

Diploma thesis review by supervisor  
**Subsolidus thermal convection as a key to understanding volatile  
evolution and internal dynamics of large icy bodies**  
author: Brendan Ninneman

**Thesis content summary:** The presented diploma thesis addresses the thermo-mechanical evolution of Titan's outer ice crust which is underlain by a gradually crystallizing ocean. The problem is mathematically described as viscous flow of heat-conducting fluid in a two-dimensional Cartesian domain. Although the physical thickness of the domain is increasing, the use of dimensionless formulation allows to solve the problem in a domain of fixed thickness. In this formulation, the physical thickness increase is incorporated through the increase of the dimensionless Rayleigh number that governs the flow efficiency.

The thesis has three chapters. In the first chapter, the developed numerical model is described in several sections. The student first briefly explains the concept of the model, then he rederives the governing equations from the continuum mechanics balance laws, introduces the standard Boussinesq approximation and dimensionless formulation, and describes the important material parameters. The next sections are dedicated to derivation of the weak formulation and description of the chosen spatial and temporal discretization. Comparison of the developed numerical code implemented in the FEM library FEniCS with the standard benchmark is also presented. The student then describes the numerical implementation of (i) the ice layer crystallization at the bottom boundary and (ii) the effect of ammonia on the crystallization temperature. Finally, he summarizes the range of expected heat fluxes that come from the moon's deep interior.

The second chapter contains the results. First, the effect of ice layer thickness and viscosity on the heat extraction efficiency is investigated in a layer of fixed thickness. Second, the evolution of a thickening ice layer is investigated in a parametric study by varying the incoming heat flux, the ice viscosity and the ammonia concentration in the underlying ocean.

In the Discussion, the student compares the obtained ocean characteristics (thickness and ammonia concentration) with the expected values and finds that only models with relatively large viscosity and/or heat flux are compatible with the available data. He also discusses some of the model assumptions and gives an outlook into the problem of volatiles transfer.

**Thesis evaluation:** The results obtained in the thesis show that large viscosity and/or interior heat fluxes are necessary to keep Titan's ocean liquid unless another mechanism is present to prevent its freezing. The developed numerical model can be further extended to account for these other effects that were neglected in the presented thesis.

The thesis text is of good quality, however the formal aspect would benefit from some improvement. At the beginning, Brendan needed help with the numerical program development, but his programming skills gradually improved and he gained more confidence towards the end of his studies. I would also like to highlight his diligence and very active communication. To conclude, **I recommend to accept the submitted thesis as the Master thesis.**

In Prague, June 24 2020  
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