Report on diploma thesis titled *"Subsolidus thermal convection as a key to understanding volatile evolution and internal dynamics of large icy bodies,"* submitted by Brendan Ninneman to Charles University, Faculty of Mathematics and Physics, for the degree of "Mgr." (Master's degree)

The content of Mr. Ninneman's thesis is concerned with numerical modeling of freezing of Saturn's moon Titan's ice crust from underlying water ocean. It is further summarized in the review by the thesis supervisor, Dr. Klára Kalousová.

The work is clearly and sufficiently motivated in the two-page introduction section. The model couples thermal convection in the ice crust, described as a Newtonian viscous fluid, to the heat flow from the underlying ocean, which controls the crystallization rate and thickening of the crust. The description of the mathematical model and numerical implementation in the 20-page "Model Formulation" section 1 is clear and complete, perhaps with the exception of specification of the initial conditions; those are only sparsely mentioned later in the text.

The subsequent 11-page "Results" section 2 systematically walks the reader through various steps in the modeling and the results. At places I find the text stylistically awkward, which slows a smooth flow and makes it more challenging for the reader to understand the presented material. However, it is an issue of the writing style, not a critique of the research. I produced an annotated PDF of the thesis with many comments of mine, which I will promptly share with the autor + supervisor, and I am ready to document my points during the defense, if needed.

I find the research subject relevant and timely; it fits well within the broader research scope of the supervisor and her local and international colleagues. The result of this work are useful in that they place constraints on and quantify the tradeoffs between the values of Titan's ice crust viscosity and the amount of ammonia in Titan's sub-crustal water ocean, of course within the assumptions of the presented model.

I recommend to accept the submitted thesis as master's thesis.

Topics for discussion

I list a number of topics for discussion during the defense. I believe that many (not necessarily all) of my questions only require brief answers.

1. Section 1.1: *"In the long term evolution of the model q_{out} will always be greater than q_{in}"* – Why is that? Is there a physics law requiring it? Could you imagine a situation whee this is not the case?
2. Section 1.2.2 on the Boussinesq Approximation: Why is neglecting those terms, as specified in your description, reasonable? Why not neglect more terms in the equations? Or less?
3. Section 1.2.3 about the Rayleigh number: You say that *"The Rayleigh number describes the driving force in the system."* In fact, it can be understood as a ratio (indeed, the dimensionless parameters which arise in scaling often represent ratios), here the ratio of driving force for convection to 'force(s)' that work against it.
4. Section 1.2.4, above equation 1.30: *"...we only use volume diffusion, as it is the linear mechanism that depends only on temperature."* What is meant by *linear* here? (Within the text it is unclear.)
5. Section 1.3.1: *"note pressure is fixed to zero at the bottom left corner of the computational domain."* Why do you need to specify pressure at one point in the domain?
6. Figure 1.7: Would you like to comment on the small discrepancy in the steady-state value of the Nusselt number between your result and the 'Blankenbach bechmark', as seen in the

figure?

7. Last sentence in section 1.3.6: *"it was decided to use the domain aspect ratio of 4 with the mesh resolution of 150x50 in the rest of the thesis."* Did you check that *in the rest of the thesis* the resulting 'crystallization rates' were similar in order of magnitude to those used in the resolution tests?

8. Discussion around equation 1.46: Ok, so with $D < 65\,\mathrm{km}$, the Δq is only matched by the crystallization of the crust. But with $D > 65\,\mathrm{km}$, Δq matches a combination of crustal and deep HPI crystallization. Is this properly accounted for in the model? (I could not tell from my reading.)

9. Last paragraph of section 2.2.2, discussion of difference between results of models shown in Figs. 2.5 and 2.6: I do not understand the argument mentioning a *constant ocean temperature.* Eqn 1.49 is for for freezing temperature, as a function of P and X, or $T = T(P, X)$. With $X = 0$, it becomes simply $T = T(P)$, i.e., freezing temperature depending on pressure, which is what you used previously. Or not??

10. If time permits, I challenge you to explain to a geophysicist the idea of a bubble function and mini-elements. (I did not understand it from the description in section 1.3.5, and I did not consult Arnold et al., 1984.)

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