Report on the Master Thesis of Marián Poppr

On representations of Chekanov-Eliashberg algebras

The Master Thesis of Marián Poppr is devoted to a forefront of the theory of Legendrian knots in $\mathbb{R}^3$, an important research domain in contact geometry, which benefits from interactions with both symplectic topology and classical knot theory. A contact structure ξ on an odd-dimensional manifold M^{2n-1} is a field (a distribution) of maximal non-integrable hyperplane ξ in the tangent bundle TM^{2n-1}. The standard contact structure ξ on $\mathbb{R}^3(x, y, z)$ is the distribution of the hyperplane $\xi \in T\mathbb{R}^3$ that annihilates the standard contact one-form $\alpha = dz - ydx$. A Legendrian knot on a contact 3-manifold (M^3, ξ) is an embedding $f : S^1 \to M^3$ such that the image $f(S^1)$ is tangent to the contact hyperplane ξ. Legendrian knots exit in abundance and they are essential tools in contact geometry. Note that all contact manifolds (M^{2n-1}, ξ) of the same dimension are locally isomorphic and therefore all invariants in contact geometry are global ones, see e.g. [Eliashberg1998]. The main focus of Poppr's Master Thesis are Chekanov-Eliashberg differential graded algebras (DGAs) attached to Legendrian knots in $(\mathbb{R}^3, \xi)$. These DGAs have been discovered by Chekanov [Chekanov2002] and Eliashberg [EGH2000] twenty years ago. Chekanov and Eliashberg proved that the DGA homotopy type (or even, stable tame isomorphism type) of a Chekanov-Eliashberg DGA is invariant under isotopy through the underlying Legendrian knot. The main task of the Thesis under consideration was to find irreducible two-dimensional representations of Chekanov-Eliashberg algebras that are graded. The importance of higher dimensional represention of Chekanov-Eliashberg algebras has been discussed in [CGRR2016].

The Master Thesis consists of Introduction, 6 chapters and bibliography. In the Introduction Poppr explains the motivation and the main task of his Thesis. Then he gives in chapters 1-5 a clear and concise exposition of the theory of Legendrian knots in $\mathbb{R}^3$, in particular the construction and properties of the Chekanov-Eliashberg DGAs with $\mathbb{Z}_2$-coefficients and $\mathbb{Z}[t, t^{-1}]$-coefficients, based on [Hans2008], [Etnyre2005], [ENS2002], [Chekanov2002], [EN2018] and [Ghiggini2012]. In the last chapter Marian Poppr solves the main task of his Thesis by classifying all graded 2-dimensional irreducible representations over $\mathbb{Z}_2$ of a Legendrian knot in $\mathbb{R}^3$, which is Legendrian isotopic to the

Legendrian knot $m(5_2)$. His result is summarized in Proposition 2.8, which is an analogue of a recent result due to C. Leverson and D. Rutherford [LR2018], who considered 2-graded representations of the Chekanov-Eliashberg DGA of a different Legendrian knot, which does not admit graded representations.

Comments and suggestions.

Marián Poppr's Master Thesis is clearly written with concise proofs to all the results. Marián Poppr demonstrates his deep understand of the theory of Legendrian knots in $\mathbb{R}^3$ and his skillful use of sophisticate techniques in the field.

I have two improvement suggestions for the writing style of the Thesis.

1) It is better to enumerate Remarks in the Thesis. For example, the Remark on p. 38 "Analogously, the existence of ..." could be linked to the second Remark in p. 35.

2) The author names should be written uniformly in the reference list. Furthermore, in the reference ENS$^+$02 of the Thesis there are only three authors, therefore I wonder why Poppr writes "John B Etnyre, Lenhard L Ng, Joshua M Sabloff, et al."

I recommend the grading 1 (excellent) for the Master Thesis of Marián Poppr.

References

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