using Akaike criterion we select SARIMA (0,1,0) (1,0,1)4. This model is also well specified since both estimated seasonal coefficients are significant, stationarity condition is satisfied

too as the roots are greater than one. The null hypothesis using Ljung-Box Q test cannot be rejected therefore the residuals behave as white noise.

The results of the forecasts are presented in Table 5.3 and Table 5.4, where forecasted observations and the actual values are shown. Mean absolute forecasting error and mean squared forecasting error for the first four periods are also indicated in the table. All errors are rather small, only the forecast of the apartment prices in the first period is over 10 percent. According to this outcome our forecast for 2020 should be also precise. The higher errors in 2009 forecast can be explained by the smallest amount of pre-forecast observations as well as by the sudden price decline due to beginning of the Great Recession. These results support the findings of Vishwakarma (2013) and Tse (1997) that ARIMA models models perform accurate short term predictions, with less precise prediction of turning points.

Table 5.3: Forecast of the apartment prices

	Forecast	Actual value	MAFE	MSFE
2009 Q1	21959,01	21052,13		
2009 Q2	21858,01	19379,20		
2009 Q3	21608,98	18977,70		
2009 Q4	21087,48	18750,18		
			0,109	0,01324
2010 Q1	18668,32	18643,86		
2010 Q2	18676,04	18551,38		
2010 Q3	18721,56	18440,41		
2010 Q4	18770,44	18366,42		
			0,011	0,00019
2012 Q1	18355,8	18329,43		
2012 Q2	18325,34	18236,95		
2012 Q3	18305,85	18199,96		
2012 Q4	18293,38	18144,47		
			0,005	0,00003
2014 Q1	18226,34	18421,91		
2014 Q2	18174,95	18588,37		
2014 Q3	18159,72	18606,87		
2014 Q4	18162,45	18680,86		
			0,021	0,00049
2020 Q1	31163,29			
2020 Q2	31721,37			
2020 Q3	32154,72			
2020 Q4	32516,02			

For the NPV analysis in the Chapter 6 we will need assumed annual growth rate in the apartment and rent prices, which we derive from the forecasted observations.

In 2008 the assumed growth rate in apartment prices and rent was -5,8 percent, while the following year it was -1.7 percent. In the year 2011 during the peak of the Great recession in the Czech Republic the growth rate was

Table 5.4: Forecast of the rent

	Forecast	Actual value	MAFE	MSFE
2009 Q1	92,11	98,12		
2009 Q2	92,96	99,08		
2009 Q3	94,19	100,62		
2009 Q4	94,76	100,99		
			0,062	0,00387
2010 Q1	103,16	107,90		
2010 Q2	105,33	107,88		
2010 Q3	107,50	108,46		
2010 Q4	109,66	108,35		
			0,022	0,00068
2012 Q1	110,55	115,23		
2012 Q2	111,55	115,72		
2012 Q3	113,29	116,28		
2012 Q4	114,59	116,42		
			0,030	0,00097
2014 Q1	118,66	119,01		
2014 Q2	118,58	118,94		
2014 Q3	119,19	119,60		
2014 Q4	119,47	119,76		
			0,003	0,00001
2020 Q1	136,02			
2020 Q2	136,78			
2020 Q3	138,39			
2020 Q4	139,02			

almost zero with -0,4 percent. The last two examined periods have positive growth rate 1,6 percent in the 2013 and 7,2 percent in 2019. Our forecast for 2019 is close to the expected growth by CNB Fait (2019).

Assumed growth rate in rent is positive in all periods. The biggest growth of 9 percent was in 2008. The next year it was 8 percent, in year 2011 the growth was 2 percent, followed by the smallest change of 0.7 percent in 2013 as the Great Recession had ended. For the year 2019 we expect to have a growth rate of 3.2 percent. Our forecast, that rent in the Czech Republic will be still increasing in the following years is in line with the findings of Somogyi (2019).

Chapter 6

Net present value

Comparing the financial aspect of tenure choice can be done by net present value approach (Tabner 2016). We will do cost benefit analysis, comparing the total cash inflows and outflows. A paper by Tláskalová (2013) compares the tenure choice on the Prague's real estate market in one term on the various scenario using different apartments' sizes. Contrarily, we will compare profitability of rent to buy decision based on one average sized apartment in the Czech Republic as whole during different stages of economic cycle.

In the simulation we will consider two scenarios. The first one is buying a real estate property with the mortgage, where the mortgage will be 80 percent of the property's price. The other option is renting an apartment and investing the money, otherwise spent as a down payment for a real estate, into government bonds. We will do five simulations using NPV. The first one will be based on the current data (2019), another one will be based on the data before the consequence of economic crisis came to the Czech real estate market (2008),the following year (2009) as the apartment 's prices started to fall, during the peak of economic crisis (2011) and the last one will be derived from the data during the end of crisis (2013). Aim of this simulation is to compare how much we really pay for the ownership and which tenure choice is more beneficial during various stages of economic cycle.

6.1 Model inputs

Real estate price Real estate price we assumed calculated with an average apartment in the Czech Republic. The average size of the apartment is 68,5

m^2. The price is derived for every each year as the average price for m^2 price /CZK multiplied by the size of the apartment[1]

Down payment Down payment was established as 20 percent of the property price. It is based on the general amount of the loan to price value that banks offer.

Furnishing costs Furnishing costs are the costs that new homeowner has to pay, after buying a property for renovation or buying a new furniture, equipment etc. We set it as 5 percent of the property price same as suggested by Tabner (2016).

Acquisition tax Acquisition tax is tax that has to be paid during the change of ownership of real estate. In the Czech Republic this tax was changed in the 2016 . Until then the tax had to been be paid by the owner, from the 2016 the buyer has to is obliged to pay it. It is set at 4 percent of the purchase price. In our simulation we have scenarios from both periods. In the simulations before the year 2016 we will count it into the formula of future resale value and in the simulations this year we will include it into total purchasing costs.

Purchase costs Purchase costs are the expenses paid during the buying process of the property. For example, it contains lawyers' fee, real estate agency fee. Tabner (2016) set is as five percent in his paper, with transfer tax inclusive. Due to changes in Czech Law regarding transaction tax we made it as a different variable, and we set the purchase costs as 1 percent of the property's price.

Selling costs Selling costs are similar to the purchasing costs but paid during the selling process of the property. It was set to 1 percent as well.

Mortgage interest rate Mortgage interest rate is the interest rate published by CNB for each year. In our simulations we will use the mortgage interest rate offered by banks, without taking in consideration interest rates by building society.

[1]The Ministry of Regional Development of the Czech Republic, 2019

Nominal discount rate Nominal discount rate is a discount of future cash flows to their present value. It is not an easy task to set this number as it is influenced by a plenty of macroeconomic variables. We decided to set it same as mortgage interest rate as suggested by Tabner (2016).

Holding period We set the holding period at 10 years as it is the optimal holding period of real estate in the Europe Baroni *et al.* (2007).

Mortgage term Mortgage term was set for 25 years because in the model the loan to value ratio is set for 80 percent, and if the term is shorter monthly mortgage payments would be too high to pay for most people with an average wage.

Rent Rent the price of rent was calculated similar as the price of real estate, as the average size apartment multiplied by the price for m^2/CZK.

Assumed real growth rate in an apartment price As a growth rate we will use firstly the forecasted values from Chapter 5 and then we will also use an inflation as the assumed real growth rate. We know the growth rate for the previous years (before 2019), but for the analysis to be consistent we will forecast also those.

Assumed growth rate in rent The same thing we will to with the assumed nominal growth rate in the rent.

6.2 Methodology

Now we are going to the describe in detail the NPV approach. Firstly, we will consider the buying scenario and derive all necessary data. After that we will compute all equations that relate to the renting. Finally we will derive the net present value as the difference between the total cash inflows and outflows.

In the buying scenario we are purchasing an apartment with the mortgage of 80 percent of the apartment's price. Therefore, the initial total purchasing costs TC consists of down payment DP, furnishing costs FC, purchasing costs and transfer tax TT in the 2019 model.

$$TC = DP + FC + PC + TT \tag{6.1}$$

We assume that the property was bought with the mortgage. Firstly, we will calculate the mortgage principal MP as the property price minus the down payment. After we have the mortgage principal, we can determine the annual mortgage payments AMP for T years using annuity formula.

$$MP_0 = P - DP \tag{6.2}$$

$$AMP = MP_0 \div \left(\frac{1 - \frac{1}{(1+r_m)^T}}{r_m} \right) \tag{6.3}$$

r_m represents the mortgage interest rate.

To calculate the present value of the mortgage payments $PVMP$ over the holding period we will use the AMP which we derived sooner and annuity formula. We assume that the mortgage interest rate is the same as the nominal discount rate r_n

$$PVMP = AMP \times \left(\frac{1 - \frac{1}{(1+r_n)^H}}{r_m} \right) \tag{6.4}$$

To derive present value of the mortgage principal payments $PVMPP$ which increases every year we will use growing annuity formula. Also we need to find the principal repayment MPP. Principal repayment is the difference between the payment and the interest paid.

$$MPP = AMP - (MP \times r_m) \tag{6.5}$$

$$PVMPP = MPP \times \left(\frac{1 - \left(\frac{1+r_m}{1+r_n} \right)^H}{r_n - r_m} \right) \tag{6.6}$$

We assumed that the mortgage interest rate is the same as the nominal discount rate r_n. Problem is that in this equation they would zero themselves. Therefore, we have to modify the r_n by adding a small value to it same way as Tabner (2016) did it in his paper. Our adjusted nominal discount rate will be $r_{n1} = r_n + 1 \times 10^{-9}$. And now we can derive the present value of the mortgage principal payments

$$PVMPP = MPP \times \left(\frac{1 - \left(\frac{1+r_m}{1+r_n} \right)^H}{r_{n1} - r_m} \right) \tag{6.7}$$

Using future annuity formula, we obtain principal outstanding MP_t at the end of holding period.

$$MP_T = MP_0 - MPP \times \left(\frac{(1 + r_n)^H - 1}{r_m} \right) \tag{6.8}$$

For calculating net present value of the resale value of the apartment $NPVP_T$, we have to find the future resale value first. For this we need assumed annual growth in real estate's prices, selling costs and principal outstanding at the end of the holding period. For the scenarios before the year 2016 we have to also include acquisition tax.

$$NP_T = P \times (1 + GP)^H - SC - TT - MP_t \tag{6.9}$$

$$PVNP_T = \frac{NP_T}{(1 + r_n)^H} \tag{6.10}$$

Where P is the price of apartment, GP is assumed apartment growth rate and SC are selling costs. Now we have the present value of the all costs connected to the buying scenario. We can derive the present value of rent. Present value of rent is derived using growing annuity formula, where GR is the assumed nominal annual growth rate of rent. Rent is denoted as monthly payment, thus we have to multiply it by 12.

$$PVR = (R \times 12) \times \left(\frac{1 - \left(\frac{1+GR}{1+r_n} \right)^H}{r_n - GR} \right) \tag{6.11}$$

And at last, we can derive net present value of homeownership as the difference of the total cash inflows and outflows. The inflows are the sum of the future resell value and rent. The outflows are present value of the mortgage payments, total costs of purchase and present value.

$$NPV = PVCI = PVCO \tag{6.12}$$

If the NPV is be positive it means that buying is the superior option to the renting.

6.3 Results and Discussion

The main objective of the thesis is to compare two different tenure choices, renting and homeownership, and show which one is more beneficial. Based on the previously stated methodology we compare NPV of both options for five various stages of the economic cycle, more precisely the year before the beginning of the great recession in 2008; the second year of the recession; during its peak in 2011; post-recession era in 2013; and for the current period - year 2019. We build two models, one with the forecasted annual growth rate of the apartment and rent prices. The second one uses the growth rate equal to inflation. After that more simulations are made in the selected periods using various variations of the variable values. We check the effects of different inputs looking for breakeven points.

6.3.1 Model 1 - analysis with forecasted assumed annual growth rate

For the first model, we used the annual growth rate for apartment prices and rent from the forecast in the previous section. We kept the holding period for 10 years, mortgage term for 25 years. All other inputs change during the years as stated in data chapter (see Table 6.2). If we take into consideration the analysis with the first model, we can see three interesting points.

- Firstly, buying is superior choice in the period before 2011 as the NPV is positive; that means before and in the early years of the Great Recession. We assume that the reason for this is that prices of real estate had declined sharply, while the rent prices were still increasing. Therefore, there is negative growth rate in the apartment prices while rent prices show positive growth rate (9 percent in 2008, and 8 percent in the following year). Combined with the negative growth rate of real estate this results in massive difference. Also compared to the periods between 2011 and 2019, the mortgage rate is high in the first two years with more than 5 percent.

- Secondly, while renting is superior option in both 2008 and 2009 there is a major difference in the NPV. Most inputs are similar, the one with the most significant difference is assumed growth rate in apartment prices, where the difference is 4.1 percent.

- Lastly, we can see that between the years 2011 and 2019 NPV had increased a lot. According to our analysis it looks like the most important factors are growth rate of both prices together with the mortgage interest rate as they are increasing during the examined periods. Mortgage interest rate has been slightly decreasing in the last 10 year, which can be also one of the reasons for NPV growth.

Table 6.1: Model 1 - NPV

	2008	2009	2011	2013	2019
Real estate price	1526180,00	1338476,58	1263801,51	1246357,50	1988507,73
Down payment	305236,00	267695,32	252760,30	249271,50	397701,55
Furnishing Costs	76309,00	66923,83	63190,08	62317,88	99425,39
Transfer tax	61047,20	53539,06	50552,06	49854,30	79540,31
Purchase costs	76309,00	66923,83	63190,08	62317,88	99425,39
Selling costs	76309,00	66923,83	63190,08	62317,88	99425,39
Mortgage interest rate	0,0564	0,0559	0,0419	0,0318	0,0236
Nominal discount rate	0,0564	0,0559	0,0419	0,0318	0,0236
Mortgage term	25	25	25	25	25
Holding period	10	10	10	10	10
Growth rate - price	-0,058	-0,017	-0,004	0,016	0,072
Growth rate - rent	0,090	0,080	0,020	0,007	0,032
Rent	5896,76	6876,73	7663,13	8155,33	9100,09
NPV	-498 745,54	-20 859,86	143 393,26	466 824,86	1 934 271,31

6.3.2 Model 2 - Analysis with the inflation as the assumed annual growth rate

In mature economies we can expect that the growth rates of both rent and apartment prices are equal to the inflation (Tabner 2016). Therefore, in our second model we will use this assumption. All other inputs are the same as in the first model. We got similar results as in the model 1 with our forecasted growth rate in apartment prices and rents. The sign of NPV is the same, negative for the first two periods, followed by positive ones. Whence, the renting is superior choice for the years 2008 and 2009. The difference compared to the first model is in the value of NPV. It is higher for periods one and three.

- In this model we can confirm the from the peak of the Great Recession are better off people who become homeowners. Presumable caused by bigger growth of real estate prices compared to rent prices (see table 1.1) Although we use the inflation as assumed growth rate for both rent and

Table 6.2: Model 2 - NPV

	2008	2009	2011	2013	2019
Real estate price	1526180,00	1338476,58	1263801,51	1246357,50	1988507,73
Down payment	305236,00	267695,32	252760,30	249271,50	397701,55
Furnishing Costs	76309,00	66923,83	63190,08	62317,88	99425,39
Transfer tax	61047,20	53539,06	50552,06	49854,30	79540,31
Purchace costs	76309,00	66923,83	63190,08	62317,88	99425,39
Selling costs	76309,00	66923,83	63190,08	62317,88	99425,39
Mortgage interest rate	0,0564	0,0559	0,0419	0,0318	0,0236
Nominal discount rate	0,0564	0,0559	0,0419	0,0318	0,0236
Mortgage term	25	25	25	25	25
Holding period	10	10	10	10	10
Growth rate - price	0,0208	-0,007	0,012	0,005	0,0257
Growth rate - rent	0,0208	-0,007	0,012	0,005	0,0257
Rent	5896,76	6876,73	7663,13	8155,33	9100,09
NPV	-98 685,74	-218 378,19	814 801,09	352 740,49	778 555,35

apartment prices, the real price of apartments and rent we use in this model increases differently.

- Years 2008 and 2009 have still negative NPV but the values differ from that of the the first model. We assume that the cause of this is the difference between assumed growth rate and inflation.
- Also, in this model the growth of NPV after year 2011 is high. Therefore, we still think that the reason behind this is the decreasing mortgage interest rate.

6.3.3 Mortgage interest rate

Earlier we assumed, that the difference in NPV between 2011 and 2019 is caused by the different mortgage interest rate. By using the same rate for all three periods (the mortgage interest rate from year 2011 - 4,1 percent) we conclude that the difference is much smaller, and we believe it is one of the main reasons behind the growth of NPV during the years after 2011.

Table 6.3: Change in the mortgage interest rate

	2011	2013	2019
NPV	255 831,83	229 534,21	372 768,31

6.3.4 Optimal holding period

Optimal holding period of apartments depends on many economic factors but one of the most important is growth rate (Baroni *et al.* 2007). We use two different types of growth rates - our forecasted values and inflation. Now we will

Table 6.4: Optimal holding period (in years)

	2008	2009	2011	2013	2019
Forecasted growth rate	17,39	10,38	6,31	3,07	1,83
Inflation rate	17,33	-	4,47	3,71	3,28

look for breakeven points in both models. Results are presented in Table 6.4. The optimal holding period between the years 2008 and 2013 is sharply declining. Since in 2008 people had to hold to the property for more than 17 years to gain profit; in 2013 it was only 1.83 years. The difference between 2013 and 2019 is relatively small.

As we can see we miss value for year 2009 in the second model. As the inflation was negative both assumed growth rates in our analysis are negative, therefore regardless the holding period, renting is superior choice. If we change the inflation rate to 0 the buying of real estate would be preferential if the holding period was at least 77,1 years.

6.3.5 Discussion

We examined the optimal tenure choice within twelve years period using NPV analysis. For the first two periods renting was the superior choice; to accumulate more wealth it is better to become a homeowner. We used 10 years holding period of real estate for the analysis. In order not to overpay in 2008, people had to wait to resell their property for more than 17 years, compared to only average 2 years in 2019. Our results support the findings of Tabner (2016), that during inflation it is better to own, while in deflation renting is superior decision and also Di *et al.* (2007) that gaining wealth by buying a property is sensitive to time.. Similar study was made by Lipán and Tláskalová (2013). Their results suggest that renting is superior choice between 2016 and 2018 respectively, but their analysis was based solely on the Prague apartment market data, and not on the figures representing the whole Czech market.

Chapter 7

Conclusion

We built ARIMAX model to find the factors determining price of apartments and rent using Box-Jenkins methodology. While most variables in our model had opposite effects on apartment prices and rent value, e.g. wage, unemployment rate, mortgage volume; other variables, such as GDP and exchange currency rate CZK/EUR had the same effect on both apartment and rent prices. Compelling finding is that the rent value had positive effect on the apartment prices and vice versa, meaning that higher prices of one tenure choice increases demand and also the prices for the other one.

The aim of the thesis was to compare the financial aspect of buying a real estate property versus renting using net present value analysis as the difference of total cash inflows and outflows. The analysis was based on the Czech real estate market evaluating various periods of economic cycle. We chose the years around the Great Recession, specifically the years 2008, when the apartment prices reached its highest level; 2009, the beginning of the recession in the Czech housing market, followed by the peak of recession in 2011 and year 2013, when the economic crisis ended. We also included the year 2019 into our analyses to show also current data.

The main contribution of the thesis is a general overview as the NPV changes amongst the years, mostly during the cycle turnovers. It is also the first NPV analysis for year 2019. Existing NPV analyses are mostly limited on the Prague real estate market therefore the approach used in this thesis is a positive and new approach aimed on the whole republic.

For the analysis we needed the assumed growth rates of the real estate and rent prices, therefore using ARIMA model we forecasted the prices for the following years. In each period we forecasted four observations, since our data

were collected quarterly. We conclude that ARIMA model performs accurate short-term predictions, with less precise prediction of the turning points. We expect the apartment price to grow by 7,2 percent and rent 3,2 percent for the year 2020.

For the NPV analysis holding period was fixed for 10 years and mortgage term for 25 years. Two models were used with different assumed growth rates. In the first model we used our forecasted values and in the second model we used inflation as the annual growth rate. In both we found that renting was superior choice in years 2008 and 2009, while for the other examined periods becoming a homeowner was better financial choice.

Negative NPV was caused by rising price of rent, while the apartment prices fell due to the Great Recession. Since the prices of real estate are increasing dramatically, becoming a homeowner is more suitable. The difference in NPV between years 2011 and 2019 is caused mainly by the mortgage interest rate and growth rate of rent and real estate prices. We found the optimal holding period for each year, and we may conclude that the optimal holding period had decreased from 17+ years (in 2008) to 2 years (in 2019).

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