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MASTER'S THESIS

Macroeconomic Uncertainty: An Exogenous Risk in Reinsurance Pricing

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Declaration of Authorship

The author hereby declares that he compiled this thesis independently; using only the listed resources and literature, and the thesis has not been used to obtain a different or the same degree.

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Abstract

The thesis focuses on the analysis of the impact of the inflation uncertainty on the reinsurance pricing, particularly on its measures of risk. Vector autoregression models are used to predict the medium-term inflation and simulate different inflation paths. The consideration of various scenarios of future inflation captured by the stochastic modelling increases the value at risk (VaR) and the tail value of risk (TVaR) of mean ceded loss to the reinsurer. The thesis founds that the inflation uncertainty measured by the stochastic inflation matters and it is important from risk management and hedging perspectives. As a result, additional loadings could be added to the price for the mitigation of the inflation risk. Although the effect of stochasticity of the future inflation is not significant on mean loss, it is the case for the risk of measures, especially for the contracts with high retention relatively to the underlying exposure.

JEL Classification	F12, F21, F23, H25, H71, H87
Keywords	reinsurance pricing, inflation forecasting, inflation risk, long-tail line of business
Title	Macroeconomic Uncertainty: An Exogenous Risk in Reinsurance Pricing

Abstrakt

Táto diplomová práca sa zamierava na analýzu dopadu inflačnej neistoty na zaistné nacenenie a hlavne na jeho miery rizika. Vektorové autoregresívne modely sú použité na predikciu strednodobej budúcej inflácie a simuláciu jednotlivých inflačných scenárií. Uvažovanie rôznych scenarií budúcej inflácie vyjardrených pomocou stochastického modelovania zvyšuje hodnotu v riziku (VaR) a aj zvyškovú hodnotu v riziku (TVaR) priemernej škody pre zaistiteľa. Práca zistila, že inflačná neistota meraná stochastickou infláciou má vplyv na cenu zaistenia a risk management zaisťovní. Navrhuje možnosť hedžovania proti inflačnému riziku zavedením nových loadingov pridaných k čistej technickej cene zmluvy. Aj keď dopad stochastickej budúcej inflácie nie je významný na priemernej škode, jej efekt je dôležitý pre správne určenie mier rizika a to hlavne pre zmluvy s vysokou prioritou. Pri nacenení týchto typov zmlúv a hlavne v neistom makroekonomickom prostredí, stochastická inflácia a zavedenie daných loadingov je vhodné na zmiernenie inflačného rizika, ktorému zaistiteľ čelí.

Klasifikace F12, F21, F23, H25, H71, H87

Klíčová slova nacenenie zaistného, modelovanie inflácie, inflačné riziko, long-tail zaistenie

Název práce Makroekonomická nejistota: Vnější Riziko v Ceně Zajištění

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Acronyms

ACF Autocorrelation Function

ARIMA Autoregressive Integrated Moving Average

CPI Consumer Price Index

ECB European Central Bank

EIOPA European Insurance and Occupational Pensions Authority

FIF Future Inflation Factor

GARCH Generalised Autoregressive Conditional Heteroskedasticity Model

GNPI Gross Net Premium Income

IBNeR Incurred But Not enough Reported

IBNR Incurred But Not Reported

MAE Mean Absolute Error

MSE Mean Square Errror

OECD Organization for Economic Cooperation and Development

PACF Partial Autocorrelation Function

RMSE Root Mean Square Error

VaR Value at Risk

VAR Vector Autoregression Model

TVaR Tail Value at Risk

Master's Thesis Proposal

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Proposed Topic:

Macroeconomic Uncertainty: Exogenous Risk in Reinsurance Pricing

Motivation:

Insurance business is the key part of financial systems. As other parts of financial system are important to transfer of money between the subjects and across the time, insurance enables reallocation of money across different possible outcomes and thus protects policyholders in case of loss. Moreover, modern financial systems providing coverages to policyholders have to be protected against extremely severe losses. And this is done by reinsurance.

The correct price of reinsurance contract is the key element for sustainable operation of reinsurance although its estimation can be challenging. To calculate the price that insurance company would pay for a reinsurance cover, reinsurance pricing is used. Underwriters, in charge of assessing the risk characteristics to decide whether the company should underwrite the risk or not, use different pricing methods such as experience pricing and stochastic modelling to calculate how much a given contract is worth. The pricing frequently relies on forming expectations about unrealized events and estimating future cash flows. The aim of pricing is to determine the expected losses to the contract and then increase it by certain percentage that represents the reinsurer's internal costs and profit margin. This amount exceeding the expected losses is called the loadings of reinsurance price. The experience pricing consists of estimating the expected ceded losses based on the historical information indexed to the valuation date. When loss developments take years since reserved and paid losses are changing over time, the business is considered to be long-tail. It is a case for motor third part liability and general third part liability.

Therefore, in order to calculate the price that a reinsurance company should ask for a given contract, the correct forecasts of ultimate losses are of a prominent importance. The forecast of losses is influenced by forecasts of inflation and discount rate. Expected future inflation is important in this context to predict the ultimate values of losses from the past.
The inflation step can be broken into two parts:

- The latest known reserves are inflated to the new valuation date year using past inflation
- The payments are inflated to the years following the valuation date using future inflation assumptions, according to the historical payout pattern and discounted to the valuation date using a risk-free rate.

Normally, fixed prediction for future inflation assumptions is used. However, when using a flat deterministic future inflation, the variance of the inflation is omitted and thus the risk may be underestimated. This thesis aims at bridging this gap by examining the effect of the stochastic future inflation on reinsurance pricing. In particular, the thesis will evaluate how the prices required from the reinsurer would change in case the inflation volatility would be included.

Hypotheses:

1. Macroeconomic uncertainty is an important factor of reinsurance pricing
2. Modelling of future inflation and its volatility could improve correct risk measures
3. The consideration of stochasticity of inflation would influence the reinsurance loadings on the market

Methodology:

As a first step, we collect the past inflation data per country and index the historical claims and exposure to the valuation date. As we want to study long tail insurance (i.e. with long payout pattern of claims), we need to take into account the future inflation of the expected amounts to be paid in the next years for each claim reported within the valuation period.

The thesis aims to assess to which degree the introduction of volatility into the future inflation would impact the results of the common risk measures considered by underwriter in the reinsurance pricing. Therefore, we will carry out the analysis by comparing outcomes using two different sets of assumptions for the future inflation:

A deterministic flat inflation curve obtained from public economic forecasts

- A set of sampled inflation paths. A vector autoregression model will be used to estimate the parameters of the stochastic dynamics of the future inflation (Ceasar, 2006) and possibly also vector error correction model. Firstly, we will estimate vector autoregression model for forecasting future inflation and then we will convert the model into stochastic process using estimated parameters.

In particular, these two sets of future inflation paths will enter two different pricing methods: (1) the experience pricing and (2) reinsurance modelling (Clark, 1996).

- Experience pricing: we use the historical losses and corresponding exposure that we index to their current values. This method assumes that the risk is well represented within the sample of losses available, which means that future losses can be fully explained by the historical. However, this assumption is very often naive.
- Reinsurance modelling: from the indexed historical claims, we estimate two independent distributions of frequency and severity. The estimation of the distribution parameters is to be done by comparing maximum likelihood and the method of moments. The two independent distributions jointly represent the distribution of future expected losses.

Finally, we will calculate empirical statistics and risk measures related to the expected losses estimated using these two methods to conclude on the impact of introducing the stochastic future inflation into the reinsurance pricing.

Expected Contribution:

To the best of our knowledge, there is so far no academic literature dedicated to study the effect of using stochastic forecast of inflation on reinsurance pricing and its measures of risks. This thesis aims to study this effect and possible contribution on the improvement of accuracy of reinsurance pricing by taking into account the stochastic inflation.

Outline:

1. Introduction – reinsurance business will be introduced and standard contracts will be explained. Also, main principles of reinsurance pricing will be discussed and compared.
2. Literature review – Relevant literature will be presented and I will also explain missing areas that should be studied.
3. Data – For reinsurance part, data provided by insurance company will be analyzed, especially their past losses, exposure to risks and future losses in order to perform correct reinsurance pricing. For inflation part, different macroeconomic variable that could have impact on forecast of inflation will be analyzed.
4. Analysis: The analysis will be performed. Firstly, the inflation will be modelled (deterministic vs. stochastic). Secondly, the obtained inflation paths will enter two reinsurance pricing methods.
5. Discussion of the results - The results from two reinsurance pricing techniques will be compared and discussed. The effect of stochastic inflation will be interpreted.
6. Conclusion: Findings will be summarized and their implications for reinsurance pricing and measured of risks of reinsurer and future research will be presented.

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1. Introduction

The objective of this thesis is the study of the effect of the stochastic modelling of the future inflation on the reinsurance pricing. Deterministic future inflation is widely used for the estimation of amount of future loss for which the reinsurer will be liable. However, with such consideration of the future inflation, the variance of the inflation is omitted which may lead to the underestimation of the risk the reinsurer faces. This thesis aims at bridging this gap by examining the change of the price of the reinsurance contract in case of the inclusion of the inflation volatility.

Insurance as a key part of the financial system enables reallocation of finance across various outcomes and protects insured people in case of a loss. Modern financial systems providing coverages to policyholders are protected against extremely severe losses through the reinsurance. By the transfer of the part of liabilities from the insurance companies, the reinsurance company provides necessary services for sustainability and protection against bankruptcy of the insurance company. Moreover, the reinsurance industry is the integral part of the risk diversification. Reinsurance treaty is an agreement between an insurance company and a reinsurance company to reduce the likelihood of the insurance company of paying a large indemnification resulting from an insurance claim in exchange of a reinsurance premium. The price of such treaty is determined by the reinsurance company as an estimate of its future liabilities by analyzing the historical development of the portfolio and past insurance claims. The reinsurance premium is set by prediction of the amount of loss for which he will be liable in the upcoming period. This is done by an analysis of the historical experience of the treaty indexed to the valuation date. As severe losses happen rarely, the historical information about this type of losses cannot be used. Therefore, probabilistic methods estimating the distributions of the loss characteristics complete the analysis.

The macroeconomic environment could also be an important determinant of the future performance of the treaty, particularly, the expected future inflation is fundamental to predict the ultimate values of future losses from the past. The main questions that the thesis aims to answer is whether the macroeconomic uncertainty is an important factor of reinsurance pricing and the existence of the added value of modelling of the future inflation and its volatility on accuracy of the risk measures. Since the correct price estimation charged for protecting the portfolio including the indemnification of the future loss is crucial for business sustainability, all available information should be incorporated to the price calculation and the inflation uncertainty is not the exception. The importance of inflation uncertainty is illustrated on non-proportional reinsurance treaty protecting the personal accident portfolio of an Austrian insurance company. As personal accident insurance claims can develop during the decades until they are closed, development of future inflation has direct effect on reinsurer's liabilities.

The thesis is structured as follows. The reinsurance business will be introduced, and main principles of reinsurance pricing will be discussed in Chapter 2. Chapter 3 discusses the literature related to the determinants of the reinsurance pricing and important role of the inflation. In Chapter 4, methodology related to forecasting future inflation from the historical development of macroeconomic time-series as well as actuarial methods used to determine the price of the reinsurance treaty will be explained. The first part of Chapter 5 will present the data provided by the insurance company necessary for the reinsurance pricing. Macroeconomic variables that could have impact on forecast of inflation will be shown in the second part of Chapter 5. Chapter 6 is devoted to the empirical analysis including forecasting of inflation and calculation of the price with deterministic future inflation as well as with stochastic future inflation. Chapter 6 ends by the discussion of the results and their possible implementation into reinsurance pricing.

2. Nature of Reinsurance Business

Reinsurance is insurance of insurance companies. More precisely, it is the transfer of part of the insurer's risk to reinsurers according to agreed conditions in exchange of a reinsurance premium. Buying a reinsurance coverage has two principal reasons. Firstly, insurers are able to limit their exposure which means also limit the retained loss arising from possible severe or frequent losses and their fluctuations. Secondly, it is a protection from catastrophic events which could ruin the insurance companies. The origins of reinsurance are the same as origins of insurance. It was in the fourteenth century in Genoa, when for the first time marine insurance was agreed (Schwepcke, 2004). Following the aim of mitigating uncertainty, people got united together with the idea "one for all and all for one". In case of marine accident of one person, the others provided financial help to his family. Later, when the trade expanded, various reinsurance companies were established. Professional reinsurance companies date nineteenth century and helped the development of insurance business by spreading the risk more effectively (Gerathewohl, 1983). Purchasing reinsurance allows insurance companies to free themselves from the undesirable part of risk exceeding their capacity or risk appetite and sell it to a reinsurer.

Functions of Reinsurance

Reinsurance cover has multiple advantages for the insurance company. Firstly, by spreading the risk among different reinsurers, the insurance company increases its stability, sustainability and protection against bankruptcy. The insurer buying the reinsurance transfers the tail of its liabilities to reinsurers which is very important in case of large severe losses such as natural catastrophes. Without purchasing reinsurance, the insurance company would not be able to indemnify all policyholders hit by the loss. Thanks to the reinsurance, the insurance company will pay only some part of the losses and the rest will be handled by the pool of reinsurers participating in the contract. The most important role of purchasing the reinsurance coverage is therefore protection against bankruptcy by transferring a part of the obligation and possibility to continue the operation of the business despite severe losses.

Secondly, the reinsurance improves stabilization of profit and loss volatility of the insurer. While insurance company is mainly operating in some line of business and with a local orientation, reinsurance companies are international companies underwriting heterogeneous risks in different part of world. This large diversification of risks makes reinsurance important part in transfer of risks. Every reinsurer will take only a part of the risk of an insurance company and thus the risk is spreading efficiently. Insurance companies are usually smaller local entities with specific orientation in respect of line of business underwritten or territorial scope where they operate. With the reinsurance, they will retain just smaller parts of each risk and this reduction in the concentration of risk is important to decrease volatility of insurer's results. Profit and loss volatility stabilization may encourage potential investors in their decision-making.

Thirdly, the reinsurance stabilizes the insurer's balance sheet by taking part of its risks and fluctuation of risk and it enlarges the insurer's underwriting capacity to take more business. As reinsurers are providers of better security, insurers can expand their business for the new products. It removes partly the technical risk which passes to reinsurer and the insurance company can acquire the new business without the unsustainable risk. The possibility of support new business in a competitive market may have positive impact on the insurer 's profit.

Lastly, due to the growing regulation of the insurance and reinsurance industry and the level of standards which the insurance company has to adhere, the solvency features are important. The solvency ratio of an insurance company is defined as a size of its capital relative to all risks it possesses. As part of risks are transferred to the reinsurer, the reinsurance improves the solvency capital relief of the insurer necessary for the sustainability of his activities.

Risk Diversification

The origin of a particular risk is coming from a policyholder who buys an insurance protection. As it is too risky for the insurance company to keep the full exposure to this risk, the insurance company prefers to buy reinsurance protection and relocates some proportion of risk to the reinsurer. By this channel, the different risks are put together and transferred to different entities. In the case of loss and indemnification of their clients, the probability of bankruptcy is lowered as the risk has been diversified. The Figure 1 illustrates the diversification process in reinsurance. The first line of the Figure 1 represents the original risk held by policyholders, for example a factory, cars, a ship. Each of these policies is considered as one risk. The arrows below represent the insurance contract transferring the risk from policyholders to insurers. The second line is the insurer who takes those risks in his portfolio of different risks and the arrows below illustrate the reinsurance contract transferring a part of insurer's portfolio to the reinsurer. In the last line, the reinsurer is shown. He is liable for a share of original portfolio.

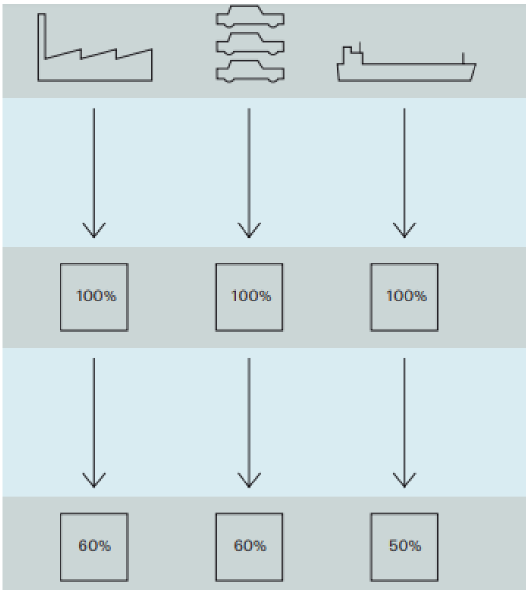


Figure 1 - Spread of risk in reinsurance

Source: Introduction to Reinsurance, Swiss Re (2002)

2.1 Types of Reinsurance

There are two main types of reinsurance - proportional and non-proportional reinsurance.

Proportional Reinsurance

In proportional reinsurance, an insurer and a reinsurer share losses and premium proportionally. It means that there is some agreed proportion of risk for which an insurer will pay a claim and receive insurance premium and for the rest, the reinsurer will pay a claim and receives the rest of the premium. There are two types of proportional reinsurance, quota share, and surplus called as variable quota share. In quota share, there is an agreed and fixed percentage – for example 50%. According to this proportion, the insurer and the reinsurer share their liability to pay a loss and also the premium received from the client.

The second type of proportional treaty is a surplus. In a surplus treaty the insurer retains a fixed amount of the maximum policy liability, measured by the insured value, or probable maximum loss, and transfers the rest to the reinsurer. In surplus treaty, the insurer stays liable for one part of the loss called a line. Then, the part exceeding the line is transferred to the reinsurer who is liable for the part above the line and below the specified multiple of lines which represents the limit of the treaty. The insurer will retain one line and cedes to reinsurer some number of line – for example nine. In this case, it means that the insurer will pay 10% of losses and collect 10% of premium and cede to reinsurer the remaining part – 90%.

In a quota share, insurer cedes the same proportion of all risks to the reinsurer – more profitable and less profitable policies for him. However, in surplus treaty, he has the right to choose the proportion on different types of risks. It is thus a great manner for balancing the portfolio and keep the profitable business. In practice the insurer pays the full loss amount and reports quarterly losses on all policies covered by the reinsurance pool to collect the money due from the losses incurred.

In proportional reinsurance, the insurer cedes the risk at original conditions with some adjustments agreed with policyholders thus the reinsurer is involved in the risk with almost the same conditions as insurer. Moreover, the absolute variation in loss experience is limited. It stabilizes the insurer's ratio between potential and actual loss and thus improves the solvency.

Non-proportional Reinsurance

Another type of reinsurance is non-proportional treaty where the reinsurer is liable for losses that exceed a specified amount. The name of this treaty is excess of loss reinsurance so-called XL treaty. It provides insurers a way to cut the probable peaks in the portfolio exposure to the acceptable level. This amount called retention or deductible is the exact amount that the insurer will always retain. If a loss exceeds the retention, the amount in excess will be ceded to the reinsurers which participate on this contract up to the limit of the treaty. In the proportional reinsurance, the risk is divided between insurers and reinsurers according to a fixed ratio (quota share) or the according to the amount insured (surplus). In non-proportional reinsurance, the loss is important to determine who will take which part of the claim. No matter what the amount insured is, the insurer will pay all claims up to the retention of the treaty. While in proportional reinsurance, insurer shares the original risk, in non-proportional reinsurance, he shares the loss.

The insurer chooses his retention in order to be able to pay those losses from his reserves and the peaks will be transferred to reinsurers. Non-proportional treaty has no implicit price and it has to be calculated by the reinsurer. He will estimate his future burden - how much he needs to cover the losses, prepare their reserves and also for the profit. The structure of XL treaty is organized into different layers. Those are different parts of the coverage and they have their own retention and limit. When capacity of one layer is completely used, the upper layer starts to be used up to the limit of the latest layer. Usually, every layer has its own price and reinsurers can participate with different shares across layers. The notation 10 xs 10 means that the retention of the layer is 10 currency units and the limit of the capacity of the layer is 10 currency units. Then the retention of the following layer is sum of the retention and the limit of the previous layer. Totally, the retention of the layer on the Figure 2 is 10 currency units and the limit are 100 currency units meaning that the insurer retains the loss up to the 10 currency units and then up to the 100 currency units, the reinsurer indemnifies the policyholder.

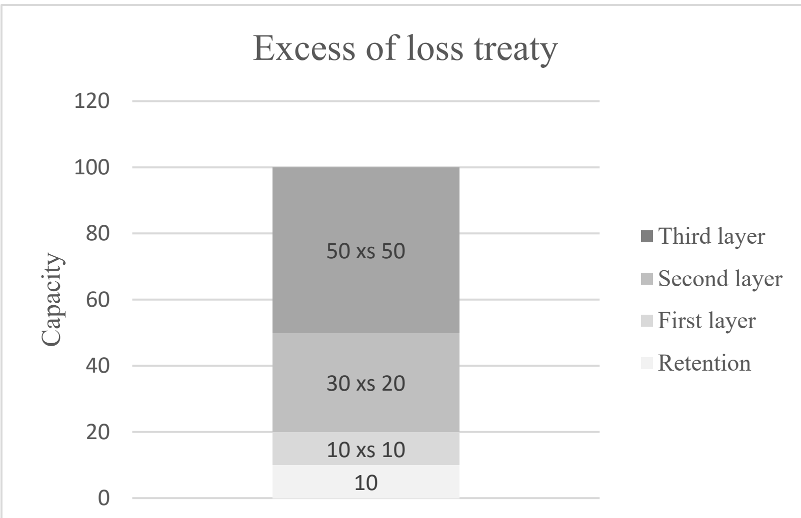


Figure 2 - Excess of loss reinsurance

The first layer has the highest probability to have a loss however only severe losses which happen rarely hit the last layer. Therefore, the price the insurer should pay for the first layer will be higher as for the others as it is the most exposed to losses.

Once a loss happens, insurer has to notify the reinsurer and then he will indemnify the insurance company. The payment for the cover – premium is paid to reinsurer on quarterly or half-yearly basis according to the terms that have been agreed between the insurer and the reinsurers when concluding the reinsurance contract.

Non-proportional reinsurance has many advantages as low administration costs and absolute limit of retained loss. Furthermore, the insurer does not cede high proportional of his income to the reinsurer and pays only small percentage as it will be explained later. For experienced companies knowing their strengths, non-proportional contracts are better as they keep profitable business and they can cede to reinsurers more risky business. On the other hand, proportional treaties are suitable for new companies and new lines of business. They are also beneficial for solvency consideration. Companies often combine proportional and non-proportional reinsurance contracts to achieve stability and good diversification.

The Table 1 summarizes the types of reinsurance contracts and their functions.

Type	Contract	Trigger	Key functions
Proportional	Surplus	Specified amount insured	Improves portfolio's homogeneity
Proportional	Quota share	Fixed ratio of loss and premium	Supports new business and decreases the capital required
Non-proportional	XL	Retention as defined above	Protection against extreme losses

Table 1 - Types of reinsurance

2.2 Lines of Business

A so-called line of business refers to a type of insurance activity. There are various lines of business for which it is possible to buy reinsurance cover. The main classification distinguishes between life and non-life reinsurance. Non-life reinsurance provides financial protection for buildings, machineries, equipment, vehicles and goods against the fire, natural catastrophes, accident and theft. The non-life includes property, casualty, credit and bond, personal accident, travel and health reinsurance. Property reinsurance covers damages to assets while casualty reinsurance protects people from claims arising from their liabilities. The credit reinsurance protects business against the risk of insolvency of their customers. Personal accident reinsurance covers claims from bodily injuries and accidental death. Health reinsurance covers expenses related to the provided health care. And finally, life reinsurance provides financial protection in case of death.

The lines of business are split into short-tail and long-tail reinsurance. It is related to time necessary for a claim to be entirely paid. If claims are usually entirely paid during the term of the policy or shortly after the policy has expired, it is called short-tail business. Property insurance is an example of short-tail business. For example, if the building burns, insurance company will be notified, it will start to investigate the amount of financial losses, reserves will be prepared and within two years the claim will be closed and paid by insurer and reinsurer. The opposite is long-tail business. The time between the loss occurrence and final settlement of claim can be years. The example is motor third part liability, general third part liability or personal accident. A car accident resulting to a damage to car, some bodily injuries and other claims arising from liability can be entirely paid after the decades. After the loss occurrence, adequate reserves are prepared both the insurer and the reinsurer. However, it can happen that after the year, the injured person becomes handicapped, the family has to pay for him expensive treatment, and they have to rebuild their house to be handicap-friendly and find different school. Moreover, those claims are often subject of trial. Third part liability claims often involve large sums of money and they can result in a lengthy court case. The long-tail liabilities result very often in high incurred but not reported (IBNR) claims. IBNR refer to reserves that reinsurer has to take into account when evaluating the loss. Those are claims that have happened but have not been yet reported to the reinsurer. Actuaries in a reinsurance company will estimate potential damage and the reinsurer will prepare the necessary reserves to compensate the claims that have not been yet reported. However, this can be different on following year and it is very important to take it into consideration when pricing the long-tail non-proportional treaty.

The line of business studied in this thesis is personal accident, one of long-tail lines of business. Personal accident insurance consists usually of an annual policy which provides compensation in the event of injuries, temporary or permanent disability or death caused by an accident. It is different from health insurance which cover diseases and provided health care. Personal accident represents a small line of business in terms of premium volume. The calculation of premium charged should take into account the long-tail feature of claims. Usually, personal accident treaties are non-proportional and they contain standard exclusions – situations in which the insured is not indemnified. It is for example the case of injuries caused by war, terrorism, nuclear or radioactive contamination. In that situation, the risk cannot be quantified properly so insurers and reinsurers prefer to avoid it. There are exceptions when also injuries caused by those causes are paid. However, the additional premium is charged to cover potential future expenses.

2.3 Concept of Reinsurance Pricing

When setting the reinsurance price, the reinsurer starts by calculating the minimum required price to participate on a given contract. The pricing of a such contract usually includes three components: the pure price which is a technical price and corresponds to the expected total loss the reinsurer will have to pay, risk loadings which is a compensation for the additional risk the reinsurer bears, and expense loadings deemed to cover the reinsurers expenses and reflecting their position in the competitive market.

The pure price and risks loadings are calculated using the most representative information of the contract future losses - its experience therefore the pricing method is called experience pricing. Based on the past losses and using statistical methods, the reinsurers calculate their best estimate of the future loss experience. This approach has two limits.

The first limit is that the future cannot be perfectly described by the past and the reinsurers do not have guarantee that past losses will identically repeat in the future. Obviously, claims characteristics will not be identical to those observed in the past, but the recent experience is the best representation of the future. Should we assume that in a given contract, a claim is reported with identical damages to what they were five years ago on another claim. The loss amount, payable by the reinsurer will be more expensive than the one paid five years ago due to the inflation. Therefore, all past losses are inflated to the current level by the past inflation in order to have them in present values. This is called the indexation of losses. The same indexation is applied to the exposure. The exposure of the contract is represented by premium collected by the insurer from the policyholders for this portfolio. It is a measure of how much people, vehicles or properties are in the portfolio and can potentially cause a loss. When the insurers ask for the reinsurance coverage, he has to provide his historical development of premium. For purpose of using the past information to forecast the future loss, the premium is used to relativize the yearly number of claims that happened in the portfolio. If a claim happened in a portfolio of thousand people five years ago, five claims are expected to happen in the current portfolio containing five thousand policyholders. This idea is called exposure adjustment and in practice, past premiums are indexed by past inflation that was in the particular year. Consequently, the premiums are comparable between them and can be used for the description of the future by the past. After indexation of losses and premium, the values from past are in the current levels and past information can be used for future forecasts. Consequently, modification of the past situation to nowadays is called As If. As loss indemnification will happen in future, future inflation will adjust the future payments.

The second limit of this approach is possible insufficiency of the information to calculate reliable statistics. Maybe the few years of experience are not a sample large enough to bring reliable forecasts. To challenge this lack of historical losses to fully describe the loss distribution, the probabilistic modelling is done. Probabilistic pricing is also classified as experience pricing as it uses historical portfolio's information but with application of probabilistic methods to complete the loss information. Especially, it is a solution for the big catastrophic losses that happen rarely, but they represent the tail of the distribution. As there are not a part of historical data but they would have big impact on the price, they have to be modelled. The loss has two characteristics. It is frequency measuring how often the loss happens and the severity which describes the amount of loss. Those two characteristics of the loss can be modelled in order to know their distribution and predict also severe losses.

The best method to predict the future loss for the reinsurer is to combine historical losses with presented adjustments and modelling of tail of severity distribution. Consequently, partial credibility is given to both experience approaches – pure experience pricing and probabilistic pricing.

To summarize, two pricing methods will be used for price determination in the empirical part. Both uses the historical information of the portfolio indexed to the current period by the known past inflation and indexed to future periods by the predicted future inflation. Firstly, pure experience pricing of the portfolio will be done. Secondly, the probabilistic modelling which better considers the tail of the severity distribution will complete the historical information and improves the accuracy of the pricing.

3. Literature Review

Inflation as the quantitative measure of the rate at which average prices of selected basket of goods is growing affects the whole economy. And the insurance and reinsurance market are not the exceptions. As price of the non-proportional reinsurance coverage is the estimate of discounted future cash flow of reinsurer, the impact of the inflation is through the amount of money that reinsurer will pay in the future. The effect of inflation on reinsurance prices was found by many scientists. D'Arcy (1990) in his study concluded that underwriting profits are positively correlated with the inflation. More scientists stated that fluctuations in the insurance premium are related to the inflation (Boubaker & Sghaier, 2014) . With the inflation, the exposure will be higher in future years. This will generate higher premium for the insurer and also for the reinsurer. In the long-term, insurance costs will move in the same direction as inflation, even though in some years insurance will exceed or lag the overall inflation rate.

Inflation represents an additional risk for the insurer and the reinsurer. As the reinsurer collects the premium for the reinsurance treaty in the current moment but his liabilities will appear in future, high future inflation represents a significant risk for him. Fortunately, there is special insurance product used to get rid of the inflation risk (Bodie, 1989).This product aiming to insure the inflation risk is equivalent to a European call option on the consumer price index. Its pricing is done with the Black Scholes formula. Policyholders purchasing this product will be sure that the benefits they receive from the insurance coverage move with general price levels, often linked to the CPI. Inflation protection is an additional product that can be added to the policy and it will represent an additional cost that can increase the price.

Inflation increases the value of future claims that reinsurer will pay in several years. Periods of accelerating inflation are especially problematic for long-tail third part liability lines of business. The possible solution how to mitigate this effect is to rise price of the insurance or the reinsurance cover due to the inclusion of payment for hedging against an inflation increase. However, this is not always possible because of the regulations or the competitive environment (Karl et al., 2010). Rarely, insurers and reinsurers can also modify insurance contracts in order to shorten the tail and avoid the uncertainty associated with the future inflation. Once prices rise, the demand for the insurance and thus for the reinsurance will decrease. Not only is the inflation problem for insurance and reinsurance market, it is also the deflation. During the period of deflation, interest rate tends to fall. This results in the difficulty for the companies to invest money and have any guarantee of an asset return (Holzheu, 2014). As the investment represents one of the most important income for the insurers and the reinsurers, they prefer the stable moderate inflation. In this equilibrium, they earn the profit from their investments but also do not lose a lot as their future liabilities arising from the losses do not increase sharply.

Ball and Staudt (2011) aimed to discover the effect of inflation on several areas of actuarial modelling and pricing. They stated that the exact extent and effect of inflation on losses is extremely uncertain and it depends on every single case. The paper raised attention to the fact that inflation is an important determinant of the price of reinsurance and should be taken into consideration by the actuarial models used for pricing of the reinsurance business. If the inflation is not taken into consideration properly, there is possible mispricing which can lead to insolvency of the reinsurer (Ravin, 2010). Moreover, an increase in the inflation rate leads to decreasing of reserve buffers which could cause financial problems of insurers and reinsurers. Companies with lower and insufficient reserving buffers may feel the biggest impact as they are less prepared to absorb the claims inflation (Galin, 2017). The CEO of Towers Perrin's reinsurance brokerage in Europe said that inflation has caused large future losses and the reinsurance industry is not able to factor this into pricing (Holbrook, 2012).

Fackler (2011) in his study of the impact of inflation on the reinsured losses focused on motor third part liability line of business which is considered to be long-tail. His research aimed to answer the question if it is possible to deduct inflation from the increasing loss affecting an insurance product. The conclusion of his research was that it depends on the tail of the severity distribution which is an important source of uncertainty. The fact that the tail is important contributor to additional uncertainty and risk shows why inflation is not considered as a major issue in some reinsurance lines of business, while being an important problem in other lines, particularly the third part liability area.

Brazauskas (2009) studied the interesting phenomenon appearing when inflation does not have impact on reinsurance claims. After loss simulation with poisson frequency and pareto severity, he concluded that losses higher than the retention are identically distributed from year to year even in the presence of inflation. On the contrary, the average loss amount before applying the retention increases with inflation. This is so-called leverage effect of the retention and it does not properly represent the increases due to inflation. The effect is caused by the fact that retention is kept unchanged. Therefore, aggregate losses higher than retention will not rise by the inflation rate because claims that were before below the retention may, with inflation, exceed the retention.

Reinsurance pricing can be influenced by the other variables, for example the interest rate. According to Swiss Re article ‘Facing the Interest Rate Challenge’ and a research paper published by McKinsey & Company in 2008 on ‘Managing Through the P&C Cycle’ (*Swiss Re*, 2012); (*McKinsey & Company*, 2008), interest rates and price of reinsurance coverage are negatively correlated. As for inflation, effect of interest rate on reinsurance prices is more significant in long-tail business. Low interest rates have an impact on both reserving financing and also general treaty performance. It can encourage unrealized investments when fixed income securities are not appropriate investment. In this way, reinsurer’s profit would rise due to the shareholders fund growth. Then the prices of reinsurance coverage could decrease (Hong, 2013). As currently, we are facing the period of depressed interest rates and they might decrease even more after coronavirus pandemics affecting the whole world currently, the reinsurers are in the difficult situation where higher yielding assets from past mature and there is no appropriate alternative (Evans, 2019).

The aggregate inflation is not the only representation of the inflation. Medical inflation can be partially included especially for pricing of health treaties but also for personal accident or motor third part liability treaties. Despite the current low rate of inflation, the medical inflation is rising and several executives of reinsurance companies pointed out the existence of systematic underpricing of the reinsurance treaties and lack of realization of extent to which medical inflation matters (Ericson & Doyle, 2004). Besides those classical determinants of reinsurance prices, recently also climate environment has impact on the reinsurers and their prices. After the years 2018 and 2019 hit by several natural catastrophes, prices tend to increase. To manage costs of those disasters, the reinsurers invest in better catastrophes modelling, better prognosis and also they need to reflect potential higher future losses caused by change of climate environment (Juggler, 2019).

According to the best practice, the past claims are indexed and developed to future, flat deterministic inflation is used to have the loss in present value (Guan & Liang, 2014). However, replacing deterministic inflation by stochastic could be more accurate and better simulate the reality. Especially, it can simulate more extreme scenarios that happen rarely but once it happens, many reinsurers can get bankrupt. With properly modelled stochasticity of the inflation, the reinsurance pricing includes extreme scenarios and unexpected bad economic environment should not hit the reinsurers significantly.

4. Methodology

The methodology part of the thesis is split into two subsections. The first one describes the approaches for studying the time-series and for their forecasting. The second one is dedicated to actuarial methods and explains the principles of the reinsurance pricing.

4.1 Future Inflation Forecasting

As introduced before, the future inflation simulation will be used in calculation of reinsurance price. For this, modelling of future inflation with time-series models will be done.

4.1.1 ARIMA Modelling

There are many studies and research papers in the area of predicting inflation. A lot of scientists use univariate time-series models as for example autoregressive integrated moving average (ARIMA) (Fritzner et al., 2002); (Pufnik & Kunovac 2006); (Junttila, 2001). Future inflation is expressed in terms of past values of inflation (the autoregressive component) and current and lagged values of error term which is assumed to be a white noise with zero expected mean and constant variance. This is the moving average component (Meyler et al., 1998).

The econometric equation of ARIMA model is:

$$y_t = c + \phi_1 y_{t-1} + \cdots + \phi_p y_{t-p} + \theta_1 e_{t-1} + \cdots + \theta_q e_{t-q}$$

Where $\phi_1, \dots, \phi_p$ measures the influence of $y_{t-1}, \dots, y_{t-p}$ on the dependant variable and $\theta_1, \dots, \theta_q$ captures the influence of $e_{t-1}, \dots, e_{t-p}$ on the dependent variable. The first part is related to autoregressive process (AR) and the second to the moving average process (MA). The ARIMA models are usually denoted as ARIMA(p,d,q) where parameters p, d, and q are non-negative integers, p is the order of the autoregressive model, d is the degree of differencing - expressing how many times data need to be differentiated in order to achieve stationarity and q is the order of the moving-average model.

The ARIMA modelling is often used for short-term inflation forecasts in case of stationary process meaning that its unconditional expected value is constant, and it has finite constant variance and covariance. Non-stationary time-series are often amended by the first differencing.

The lag length of the model can be calculated by autocorrelation function (ACF) and partial autocorrelation function (PACF).

Generally, scientists rely on the information criteria that use the trade-off between the number of lags and parsimony of the model. During the estimation of the optimal ARIMA models, the following criteria are minimized. Schwarz criterion (SC) is characterized by the equation:

$$SC = -\frac{2L}{T} + \frac{k \log{(T)}}{T}$$

where $L = \frac{T}{2(1+\log(2\pi)+\log\left(\frac{RSS}{T}\right))}$ is the log likelihood of the data, T is the number of observations used in the data, k is the number of observatory variables plus one and RSS is the residual sum of square. Akaike information criterion (AIC) is characterized by the equation:

$$AIC = -\frac{2L}{T} + \frac{2k}{T}$$

As stated above, the objective of the estimation is to find the lag length of parameters for which those criteria are minimized meaning that the model suits the data well. However, those two criteria are used for two different purposes. While the AIC attempts to approximate models to the reality, the SC tries to find the perfect fit and penalizes models more for having more parameters. There are a lot of different criteria. Finally, the best approach is to use various criteria together in model selection.

4.1.2 VAR Modelling

As stated, with those univariate models, inflation is modelled just in function of its past values. However, in multi-variate models, it depends on more macroeconomic variables (Zivot & Wang, 2006); (Ahmed & Abdelsalam, 2017). The vector autoregression models (VAR) are often used to predict future inflation. It chooses relevant economic variables and identifies how those variables and their lagged value are correlated. Then, according to movement of those variables, predictions to the future can be done. VAR models are used for prediction and also for analyzing the effects of policies done.

VAR models contain one equation per variable in the system. The right-hand side of each equation includes a constant and lags of all variables in the system. Bivariate case of VAR equation of the first order will be presented as an example. The system of VAR equations is defined as

$$y_{1,t} = c_1 + \phi_{11,1} y_{1,t-1} + \phi_{12,1} y_{2,t-1} + e_{1,t}$$

$$y_{2,t} = c_1 + \phi_{21,1} y_{1,t-1} + \phi_{22,1} y_{2,t-1} + e_{2,t}$$

where $e_{1,t}$ and $e_{2,t}$ are white noise processes that might be contemporaneously correlated. The coefficient $\phi_{ii,l}$ expresses the impact of the l^{th} lag of variable y_i on itself and the coefficient $\phi_{ij,l}$ expresses the impact of the l^{th} lag of variable y_j on the variable y_i (Harvey, 1990). This is so-called reduced form and there is no problem of endogeneity.

The original system called structural equation is defined as

$$y_{1,t} = \beta_{10} + \beta_{12} y_{2,t} + \gamma_{11} y_{1,t-1} + \gamma_{12} y_{2,t-1} + \varepsilon_{y_{1,t}}$$

$$y_{2,t} = \beta_{20} + \beta_{22} y_{1,t} + \gamma_{21} y_{1,t-1} + \gamma_{22} y_{2,t-1} + \varepsilon_{y_{2,t}}$$

where two dependent variables are regressed on their own lags, on the other's dependent variables lags and on contemporary values of other dependent variables which enables the analysis of relations between them. For this estimation, it is not possible to use ordinary least square (OLS) approach due to the endogeneity of independent variables and the bias of simultaneity. Each equation of the reduced form is estimated using OLS approach. Then structural form is recovered from the reduced form, with identification restriction that is imposed to the system of equations. The most popular procedure is the recursive identification called as Choleski identification.

$$y_{1,t} = \beta_{10} + \gamma_{11} y_{1,t-1} + \gamma_{12} y_{2,t-1} + \varepsilon_{1t}$$

$$y_{2,t} = \beta_{20} + \beta_{21} y_{1,t} + \gamma_{21} y_{1,t-1} + \gamma_{22} y_{2,t-1} + \varepsilon_{2t}$$

This restriction means that we imposed $\beta_{12} = 0$ and thus $y_{2,t}$ does not affect $y_{1,t}$ contemporaneously which is quite strong assumption. Therefore, the ordering of the variables matters for identification of structural shocks and coefficients. The implication of Choleski decomposition is that the variable ordered first has contemporary effect only on its own, but the variables after are influenced contemporaneously by the previous variables.

The identification is necessary for the estimation. As the reduced form has lower number of parameters than the structural form, the structural form is not identified. It means that residuals in the reduced form are linear combination of structural combination of individual variables:

$$e_t = \frac{1}{1-\beta_{12}\beta_{21}} \begin{pmatrix} 1 & \beta_{12} \\ \beta_{21} & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{y_{1,t}} \\ \varepsilon_{y_{2,t}} \end{pmatrix}$$

Let $Y_t = (y_{1t}, y_{2t,\dots} y_{nt})$ denote a $(n \times 1)$ vector of time-series. The p-lag VAR(p) model has form

$$Y_t = c + \pi_1 Y_{t-1} + \pi_2 Y_{t-2} + \cdots + \pi_p Y_{t-p} + e_t$$

where $t = 1, \dots, T$, π_i are $(n \times n)$ coefficient matrices and e_t is a $(n \times 1)$ zero mean white noise vector. The general VAR(p) model has the same properties as bivariate case of first order presented above. However, it is not possible to express residuals in a simple formula as above for the general multi-variate VAR model that will be used in the empirical analysis.

After theoretical features of VAR model, practical aspects as estimation and diagnostic tests will be discussed. For the choice of the lag length neither the ACF nor the PACF can be used in this case. We should consider not only autocorrelations in residuals but also cross-correlations of residuals across equations. Therefore, the best fit is found with information criteria which are minimized. Often, those three information criteria are considered:

Akaike information criterion:

$$AIC = T\log(\Sigma) + 2m$$

Hannan-Quinn information criterion:

$$HQ = T\log(\Sigma) + 2(\log(\log(T))\,m$$

Schwarz information criterion:

$$SC = Tlog(\Sigma) + \log(T)\,m$$

where T is the number of observations, Σ refers to the determinant of variance covariance matrix of residuals and m is the number of parameters in all equations. Another possibility is likelihood ratio test which compares unrestricted and restricted model with less lags. The likelihood ratio in defined as

$$LR = T(\log(\Sigma_r) - \log(\Sigma_u))$$

where the Σ_r is the determinant of variance covariance matrix of the restricted model with p_2 lags and Σ_u is the determinant of variance covariance matrix of the unrestricted model with p_2 lags for $p_1 > p_2$. This likelihood ratio statistics has χ^2distribution with n degrees of freedom, where n is the number of restrictions that is $(p_1 - p_2)N^2$ where N is the number of variables.

To avoid spurious relations between variables, only stationary time-series should be used in VAR models. The stationary process is not trending over time, it fluctuates around its mean with constant variance. The identification of stationarity starts by plotting the variables and analysis of correlogram. The formal test is the Dickey-Fuller test where the null hypothesis is that there is presence of unit root. Augmented Dickey-Fuller test controls also for other lags. Also, the ACF and the PACF are used to decide whether the time-series is stationary or not.

Stability of the VAR as a whole is more important than stationarity of individual variables. Stability of the system is investigated using eigenvalues of the matrix of coefficients. The characteristic roots need to be outside the unit circle. In practice, inverse roots are preferred for graphical analysis, so they need to be within the unit circle.

In practice following steps should be taken when estimating the VAR model. Firstly, the stationarity of variables is checked and if it is violated, the variables are transformed in order to get rid of the unit root. The stability of the whole system is checked with the values of inverse roots. If the unit root in present in the data, it is possible to include linear trend as exogenous variable and estimate the amended model. Before differencing the variables, it can be useful to check for cointegration. Cointegration is the existence of long-run relationship between the variables that move together in the time. By this movement, they eliminate the unit root even both variables are not stationary. If the cointegration is rejected, VAR model is estimated in first differences. If cointegration is non-rejected, there is a stable long-term relationship among variables and the vector error correction model fits well. It associates the short-term dynamics expressed as first differences to deviations from long-term cointegrating relationships.

Also, there are some problems that may happen when estimating VAR models. As researchers seek to include all explanatory variables that could have impact on dependent variable, and also their past value, it may lead to over-parametrization. Consequently, multicollinearity between the different lagged variables may appear. A solution for that may be the Bayesian vector autoregression (BVAR). Gilbert has suggested to combine VAR models and state space model reduction techniques in determining the model (Gilbert, 1995).

As many determinants can influence development of inflation, VAR model is a good choice for forecasting inflation and it has been in many previous studies. As stated above, the variables of interest are determined by their past values and past values of other variables in the model. Consequently, VAR models can be used for forecasting the complete set of variables *(Bank of England*, 1993). An important advantage is that VAR models do not need as many assumptions as different models do. It is a case of classical macroeconomics models evaluating the policy changes or trends in exogenous variables. For them, assumption that the estimated form of the model remains unchanged over the forecasting period is crucial. By VAR models, all variables are forecasted within the model (Clark, 2006).

The list of the variables in the model should not be very long to avoid problem of overfitting. Previous researches suggest that variables which represent domestic demand pressures as unemployment or growth of production should be included. Moreover, external inflationary pressures such as changes in the nominal exchange rate and monetary variables contribute to forecast of inflation. Monetary variables are the aggregates M0 and M4, the measures of short-term interest rates and credit-quality spread which is the difference between commercial and government yields. Then the nominal exchange rate, world commodity index, world consumer prices and world oil prices are included. Finally, GDP with its components, output of production industries, retail sales, unemployment and the Gallup index of consumer confidence are used to forecast inflation. All those variables were present in the VAR model in Bank of England (*Bank of England*, 1993). Different models use some other additional variables, but the core variables are included in all monetary VAR models used to predict inflation.

The monetary VAR study done by Castelnuovo and Surico (2006) stated that response to monetary policy is limited by the weak central bank responses to inflation and concluded necessity to control for long-term inflation expectations. Stock and Watson (2001) analyzed the monetary policy in the United States economy by VAR model including unemployment rate, inflation rate and interest rate. The study pointed out the accurate forecasting by VAR models but also identified the inference as its weakness. Similar VAR model aiming at forecasting inflation was estimated also by Bank of Richmond. The researchers estimated several smaller models with inclusion of dummy variable capturing political decisions. Their study revealed that monetary policy changes were responsible for persistent shift in the inflation rate (Webb, 1994). The study describing the Swedish economy stated that the Bayesian VAR model outperforms the other models in forecasting. The authors concluded that besides classical variables describing the domestic economy, also foreign variables mainly related to Euro area included in the model have significant impact on inflation development (Iversen et al., 2016).

4.2 Reinsurance Pricing

The second part of methodology is devoted to the presentation of actuarial principles and main methods for pricing the reinsurance contracts used in the empirical part.

As mentioned, price of the proportional treaties is given by the percentage according to which insurer and reinsurer share losses and premiums. However, for the non-proportional reinsurance, the price that the reinsurer charges for being liable for future losses of the insurance company is calculated by the reinsurer. This process done by an actuary or an underwriter is called pricing or quotation of the treaty. Since in reinsurance every program is tailor-made in comparison with insurance, the reinsurer cannot rely on average and every contract has to be individually priced. For this, various data are needed from the insurer. He has to provide his collected premium from the policyholders for this particular portfolio and its loss history. The collected premium is called gross net earned premium income and serves as a base for the calculation of price. This premium is net of any other reinsurance that can be related to this treaty but gross of any other expenses (Wehrhahn, 2009).

The objective of the reinsurance pricing is to estimate mean ceded losses to the reinsurer, sometimes called as recoveries of the reinsurance treaty. This part of losses represents reinsurer's future obligations thus he will be obliged to pay it. Then, the different loadings are added to cover expenses and guarantee the profit of reinsurer. The additional costs are for example fixed and variable administration costs, loadings for underwriting expenses, commissions and general expenses allocated to the policy. Finally, loading for the profit for reinsurer is added. It is an allowance for profit based on the company's guidelines or required return on capital. Price adjusted like this is then offered to the insurer and it can be accepted, or parties can start to negotiate. The reinsurer should never accept risk transfer with price inferior to mean ceded losses.

Let μ be expected losses for next year, l_R loadings representing risk aversion of the reinsurer, l_c loadings related to management and other general costs of the company and R is the risk measure. Then the price P of reinsurance cover is defined as

$$P = (\mu + Rl_R) * (1 + l_c).$$

It means that once μ is estimated, the loadings related to risk and general costs of the reinsurer are added. As the reinsurer is risk averse, he asks for higher premium when he is in riskier situation and faces higher possible losses. The variable R is the measure of risk reinsurer faces. In most cases, it is value at risk (VaR), tail value at risk (TVaR), standard deviation or return period.

Value at risk is the minimum loss that is likely to happen in the particular year for a particular level of probability. Let X be the annual loss and p level of probability. Then the VaR is the loss π that has a corresponding annual exceedance probability of p.

$$VaR_{1-p}(X) = \pi$$

On the Chart 1, there is representation of the VaR. Based on stochastic simulation, 1 000 000 EUR is estimated to correspond to the VaR at 90%. This VaR corresponds to an annual aggregate loss level that will be exceeded 10% of the time.

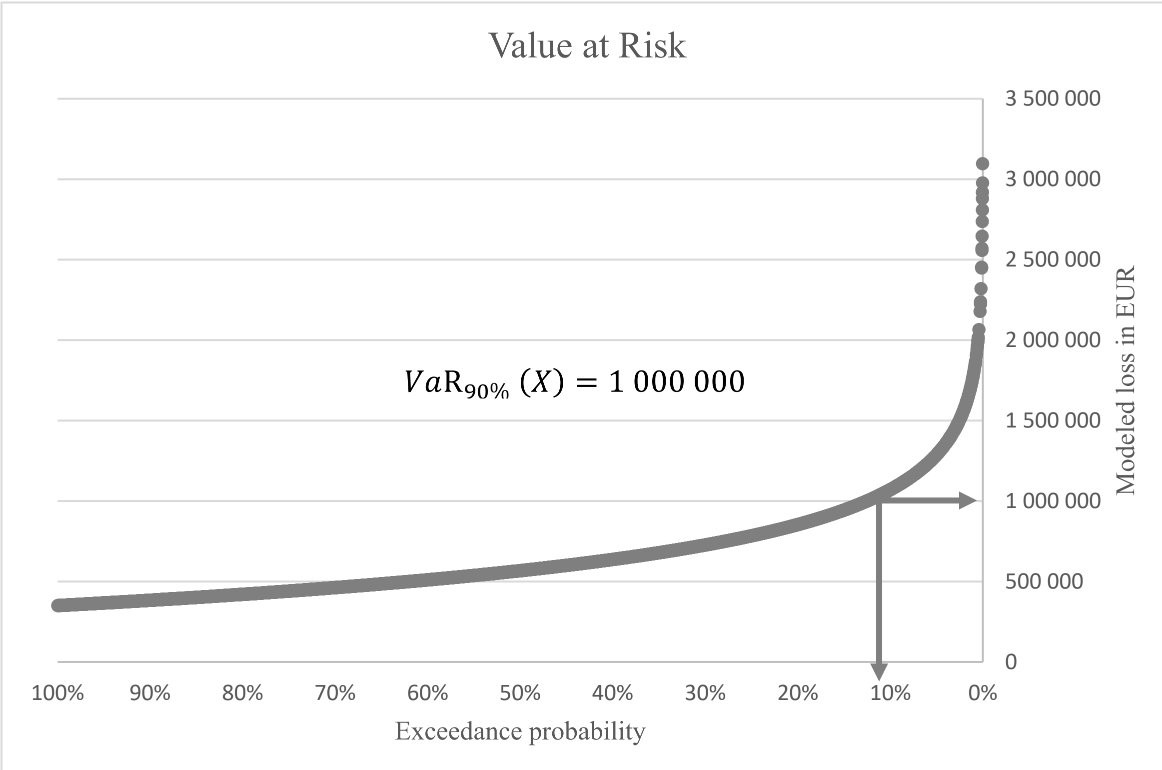


Chart 1 - Value at Risk

The VaR is one of the most used risk measures in reinsurance and it is also the input for the Solvency II calculations. It enables the reinsurer to know how big the loss can be in extreme case and how much of capital and reserves should be allocated to the losses. If VaR is high, also the risk loading will be higher.

The second risk measure is tail value at risk. The reinsurers are often interested in the tail of the loss distribution. However, it is also the most difficult to estimate how losses behave in the higher percentile. TVaR is an expectation of the potential loss remaining after applying the VaR. TVaR is a more conservative way of measuring tail risk than the VaR. TVaR is always greater than or equal to the VaR for a given probability. It measures how big the average loss can be given that the VaR is exceeded. It can be the case of some extreme natural catastrophes lying outside of the confidence interval for the mean expected losses. TVaR is often chosen in situations where outcome distributions are skewed, they have fat tails or are non-normal.

The third of the mostly used risk measures is the return period or sometimes called recurrence interval. In practice, it is mainly used for natural catastrophes, but it can be applied to all risks. It is the inverse of the probability. For example, if the return period of some event is 100 years or probability of occurring is 1% in one year then, in any given year, there is a 1% chance that the loss will happen, regardless of when the last similar event happened.

The Table 2 shows mostly used recurrence intervals and their interpretation in terms of probability.

Recurrence intervals (years)	Probability of occurring in any given year	Chance of occurrence in any given year (%)
100	1 in 100	1
50	1 in 50	2
25	1 in 25	4
10	1 in 10	10
5	1 in 50	20
2	1 in 25	50

Table 2 - Recurrence intervals

Also, the standard deviation can be used as measure of risk. All those four risk measures are used to evaluate the risk and then the reinsurer asks for corresponding loading to compensate the risk he faces (Artzner, 1999).

The description of the mean ceded loss μ estimation will follow. The process is called reinsurance pricing or quotation of the contract. They are two main methods for pricing. One is exposure pricing and the second is experience pricing, shortly introduced in the Chapter 2.3. Exposure pricing analyzes the distribution of the risk in the insurer's portfolio and calculates the possible loss arising from this portfolio. Experience pricing is either burning costs method or frequency and severity modelling. The burning costs method uses only historical information of losses to predict the future behaviour and the frequency and severity modelling relies on history and completes the tails of the distribution by probabilistic method. The thesis is devoted only to the experience methods and their details will be explained in next subsection.

Experience pricing is one of the most used pricing methods in order to determine the price of non-proportional coverage. The reinsurer receives so-called renewal information from the insurance company containing the general information about the portfolio, their insurance products, underwriting policies and plans for future. It must contain historical development of gross net premium income (GNPI). The GNPI represents the amount of money that the insurer collects every year from policyholders for the given treaty. The loss history is very important part of the renewal information and together with the GNPI will be used to calculate the price for the treaty. Insurer asking for the reinsurance coverage specifies the type of treaty, line of business, retention, limit of coverage and territorial scope that enable the reinsurer to quantify the risk and estimate the price.

Burning Costs

The method of burning costs in the simplest technique of the experience rating. It provides an initial idea about the price and the performance of the treaty relying on the assumption that the best prediction of future is history adjusted by information that are available at the time of calculation. The GNPI development and the loss history is necessary for this. The insurer reports losses in excess of a threshold. This threshold is in practice corresponding to 50% to 70% of the treaty retention. Losses below the threshold are not necessary for the reinsurer analysis since they do not reach the treaty retention. However, a loss with incurred amount being 90% of the treaty retention could have an impact on the analysis as this loss can reach the treaty retention after indexation. As a result, the minimum threshold corresponds to the treaty retention amount decreased by the maximum indexation factor which will be used in the reinsurance pricing analysis. Burning costs is the ratio of indexed losses adjusted over indexed premium. Only the part of indexed losses which is higher than retention is used for calculation. For example, if the coverage requested by the insurer is XL treaty with retention 15 000 000 EUR XS 2 000 000 EUR, 2 000 000 EUR is deducted from every loss and the maximum the reinsurer pays is 15 000 000 EUR.

The reinsurer chooses the manner of appreciation of claims. A wage index or a consumer price index (CPI) are commonly used. For example, housing price index can be used for property treaty and medical inflation for health treaty (David, 2009). Those indexes can be also combined in order to describe how the situation on the market changes by the most reliable manner.

The first step of burning costs method is indexation of losses illustrated in the Table 3. In this example, layer with structure 15 000 000 EUR xs 2 000 000 EUR is priced. The loss to layer in the second column is already decreased by the 2 000 000 EUR. In the fourth column, indexed losses are comparable among them. The year 2016 is an outlier, maybe due to some natural catastrophe. Two last years are very often not developed as there is delay in reporting the claims so the loss from 2019 is not complete and maybe also that from 2018.

Year	Loss to layer	Inflation index	Indexed loss
2014	1 634 160	1,12	1 830 259
2015	1 250 011	1,10	1 375 012
2016	12 599 700	1,05	13 229 685

Year	Loss to layer	Inflation index	Indexed loss
2017	2 789 505	1,03	2 873 190
2018	1 856 966	1,02	1 894 105
2019	985 200	1,00	985 200

Table 3 - Burning costs example 1

The same indexation is done with the GNPI as shown in the Table 4.

Year	GNPI	Inflation index	Indexed GNPI
2014	12 158 930	1,12	13 618 002
2015	13 589 647	1,10	14 948 612
2016	13 968 451	1,05	14 666 874
2017	14 859 612	1,03	15 305 400
2018	18 475 811	1,02	18 845 327
2019	20 125 841	1,00	20 125 841

Table 4 - Burning costs example 2

Finally, the burning costs are calculated as ratio of indexed loss to indexed GNPI in the Table 5.

Year	Inflated loss	Indexed GNPI	Burning costs
2014	1 830 259	13 618 002	13,44%
2015	1 375 012	14 948 612	9,20%
2016	13 229 685	14 666 874	90,20%
2017	2 873 190	15 305 400	18,77%
2018	1 894 105	18 845 327	10,05%
2019	985 200	20 125 841	4,90%

Table 5 - Burning costs example 3

In the Table 6, pure average of burning costs from all years equals 24,43%. That means that reinsurer should ask 24,43% of the GNPI collected by the insurer from his policyholders corresponding to the period for which he buys the reinsurance coverage as price for reinsurance coverage. However, this is just pure technical rate corresponding to the expected ceded loss. As mentioned, claims from two last years can increase thus we should exclude them from the calculation. Then, the average burning costs equal 32,90%. When excluding the outlier, the burning costs dropped sharply to 13,80%. The reasons of excluding a year must be serious. In this example, it is preferred not to omit completely this year because this year can happen every decade or more. Therefore, the burning costs were increased to 15%. And finally, 15% was divided by 0,9 to include loadings and profit margins in the rate. Thus, the final rate including loadings is 18,75% which means the price charged for the treaty should be 18,75% of the estimated GNPI for the next year.

Pure average	24,43%
Excluding last years	32,90%
Excluding outlier	13,80%
Final burning costs	15,00%
Including profit	18,75%

Table 6 - Burning costs example 4

It was the simplest example how to price non-proportional treaty with burning costs approach using loss history and development of the GNPI. The disadvantage of burning costs is the absence of losses to higher layers. For example, if we want to price the second layer of the treaty with structure 13 000 000 EUR xs 17 000 000 EUR, the price will be zero as no loss is higher than the retention of this layer. Reinsurance companies can use for upper layers without historical information capital model where the general rule is that rate on line (RoL) should be at least 1% (Schwepcke, 2004). The rate on line is defined as the ratio of premium paid over the limit of reinsurance contract. As if there are no historical losses, it does not mean that they will not be there in next year. Price of the layer cannot be 0% but according to this rule it will be 1% of limit of this layer. If the insurer needs capacity of 13 000 000 EUR, he should pay at least 130 000 EUR for this layer.

Probabilistic Method

Another type of experience pricing is the probabilistic method. Instead of considering just one scenario like in burning costs method, probabilistic pricing can be more appropriate as it simulates all possible events that could be expected in sufficiently large period (*Mapfre*, 2013). This stochastic loss modelling builds a simplified mathematical description of the losses and their characteristics as the frequency and the severity. The aim of this method is to estimate separately the number of losses in next period - frequency of loss and average amount of loss - severity of loss.

The stochastic modelling is based on the collective risk model (Klugman et al., 2008), which models the total losses over a year as the sum of all individual losses. Today, collective risk modelling is the most used stochastic model for calculating prices in non-life reinsurance (Albrecher et al., 2017).

The total loss is defined by the random variable S expressed as:

$$S = \sum_{i=1}^{N} X_i$$

where N and X_i respectively stand for the number of claims and the size of the claim i. The model is based on two assumptions:

- $(X_i)_{i=1\ldots N}$ are identically independently distributed random variables. This assumption supposes that each claim has the same severity distribution.
- $\forall i \leq N, X_i$ and N are independent. This means that the number of claims incurred does not depend on the claim size.

The number of individual losses is distributed according to a given discrete distribution such as a poisson distribution or negative binomial distribution. The individual losses are assumed to be independent and identically distributed from the same loss severity distribution such as lognormal distribution, exponential or pareto distribution. The chosen theoretical distribution is in practice shifted by a threshold representing the lowest limit of losses used for the pricing. The analysis and calibration of both severity and frequency distributions are done on historical claims indexed to their as-if version.

The severity is modelled as the first loss characteristic. The severity modelling is done two steps: the analysis of IBNeR and the calibration of a severity model. The IBNeR (Incurred But Not Enough Reported) losses are expressed by the ratio of incurred amount of losses in two consecutive years. The IBNeR analysis aims to find the pattern of under-reserving or over-reserving in historical losses and the correct value of claim is settled. Since the recent incurred losses are often not paid in full, an analysis of the reserve accuracy is important to correctly estimate the ultimate paid amount for each historical large claim. The second step is the selection and calibration of a severity model. A severity model is fitted to the inflated and IBNeR rescaled past loss amounts by searching for the theoretical distribution that fits the best the empirical one. In practice, the selection of a severity model consists of selecting the distribution function and calibrating its parameters. The calibration of the parameter is usually based on statistical methods such as the maximum likelihood and the method of moments. The Chart 2 illustrates the common shape of a cumulative distribution function (cdf) of a severity distribution where the loss amount is expressed as a multiple of the retention level. The more concave the severity distribution is, the more extreme events it can generate. In this example, there is 50% probability that a loss which will reach the retention, and a claim will be reported to the reinsurer.

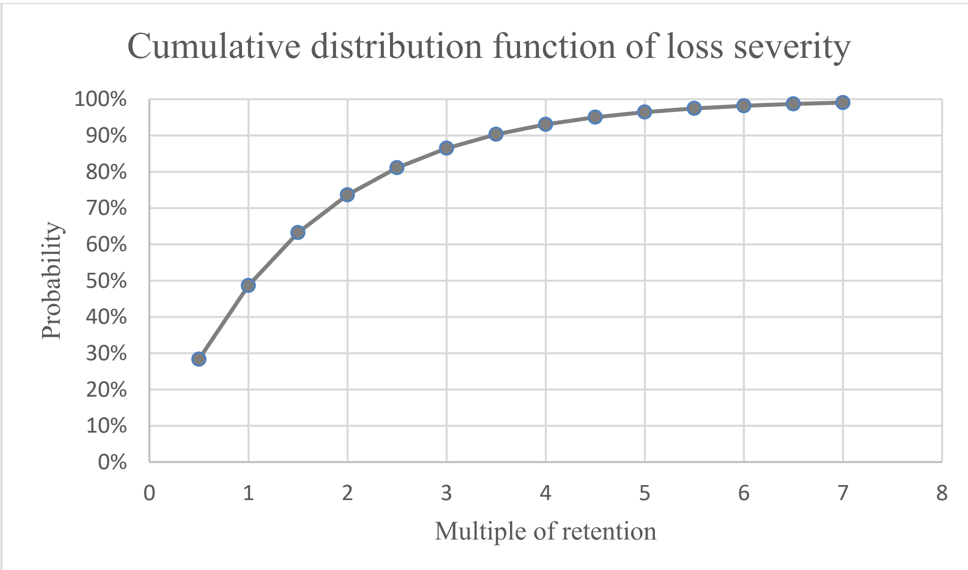
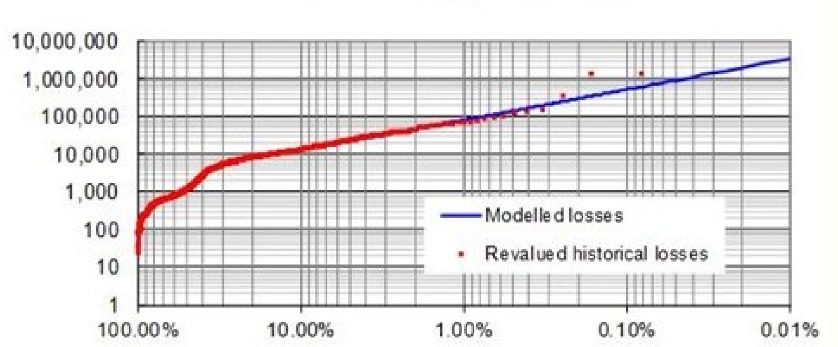


Chart 2 - Cumulative distribution function – severity of loss

An example of severity modelling is shown on the Chart 3. As there is sufficiency of known losses lower than 100 000 EUR, and since the shape of the bottom of the distribution is non-standard, the bottom of the severity distribution is empirical. As the large losses happen rarely, the tail of the empirical distribution is very incomplete and extreme events under-represented. For this reason, the bottom part of the empirical distribution is combined with a theoretical distribution for the tail. Empirical distribution is fitted and the concrete theoretical distribution is determined by searching the best similarity between them. This is called a distribution mixture. Once convenient theoretical distribution is found, its parameters are determined by maximum likelihood



Probability that a given loss exceeds a given amount

Chart 3 - Severity of the loss

Source: Pricing in General Insurance, Parodi (2014)

The frequency modelling is also done in two steps: the calculation of an IBNR pattern and the selection and calibration of a frequency distribution. Incurred But Not Reported (IBNR) are claims that have already occurred but the reinsurer does not have the invoice due to the delay in reporting. The frequency analysis is based on the number of reported claims, which can be incomplete for recent years due to the IBNR claims. The reinsurer will do IBNR adjustment to estimate the ultimate number of claims. Such adjustments are based on an IBNR pattern constructed by triangle development techniques such as chain ladder development using the link ratios as illustrated in the Table 7 (Parodi, 2016).

In the triangulation shown in the Table 7, there are cumulative numbers of reported losses per underwriting year and per year of reporting development year. The underwriting year (UW year) refers to the period where the reinsurance treaty was in force. Years of reporting or sometimes called as development years refer to years when loss is reported. For example, 29 losses with date of loss in 2010 were reported at the end of the year 2010, additional 59 losses incurred in underwriting year 2010 were reported by the end of 2011, etc. For the last year 2019, the reinsurer knows about only 42 reported losses and it is probable that this number will be growing in the future. The link ratios measure the average yearly change of the reported claims (Walhin et al., 2001). In this example, there is 74% growth of reported claims between the first and the second year. After the fifth year, the link ratios are close to one as there is almost no new claim reported. The projection of reported number of claims to ultimate number of claims using link ratios is called the chain ladder method. An IBNR pattern is constructed as the product of the link ratios. Sometimes the link ratios become smaller than one due to the fluctuation of the loss reserve below the threshold of reporting. Therefore, it may happen that lower cumulative amount of losses is reported in successive period.

UW year	Year of reporting 1	2	3	4	5	6	7	8	9	10
2010	29	88	155	228	309	333	355	373	371	371
2011	30	69	112	164	239	247	300	274	274	
2012	29	74	128	198	198	198	250	201		
2013	41	111	231	387	477	477	477			
2014	30	30	45	45	60	70				
2015	24	49	85	124	120					
2016	28	71	110	100						
2017	120	130	145							
2018	85	105								
2019	42									
Link ratios	1,74	1,62	1,43	1,22	1,03	1,10	0,93	0,99	1	

Table 7 - Number of claims reported and link ratios

In practice, for the last few years, the reinsurer does not rely on the number of claims exceeding the retention, but these values are rescaled by the IBNR pattern to reflect the claims, incurred in recent years that could be reported after the pricing date.

Once number of losses are projected to their ultimate value by multiplication by link ratios, the number of losses of each underwriting year is adjusted for changes in exposure between this underwriting year and the reinsurance contract period. If there is for example 10% growth in GNPI, 10% more losses are assumed for the final loss frequency calculation, as it is assumed that there will be 10% more claims than observed in the past attributable to the policies corresponding to the 10% of GNPI growth.

Besides IBNR adjustment of the frequency, also the Bornhuetter-Ferguson method is widely used. This approach adjusts undeveloped last years. The method uses already known losses in addition to a priori expected losses that would happen in this year according to average value of losses happened in other years. Known losses are added to a priori expected losses multiplied by an estimated part of unreported losses. The part of unreported losses is estimated by observing historical claims experience (Bornhuetter & Ferguson, 1972). The frequency by Bornhuetter-Ferguson method is thus calculated as

$$BF\ frequency = relative\ frequency + BF\ factor * (1 - \frac{1}{IBNR})$$

where relative frequency is number of claims in the given year adjusted by the change of exposure and BF factor is the average of relative frequency after the application of the IBNR of the developed years.

Finally, the empirical frequency distribution is compared with theoretical ones. In most cases, one of these three distributions are commonly used: binomial - for which the variance is lower than the mean, poisson distribution - for which the variance is equal to the mean and negative binomial model - for which the variance is larger than the mean (Cruz et al., 2015). The parameters are calibrated with the method of moments. The empirical moments are estimated on the IBNR and exposure adjusted historical number of reported claims per underwriting year.

Often, a threshold is applied to the reported claims and the pricing is done only with the part of the losses exceeding this threshold. Indeed, in non-proportional, the reinsurer is liable for large claims only. As a result, the severity and frequency distributions shall capture the best the large losses behaviour and characteristics. In practice, the threshold corresponds to 50% to 70% of the retention amount. Losses lower than threshold are not relevant for the analysis therefore the theoretical distribution can be shifted the threshold.

The loss ceded to the reinsurer is estimated from both the modelled frequency, severity distributions and the characteristics of the reinsurance contract. The most used method is Monte Carlo simulation by which the losses are randomly simulated according to the theoretical severity distribution. Mean and measure of risk are calculated from simulated losses.

After simulation of severity and determination of the frequency of one loss, the characteristic of aggregate loss S will be calculated. Suppose the distributions of N and X, respectively called the frequency and the severity distributions are known, the expected value and variance of the aggregate losses S is calculated as:

$$\mathbb{E}[S] = \mathbb{E}[NX] = \mathbb{E}[N]\mathbb{E}[X]$$

using the underlying assumptions that $(X_i)_{i=1\ldots N}$ are identically independently distributed and X and N independant.

Using the conditional variance formula,

$$Var[S] = \mathbb{E}[Var(S|N)] + Var(\mathbb{E}(S|N)] = \mathbb{E}\left[Var(\textstyle\sum_{i=1}^{N} X_i \,|N)\right] + Var\left(\mathbb{E}(\textstyle\sum_{i=1}^{N} X_i \,|N)\right]$$

By mutual independence:

$$\mathbb{E}\left[Var(\textstyle\sum_{i=1}^{N} X_i \,|N)\right] + Var\left(\mathbb{E}(\textstyle\sum_{i=1}^{N} X_i \,|N)\right] = \mathbb{E}\left[\sum_{i=1}^{N} Var(X)\right] + Var\left(\sum_{i=1}^{N} \mathbb{E}[X]\right)$$
$$= \mathbb{E}[NVar(X)] + Var(N\mathbb{E}[X])$$

As $Var(X)$ and $\mathbb{E}[X]$ are constants,

$$Var[S] = \mathbb{E}[N]Var(X) + Var(N)\mathbb{E}[X]^2$$

With those formulas, the expected value of aggregate loss $\mathbb{E}[S]$ and its variance $Var[S]$ will be determined. The final price $P[S]$ will be calculated according to the formula for the price using the loadings presented before. Besides the modelling the various outcome and better simulation of the reality, the frequency and severity method is able to price also unused capacity – layers where loss history is not available in comparison with burning costs and therefore, we obtain full distribution of aggregate claims.

4.3 External Factors in Reinsurance Pricing

As the price of reinsurance coverage is present value of future cash-flows, one should consider also external variables that can influence pricing. Mainly, it is inflation and discount factor as they appear in the formula for reinsurance price. Inflation affecting the reinsurer is decomposed into past inflation used to adjust past claims and future inflation adjusting the amount of future losses. As those payments will be done in future, they need to be discounted by discount factor calculated with risk-free interest rate with corresponding maturities.

The insurer and the reinsurer face in practice inflation risk. As already mentioned, past inflation is used to adjust past claims to be comparable with the others in experience pricing methods Those claims behave according to some payment pattern, there are changes in paid loss but also in reserved loss. When pricing a treaty for next year, a loss from current year will also develop according some pattern. As some part of this loss will be paid in ten or even twenty years, the reinsurer faces the inflation risk. In present moment, the "correct" value of the future loss is not known. If at the moment of loss settlement inflation is higher, and this fact was not taken into consideration, the estimated loss can be lower than it is in reality. If the reinsurer charged lower price, it would negatively influence his profit.

An example of a loss development will follow to highlight the long-tail aspect of the loss and importance of the inflation. There are three types of losses. The paid loss is the part of the loss that has been already paid to a policyholder. During some years, the cumulative paid loss increases, during the others stays the same. The reserve is the amount calculated by the actuarial methods and it is the estimation of amount to be paid in the future. The incurred loss is the sum of paid loss and reserve.

In the Table 8, a loss which happened in 2012 is shown during the years 2012 - 2019. It is split into paid loss, reserve and incurred loss. In the year of loss occurrence, only a small portion of the loss was paid to the policyholders and the reserve was calculated. However, during the investigation new reasons to pay more were found. The cumulative amount paid to the policyholder was 480 599 EUR and the reserve increased up to the 215 125 EUR. From the sixth development year corresponding to the year 2017 the loss is almost paid and the development is almost finished. The reserve is approaching zero after the sixth year as expected. Usually, the incurred amount is not changing a lot unless in the situation of under-reserving or over-reserving

UW year	Type	Development year							
2012		1	2	3	4	5	6	7	8
	Paid loss	42 180	480 599	490 000	582 054	603 031	603 031	603 110	603 110
	Reserves	10 250	215 125	215 125	187 564	120 411	604	40	20
	Incurred loss	52 430	695 724	705 125	769 618	723 442	603 635	603 150	603 130

Table 8 - Development of losses

The pattern illustrating the development of paid loss, reserve and incurred loss is drawn on the Chart 4. On the horizontal axis, there are development years and vertical axis represents amount of loss. The biggest increase in all types of loss is between the first and the second development year. From the sixth year, the loss is almost constant.

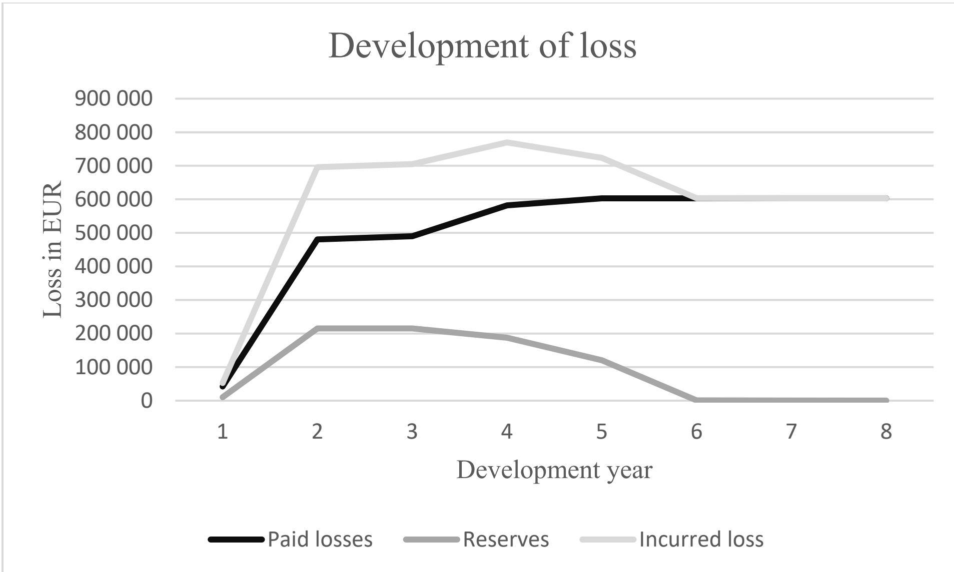


Chart 4 - Development of loss

This was just one loss but in practice all reported losses are aggregated and an aggregate payment pattern enabling to see how losses develop is constructed and used to develop recent loss to the ultimate value. For the loss which happened in the current year, according to that pattern most of the money will be paid in future with biggest part during the second and third year. The information about the amount of loss to be paid in each year is important for the reinsurer due to the inflation effect. For the loss which happened in 2019, the biggest part is estimated to be paid in 2020 and 2021.

The expected ultimate value of a loss is calculated as the discounted sum of all future payments indexed at the date the payment is due. This indexation is in most cases an inflation index, such as CPI or wage index. As reinsurer 's future liabilities are influenced by the inflation, he estimates the amount of loss paid in each year. In practice, flat deterministic inflation forecast enters in the price estimation. In this thesis, the future stochastic inflation will be modelled and then used for calculation of reinsurance price. The reinsurer is rewarded by a fixed premium at the beginning or during the reinsurance period, but he pays an uncertain inflation impacted loss payments later. The inflation risk must be incorporated in the calculation of the price of the coverage and stochasticity of future inflation could be adequate way for the inflation risk description. The price is calculated from the formula mentioned before $P = (\mu + Rl_R) * (1 + l_c)$. When considering stochastic inflation, the mean ceded losses μ should change a bit as some losses would exceed the retention after application of future inflation. However, the part of loading related to risk l_R and the measure of the risk R would experience larger increase. As modelling of inflation will be the source of an additional uncertainty, price of the reinsurance coverage might be higher.

5. Data

This part of thesis is devoted to a presentation of real economic data used to price the personal accident reinsurance treaty for the year 2020 covering an Austrian insurance company's portfolio. The chapter is split into two main subsections. Firstly, the reinsurance data received from the insurance company called the renewal information will be presented. All the original data contained in this thesis may have been rescaled by an undisclosed factor for confidentiality purposes. As the original proportionality between variables is unchanged, such adjustments have no impact on this thesis calculations and results. Secondly, the macroeconomic variables describing the Austrian economy will be explained.

5.1 Reinsurance Data

Exposure

The content of a renewal information is standardized in order to give to the reinsurer the best idea about the portfolio and the insurer itself. The GNPI development during last year is crucial because it enables the reinsurer to see portfolio changes from the quantitative point of view. The price is adjusted to account for nonstandard development in time. In order to price the portfolio, the reinsurer should understand the development of the portfolio as every increase in exposure can mean significant change in the risk. If the reinsurer participates in this contract more years, he can compare prices or made other adjustments thanks to the knowledge of its past development. In the Table 9, inflation of exposure is done. The last column measures the yearly growth rate of indexed GNPI with the average growth of 3%. The source of inflation data is OECD and the total CPI is used.

Year	GNPI	Inflation rate	Indexed GNPI	Aggregate growth rate
2006	59 112 891	1,70%	77 050 281	
2007	63 456 491	2,20%	81 329 318	1,05
2008	68 355 078	3,20%	85 721 737	1,05
2009	70 678 380	0,40%	85 886 928	1,00
2010	73 421 824	1,70%	88 865 244	1,03
2011	75 556 600	3,50%	89 920 399	1,01
2012	77 185 336	2,60%	88 752 433	0,98

Year	GNPI	Inflation rate	Indexed GNPI	Aggregate growth rate
2013	80 299 381	2,10%	89 993 326	1,01
2014	85 559 615	1,50%	93 916 345	1,04
2015	92 461 594	0,80%	99 992 562	1,06
2016	98 185 412	1,00%	105 339 865	1,05
2017	101 930 473	2,20%	108 275 065	1,02
2018	107 432 673	2,10%	111 663 157	1,03
2019	112 986 534	1,80%	115 020 292	1,03
2020	114 752 447		114 752 447	0,99

Table 9 - Indexation of exposure

Source: Author's calculations & OECD (2020)

From the year 2006 up to the year 2019, the premium collected by the company is rising with more important growth between the years 2013 and 2019. The indexed GNPI illustrates the real monetary value of the premium. It was rising from the year 2006 up to the year 2019 except for the year 2012 and 2020. As for non-indexed GNPI, the growth is more visible since the year 2013.

Risk Profile

Risk profiles are also the crucial part of renewal information as they describe the distribution of the risk in the portfolio. All policies are split according to the sum insured into intervals of sum insured called the bands. The number of risks within each band of sum insured is illustrated together with the premium collected for those risks. The accumulated sum insured is obtained as the multiplication of all risks within the band of sum insured and the respective sum insured. The reinsurer can compare the information from last years which enable him to understand the development of portfolio. Risk profile is particularly important for exposure pricing which estimates a loss based on the premium and limits exposed from the risk profiles. The exposure pricing approach uses a severity distribution based on whole market to estimate losses to particular layers (Clark, 2014). Other data requirements are underwriting limits of the insurance products as the reinsurer needs to know the maximum sum insured which implies the maximum loss amount. Sometimes, the complete list of the insured and their policies is sent to the reinsurer.

On the Table 10, the risk profile for the death is captured. As the bands of sum insured are growing, the company has lower amount of the policies and the highest reinsurance premium from policies is collected within the lowest sum insured. The highest sum insured in the portfolio does not exceed 4 000 000 EUR. The split into different bands of sum insured enables the reinsurer to see the distribution of risk he faces.

Sum insured		DEATH		
From	to	No. of Insureds	Accumulated SI	Accumulated Premium
0	50 000	95 044	1 384 011 217	26 474 353
50 001	100 000	8 514	550 336 076	4 070 885
100 001	200 000	1 431	194 911 738	953 805
200 001	300 000	244	59 539 617	221 820
300 001	400 000	96	33 037 267	105 249
400 001	500 000	42	18 992 717	44 002
500 001	600 000	28	15 155 374	35 956
600 001	700 000	18	11 951 042	19 619
700 001	800 000	18	13 046 808	22 595
800 001	900 000	8	6 619 704	12 527
900 001	1 000 000	3	2 701 376	4 601
1 000 001	2 000 000	11	14 403 768	26 869
2 000 001	3 000 000	3	8 079 068	12 943
3 000 001	4 000 000	1	2 275 313	1 732
4 000 001	>	0	0	0
Total		**105 461**	**2 315 061 084**	**32 006 953**

Table 10 - Risk profile

Source: Renewal information of an Austrian company (2019)

Losses

Historical losses in this portfolio are necessary input into experience pricing. The loss is characterized by date of loss, underwriting year, name of loss, its schedule indicating if the loss closed or it still develops and its paid value, reserved value and incurred value. There are 266 reported losses in the portfolio chosen. The oldest happened in 2001 and the biggest one equals to 1 699 456 EUR. As older losses have been reported more time than the new ones, the losses are in form of triangulation where on the vertical axis, there are underwriting years and development years are on the horizontal axe. As mentioned in the Chapter 4.2, the insurer reports only losses exceeding a fixed threshold relative to the incurred loss, in this case it is 250 000 EUR corresponding to 50% of treaty retention. The losses smaller than 250 000 EUR are not relevant for the reinsurer as they are fully retained by the insurer. The cumulative distribution function of all reported incurred losses can be seen on the Chart 5. Losses have been reported if the incurred amount once exceeded 250 000 EUR. The latest value reported in 2019 was taken for constructing the Chart 5, therefore it also contains losses which are now lower than 250 000 EUR due to reliefs of reserves but have once exceeded 250 000 EUR during the years reported. This is captured by the inflexion point at 200 000 EUR and 13% and it confirms the need of analyzing losses higher than reporting threshold of 50% of retention. The cumulative distribution function of losses exceeding the 250 000 EUR does not have inflection point and is smoother.

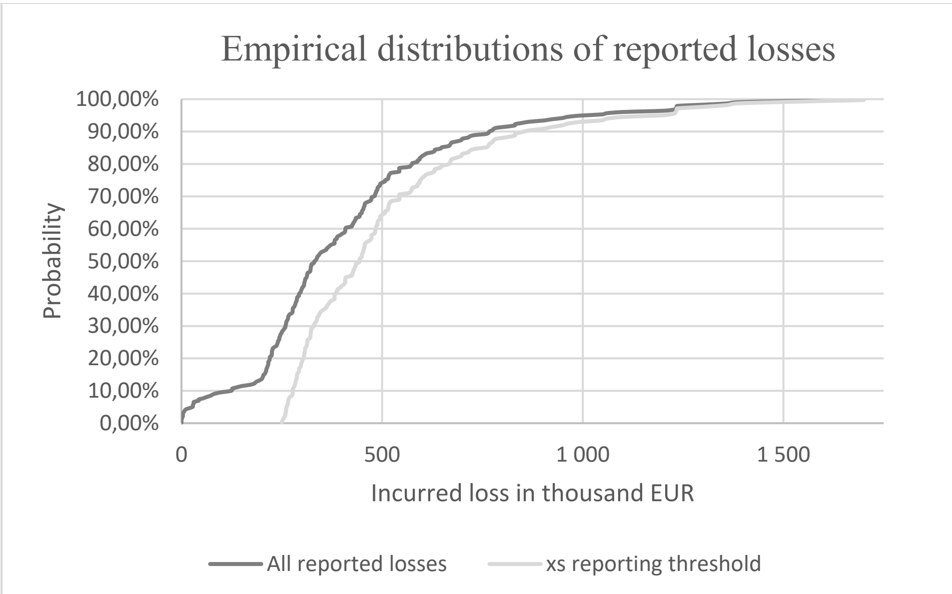


Chart 5 - Distribution of reported losses

On the Table 11, there is example of the one particular loss which happened on the 3rd January 2013 in Austria and was reported to the insurer on the 13th December 2013 which represents a usual delay in reporting. In the first development year, nothing was paid to the insured, but the reserve of 1 499 000 EUR was created. In the second development year, 193 128 EUR was paid to the insured and no additional reserve was created. Then, indemnification and reserve creation continued in the standard way and in 2018, 377 993 EUR was paid to the insured and the reserve has decreased to 1 084 307 EUR. The similar development should follow also in the future and the loss will be slowly fully paid and the reserves will drop to zero. However, some new damage related to loss can happen and the reserve may double and even it can decrease to zero without paying more to insured and the claim may be closed. The example illustrates well the long-tail of the personal accident treaty.

UY	Claim ID	Insured	Date of Loss	Date of 1st Report	Currency	Country of accident	Schedule	2013	2014	2015	2016	2017	2018
2013	1804639782	Z. S.	03.01.2013	13.12.2013	EUR	AUT	**Claims Paid**	0	193 128	203 100	326 752	347 282	377 993
							Reserve	1 499 000	1 296 872	1 281 900	1 147 748	1 115 718	1 084 307
							Total -Claims Incurred	1 499 000	1 490 000	1 485 000	1 474 500	1 463 000	1 462 300

Table 11 - Example of a loss

Source: renewal information of an Austrian company (2019)

5.2 Inflation Data

As the chosen treaty is from the portfolio of an Austrian insurance company, the macroeconomic variables describing the Austrian economy will be used. With those data, the simulations of future inflation will be built and later it will be used as input to the reinsurance part of analysis. The choice of the variables which might have impact on inflation is done according to the economic intuition and based on inflation modelling literature review. Concretely, GDP, monetary aggregates, short-term interest rates, unemployment, EUR to USD exchange rate, outstanding loan, 10 years government bond yields, and total production index are chosen. All those data are quarterly, starting from Q1 2005 and ending at Q1 2019. GDP and unemployment are seasonally adjusted to get rid of the short-term fluctuations.

Inflation measured by consumer price index (CPI) was downloaded from OECD database and it is defined as the change in the prices of a basket of goods and services purchased by the households. Unemployment was taken from FRED database and it is seasonally adjusted because of its short-term fluctuations. Monetary aggregates M1, M2 and M3 representing different forms of money supply in Eurozone were downloaded from the database of the European Central Bank. Interest rate is the 90 days interbank rates for Austria. Exchange rate between Euro and United States Dollar has also the FRED as the source. The seasonally adjusted GDP was downloaded because of its short-term fluctuation. Total credit to private non-financial sector, adjusted for breaks is taken as proxy for outstanding loan of the economy. As through the investment channel companies ‘s reaction to changes in the cost of credit directly influences the price of products and services, this variable could be determinant for the inflation. Bond yield is sovereign bond year for 10 years maturity. Production index describes similarly as GPD the activity of the economy and it will be used in some model versions below instead of GDP for the robustness check. It is defined as production of total Austrian industry. The Table 12 summarizes the time-series.

Variable	Mean	Standard deviation	Minimum	Maximum
Inflation	1,90%	0,84%	0,03%	3,73%
Unemployment	5,27%	0,55%	4,09%	6,26%
M1	5 273 741 379	1 552 295 375	2 964 000 000	8 490 000 000
M2	8 805 431 034	1 698 882 017	5 625 000 000	11 943 000 000
M3	9 691 793 103	1 529 995 319	6 575 000 000	12 571 000 000
Interest rate	1,20%	1,66%	-0,33%	5,02%
Exchange rate	0,79	0,08	0,64	0,95
GDP	76 507 534 483	4 013 631 864	68 240 200 000	84 783 500 000
Outstanding loan	15 298	1 401	11 324	17 533
Bond yield	2,37%	1,49%	0,03%	4,80%
Production index	98,33	8,03	81,74	117,97

Table 12 - Summary statistics of time-series

Source: Author's calculations & various sources as seen above

Before the analysis, two monotonic transformations were done with the time-series. The natural logarithmic transformation was applied on the variables representing accumulated levels. Secondly, the non-stationary variables were taken in first differences either their simple or logarithmic form to achieve stationarity of time-series and for better economic interpretation of the quarterly changes of those variables.

6. Empirical Analysis

The empirical analysis is aimed at studying the effect of the inflation uncertainty on the reinsurance pricing. With widely used deterministic future inflation, the variance of the inflation is omitted which may lead to the underestimation of the risk the reinsurer faces. The main hypothesis of the thesis is that the use of stochastic future inflation instead of deterministic flat future inflation improves the accuracy of the reinsurance pricing. As the distribution of future inflation is needed as an input to reinsurance pricing, the vector autoregression model will be estimated to forecast medium-term inflation paths. Later, the scenarios of future inflation will be the input for the price estimation of the reinsurance contract enabling the comparison of the two approaches and discussion of the added value of the stochastic modelling.

6.1 Inflation Forecasting

The objective of this chapter is the prediction of the future development of the inflation. An econometric model will be used to study historical macroeconomic variables and forecast their future development. Due to the long-tail aspect of the claim development, medium-term and long-term forecasts are needed. Widely used models to forecast the mean and the variance of the time-series such as ARIMA-GARCH will not be helpful as they are adequate for short-term forecasting. Therefore, the vector autoregression models are more appropriate.

The choice of variables has been done according to past studies aiming at forecasting the inflation and economic theory (Jansen, 2004); (Meyler et al.,1998). The graphical representation of the variables can be seen on the Chart 6.

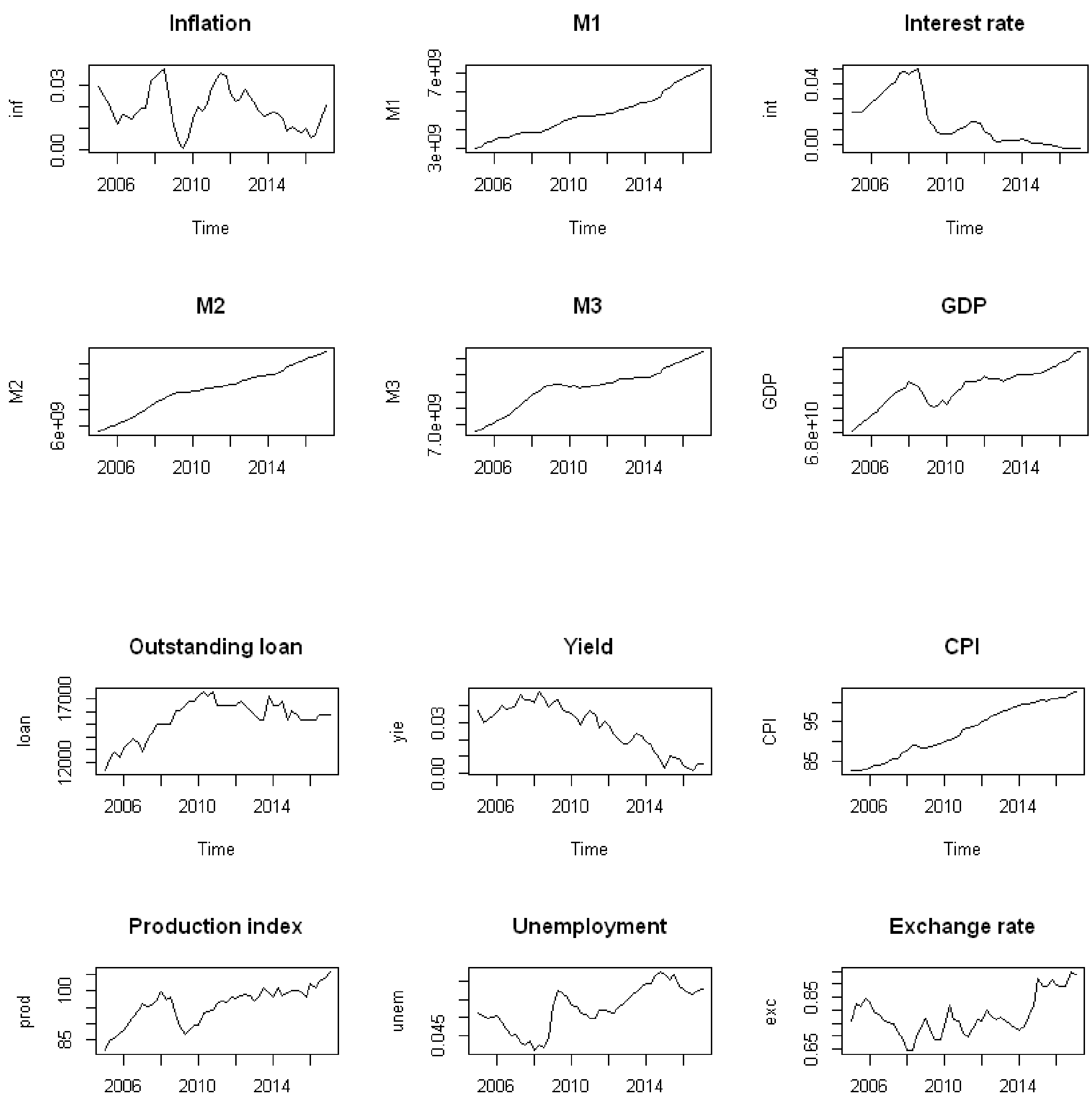


Chart 6 - Development of time-series

The variables do not seem to be stationary as many of them contain the time trend. There are significant jumps during the financial crisis where production, interest rate, GDP and inflation decreased sharply. On the other hand, unemployment increased. The stationarity has been tested with Augmented Dickey-Fuller test and it has been discovered that M1, M2, M3, GDP, outstanding loan, production index and exchange rate are not stationary. The variables unemployment and yield are marginally stationarity with p-value close to 5%. For this case, variables in level as well as in the first difference will be considered for the model specification.

The presence of cointegration the between variables in levels was checked and denied so we decided for VAR model and not for the vector error correction model. The variables containing unit root were transformed to first differences to achieve stationarity. The logarithmic differencing was applied on the monetary aggregates, GDP, outstanding loan and production index. Logarithmic transformation is a variance-stabilizing transformation and has a good interpretation as the differences in logarithms become continuously compounded returns. Stationary variables including inflation, short-term interest rate or unemployment stayed in levels in order to do not lose long-term information. The Chart 7 illustrates the development of time-series after differentiation.

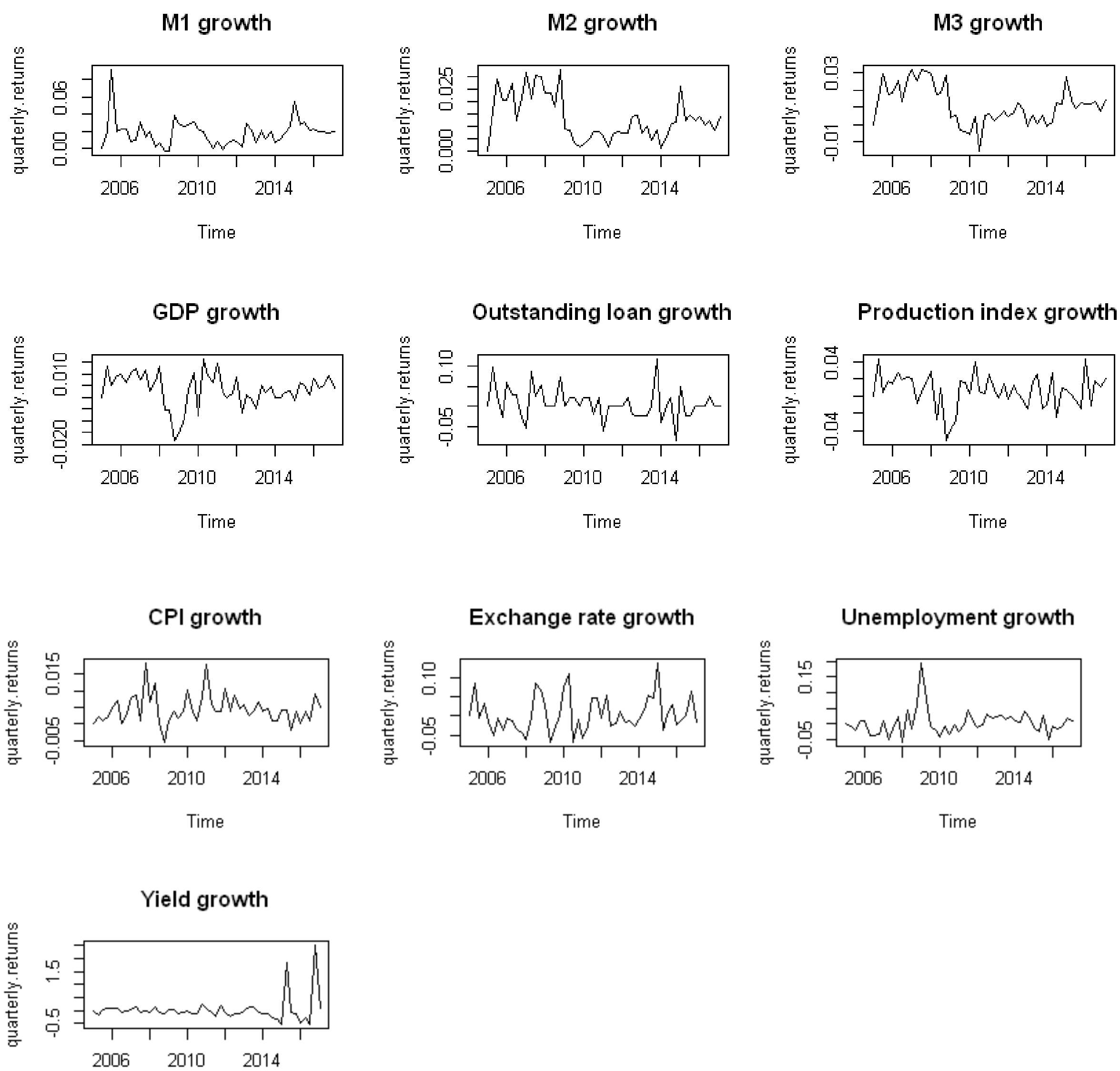


Chart 7 - Development of returns

The ordering of variables is important feature of VAR models and Choleski recursive identification is used. Ordering the variables from the most exogenous to the least exogenous according to the economic intuition and the literature review presented in the Chapter 4.1.2 is crucial as the variable influences the others ordered after also contemporaneously but the least one has the impact on the previous one only with the lag. As suggested by Mojon and Peersman (2001), the standard way of ordering the variables is as follows: unemployment, GDP, inflation, M1, interest rate, yield, exchange rate. Most endogenous variables are interest and exchange rate that are affected contemporaneously and with lag by the others. For example, the national bank sets interest rate considering target price level in two years and GDP. Output and price level react to changes in monetary policy only with lag. Exchange rate reacts to interest rate contemporaneously and it accounts for supply shocks. The unemployment is set as the most exogenous variable as it is influenced only with lags by the others.

More studies have found that monetary policy has no immediate effect on output and inflation (Bernanke & Blinder, 1992). Although it is difficult to investigate into long-term restrictions that identify only monetary policy shocks, Keating (1992) and Walsh (1993) stated that monetary policy cannot have any long-term effects on real variables such as GDP, the interest rate or the real money supply.

Production index was in some model specifications used instead of GDP and the outstanding loan instead M1 as M1 is determined by loan in the economy. It seems that replacement of variables does not change the results in the significant way thus the models are robust. With those variables, several VAR model specifications were estimated and compared. Granger causality test was used to identify if a given variables has Granger effect on the others which means that their past values contribute to the variable of interest. Variables without Granger causality were dropped from the model.

After ordering the variables from the most exogenous to the least exogenous, the lag length is determined by minimizing information criteria but also considering that parsimonious model is needed for forecasting. Then, the diagnostic analysis of residuals is done to find the most suitable model. Also, stability of the model is checked and tests for autocorrelation and heteroscedasticity were done. The most suitable model is estimated and used for forecasting. The data from Q1 2005 to Q1 2017 were used for estimation and choice of model and the period between Q2 2017 and Q2 2019 was used to in-sample forecast and evaluation of model. Various accuracy measures including RMSE, MSE, MAE were computed and compared. Finally, the out-of-sample forecast was done for next quarters.

More models differing in number of lags, variables used and in form of variables – levels or first differences were estimated. They are summarized and compared in the Table 13. According to the information criteria, the best number of lags was four, therefore one year is enough for explaining the current data. The model 4 was excluded from the choice as it is not stable with inverse root higher than one. The model 2 was suitable for the data the test of Granger causality was done. Variables without explanatory power were dropped from the model and we arrived at the model 5. In the model 6, GDP and M1 were replaced by the variables containing the similar information. Then, the models were compared by their predictive power and accuracy of the forecast. The model 5 seemed ideal but some variables were only marginally Granger causal. After their deletion, the simplest model 7 containing only growth of GDP, inflation, interest rate and growth of exchange rate has been obtained. The model 7 seems to be the most appropriate due to the lowest RMSE and its simplicity. Therefore, this model will be used for forecasting of inflation.

Model	RMSE	Detail	Lags	Comment
varmodel2	0,0407	unem, GDP_r, inf, M1_r, int, yie, exc_r	4	
varmodel3	0,0381	unem, GDP_r, inf, M1_r, int, yie, exc_r	2	
varmodel4	0,0413	unem_r, GDP_r, inf, M1_r, int, yie+r, exc_r	4	explosive
varmodel5	0,0367	unem, GDP_r, inf, M1_r, exc_r	4	some variables excluded due to the marginal Granger causality
varmodel6	0,0481	unem, prod_r, inf, loan_r, int, yie, exc_r	4	robustness - GDP replaced by production index, M1 by outstanding loan
varmodel7	0,0218	GDP_r, inf, int, exc_r	4	additional variables excluded due to the marginal Granger causality

Table 13 - Summary of VAR models

The estimated model is stable with the highest inverse root equal to 0,9512. Even though the stationarity of several variables in some models was not achieved even after differencing, the VAR model can be used with non-stationary data as long as it is stable as a whole (Lütkepohl, 2005). Moreover, this combination and ordering of variables corresponds to the economic theory. After model estimation, the diagnostic tests of the model residuals were done. They seem to be not far from white noise as seen on the Chart 8. However, there is still some dependence that could be further handled for example by volatility modelling.

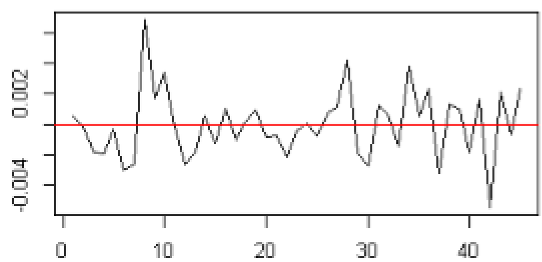


Chart 8 - Residuals of inflation

There is no autocorrelation left in the residuals. It can be seen on the Chart 9 showing the ACF and the PACF for residuals as well as for squared residuals representing the second moment.

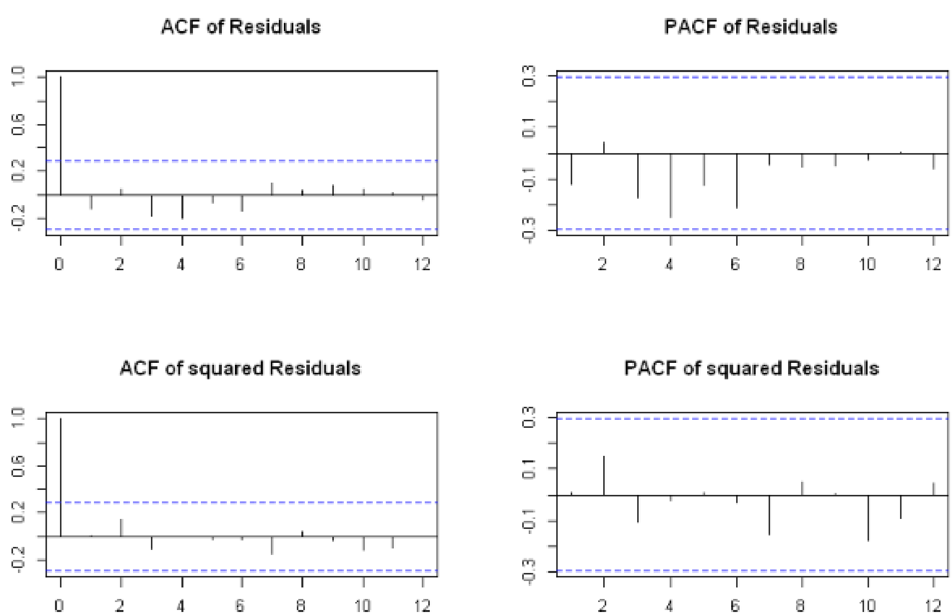


Chart 9 - ACF & PACF of residuals

The stability of the model and serially uncorrelated errors allow for providing orthogonal impulse response functions. Those functions are also the important criterion for the choice of VAR model describing the interaction within the variables in the system. They represent the evolution of one variable after the shock into another and they are drawn in order to evaluate the models qualitatively (Lütkepohl, 2016). Our interest was mainly about the effect of interest rate on the other variables in order to study the influence of monetary policy shock into the economy. The impulse responses are not the objective of the thesis, they rather serve as controlling mechanism for the final model.

Firstly, we investigate in the presence of price puzzle, the common phenomenon in the VAR literature firstly described by Sims (1992). According to Sims, the restrictive monetary policy is followed by slight increase in the prices and not by the decrease what could be expected by the standard economic theory. Sims stated that price puzzle is the result of the imperfection in the control of information that the central bank could have about the future development of inflation. The reaction of inflation in captured on the Chart 10. The effect is the biggest after six quarters. Also, it is the only one moment where whole confidence interval is below zero. Firstly, after shock into monetary policy the inflation decreases, then the effect is positive but not significant. The decrease of inflation after shock into interest rate confirmed the absence of price puzzle effect and expected reaction of the economy to restrictive monetary policy.

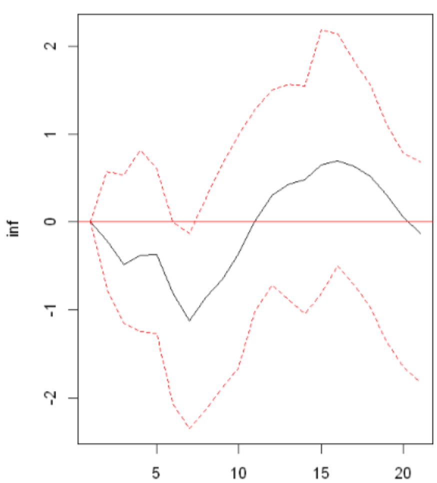


Chart 10 - Impulse response

Unfortunately, the other impulse responses are not significant. The shocks into monetary policy do not influence so much Austrian economy as we would think. Therefore, assuming the model is correct would indicate non-functionalities in the transmission of the monetary policy. Results which do not confirm the effectiveness of the standard monetary policy under zero lower bound and possible unconventional policies have been already discovered by some researchers (Fiore & Tristani, 2019). The limitation of the observations and inclusion of the time-series during the financial crises that have different behaviour than the rest is also a possible explanation. All impulse responses can be found in the Appendix A.

Forecasting of inflation is performed for next 10 years and the obtained deterministic path of inflation that can be seen on the Chart 11. The future inflation fluctuates around the target inflation of 2% and the volatility decreases over time. During 2021, according to the model, there should be a visible decrease of the inflation what may signify the weakening of the economy. On the contrary, there are several advantages of lower inflation as higher international competitiveness due to the increasing exports, better investment environment for firms and increase in the rates of return. The minimum inflation should be reached in the fourth quarter of 2022 and it should be 1,30%. Later, in the 2023, the inflation shall slowly move slightly above 2%. The longer-term inflation forecast needed for our further analysis of reinsurance pricing may be uncertain.

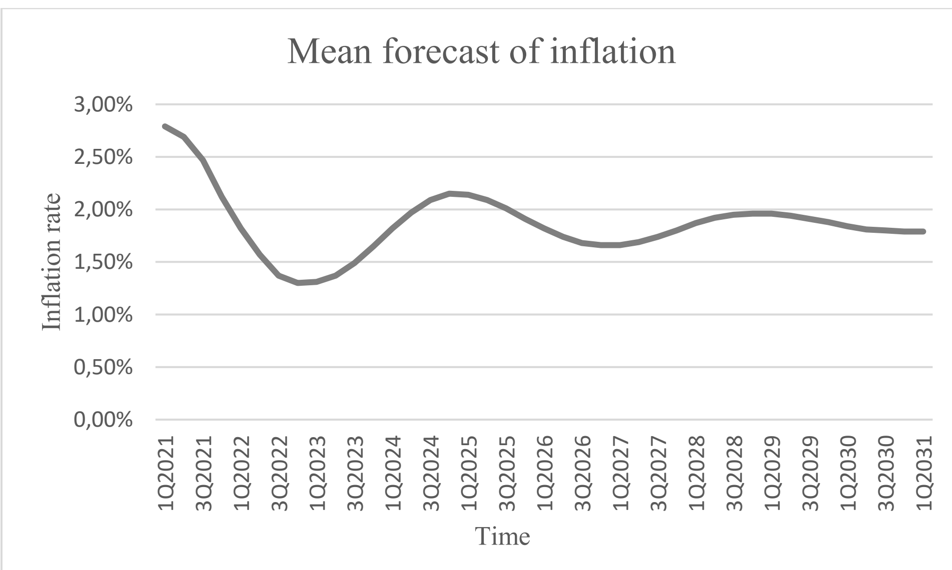


Chart 11 - Mean forecast of inflation

As the objective of this thesis is to study the effect of stochastic future inflation on reinsurance pricing, stochastic simulation of inflation is done. The independent and identically distributed shocks for 48 quarters for 4 variables are drawn. With the use of estimated covariance matrix from VAR model, shocks are transformed in order to correspond to estimated relations within final VAR model based on Choleski decomposition. Those simulated shocks are dynamically input into estimated VAR equations and we obtain 1 000 simulations for each variable. Mainly, we are interested in the simulations of the inflation illustrated on the Chart 12. For better visualization, only 10 simulations instead of 1 000 were used. On the horizontal axis, the quarters are used as the units.

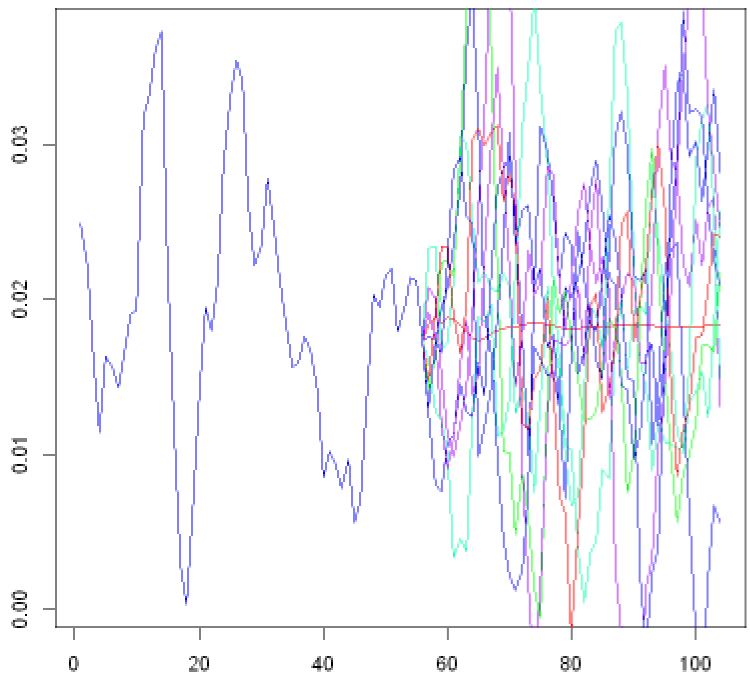


Chart 12 - Simulation of inflation paths

Those simulated time-series will be used in the second part of the analysis devoted to reinsurance pricing.

To conclude, macroeconomic time-series from Austria were used to predict the inflation paths by VAR models. The four variables with the best predictive power are inflation, interest rate, GDP and exchange rate. Results of the model show the existence of relation between inflation, interest rate and exchange rate and GDP. The existence of price puzzle has been denied by studying the effect of monetary policy shock. The deterministic forecast for next quarters is quite stable and fluctuates around the target inflation of 2%. The stochastic simulations allowing for different scenarios are centred about the forecast but with higher variance.

Our choice for the inflation was total CPI which is general and widely used measure of the speed of increasing prices. As the personal accident treaty covering death, permanent disability and partly the wage compensation will be studied in next subsection, the general CPI suits well for description of the inflation of future claims. Another alternative could be also wage index mainly for professional indemnify and workers ‘compensation reinsurance, medical inflation for health reinsurance or housing index for property reinsurance.

The problematic issue of the VAR model is the interest rate which is decreasing during last years and from the second quarter of 2015 it becomes negative. Sovereign bond yield as risk-free interest rate tends to be negative in many EU countries. The possible improvement could be the use of shadow rates. As interest rates close to zero causes many economic models to stop working, Wu and Xia described the use of so-called shadow rate that replace the negative interest rates and make the models functional again. The shadow rate uses the movements of various benchmark data (Wu & Xia, 2015). The modelling of shadow interest rate could improve the models, but it is out of scope of the thesis. Despite the use of standard interest rate, the VAR model is stable, which is the necessary condition for functionality of the model and forecast.

6.2 Parameters and Introduction to the Pricing

In this part of analysis, the calculation of the reinsurance price for the personal accident treaty covering risks and events will be done. The personal accident treaty used as illustration has the retention equal to 500 000 EUR and limit equal to 39 500 000 EUR. Therefore, the reinsurer will pay each and every loss exceeding 500 000 EUR but not more than 39 500 000 EUR per loss. The reinsurer participating in this treaty will indemnify the insurance company in case of accident of one person but also for catastrophic events when at least two persons are involved. In our analysis, experience pricing using the historical information will be used. The objective of this part is to estimate the amount of loss that the reinsurers will have to pay to the insurer during the next year which corresponds to the minimum sustainable price of the treaty. The loadings will be added later to this pure price in order to provide an adequate margin.

As already explained, personal accident is long-tail line of business, which means that claims are developing during the years and are fully paid in approximately ten years. Therefore, the reinsurer has to take this feature into consideration when pricing the treaty and incorporate the effect of the inflation.

Firstly, the parameters of the treaty will be identified. Secondly, the losses will be indexed by the inflation. Payment pattern, IBNR and IBNeR will be constructed in order to develop recent losses to the ultimate values. Then, future inflation factor measuring the impact of the future inflation on losses will be applied. Pricing with burning costs and frequency and severity method will be done with developed losses in as if status. Once loss characteristics are determined by frequency and severity modelling, Monte Carlo simulation will randomly simulate the losses and recoveries of the treaty will be calculated by applying the structure of the treaty. Finally, the price will be calculated from simulated losses and simulated risk measures by application of loadings. The prices calculated from the forecast of inflation and from stochastic simulation of inflation will be compared.

Parameters

As the first step, all the parameters that will appear in the calculation are presented. The retention of the chosen treaty is 500 000 EUR and the limit of the treaty is 39 500 000 EUR. As in excess of loss treaty, the reinsurer is liable only each and every for loss higher than retention, he is interested in losses higher than the retention. As a lower loss which happen years ago can exceed the retention after indexation, the retention is not sufficiently low limit for studying the losses. Therefore, a threshold above which the losses are studied is established. The best practice is to take 60%-70% of the retention. In our case, it is 350 000 EUR. Therefore, from 266 reported losses from the insurer, only 162 are relevant for the further pricing as reinsurer can become potentially liable for them.

As the price of reinsurance coverage is discounted sum of the future cash-flow, the interest rate of government bonds of different maturity will be used. The source is EIOPA database and the yields correspond to the December 2019 since pricing for the 2020 treaty is done. They are illustrated in the Table 14 and maturity is measured in years.

Maturity	1	2	3	4	5	6	7	8	9	10	11
Yield	-0,42%	-0,39%	-0,34%	-0,29%	-0,23%	-0,16%	-0,08%	-0,02%	0,05%	0,11%	0,16%

Table 14 - Yield of government bonds

Source: Author's calculations & EIOPA (2019)

Inflated Losses

The next input into calculation are losses in form of loss triangulation showing paid, reserved and incurred loss as in the Table 11. The status of each loss is reported yearly. The majority of 162 studied losses is still open, only a few losses from the first years are closed. Therefore, considering their development to the ultimate value when determining the price is important. As explained in the Chapter 4.2 the losses have to be expressed in current real terms thus, we inflate the triangulation. The paid amount in each year is incremental therefore the inflation indexes are applied on each and every increment according to the respective payment year. In practice, the additional payment done in given year is multiplied by the inflation rate calculated as ratio of two inflation indexes of two different years. For reserved amount, which is cumulative we use multiplication of the reserve by difference between two inflation indexes. The indexed incurred loss is the sum of the indexed paid loss and the indexed reserve. See Appendix B for detailed information about indexation.

IBNR, IBNeR and Payment Pattern

As prediction of the development of losses into future is needed, the past losses information will be used, and the payment pattern will be constructed from old losses whose development is known. The payment pattern is the proportion of paid loss over incurred loss during the years. This pattern will be later applied on all losses to be able to see future obligations of the reinsurer. Two different approaches were used to calculate payment pattern. The first consists of division of total paid losses by total incurred in each development year. It is named the weighted payment pattern. The non-weighted approach uses average ratios of paid and incurred loss in each development year and each year of occurrence.

The Table 15 illustrates one empirical payments pattern, one fitted and then the final choice being the combination of empirical and fitted. Due to the lack of the old losses that have at least fifteen development years, the payment pattern in the development years 15 – 18 is not strictly non-decreasing and the payment pattern does not reach 100% at the end of studied time period as it should. Therefore, the fitted pattern is constructed as linear extrapolation of points belonging to empirical pattern. The final pattern is combination of the empirical for first five years where there are enough data for empirical pattern. From the sixth development year up to the tenth, fitted pattern is chosen. After eleven years in average, losses are completely paid and the reserves are zero. It corresponds to the theory where personal accident claims develop during the first decade. This payment pattern constructed from the historical losses will serve as a rule according to which also future losses shall develop.

Development year	Empirical	Fitted	Final
1	12,1%	15,4%	12,1%
2	42,7%	40,3%	42,7%
3	58,7%	54,8%	58,7%
4	66,9%	65,1%	66,9%
5	72,6%	73,1%	72,6%
6	75,9%	79,7%	79,7%
7	77,9%	85,2%	85,2%
8	79,7%	90,0%	90,0%
9	80,5%	94,2%	94,2%
10	81,0%	98,0%	98,0%
11	77,9%	100,0%	100,0%
12	80,3%	100,0%	100,0%
13	81,3%	100,0%	100,0%
14	82,4%	100,0%	100,0%
15	84,5%	100,0%	100,0%
16	70,4%	100,0%	100,0%
17	61,3%	100,0%	100,0%
18	59,2%	100,0%	100,0%

Table 15 - Payment Pattern

The Chart 13 represents the payment pattern. The dots are empirical payment pattern and dashed line is fitted pattern. The full line in first five years and the dashed line form the final payment pattern.

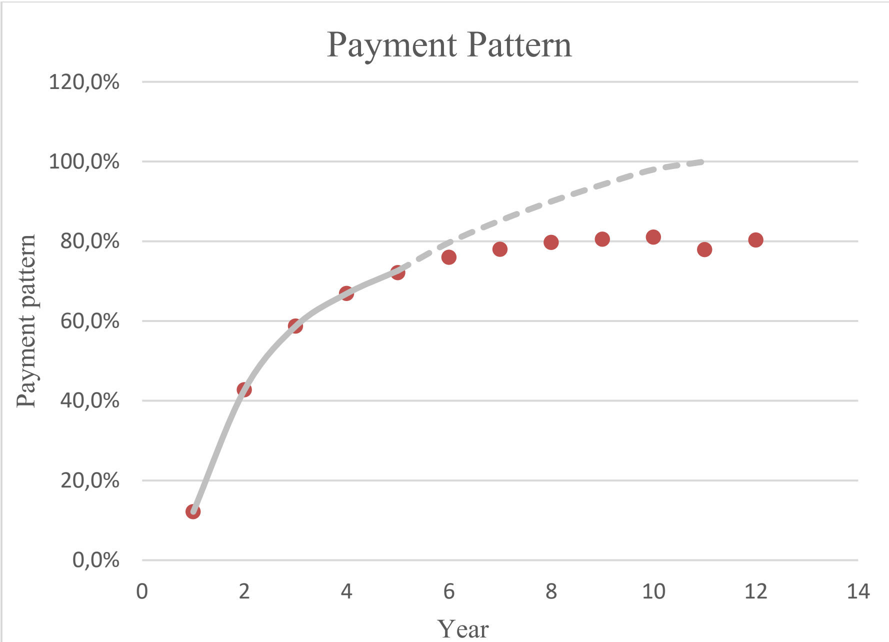


Chart 13 - Payment Pattern

IBNR and IBNeR are closely related to payment pattern. IBNR means "Incurred But Not Reported" losses and describes the claims that have already occurred, but they were not officially reported to the reinsurer. As these claims still have to be paid in the future to the insurers, the reinsurer needs to form its reserves to cover its costs. Since the reinsurer knows neither how many of these losses have occurred, nor the severity of each loss, IBNR is just an estimate. The total amount of loss reported in two or three last years is usually smaller than in years before. It is caused by delay in reporting losses and it has to be considered when pricing the treaty. IBNR is an adjustment of the amount of loss reported in last years by the certain factor.

The IBNR calculation is illustrated in the Table 16. In the first column, there is number of incurred claims exceeding the threshold that happened in the particular underwriting year and has been reported until the end of this year. The difference between value in the first and the value in the second column indicates how much claims have been reported one year after the year of occurrence of loss. For example, for the claims which happened in 2001, one was reported in 2001, the second loss in 2002 and the third in 2017 which happens rarely. For most years, almost all losses were reported until the third year after the loss occurrence.

The row below the triangulation named Empirical represents the link ratios. For example, division of sum of the column 2 by the sum of the column 1 equal to the link ratio 2. The link ratio is the factor by which the number of reported claims increases in every year. Two times more losses are reported one year after the date of loss than in the year of loss occurrence. It is 1,22 time more between the second and the third year and then the number fluctuates around one. As this effect is considered in next years as insignificant for the analysis, the chosen pattern is in the row named Fitted. As we need to know the ultimate number of claims, the product of fitted link ratios will be used to this development. For example, five claims which occurred in 2019 were reported till the end of the year. It is expected to be two times more in 2020 and 1,22 more in 2021. The ultimate number of losses in the end of the development period is expected to be 5*2*1,22. The third row named Product illustrates the factor by which the number of loss in respective column needs to be multiplied to achieve ultimate number of losses in the end of studied period. All undeveloped numbers of claims will be multiplied by this value in order to obtain the ultimate frequency. From the third year, the effect of the IBNR on loss frequency is neglected as it does not provide any additional information. For development years higher than three, empirical link ratios are smaller than one and rather come from the reserve fluctuation from the same claim which fluctuates above and below the threshold. It can be seen in the row illustrating the claims which happened in the underwriting year 2008. Between the fourth and the fifth development year, the reserve of some claims decreased, then in the following year it increased and later decreased one more time. Therefore, the empirical link ratios can be smaller than one due to the reserve fluctuation below the threshold even it has exceeded the threshold before.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2001	1	2	2	2	2	3	3	3	3	2	2	2	2	2	2	2	3	3
2002	2	4	5	5	5	5	5	5	4	5	5	3	3	3	3	4	3	0
2003	1	1	3	3	3	3	3	3	3	2	2	2	1	1	2	2	0	0
2004	1	7	7	8	8	8	8	8	8	7	7	7	7	7	7	0	0	0
2005	2	2	3	4	5	5	6	5	5	5	5	6	6	6	0	0	0	0
2006	2	5	4	5	5	5	5	4	4	4	6	6	6	0	0	0	0	0
2007	3	10	9	9	7	8	7	7	7	8	8	8	0	0	0	0	0	0
2008	5	10	15	16	12	13	13	12	12	11	11	0	0	0	0	0	0	0
2009	6	12	15	14	15	13	13	13	12	12	0	0	0	0	0	0	0	0
2010	6	10	9	9	9	10	10	8	7	0	0	0	0	0	0	0	0	0
2011	6	9	21	15	15	15	14	13	0	0	0	0	0	0	0	0	0	0
2012	4	11	7	6	6	6	6	0	0	0	0	0	0	0	0	0	0	0
2013	9	14	17	16	13	13	0	0	0	0	0	0	0	0	0	0	0	0
2014	5	8	8	7	8	0	0	0	0	0	0	0	0	0	0	0	0	0
2015	4	7	10	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2016	1	6	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2017	5	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2018	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Empirical	2,00	1,22	0,95	0,95	1,02	0,99	0,93	0,96	0,97	1,05	0,97	0,92	1,00	1,08	1,14	1,00	1,00	
Fitted	2,00	1,22	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00
Product	2,44	1,22	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	1,00	

Table 16 - IBNR triangulation

The Chart 14 illustrates the chosen fitted IBNR pattern. On the horizontal axis, there are years after occurrence of loss and vertical axis measures the IBNR. The dots stand for empirical IBNR and the curve represents the final fitted IBNR.

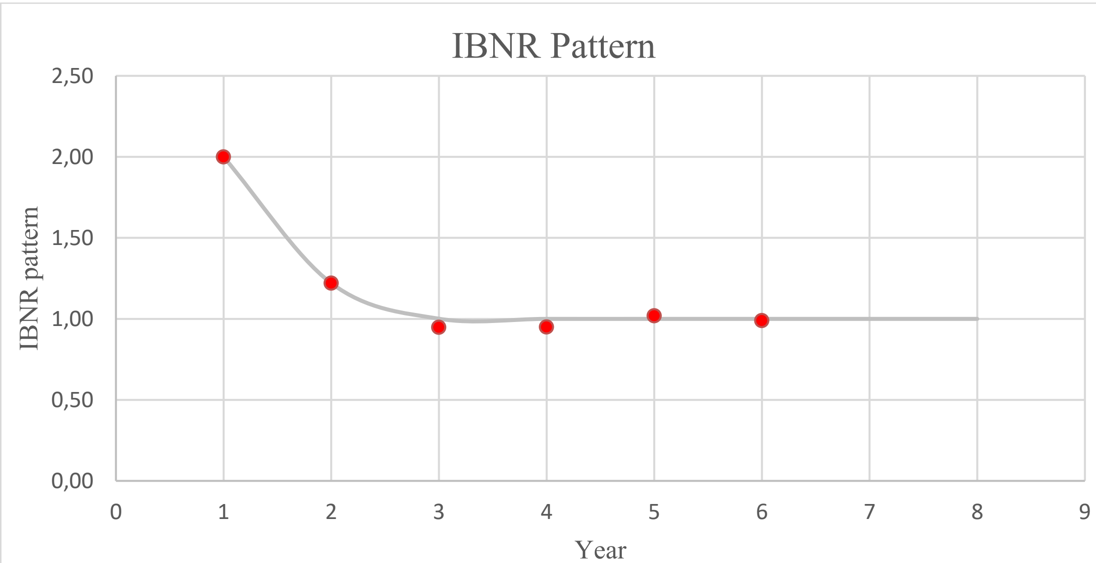


Chart 14 - IBNR Pattern

IBNeR means "Incurred But Not enough Reported" losses. The incurred loss should remain almost the same during the loss development. The IBNeR analysis serves as a check if the reserving is done properly by the insurance company. The IBNeR are the claims that may be underreported or overreported meaning that the reported claim is less or more than the actual amount respectively. IBNeR are measured as a ratio of yearly changes in incurred loss exceeding the chosen threshold. The IBNeR calculation is done in the Table 17. In the first column of the triangulation, there is the ratio between the incurred loss in the second year after loss occurrence and the years of loss occurrence. All IBNeR are very close to one. However, there are more cases when the IBNeR is below one than above one meaning that the insurance company is overreserving. As empirical IBNeR pattern as well as fitted IBNeR pattern fluctuate around one, it will not influence pricing.

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Empirical	0,96	0,98	0,97	0,99	0,97	1,00	0,99	0,99	1,01	1,00	1,00	1,00	1,00	0,99	1,07	1,00	0,99	1,00
Fitted	0,96	0,97	0,98	0,98	0,99	0,99	0,99	1,00	1,00	1,00	1,00	1,00	1,00	1,01	1,01	1,01	1,01	1,01
2001	0,72	0,99	0,97	0,99	0,99	0,99	0,99	0,99	1,04	0,99	0,99	0,98	0,99	0,99	1,02	1,03	0,99	1,00
2002	1,17	1,02	1,00	1,15	0,87	0,99	0,99	0,90	1,00	1,01	0,99	1,00	0,99	0,98	1,16	0,99	1,00	
2003	1,01	1,01	0,98	0,98	0,99	0,98	1,00	0,99	1,00	0,96	0,99	0,98	1,01	1,02	0,98	1,00		

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
2004	1,12	0,98	0,98	0,99	0,76	1,00	0,99	0,85	0,98	1,00	0,99	1,00	1,02	0,99	1,00			
2005	0,98	1,01	0,90	0,96	1,00	1,02	0,95	0,99	1,00	1,00	1,00	1,00	1,00	1,00				
2006	1,09	0,86	1,02	1,00	1,00	1,01	0,90	1,00	1,00	1,13	1,01	1,00	1,00					
2007	0,98	0,98	1,00	0,93	1,01	0,98	1,00	1,04	1,11	0,99	0,99	1,00						
2008	0,91	0,89	0,95	1,09	0,98	1,02	0,97	1,01	0,92	0,99	1,00							
2009	0,93	1,08	1,02	1,00	0,95	1,00	1,00	1,00	1,04	1,00								
2010	0,94	0,86	1,03	0,95	1,04	1,00	1,01	0,99	1,00									
2011	0,89	1,06	0,87	1,00	0,99	0,97	1,00	1,00										
2012	1,06	0,90	0,97	1,00	0,96	0,99	1,00											
2013	0,88	1,05	1,02	0,97	0,93	1,00												
2014	1,03	0,92	0,88	1,00	1,00													
2015	0,99	0,97	1,00	1,00														
2016	1,04	1,01	1,00															
2017	0,94	1,00																
2018	1,00																	

Table 17 - IBNeR triangulation

The Chart 15 summarizes the IBNeR pattern. The dots show empirical IBNeR and the curve fitted IBNeR.

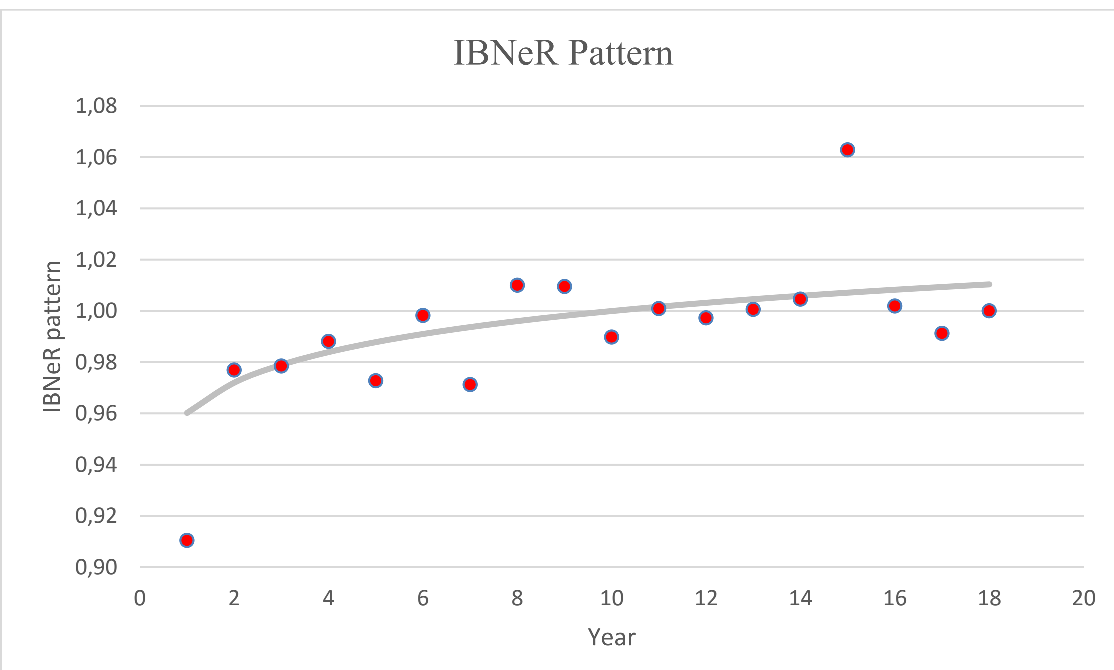


Chart 15 - IBNeR Pattern

Future Inflation Factor Analysis

Future inflation factor is the part of the analysis introducing the future inflation into pricing. The output of the VAR model from the first part of analysis is the inflation forecast and 1 000 stochastic simulations. Those paths were transformed from quarterly data to yearly assuming that the loss is paid in the middle of the year. The forecast from VAR analysis is quite stable with average inflation of 1,90%. The illustration of inflation forecast including all quarters is the Chart 11 already presented.

As the price of the reinsurance treaty is present value of all future cash-flows, discounting will be used to achieve present value. Inflation as well as risk-free interest rate determine the future amount of reinsurer's liabilities. The risk-free interest rate with different maturities has been transformed to discount factor by applying the following formula

$$d_t = (\frac{1}{1 + r_t})^t$$

where T is the maturity and r_T is the respective risk-free rate and it will be used for discounting loss recoveries that the reinsurer will pay.

The effect of future inflation is purely proportional notwithstanding of the amount of loss incurred. The proportional effect of the future inflation factor is calculated from the payment pattern and the future inflation. For instance, below in the Table 18, we can see the calculation of future inflation factor. The second column is payment pattern which has been already specified before. Incremental paid amount in the third column is yearly growth of payment pattern multiplied by the loss amount, in this case one unit. The reserve is calculated as (100% - payment pattern). The forecast of inflation is used in the example. The paid inflated incremental amount is calculated as (amount paid incremental *(1+inflation)). Paid inflated cumulative amount is just the accumulation of inflated incremental amount of paid loss. The same calculation holds also for inflated reserve. Finally, the incurred inflated loss is the sum of the paid inflated loss and the inflated reserve. The future inflation factor is the ratio of inflated incurred loss in the last column by original amount of loss being the unity. Therefore, the future inflation factor corresponding to inflation forecast equals to 1,082. The future inflation factor noted as FIF is defined as:

$$FIF = \sum_{t=2021}^{2031} (PP_t - PP_{t-1}) \prod_{j=2021}^{t} (1 + i_j)$$

where PP_t is the cumulative payment pattern with the condition $PP_{2020} = 0$, i_j is the inflation during the year t.

If the amount of loss is 1 000 currency units, inflated incurred amount will be 1 082 currency units. As consequence, we can state that the amount of loss does not influence the inflated incurred amount of the loss. Therefore, we will use different future inflation factors determined only by payment pattern, being the same for all simulations and future inflation path. Therefore, the future inflation path will be the only variable affecting future amount of incurred loss.

Year	Payment Pattern	Amount paid (incr)	Reserve	Inflation	Paid inflated incr	Paid inflated cumulative	Reserve inflated	Incurred inflated
2021	0,121	0,121	0,879	2,69%	0,124	0,124	0,903	1,027
2022	0,427	0,306	0,573	4,30%	0,319	0,443	0,598	1,041
2023	0,587	0,160	0,413	5,73%	0,169	0,613	0,437	1,049
2024	0,669	0,082	0,331	7,82%	0,088	0,701	0,357	1,058
2025	0,726	0,057	0,274	10,07%	0,063	0,764	0,302	1,065
2026	0,797	0,071	0,203	11,99%	0,080	0,843	0,227	1,071
2027	0,852	0,055	0,148	13,88%	0,063	0,906	0,169	1,074
2028	0,900	0,048	0,100	16,07%	0,056	0,962	0,116	1,078
2029	0,942	0,042	0,058	18,32%	0,050	1,011	0,069	1,080
2030	0,980	0,038	0,020	20,46%	0,046	1,057	0,024	1,081
2031	1	0,02	0	22,64%	0,025	1,082	0,000	**1,082**
FIF					**1,082**	**1,082**		

Table 18 - Calculation of the future inflation factor

As described in the example below, each of 1 000 simulations is characterized by its future inflation index. The mean forecast has future inflation index equal to 1,082 and the average future inflation index of all simulations is 1,076. The distribution of the future inflation factor can be seen on the Chart 16.

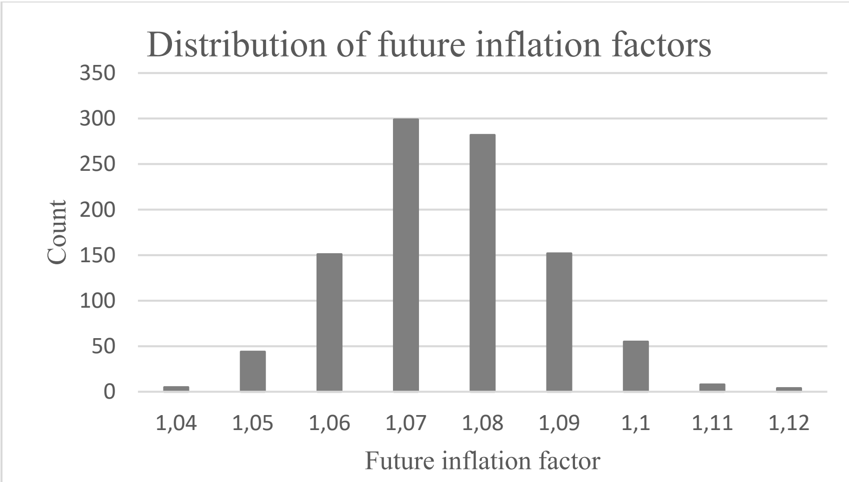


Chart 16 - Distribution of future inflation factor

6.3 Calculation of Price with Deterministic Inflation

The Chapter 6.2 introduced all necessary parameters for the pricing. Indexation of losses was done and payment pattern, IBNR and IBNeR developed losses to the ultimate value. As losses are now in as if status, the pricing firstly by burning costs method and secondly by frequency and severity modelling can follow.

Burning Costs Calculation

Burnings costs are important price benchmark comparing inflated losses and inflated premium. The indexation being the first step is shown in the Table 19 for five highest incurred losses ranked according to their latest value after inflation. The third column represents the reported value of loss. The fourth column illustrates the loss inflated to current moment by past inflation. This value is multiplied by future inflation factor of 1,082 and we obtain the fifth column. The recovery after future inflation of the treaty is calculated by subtracting the retention and respecting the capacity of the treaty from the latest value after future inflation. As this amount will be paid in the future, discounting is applied. The discount factor is defined as

$$DF = \frac{\sum_{t=2021}^{2031}(PP_t - PP_{t-1})}{(1+r_t)^t}$$

where PP_t is the cumulative payment pattern with the condition $PP_{2020} = 0$ and r_t is the risk-free interest rate with maturity t. After numerical application, discount factor equals 1,006. After those steps, losses are in the appropriate form for the burning costs analysis.

Rank	Year of occurrence	Latest before inflation	Latest inflated to date	Latest after future inflation	Discount Factor	Recovery after future inflation and discounting
1	2012	1 699 456	1 813 050	1 960 980	1,006	1 470 162
2	2004	1 359 781	1 809 618	1 957 267	1,006	1 466 426
3	2016	1 523 683	1 562 150	1 689 608	1,006	1 197 084
4	2015	1 384 439	1 439 041	1 556 455	1,006	1 063 094
5	2017	1 301 423	1 345 360	1 455 130	1,006	961 132

Table 19 - Burning costs – inflation of losses

The recoveries after future inflation are aggregated together according to the year they belong to and the Table 20 illustrates indexed GNPI and recoveries after future inflation adjusted by the exposure. The average recovery of all years is 2 748 284 EUR and standard deviation 838 891 EUR.

Year	Indexed GNPI	Recovery - after future inflation and exposure adjustment
2006	77 050 281	91 438
2007	81 329 318	2 275 064
2008	85 721 737	2 083 829
2009	85 886 928	3 728 306
2010	88 865 244	600 715
2011	89 920 399	2 401 862
2012	88 752 433	2 556 888
2013	89 993 326	4 177 596
2014	93 916 345	2 938 897
2015	99 992 562	3 254 620
2016	105 339 865	3 108 242
2017	108 275 065	3 435 434
2018	111 663 157	2 262 890
2019	115 020 292	2 460 554
2020	114 752 447	0
From	2010	
To	2019	
Average		2 748 284
St.Dev		838 891

Table 20 - Burning costs calculation

The Table 21 illustrates the detail of recovery calculation for the year 2019. Discounted recoveries for those four losses that happened in this year are calculated and they are adjusted by the growth of indexed GNPI and IBNR factor. The recovery of 2019 equals 2 460 554 EUR.

Year	Incurred after future inflation	A: Discounted recovery after future inflation	B: Growth of indexed GNPI between 2019 and 2020	C: IBNR	D: Recovery - after future inflation and exposure adjustment
2019	1 167 380	671 574	0,9977	2,44	1 635 280
2019	836 806	338 922	0,9977	2,44	825 274
2019	467 301	0	0,9977	2,44	0
2019	369 956	0	0,9977	2,44	0
D=A*B*C					**2 460 554**

Table 21 - Recovery calculation example

Recoveries for each year are calculated and illustrated in the Chart 17. The average value equals 2 748 284 EUR representing the minimum sustainable price of reinsurance treaty for next year. This price is not final as neither loadings nor profit margin have been added yet.

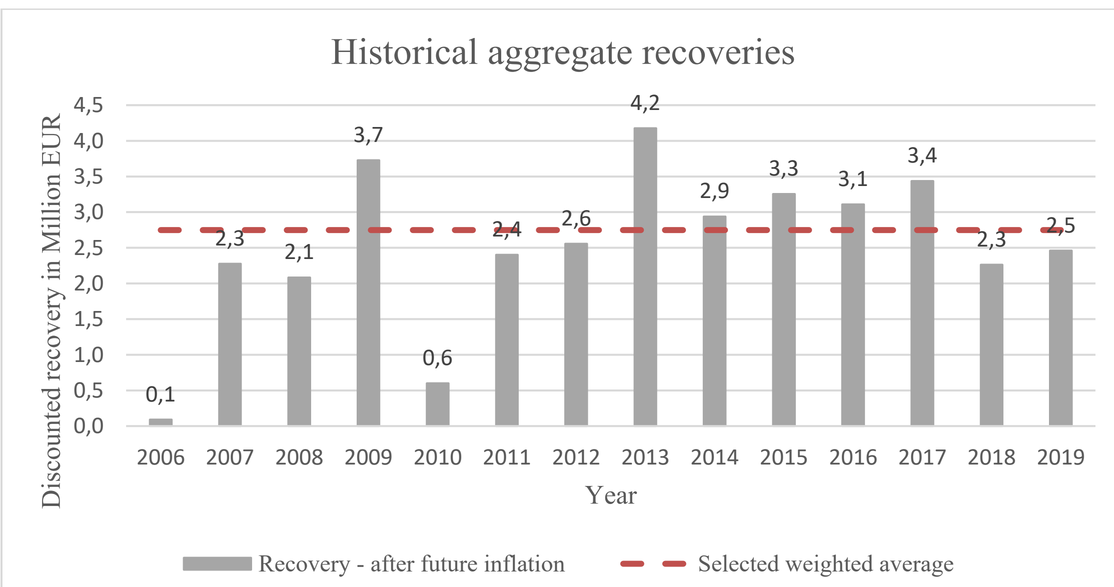


Chart 17 - Historical aggregate recoveries

Frequency and Severity Modelling

The objective of the frequency and severity modelling is to determine the average number of losses in next year and theoretical continuous distribution that will be followed if the loss happens. Frequency and severity approach models distributions of loss characteristics. This method is more complex than burning costs as historical information are completed by probabilistic method. Essentially, the ultimate loss amount based on assumed severity distribution is simulated with consideration of the stochastic inflation from previous steps to allow the impact of an increasing uncertainty in the future inflation.

Firstly, the severity of loss is modelled. The losses are ranked from the highest according to the latest value after future inflation and empirical cumulative distribution function is fitted from those losses with a small adjustment - the probability of occurrence of the lowest loss is divided by two in order to decrease the highest empirical cdf below 100%. As it is possible next year to have higher loss than the historical maximum, the loss 1 960 980 EUR corresponds rather to 99,69% of empirical cdf instead of 100%. The Table 22 illustrates the five highest losses and five lowest losses ranked according to the latest value after future inflation.

Rank	Year	Empirical CdF	Latest before inflation	Latest inflated to date	Latest after future inflation	Exponential distribution
1	2012	99,69%	1 699 456	1 813 050	1 960 980	99,4%
2	2004	99,07%	1 359 781	1 809 618	1 957 267	99,4%
3	2016	98,46%	1 523 683	1 562 150	1 689 608	98,6%
4	2015	97,84%	1 384 439	1 439 041	1 556 455	97,9%
5	2017	97,22%	1 301 423	1 345 360	1 463 976	97,0%
158	2013	2,78%	314 830	330 778	357 767	2,5%
159	2009	2,16%	285 157	327 930	354 687	1,5%
160	2018	1,54%	321 644	327 435	354 151	1,3%
161	2012	0,93%	293 533	325 017	351 536	0,5%
162	2011	0,31%	305 855	324 405	350 874	0,3%

Table 22 - Severity modelling

The empirical cdf is drawn on the Chart 18 with the latest value of loss after future inflation on the horizontal axis. It is visible that lower part of the cdf is well explained thanks to the sufficiency of loss data. However, the losses above 1 500 000 EUR are rare, the tail of empirical distribution is not very informative and cannot be used for the loss simulation. Therefore, it is necessary to find theoretical continuous distribution that fits the best our data.

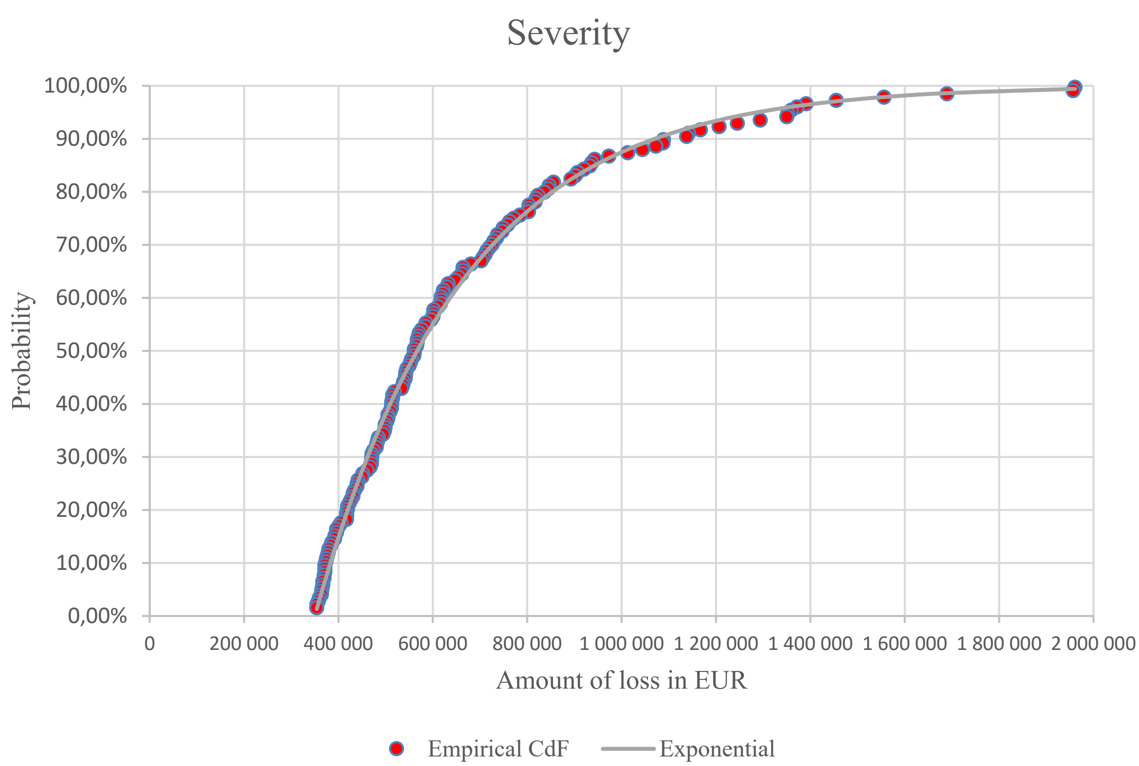


Chart 18 - Severity of losses

Empirical cdf will be replaced by the theoretical cdf of the shifted exponential distribution with the parameter $\hat{\lambda}$. See Appendix C for calculation of the parameter by maximum likelihood estimation.

By numerical application to the formula derived in the Appendix C, $\hat{\lambda}$ is found to be $3{,}19 * 10^{-6}$. The inverse of $\hat{\lambda}$ is equal 312 711 EUR which is the average of losses conditional on exceeding the threshold 350 000 EUR. As shifted exponential distribution fits the best our losses, Q-Q plot is done to confirm the idea. On the Chart 19, it is visible that our empirical distribution and shifted exponential distribution are similar for all quantiles. On the contrary, it is not the case for the other distributions.

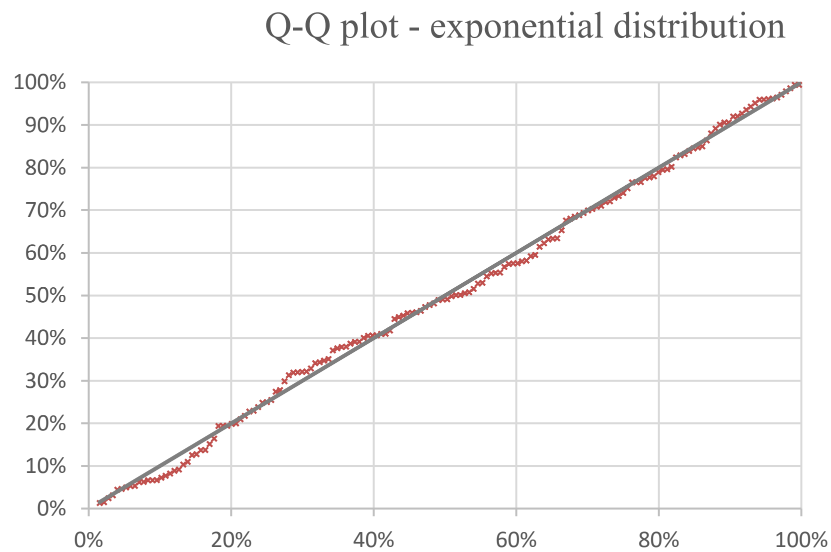


Chart 19 - Q-Q plot of exponential distribution

Finally, it has been found that severity of our losses has shifted exponential distribution with s = 350 000 EUR and with $\frac{1}{\lambda}$ equal to 312 711 EUR. Therefore, the threshold will be exceeded in average by 312 711 EUR.

Specification of the frequency of the loss is the next step. For the calculation, the Table 23 will be followed. The second column named Indexed GNPI summarizes the premium received by the insurance company. The IBNR is taken from previous steps. The number of claims exceeding the threshold after applying future inflation is shown in the fourth column. Because of the IBNR, numbers from the years 2018 and 2019 cannot be trusted and they need to be transformed. The three last columns measure the frequency with different adjustments. Relative frequency is the number of claims exceeding the threshold from the third column multiplied by the ratio of the inflated GNPI change from that year to the year 2020. Consequently, the years with higher exposure have also higher relative frequency of loss. Relative frequency after applying IBNR is different only for the years 2018 and 2019, where relative frequency is multiplied by the IBNR factor. The third approach adjusts two last years according to Bornhuetter-Ferguson method. By numerical application to the formula

$$BF\ frequency = relative\ frequency + BF\ factor * (1 - \frac{1}{IBNR})$$

where BF factor is the average of relative frequency after the IBNR from the year 2006 until 2017 equal to 15,76, BF frequency is obtained. It equals 11,39 for the year 2018 and 13,36 for 2019.The reinsurer should consider both the IBNR and the Bornhuetter-Ferguson method adjustments as they use all historical information for the prediction of the number of loss in undeveloped years. Finally, the average and the standard deviation are calculated from frequencies from the year 2006 up to the year 2019. The best prediction of the frequency in the next year is 15,28 losses.

Year	Indexed GNPI	IBNR	# xs threshold (after future inflation)	Relative frequency	Relative frequency after IBNR	BF frequency
2006	59 112 891	1,00	6	11,65	11,65	11,65
2007	63 456 491	1,00	8	14,47	14,47	14,47
2008	68 355 078	1,00	11	18,47	18,47	18,47
2009	70 678 380	1,00	15	24,35	24,35	24,35
2010	73 421 824	1,00	8	12,50	12,50	12,50
2011	75 556 600	1,00	17	25,82	25,82	25,82
2012	77 185 336	1,00	8	11,89	11,89	11,89
2013	80 299 381	1,00	15	21,44	21,44	21,44
2014	85 559 615	1,00	9	12,07	12,07	12,07
2015	92 461 594	1,00	9	11,17	11,17	11,17
2016	98 185 412	1,00	12	14,02	14,02	14,02
2017	101 930 473	1,00	10	11,26	11,26	11,26
2018	107 432 673	1,22	8	8,55	10,43	11,39
2019	112 986 534	2,44	4	4,06	9,92	13,36
2020	114 752 447	1,00	0	0,00	0,00	0,00
From	2006		Average	14,41	14,96	15,28
To	2019		St.Dev	6,08	5,34	5,10
To (BF)	2017					
BF Factor	15,76					

Table 23 - Frequency calculation

The Chart 20 illustrates the frequency. The points are relative frequencies in each year, the red line matches frequency by Bornhuetter–Ferguson method which is different only for two last years and the grey dashed line represents the prediction of frequency for the next year. Clearly, it is mean value of the historical frequencies with some adjustments.

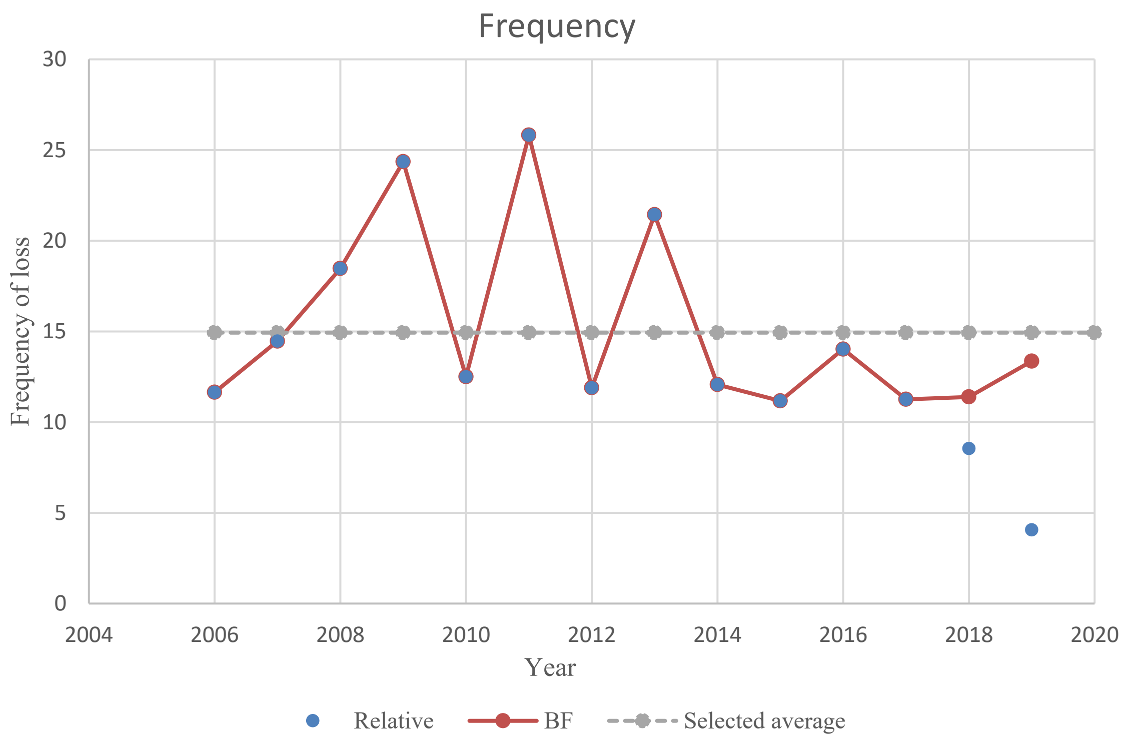


Chart 20 - Frequency

To summarize the frequency and severity modelling analysis, it has been found that the number of losses in next year should be in average 15,28 and if the loss happens, it should follow exponential distribution shifted by the threshold.

Loss Simulation

In this part of analysis, the Monte-Carlo simulation of losses will be done in order to calculate the mean recoveries of the reinsurance programme. 5 000 simulations are done from the shifted exponential distribution and for each trial, cumulative distribution function F(x) is randomly generated. The loss is calculated as

$$\frac{-\ln\left(1-F(x)\right)}{\hat{\lambda}} + 350\,000.$$

Then the recovery of reinsurance treaty is calculated as loss which happened minus the retention, in our case 500 000 EUR. This process can be seen in the Table 24 for first 5 simulations.

Simulation	F(x)	Loss	Recovery
1	0,87	987 785	487 785
2	0,88	1 004 824	507 996

Simulation	F(x)	Loss	Recovery
3	0,05	367 194	0
4	0,39	502 936	2 954
5	0,54	596 166	96 771

Table 24 - Monte Carlo simulation of losses

The Table 25 shows the moments of severity for the shifted exponential distribution. The mean is the $\hat{\lambda}$ parameter, in our case 312 711 EUR. Mean loss and mean recovery are the simple averages of the simulated loss and recovery from the Table 24 respectively. Standard deviation is standard deviation of recoveries from the Table 24.

Parameter mean	Mean loss	Mean recovery	St.Dev recovery
312 711	660 037	192 077	280 231

Table 25 - Moments of severity

The Table 26 represents the moments of frequency. The mean frequency was calculated in previous step and it is equal to 15,28 with standard deviation 5,10.

Mean	St.Dev
15,28	5,10

Table 26 - Moments of frequency

The distribution of the frequency is discrete. From the set of suitable discrete distributions, the poisson, negative binomial and binomial distributions have been chosen. The mean in our empirical distribution is lower than the variance therefore according to the Chapter 4.2 the negative binomial distribution is chosen, and the parameters of distribution are estimated by the method of moments. Let’s N be the frequency of loss, so its expected value is defined as

$$\mathbb{E}[N] = r\beta$$

where r and β are the parameters of the negative binomial distribution.

Variance of N is defined as

$$Var(N) = r\beta(1+\beta).$$

By solving for two parameters from those two equations, the results can be found in the Table 27.

Frequency	Parameter β	Parameter r	Mean Recovery (E[S])	St.Dev Recovery (St.Dev(S))
Negative Binomial	0,70	21,7	2 934 180	1 469 521

Table 27 - Parameters of negative binomial distribution and aggregate loss

Currently, the frequency and severity of one loss are known for one loss. However, it has to be generalized for the whole portfolio and the calculation of the characteristics of aggregate loss, noted as S follows according to the formulas presented in Chapter 4.2.

The expected value of recovery $\mathbb{E}[X]$ equals to 192 077 EUR according to the Table 25. After numerical application into the following formula

$$\mathbb{E}[S] = r\beta\mathbb{E}[X]$$

and

$$Var[S] = \mathbb{E}[N]Var(X) + Var(N)\mathbb{E}[X]^2$$

the expected value of the aggregate loss for the reinsurer equals 2 934 180 EUR and the standard deviation equals 1 469 521 EUR. Therefore, by severity and frequency modelling the reinsurer should be liable in next year for 2 934 180 EUR of aggregate loss which can be seen in the Table 27. The expected aggregate loss is the minimum sustainable price of the reinsurance treaty as the reinsurer will indemnify this amount of losses in next period. The result is slightly higher as the result of the burning costs. It is because the frequency and severity modelling relies on historical losses, but the tail of the severity distribution is modelled by the theoretical distribution. Severe losses were not considered in burning costs approach, but they are in frequency and severity modelling. With those two approaches, the expected recovery of a loss in next period is determined. The Chapter 6.4 will calculate the price by application of loading to the expected loss and expected risk measure, in our case expected standard deviation.

6.4 Calculation of Price with Stochastic Inflation

Price Simulation

After the simulation of losses and the calculation of the expected mean recovery for which the reinsurer will be liable in next year, loadings will be added to the pure mean recovery. The stochastic future inflation has not been used until now. The simple deterministic path of inflation was the input for the calculations represented by the future inflation index of 1,082.

The Table 28 shows the calculation of the mean recoveries and standard deviation by the method of burning costs and frequency and severity modelling using the stochastic inflation taken from the VAR model and transformed to the future inflation factor. The price is calculated for first five simulations out of 1 000. For each future inflation factor, the mean and standard deviation of burning costs are calculated as explained in the Chapter 4.2 and the third and the fourth columns indicates the mean and standard deviation respectively. The two last columns represent the average and standard deviation calculated by the severity and frequency approach using different future inflation factors. The average as well as standard deviation are higher for frequency and severity modelling than for burning costs method.

Scenario	FIF Factor	Burning Costs Average	Burning Costs St.Dev	Modelling Average (E[S])	Modelling St.Dev (S)
Forecast	1,082	2 748 284	838 891	2 934 180	1 469 521
1	1,069	2 672 987	823 004	2 910 923	1 465 608
2	1,049	2 552 797	798 758	2 698 204	1 370 678
3	1,082	2 753 855	840 066	2 893 120	1 465 801
4	1,081	2 743 707	837 927	2 972 504	1 503 879
5	1,069	2 669 628	822 310	2 926 069	1 499 309

Table 28 - Mean and standard deviation using stochastic inflation

As explained in the Chapter 4.2, the price of reinsurance cover is defined as

$$P = (\mu \ + R l_R) * (1 + l_c).$$

Usual loadings for the reinsurance business are taken such as 25% for risk aversion and 15% for management and administration costs. As all variable are known, the final price can be calculated as shown in Table 29. It contains the calculations for the first five scenarios.

The decision to rely on historical experience captured by the burning costs method as well as on the information from modelling is the 50% credibility factor for each method and the final price is done by combining those two approaches. The prices in the third and the fourth columns of the Table 29 are calculated from the means and standard deviations from the Table 28 and loadings specified above according to the formula for the price. Finally, the fifth column is combinations of both prices with 50% credibility for the each. Naturally, with increasing future inflation factor meaning higher uncertainty due to the inflation, the reinsurer will charge more for providing reinsurance coverage. The average final price of all simulations is 3 477 652 EUR. Therefore, this should be the price of the treaty which takes into consideration profit loadings and different inflation scenarios.

The price with deterministic inflation equals 3 466 517 EUR. The prices change substantially according to the different inflation scenarios which means that the effect of the inflation uncertainty on reinsurance price matters. Consequently, it is important for the reinsurer to be protected against inflation risk and uncertainty caused by the future inflation. In next section, this effect will be quantified and explained by the measures of risks.

Scenario	FIF Factor	Price (Burning Cost)	Price (Stochastic modelling)	Selected Price
Forecast	1,082	3 305 235	3 627 799	3 466 517
1	1,069	3 215 904	3 600 379	3 408 141
2	1,049	3 073 503	3 339 376	3 206 439
3	1,082	3 311 845	3 579 938	3 445 892
4	1,081	3 299 806	3 677 798	3 488 802
5	1,069	3 211 921	3 623 610	3 417 766

Table 29 - Price calculation with profit loadings

Results

In this part, the effect of inflation uncertainty on different risk measures will be presented. After 1 000 price simulations according to different simulated inflation scenarios, risk measures were calculated. We are interested in 99,50% value at risk as this value is used in Solvency II calculations. According to the Table 30, the VaR of price equals 3 775 667 EUR, which represents an 8,90% increase in comparison with central scenario ‘s price. This situation is estimated to come once every 200 years and these losses are included in the calculation of the Solvency II. Therefore, loading the premium by an additional 8,90% corresponds to be hedged for the worst scenario within 200 years. The standard deviation is 111 284 EUR, which indicates 3,20% growth.

Therefore, if the reinsurer charges additional 3,20% of the price central estimate, he is protected against inflation risk, particularly against severe losses coming once per 200 years. The tail value at risk equals 3 813 791 EUR and the impact is measured to be 10,00%.

Risk measure	Value	Impact
VaR 99,5%	3 775 667	8,9%
St.Dev	111 284	3,2%
TVaR 99,5%	3 813 791	10,0%

Table 30 - Risk measures

By ranking final prices from the price simulation according to different simulated inflation scenarios from the highest to the lowest, the worst scenarios can be seen. The price asked by the reinsurer should be 3 856 717 EUR to be protected against inflation risk for the highest future inflation factor. The first five scenarios are illustrated in the Table 31 and their impact is calculated in the fourth column. It represents the relative growth of the price with stochastic inflation to the price with deterministic inflation and in the highest case, it can reach 11,3%.

Rank	Selected Price	CdF	Impact	Rounded
1	3 856 717	100,0%	11,26%	11,3%
2	3 821 255	99,85%	10,23%	10,2%
3	3 816 725	99,75%	10,10%	10,1%
4	3 790 201	99,65%	9,34%	9,3%
5	3 784 059	99,55%	9,16%	9,2%

Table 31 - Impact of stochastic inflation

The impact is rounded to one decimal place and its histogram is shown on the Chart 21. It is quite platykurtic with heavy tails and positive skewness. The fat right tail cannot be neglected as it represents potential risk for the reinsurer.

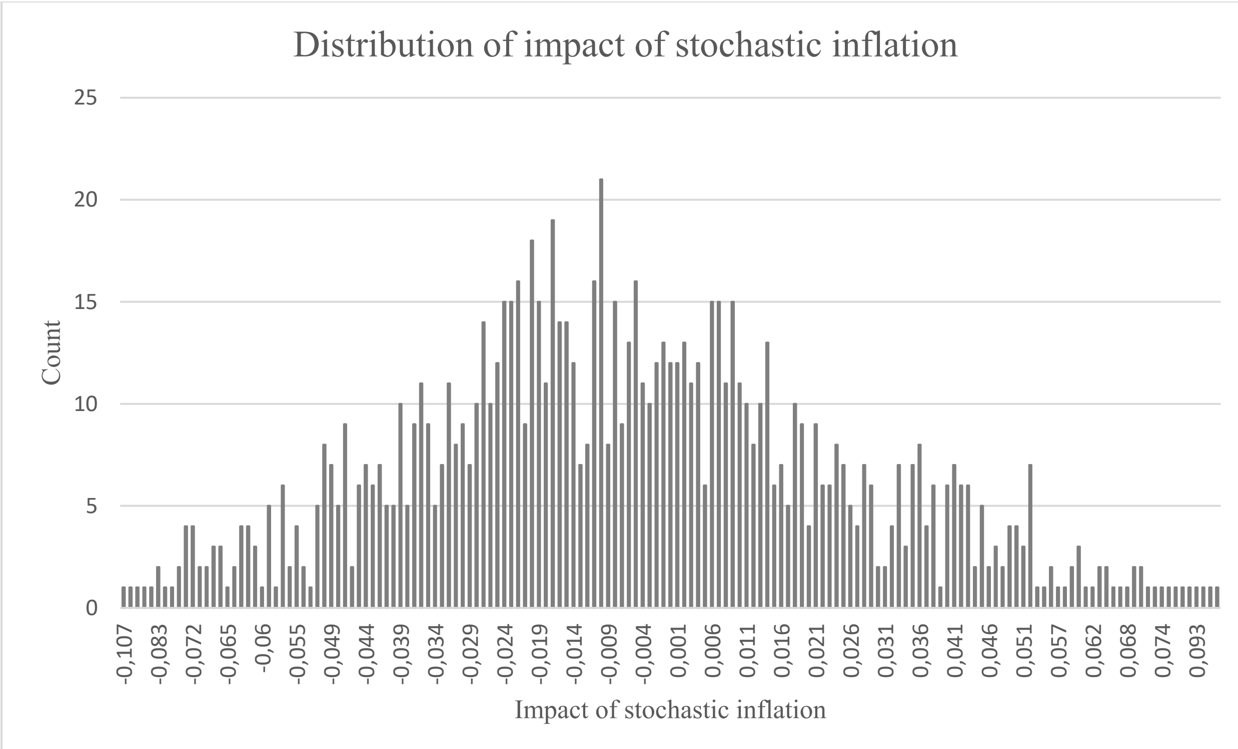


Chart 21 - Distribution of impact of stochastic inflation

As the reinsurer faces an additional risk which has been specified in the analysis, according to our results, he should ask for the additional loading. The amount depends on the risk appetite of the reinsurer, but he could ask for one standard deviation which is 3,20% of central price as protection against the inflation. This loading represents a hedging against inflation uncertainty. The inflation uncertainty captured by the stochastic simulations matters and has significant impact on the reinsurance prices. After quantification of this uncertainty, the incorporation of additional loading could be the solution for mitigation of inflation risk, especially for long-tail lines of business. This shall be eventually considered by risk managers of the reinsurance companies when monitoring and managing the risks. The inflation risk should not be neglected and maybe reinsurers could consider its incorporation into underwriting guidelines setting the underwriting strategy and loadings.

Finally, this analysis introduced the stochastic modelling of the future inflation and its effect on the reinsurance pricing, mainly on its measures of risks and possible way of the hedging against the inflation risk that threatens the reinsurers.

6.5 Discussion and Criticism of Results

Non-proportional Effect of the Retention

One of the most essential remarks of the empirical part is the existence of non-proportional effect caused by the application of retention for recovery calculation. The impact of stochastic inflation is larger on losses close to the retention of the treaty in comparison with large losses. There is a positive correlation between the future inflation factor and the recovery. However, the relative growth is higher for smaller losses. When future inflation factor increases and smaller loss around 600 000 EUR happens, the relative growth between the recovery with lower future inflation factor and recovery with higher inflation factor is large. On the contrary, for losses far from the retention, as for example 1 000 000 EUR, the relative growth between two recoveries with different future inflation factor is smaller. Indeed, the impact of future inflation factor depends on the loss and the retention of the treaty. To achieve smaller impact by higher future inflation, the possible hedging for the reinsurance company is to decrease the retention and thus decrease part of the loss that the insurer will retain. However, decreasing retention will increase the price for that reinsurance coverage as there will be more losses higher than the retention. The agreement of the reinsurer and the insurer should find a balance for proper retention determination.

This non-proportional effect can be seen also if the retention was increased to 1 000 000 EUR. The impact on the risk measures is presented in the Table 32. It is evident that an increase in the retention leads to the bigger impact on risk measures. It is because recoveries after the application of the retention are smaller and thus the effect of stochastic inflation is more significant for the smaller losses.

Risk measure	Value	Impact
VaR 99,5%	973 851	14,3%
St.Dev	43 962	5,2%
TVaR 99,5%	989 562	16,1%

Table 32 - Risk measures for higher retention

For the confirmation of this idea, the retention has been changed to 350 000 EUR and the results are presented in the Table 33. With smaller retention, the recoveries are higher, and the non-proportional effect of the retention causes smaller effect of stochasticity of inflation on the measures of risk. Therefore, if the reinsurer charges additional 2,4% of the price central estimate, he is protected against inflation risk, particularly against deviation of inflation coming once per 200 years.

Risk measure	Value	Impact
VaR 99,5%	5 805 115	3,3%
St.Dev	137 003	2,4%
TVaR 99,5%	5 921 384	5,4%

Table 33 - Risk measures for lower retention

The sensitivity analysis of increasing the retention capturing the non-proportional effect of the retention is represented by the Table 34. The objective of this calculation is generalization of the treaty type and investigation of the effect of change in the retention on the price. After an increase in the retention up 1 500 000 EUR subject to the loss history available, the mean loss as well as standard deviation decrease.

Threshold	245 000	350 000	525 000	700 000	875 000	1 050 000
Limit	36 000 000	36 000 000	39 250 000	39 000 000	38 750 000	38 500 000
Retention	350 000	500 000	750 000	1 000 000	1 250 000	1 500 000
Minimum	5 029 829	3 096 704	1 457 385	723 720	322 013	120 470
25%	5 328 724	3 374 552	1 604 754	804 179	368 133	141 286
Median	5 409 699	3 438 936	1 652 027	828 397	388 955	151 836
Average	5 418 903	3 445 807	1 655 616	833 128	390 532	151 585
St.Dev	137 003	111 284	73 977	43 962	29 632	13 414
75%	5 505 078	3 512 288	1 704 494	859 187	409 031	161 267
Maximum	5 948 094	3 856 717	1 888 435	999 233	481 336	206 498
Range	918 265	760 013	431 050	275 513	159 323	86 028

Table 34 - Sensitivity analysis

The regression between the retention and the coefficient of variation defined as the ratio of standard deviation over the mean is done. The extent of variability of data in a chosen sample in relation to the mean is captured on the Chart 22. In this case, the coefficient of variation indicates the additional price relative to the mean price asked for the inflation risk by the reinsurer. The exponential relation between the retention and coefficient of variation confirms the non-proportional effect of the retention. Additional 3,2% of the mean price is enough for current retention of the treaty but if the retention increases to the 750 000 EUR due to the occurrence of a high loss, the reinsurer shall request 4,5% of the mean price as a compensation for the inflation risk. Therefore, the treaties with high retention are more sensible to future inflation fluctuations. As there are only three historical losses exceeding 1 500 000 EUR, our sensitivity analysis is limited. However, it is assumed that the future inflation uncertainty could have even higher impact on the reinsurance prices for higher retention.

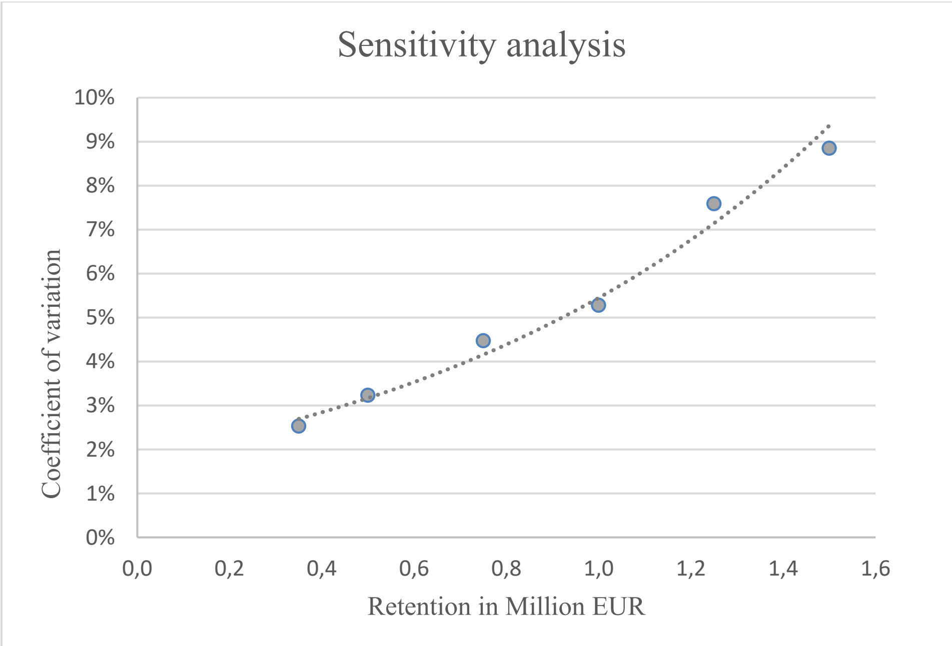


Chart 22 - Sensitivity regression

Comparing the mean recoveries with and without inclusion of the stochastic inflation, its effect on the reinsurance pricing seems rather small due to the competitiveness of the reinsurance business. The reinsurer cannot increase its price a lot in order not to lose his clients. However, it is interesting to isolate the part of risk attributable to inflation for the risk management purposes. The main idea of the analysis is that the effect can be neglected in average but for treaties with high retention, the stochasticity and the different inflation scenarios should be incorporated into pricing. We can distinguish two cases of layers. The first case is so-called working layer which means that the reinsurer assumes a substantial amount of losses to happen to this layer. Often those layers have low retention and the impact of stochasticity of inflation is not significant. The second case are the layers with high retention that are hit rarely by losses protecting the insurer in case of big catastrophic losses once a long time. In this case, losses indexed by the future inflation can reach the retention that has not been exceeded before. It means that impact of the stochasticity of the future inflation is more important and it confirms the discovered non-proportional effect of retention. The treaty with high retention agreed especially in uncertain inflation environment can have harmful consequences on the profit of the reinsurer without considering the effect of stochastic inflation.

Discussion on Validity of Assumptions

Although the empirical part of the thesis tried to describe the reality in the best way, it has partially relied on the assumption that the future can be explained by the past. From all past losses that happened, the payment pattern has been constructed. According to this rule, it has been stated how the losses will be paid in the future and the assumption that each open loss or each loss that will happen in next years will follow the same pattern has been made. Nevertheless, some claims are usually paid sooner and more complicated losses needing the involvement of the court can last much longer. For the simplification, it has been decided to use the average payment pattern. However, for better simulation of the possible future development, more payment pattern could be used or even simulated.

Another assumption that was taken into consideration was that the growth in the GNPI will lead to the same growth of frequency of claims. This general rule is correct in the average. Nevertheless, some new business has higher potential frequency of the loss than the other. The risk associated with military group working in countries with war conflict is not comparable with that of administration workers of military group. The insurance company provides sometimes the information about the content of their business, but this is not the rule, so the reinsurer is obliged to assume that all business has the same probability to be hit by the loss.

Furthermore, we confided in the general rule about the choice of threshold of claims above which the analysis is done. According to best practice, it should be between 60% and 70% of retention that has been followed and sufficient number of claims was respected to be able to carry out the reliable analysis. However, there is the sensitivity of results when changing the threshold.

Indexation Clause

As inflation risk can have harmful consequences on future liabilities of the reinsurer, there is possible solution to mitigate the increase of the future inflation. An indexation clause is a clause present in the majority third part liability reinsurance treaties. It distributes the effect of inflation on increased losses, which tends to fall on the reinsurer, between the insurer and the reinsurer. Majority of European companies use widely the clause, but it is still not the case in the United States. The indexation clause ensures the redistribution of the growth of the losses due to the inflation by the adjustment of the retention and the capacity of the treaty in accordance with the inflation. Consequently, the liabilities of the reinsurer rise but also those of the insurer because of the increase of the retention. Currently, indexation clause is not used in personal accident treaties. However, as this thesis showed, the importance of inflation uncertainty on the reinsurer’s profitability is significant. Therefore, indexation clause should be eventually introduced also to this line of business and maintained in all long-tail lines of business.

Medical inflation measuring the increase in costs of medical care can be also the contributor to the additional uncertainty of the reinsurer. Particularly, this type of inflation index should be used for health reinsurance treaties. As the empirical analysis was done on personal accident contract, medical inflation could reinforce the effect of rising claims but should not replace the general CPI. For better accuracy of the results, the inflation index used could contain the CPI as well as medical inflation. Unfortunately, due to the unavailability of medical inflation data appropriate for the analysis, the total CPI index was taken as measure of the inflation in the economy.

Possible Extension and Generalization

Our findings that additional loadings should be introduced in order to incorporate the inflation risk into pricing could be generalized to other contracts. By studying more contracts, the rule could be generalized and then used in the reinsurance industry, especially in periods with high inflation uncertainty. Personal accident line of business is long-tail line, but the development of claims is not as slow as in third part liabilities treaties. It is assumed that the effect would be even higher for motor third part liabilities contracts where payment pattern lasts for 15-20 years in average. Therefore, introducing the stochasticity of inflation into such reinsurance pricing could lead to higher uncertainty and the measures of risk would change even more. However, the modelling of stochastic inflation would be more difficult as forecasts for 15-20 years cannot be believed a lot. Nevertheless, it would clarify the approach of necessity of inclusion of new loadings based on the line of business.

The problematic issue in the stochastic inflation is the use of the future inflation factor. For the simplicity, the future inflation factor that supposes the amount by which losses are inflated to the future was used. However, this can be slightly different in the reality and the future inflation factor can be different in the different development years. Secondly, the future inflation factor has been calculated with the use of inflation path from 2021 until 2031. We admit that VAR models have their limits and forecasts for 10 years can be less reliable. The interesting remark in the inflation path is the high volatility. Volatile inflation trajectory would represent the situation in which economic cycle is very volatile and significant what is not the case of the Austrian economy. Therefore, the use of models able to the correct correlation between inflation rates in two successive years could be more suitable.

7. Conclusion

In this thesis, the effect of the macroeconomic environment on the reinsurance business is studied. In particular, the thesis focuses on the effect of inflation on the reinsurance pricing. Inflation increases the future obligations of the reinsurance company as it receives the reinsurance premium in the beginning of the period and its liabilities in form of insurance claims indemnification appear later. This thesis introduces the stochastic modelling of the future inflation and its effect on the reinsurance pricing, mainly on its measures of risks and possible way of the hedging against the inflation risk that threatens the reinsurers. The personal accident reinsurance treaty protecting the portfolio of an Austrian insurance company in case of accidental death and permanent disability of its policyholders is studied in the empirical part. Its claims may develop during the decades therefore the impact of future inflation was incorporated into the price estimation. Since price is defined as discounted future cash-flows of the reinsurer, future losses representing its cash-flows have been adjusted by the inflation forecast in order to represent the future liabilities correctly.

To the best of our knowledge, there is so far no academic literature devoted to study the effect of stochastic modelling of inflation on reinsurance pricing and its measures of risks. This thesis aims at bridging this gap by evaluating how the prices would change in case the inflation volatility is included. In comparison with previous researches, the flat deterministic inflation forecast was replaced by the stochastic modelling. Considering multiple scenarios enables the reinsurer to be prepared also for the extreme years. The adequate reserving of losses and sufficient capital buffer to maintain the solvency can mitigate the impact of extreme years hit by the severe losses. The introduction of the stochastic modelling of inflation has proven the existence of the effect on the risk measures, especially value at risk, standard deviation and tail value at risk. Those measures indicating the possible loss for the reinsurer increased by 8,9%, 3,2% and 10,0% respectively for 99,5% percentile. Therefore, if the reinsurer charges this additional amount of the price central estimate, he is protected against inflation risk, particularly against inflation deviation coming once per 200 years. The thesis specifies the part of the risk attributable to the inflation uncertainty and its implementation could improve risk management of the reinsurance companies. Although on average the inflation risk impact on the premium level is not significative, the risk measures of the premium necessary to cover inflation deviations has been found to be more significant, especially for the contracts with high retention. This means that for those type of reinsurance contracts, the stochasticity of inflation should be considered, especially in the uncertain macroeconomic environment in order to ensure the sustainable business and protection of the reinsurer’ s profit.

The thesis found that the additional loading representing the mitigation of the inflation risk could be included to the final price to improve its accuracy. Since the losses in the different lines of business are closed earlier or later, the lines of business are exposed differently to the risk of future inflation. Therefore, the loadings should be related mainly to third part liability or personal accident treaties where the development of the insurance claim takes long time and the treaties are subject to higher degree of inflation uncertainty. However, the introduction of the additional loadings can be difficult to happen due to the competitiveness of the reinsurance business. Nevertheless, it is important for the reinsurer and for reinsurance risk managers to know the part of risk attributable to the inflation in order to be able to mitigate it. The main result of the analysis is that the effect can be neglected in average but for treaties with high retention, the stochasticity of inflation and the different inflation scenarios should be considered and eventually incorporated into the pricing.

Using different treaties from various lines of business, we may come to different patterns in results. Therefore, it might be interesting to study more treaties in order to generalize the results and examine the different impact of stochasticity of inflation on their measures of risks. Moreover, the different types of inflation could be used as for example partial incorporation of medical inflation forecast in the personal accident and the health reinsurance contracts. All these possible amendments of the analysis are left for future research.

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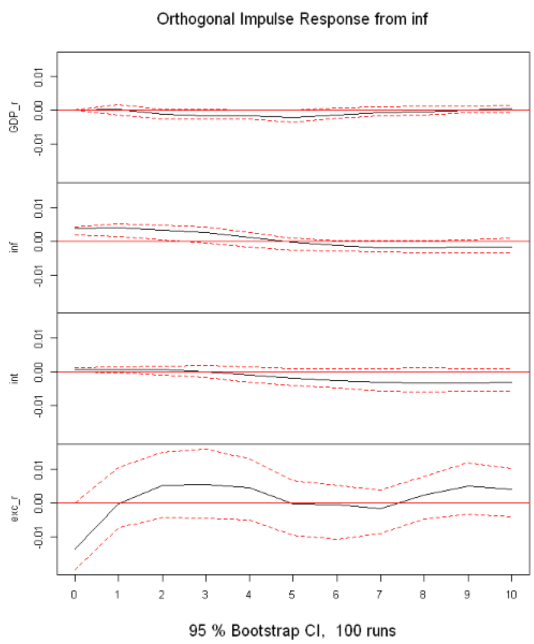
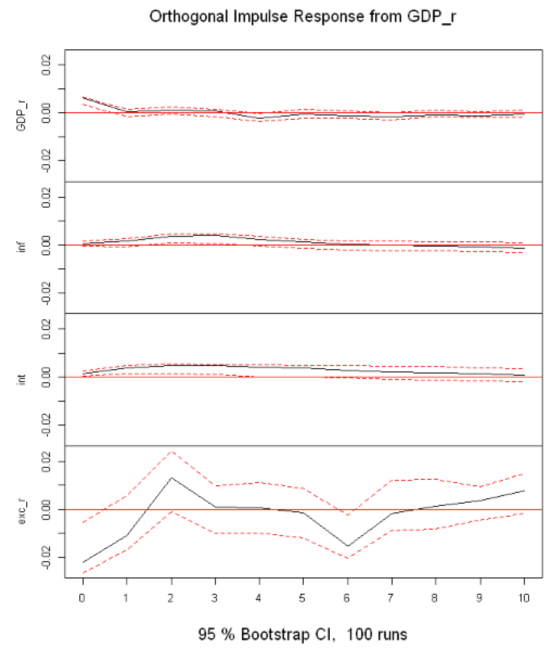
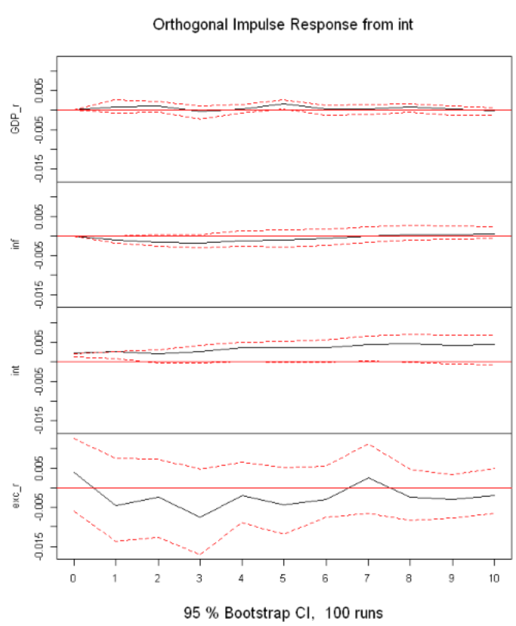
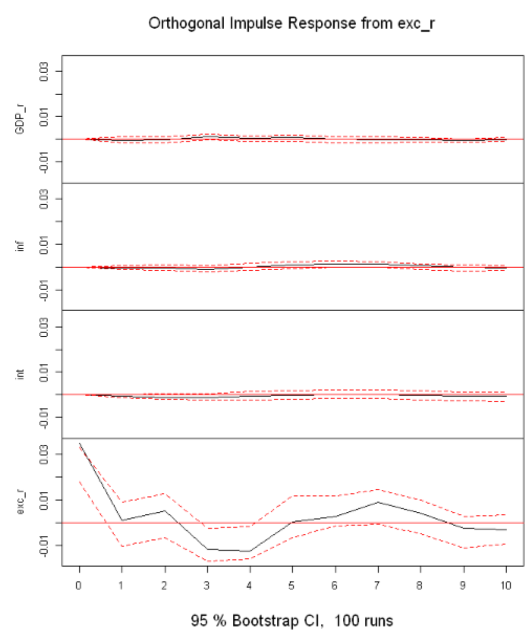
Appendix A: Impulse Responses









Appendix B: Indexation of losses

We note:

- $T_{j,t}$ (respectively $P_{j,t}$, $R_{j,t}$) non-inflated total incurred (respectively cumulative paid amount and reserve) of a loss incurred in year j at the end of development year t
- $T^i_{j,t}$ (respectively $P^i_{j,t}, R^i_{j,t}$) inflated with past inflation total incurred (respectively paid amount and reserve) of loss incurred in year j at development year t
- I_t inflation index at the end of year t

$\forall t \geq j, T_{j,t} = P_{j,t} + R_{j,t}$ and $T^i_{j,t} = P^i_{j,t} + R^i_{j,t}$

The non-inflated amount paid during year 1 if $P_{j,j}$. The amount paid during development year $t > j$ is $P_{j,t} - P_{j,t-1}$

$\forall t \geq j$ for the analysis year T the inflated and non-inflated total incurred amount of a loss incurred in year j are linked by the below formulas:

$T^i_{j,t} = P^i_{j,t} + R^i_{j,t}$ where $P^i_{j,t} = P_{j,j} \frac{I_T}{I_j} + \sum_{k>j}^{t} (P_{j,k} - P_{j,k-1}) \frac{I_T}{I_k}$ and $R^i_{j,t} = R_{j,t} \frac{I_T}{I_t}$

Appendix C: Calculation of the parameter of the shifted exponential distribution

The cumulative distribution function of the exponential distribution X of parameter $\lambda > 0$ is defined as:

$$\forall x \geq 0, F_X(x) = 1 - e^{-\lambda x}$$

The exponential distributions mean, and variance are given by the below formulas:

- $\mathbb{E}[X] = \frac{1}{\lambda}$
- $Var[X] = \frac{1}{\lambda^2}$

From the exponential distribution, the shifted exponential X^sdistribution as $X^s \sim X - s$ is defined. Its cumulative distribution function is given by:

$$\forall s \geq 0, \forall x > s, F_{X^s}(x) = F_X(x - s)$$

In our practical case, s is fixed and it corresponds to the analysis threshold of 350 000 EUR.

$$\forall x > s, F_{X^s}(x) = 1 - e^{-\lambda(x-s)} \text{ and } f_{X^s}(x) = \frac{dF_{X^S}}{dx}(x) = \lambda e^{-\lambda(x-s)}$$

For the parametrization of the distribution, the parameter λ is calculated so that the distribution fits the data best using the maximum log-likelihood.

Let's note the n large losses exceeding the threshold s of EUR 350 000 $x_1, x_2, \dots, x_n$.

For the shifted exponential distribution, $\hat{\lambda}$ the maximum likelihood estimator of λ is given by:

$$\hat{\lambda} = argmax_{\lambda>0} \hat{\mathcal{L}}_n(\lambda, x)$$

$$\text{where } \mathcal{L}_n(\lambda, x) = \prod_{i=1}^{n} f_{X^s}(x_i)$$

For the shifted exponential distribution, it is convenient to work with the natural logarithm of the likelihood function. The log-likelihood is defined as: $\ell(\lambda, x) = \ln \mathcal{L}(\lambda, x)$.

Since the logarithm function is monotonic the parameter λ of $\ell_n(\lambda, x)$and $\mathcal{L}_n(\lambda, x)$ are identical.

$$\ell_n(\lambda, x) = \ln \mathcal{L}_n(\lambda, x) = \ln \prod_{i=1}^{n} f_{X^s}(x_i) = \sum_{i=1}^{n} \ln f_{X^s}(x_i) = \sum_{i=1}^{n} \ln \lambda e^{-\lambda(x_i - s)}$$

$$= n \ln \lambda - \lambda \sum_{i=1}^{n} x_i + n\lambda s = n \ln \lambda - \lambda n \frac{1}{n} \sum_{i=1}^{n} x_i + n\lambda s = n \ln \lambda - \lambda n \bar{x} + n\lambda s$$

where $\bar{x}$ is the sample mean $\frac{1}{n} \sum_{i=1}^{n} x_i$.

$$\frac{d\ell_n(\lambda, x)}{d\lambda} = \frac{n}{\lambda} - n\bar{x} + ns$$

Thus $\hat{\lambda}$ is such as $\frac{1}{\hat{\lambda}} - (\bar{x} - s) = 0$ thus $\hat{\lambda} = \frac{1}{(\bar{x} - s)}$