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On Extended Mimetic Gravity

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I declare that I carried out this master thesis independently, and only with the cited sources, literature and other professional sources.

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Title: On Extended Mimetic Gravity

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Abstract: We consider a novel extension of the recently proposed mimetic gravity. The latter is a scalar-tensor theory which is able to describe dark matter on cosmological scales. Moreover, this theory can be considered as a low energy limit of the projectable Horava-Lifshitz gravity. The proposed novel extension directly couples gradients of the mimetic scalar field to the curvature tensor. These couplings introduce into the energy momentum tensor an anisotropic stress which is non-vanishing even at the first order in perturbations around a cosmological background. Further we show that such terms modify the formula for the speed of sound of scalar perturbations and even more importantly change the speed of propagation for the gravitational waves. The appearance of the anisotropic stress and the consequent nontrivial speed of propagation of the gravity waves are new phenomena which were not present in the previously studied mimetic models. Furthermore, we demonstrate that the effective Newton's gravitational constant in the background Friedmann equations is shifted in the presence of the novel couplings of the mimetic scalar field. We calculate the quadratic action for scalar and tensor perturbations and briefly discuss possible instabilities. Finally we consider the current observational bounds on the model.

Keywords: mimetic gravity, modified gravity, gravitational waves, tensor-scalar theory

Název práce: Rozšířená mimetická gravitace

Autor: Pavel Jiroušek

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Abstrakt:

V této práci zkoumáme dvě nová rozšíření nedávno objevené mimetické gravitace. Mimetická gravitace je tensorová-skalární teorie schopná popisu temné hmoty na kosmologických škálách. Tato teorie lze být považovánu za nízkoenergetickou limitu projektovatelné Hařavovy-Lifshitzovy gravitace. Tato nová rozšíření představují přímou vazbu gradientu mimetického pole na tenzor křivosti. Tyto vazby jsou zdrojem anizotropního napětí v tenzoru energie a hybnosti, který je nenulový i v prvním řádu perturbací na kosmologickém pozadí. Dále ukážeme, že nově zavedené členy ovlivní rychlost zvuku skalárních kosmologických perturbací, ale především ovlivní i rychlost šíření gravitačních vln. Anizotropní napětí a netriviální rychlost propagace gravitačních vln jsou nové vlastnosti, které se v předchozích modelech mimetické hmoty nevyskytovaly. Dále demonstrujeme, že přítomnost mimetické hmoty změní effektivní Newtonovu konstantu na úrovni Friedmanových rovnic pro kosmologické pozadí. Odvodíme kvadratickou akci pro skalární i tenzorové perturbace a krátce diskutujeme možné nestability. Též diskutujeme omezení našeho modelu z pozorování.

Klíčová slova: mimetická gravitace, modifikovaná gravitace, gravitační vlny, tensorová-skalární teorie

Content

Introduction

Despite the fact that Einstein's general relativity has been a tremendous success, it has been clear for some time that it cannot be the final theory of gravity. In the face of extremely high energies and matter densities that were present in the early universe, one cannot ignore the quantum nature of the world, and thus is in need of a unified theory of quantum gravity. General relativity breaks down at Planck scale, and therefore it is only natural that we seek out alternatives to general relativity as the theory of gravity. While this is fairly obvious, probing such high energies does not seem to be in our reach in near future. However, there is another strong evidence that puts Einstein's theory in question. To describe the universe, general relativity requires a substantial portion (95%) of its matter content to be made of the so called dark sector, which consists of dark energy (69%) and dark matter (26%)[1]. While we cannot dismiss the existence of such substances, it leads many to speculate whether this is a signal that general relativity has reached its limits. Note that the behavior in question concerns low energies and extremely large scales corresponding to billions of light-years as opposed to the high energetic limit mentioned above. These hints gave rise to a plethora of modified theories of gravity that are able to produce the effects similar to dark energy and dark matter. A big motivation for such studies nowadays is the advancement in observational cosmology which allows us to test such theories or constrain possible parameters.

A large class of modified theories of gravity consists of models where general relativity is supplemented by additional degrees of freedom that are usually made of scalar fields non-trivially coupled to standard gravity. Such theories are called tensor-scalar theories (TeS) [2] and mimetic gravity [3] falls within such category. Even though scalars are relatively rare in nature so far (first observed scalar particle is the Higgs boson discovered only few years ago [4]), there are numerous reasons why their addition to the gravitational sector is desirable. For example, scalar fields satisfy the cosmological principle and can have homogeneous and isotropic classical backgrounds relevant to cosmology. Furthermore, such fields can form condensates, which is desirable for a classical configuration applicable to the very large scales. Scalar fields also provide a natural framework for the description of accelerated expansion of the Universe as it was realized through the study of inflation [5], [6] and dark energy (the quintessence) [7], [8]. In quintessence models scalar fields showed much promise as they offer a possible answer to the coincidence problem [8].

Famous progenitor to TeS theories is the theory of Brans and Dicke [9], which sought to reconcile Mach's principle with general relativity by adding a scalar field in a non-trivial way. One of the required properties of such theory is the varying Newton's gravitational constant. At first this seems as a downside; however, such behavior was hypothesized by Dirac in the last century following his observation of the apparent coincidence of the ratio of the electric and gravitational force between an electron and a proton and the age of the universe (multiplied by the speed of light) to the diameter of an electron [10]. This, as it turns out, is not a very special feature because many TeS theories produce some effective modifications of gravitational constant. Further motivation for scalars in gravity comes from the realm of string theory, where gravitons are partnered up with a scalar field called the dilaton [11]. Unfortunately, the effective 4-dimensional theory predicted by the string theory is not yet known in detail.

An interesting possibility, which makes the landscape of various TeS much richer, is to add HD (higher derivative) terms into the Lagrangian. Such extensions often result in addition of a new degree of freedom which is typically unstable as is described in the Ostrogradsky's theorem [12]. The need to avoid these instabilities leads to severe limitations of the space of theories that can be considered. However, there are special cases where this lethal fate can be avoided. Well known examples include the Galileon models in Minkowski space [13] and their covariant extension [14], which have later been expanded by additional Lagrangians [15] to give a set of TeS theories that give rise to second order equations of motion in both the scalar and the gravitational sector. After that, it was proven [16] that these extended Galileons are equivalent to the most general theories with second order equations of motion introduced by Horndeski [17]. Later it has been realized that there are other options that bypass the Ostrogradsky instability, namely degenerate or constrained Lagrangians. Degeneracies in physics naturally arise from gauge symmetries. A system with such symmetry reduces to the constrained case after gauge fixing. Mimetic gravity falls within such category.

The original idea of mimetic matter was proposed in [3] and since then it has attracted considerable attention we e.g. [18], [19], [20], [21], [22]. The name originates from the fact that it was able to mimic dark matter by modifying standard gravity. Later it has been shown that such model can be reformulated as irrotational pressureless fluid [23], [21], [22]. Another important feature of the mimetic gravity is that it allows for a manifestly Weyl-invariant formulation. Moreover, it turned out [24] mimetic gravity is closely related to the low energy limit of a branch of the projectable Horava-Lifshitz gravity [25], [26]. The latter is a power counting renormalizable but not Lorentz-invariant theory of gravity. Furthermore a modification of such model (in general any model of irrotational pressureless fluid) allows us to mimic any type of matter on the background level [27]. Such versatility of course comes with a price and therefore mimetic gravity has some unfixed parameters and in principle an unfixed shape of its potential term. However, the present parameters correspond to measurable quantities, and thus the theory can be constrained. Inclusion of higher derivative terms in the action allowed mimetic matter to obtain non-zero sound speed in the background of FRW universe as was shown in [18]. Our work builds upon these ideas and it probes similar possible modifications that contain non-minimal couplings to curvature, in hopes that new effects will reveal interesting possibilities for the future of this model. In [19] it was explained that the higher derivatives (HD) would naturally appear in a gradient expansion formalism. Indeed, original mimetic matter corresponds to a pressureless perfect fluid whereas the HD correspond to departures from this simple perfect fluid picture. Thus mimetic matter with HD can be considered as imperfect DM [19] and the latter can be phenomenologically interesting [28]. Our setup considers further departures from the perfect fluid DM description. We achieve this by introducing HD operators of a more general structure and allow for a direct coupling of the mimetic field to curvature. The latter coupling would break the weak equivalence principle, but this breakdown was already present on the level of EoM in the previous constructions.

The first chapter of this thesis aims to lay out some very basic concepts of standard cosmology like the Friedmann models as well as to give a review of the machinery for analyzing cosmological perturbations. A short review of key aspects of dark matter is also included. Much of the notation conventions are developed throughout this chapter.

In the second chapter, we give an introduction to concepts like inflation and quintessence to give the reader some context to extensions of the standard model of cosmology.

Third chapter is fully fledged to the actual subject of mimetic gravity (or mimetic matter). In this chapter we review much of the major features and properties of the model based on the latest research prior to this thesis.

In the fourth and final chapter, we introduce our modifications and derive their effects on the behavior of mimetic matter and gravity itself. We also discuss how these modifications expand the preceding model. Much of the work behind this thesis lies in lengthy calculations of technical nature. For that reason, we have decided not to include much of the details of the calculations in the final chapter and instead move them to the appendixes.

We work in the mostly negative signature $(+,-,-,-)$. Space-time indeces are denoted by lowercase Greek letters while space indeces are lowercase Latin. We follow Einstein's summation convention. Note that we will often sum over two covariant Latin indeces as it is more convenient to work with only covariant objects. To simplify equations we work in reduced Planck units in which

$$c = 8\pi G = \hbar = 1. \tag{1}$$

1. Standard model of cosmology

Physical cosmology studies the universe at its largest scales. It aims to describe its dynamics, origin and its ultimate faith. Contemporary cosmology lies on the shoulders of Einstein's general relativity that allowed a dynamic description of the universe. Building upon this framework Friedmann discovered an expanding solution of Einstein's equations that was later shown to be unique by Robertson and Walker. This solution was then correctly connected with the observations of Hubble by Geoger Lemaître. This metric is now called FLRW metric (or FRW model) in their honor.

FRW model has become the backbone of cosmology as it accurately describes the coarse grained structure of the universe. Another ingredient to cosmology is thermodynamics. Combined with the standard model of particle physics, it has been able to provide accurate predictions of the chemical elements found in the universe based on the hot early universe that is predicted by the FRW models. The last major constituent is the study of cosmic perturbations, which studies the growth of inhomogenities and thus provides a framework to address the issue of formation of galaxies and larger objects (clusters, supercluster etc.).

Having all these tools, it has become clear that our universe does not really fit the description. Not unless one adds additional types of matter and energy now called dark matter [29],[30] and dark energy [31]. One of the first signals toward their existence lies with the discovery of inconsistencies of the galactic motion [32]. To clear this discrepancy scientists have postulated the existence of dark matter to make up for it. At the end of the 20th century a major breakthrough occurred when it was discovered that the universe's expansion is accelerating, which called for dark energy [31]. While it is unclear where this dark sector originates, it was fairly easy to incorporate its basic features into the existing models. Namely a nonzero cosmological constant was posed and a non-specified dust-like matter was added into the theory. Together with the above ingredients, this constitutes Λ-CDM model (Λ - cold dark matter) often refered to as the standard model of cosmology.

In the following sections, we will introduce the FRW models and the subject of cosmological perturbations since they are relevant to our research. Our introduction is mainly based on the textbook of Mukhanov [6] supplemented by other sources that are cited throughout the chapter. For a more detailed introduction to cosmology please refer to this book or other textbooks [33], [34].

1.1 FRW metric

FRW models describe a universe which is completely homogeneous and isotropic. Moreover it contains a privileged class of time-like world-lines (fundamental observers), that define an integrable foliation of the space-time into space-like hypersurfaces that are everywhere orthogonal to the world-lines. In accord with the assumption of isotropy and homogeneity, these hypersurfaces are of constant curvature.

Construction of a metric with the above properties is fairly easy. Let us go through the steps. There exists a time-like coordinate t which measures the proper time of the fundamental observers. We can always shift the time coordinate so that the spatial slices are hypersurfaces of constant t. These slices are 3-dimensional manifolds of constant curvature, and as such they can be covered by standard polar coordinates. Thanks to the orthogonality condition mentioned above, the mixed spatial and time components of the metric vanish in these coordinates. Metric constructed in this way has the following general form

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2(\theta)d\varphi^2)\right], \tag{1.1}$$

where $a(t)$ is the so called scale factor, whose dynamics will be determined from EFE (Einstein field equations). $(r,\ \theta,\ \varphi)$ are polar coordinates of the spatial slices and k measures the spatial curvature, while its sign $+1$, 0 and -1 corresponds to closed, flat and opened universes respectively [35].

For future reference let us write down two other forms of the metric that are commonly used in literature and will be used in the rest of this work. First of these can be obtained by introducing a new spatial variable $d\chi = dr/\sqrt{1-kr^2}$.

$$ds^2 = dt^2 - a^2(t)\left[d\chi^2 + f_k(\chi)^2(d\theta^2 + \sin^2(\theta)d\varphi^2)\right], \tag{1.2}$$

where

$$f_k(\chi) = \begin{cases} \sin(\sqrt{k}\chi)/\sqrt{k} & \text{for} \quad k > 0 \\ \chi & \text{for} \quad k = 0 \\ \sinh(\sqrt{-k}\chi)/\sqrt{-k} & \text{for} \quad k < 0. \end{cases}$$

Another important coordinate is the conformal time

$$d\eta = dt/a, \tag{1.3}$$

in which the metric has the following form

$$ds^2 = a^2(\eta)\left[d\eta^2 - d\chi^2 - f_k(\chi)^2(d\theta^2 + \sin^2(\theta)d\varphi^2)\right]. \tag{1.4}$$

Using conformal time the metric takes the form of conformal transformation of maximally symmetric 4-dimensional metric. While using conformal time it is customary to use $'$ to denote time derivatives as opposed to the standard overdot for the above forms of the metric. Often we will work in flat FRW universe ($k = 0$) in such case we will use Cartesian coordinates

$$ds^2 = dt^2 - a^2(t)dx^i dx^j \delta_{ij} \tag{1.5}$$

instead of polar ones.

The next step is to determine the dynamics. Let us focus on the curvature side of the Einstein's equation. The Einstein tensor for this metric is

$$\begin{aligned} G^0_0 &= 3H^2 + \frac{3k}{a^2}, \\ G^i_j &= 2\dot{H} + 3H^2 + \frac{k}{a^2}, \end{aligned} \tag{1.6}$$

where $H = \dot{a}/a$ is the Hubble parameter.

The matter sector of EFE has to possess the same symmetries as the metric (1.1). This severely restricts all matter fields that can be present in the theory. First of all, no field can have any spatial dependence once expressed with respect to the coordinates (1.1). Vectors can only point in direction of u^μ (coordinate vector of time t), and tensors of rank 2 exist only in diagonal form with all spatial entries equal. This constrains the form of the stress-energy tensor to be

$$T^\mu_\nu = \rho u^\mu u_\nu + (u^\mu u_\nu - \delta^\mu_\nu)p. \tag{1.7}$$

Quantities ρ and p are functions of time, and for the fundamental observers they play the role of energy density and pressure respectively.

From EFE we get the Friedmann equations

$$\begin{aligned} 3H^2 + \frac{3k}{a^2} &= \rho, \\ 2\dot{H} + 3H^2 + \frac{k}{a^2} &= -p. \end{aligned} \tag{1.8}$$

There is another Friedmann equation (fluid equation) that can be readily obtained from the conservation law for stress-energy tensor:

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{1.9}$$

This does not provide a new information about the system as the above formula is not independent from the set (1.8); however, its form is fairly useful.

Since we have three unknown functions of time and only two independent equations, we deal with an under-determined system. In order to close it, we need one additional piece of information that would characterize the type of matter involved. This is the equation of state for matter. Let us consider a linear barotropic equation of state that is characteristic for different cosmic fluids.

$$\frac{p}{\rho} = w = \begin{cases} \frac{1}{3} & \text{for} \quad \text{radiation or relativistic matter} \\ 0 & \text{for} \quad \text{dust} \\ \text{-1} & \text{for} \quad \text{cosmological constant} \end{cases} \tag{1.10}$$

Using assumption (1.10), we can integrate equation (1.9) to obtain

$$\rho = \rho_0 \left(\frac{a_0}{a}\right)^{3(1+w)}. \tag{1.11}$$

The factors a_0 and ρ_0 correspond to the scale factor and energy density at some reference time. In our universe the energy density is composed of a mixture of matter types

$$\rho = \rho_R + \rho_M + \rho_\Lambda, \tag{1.12}$$

where R and Λ stand for radiation and cosmological constant respectively. The subscript M marks ordinary baryonic matter and cold dark matter, which are both characterized by the equation of state for dust. Using this decomposition, one can rewrite the first Friedmann equation (1.8) in terms of the density parameters.

$$\begin{gathered} 1 + \frac{k}{H^2 a^2} = \frac{\rho_R}{3H^2} + \frac{\rho_M}{3H^2} + \frac{\rho_\Lambda}{3H^2} \\ 1 - \Omega_k = \Omega_R + \Omega_M + \Omega_\Lambda \end{gathered} \tag{1.13}$$

Current measurements yield the value $\Omega_k = 0.000 \pm 0.005$ [1] suggesting that universe is flat to a high degree of accuracy.

1.1.1 Solutions for universe dominated by different matter fields

We consider a universe that is filled by matter with equation of state (1.10) characterized by w that dominates over other energy contributions, that is we neglect the others.

As a first case let us consider a flat space $k = 0$. Using the first Friedmann equation (1.8) and the result (1.11) we obtain an equation for the scale factor that has the following solution (assuming $w \neq -1$)

$$a(t) = a_0 \Big(\frac{1}{2} \rho_0 (1+w)^2 t^2 \Big)^{\frac{1}{3(1+w)}}. \tag{1.14}$$

For $w = -1$ this formula breaks down, and we need to treat this case separately. The solution is

$$a(t) = a(0) e^{Ht}, \tag{1.15}$$

where $H = \sqrt{\rho_0/3}$ is the Hubble parameter, which is constant in this case.

For the case of $k \neq 0$, the analysis is a little bit more involved since the solution cannot be expressed in a closed form. Instead, we will find a parametric solution parametrized by the conformal time η introduced earlier. Let us rewrite the first Friedmann equation (1.8) and the fluid equation (1.11) in terms of conformal time

$$a'^2 + ka^2 = \frac{\rho}{3} a^4, \tag{1.16}$$

$$\rho' + 3\mathcal{H}(\rho + p) = 0. \tag{1.17}$$

By differentiating the first equation with respect to conformal time and expressing the derivative of energy density from the second equation, we get

$$a'' + ka = \frac{1}{6}(\rho - 3p) a^3. \tag{1.18}$$

For radiation the pressure is $3p = \rho$, and the right side vanishes, which makes the equation easy to solve. The solutions are

$$a(\eta) = C_r \begin{cases} \sin(\sqrt{k}\eta) & \text{for } k > 0 \\ \sinh(\sqrt{-k}\eta) & \text{for } k < 0, \end{cases} \tag{1.19}$$

where C_r is an integration constant that depends on the energy density of radiation at some reference time. For the time t we get

$$t(\eta) = C_r \begin{cases} (1 - \cos(\sqrt{k}\eta))/\sqrt{k} & \text{for } k > 0 \\ (1 + \cosh(\sqrt{-k}\eta))/\sqrt{-k} & \text{for } k < 0. \end{cases} \tag{1.20}$$

For dust we have the equation of state $p = 0$. From integration of the fluid equation (1.9), we obtain $\rho(\eta) \propto a^{-3}$, and therefore the right hand side of (1.18) is some constant C_d. With that in mind, the solution can be easily found to be

$$a(\eta) = \frac{C_d}{k} \begin{cases} 1 - \cos(\sqrt{k}\eta) & \text{for } k > 0 \\ 1 - \cosh(\sqrt{-k}\eta) & \text{for } k < 0. \end{cases} \tag{1.21}$$

For the time t we get

$$t = \frac{C_d}{k} \begin{cases} \eta - \sin(\sqrt{k}\eta)/\sqrt{k} & \text{for } k > 0 \\ \eta - \sinh(\sqrt{-k}\eta)/\sqrt{-k} & \text{for } k < 0. \end{cases} \tag{1.22}$$

For the cosmological constant $w = -1$ the solution is just the de Sitter space. The parametric solutions for closed universe describe a circle for radiation and a cycloid for matter. Therefore, in such scenarios the universe expands and then re-collapses. Important fact about all of these solutions is that they can be traced back in time to $a \to 0$. Except for the cosmological constant domination, $a = 0$ occurs in finite past. This suggests that the universe was once very small and therefore very hot.

1.1.2 Cosmic distances and horizons

In cosmology there are several noteworthy horizons and distances that are important in understanding the cosmos.

Particle horizon is the maximal distance that particles could have traveled since the beginning of the universe. If we fix the angular coordinates θ and φ and assume that no particle can move faster then light ($dt^2 = d\chi^2$), we obtain

$$d_p(t) = a(t)\chi_H = a(t)\int_0^{\chi_p} d\chi = a(t)\int_{t_0}^{t} \frac{dt}{a(t)}. \tag{1.23}$$

We define a similar type of horizon specially for photons, the so called **optical horizon**. We have to keep in mind that photons could not move freely until the recombination occurred. Thus the lower bound of the integral (1.23) has to be changed to the time of recombination

$$d_o(t) = a(t)\int_{t_r}^{t} \frac{dt}{a(t)}. \tag{1.24}$$

Event horizon surrounds the region that will ever be able to receive signals from us. That is

$$d_e(t) = a(t)\int_{t}^{t_{max}} \frac{dt}{a(t)}. \tag{1.25}$$

t_{max} corresponds to the final moment of time. If the universe will expand forever, then $t_{max} = \infty$.

Two points lying on a single spatial slice have a **proper distance** equal to their physical distance given by the metric (1.1). The **comoving distance** is the same with the factor a divided out. Unlike proper distance, comoving distance does not change with time. Note that galaxies in cosmology play the role of the fundamental observers, they move along the fundamental geodesics, and thus their relative comoving distance remains unchanged.

The Hubble parameter has the units of speed per distance, and thus its reciprocal value defines one of the characteristic scales of FRW model, the so called Hubble scale or Hubble distance H^{-1}. This distance defines a boundary (Hubble horizon) between points that recede slower and faster then the speed of light at a given time for a given observer. Let us discuss this in a little more detail. Once a galaxy crosses the Hubble horizon (with us in the center), any light signal it releases after that point in time will never reach as. This does not mean that such galaxy disappears from our telescope since the light that already travels to us will not vanish. However, we will not be able to receive any information about it from the period after it crosses.

1.2 Cosmological perturbations

While the universe is homogeneous and isotropic on the largest scale, it produces nonlinear structures on smaller scales in the form of galaxies, clusters and superclusters of galaxies and so on. These structures are formed from slight inhomogenities in the early universe and are then amplified by the gravitational instability. Gravitational instability is a central feature of gravity that is already present in the Newtonian limit. However, in order to study how the large scale structure forms from the initial inhomogenities in a relativistic theory, we have to understand the theory of cosmological perturbations.

In the original FRW model, there is a preferred choice of coordinates that is given by the nature of the underlying geometry. If we consider small perturbations around this metric, this seizes to be the case, and we have to consider a larger class of coordinates that can be obtained by small changes of the coordinate system. As we will see, these small coordinate transformations can account for some of the perturbations. If this happens, it means that the original perturbation was fictitious and unphysical. So we see that the physical meaning of perturbation gets a little obscured by the diffeomorphism invariance, on the other hand it gives us freedom to choose coordinates (gauge) that is tailored to address certain problems, thus making the calculations easier.

Let us now look closer at this problem. We consider two sets of coordinates x^μ and y^μ, which are connected by an infinitesimal coordinate change

$$y^\mu = x^\mu + \epsilon\xi^\mu, \tag{1.26}$$

where ϵ is an infinitesimal parameter. We will denote tensors with respect to the y^μ coordinates with a tilde above. Additionally we have a perturbed metric $g_{\mu\nu}$ around the FRW metric $f_{\mu\nu}$. Its components can be expressed with respect to both coordinates and are related by the standard formula

$$\tilde{g}_{\mu\nu}(y) = \frac{\partial x^\rho}{\partial y^\mu}\frac{\partial x^\sigma}{\partial y^\nu} g_{\rho\sigma}(x(y)). \tag{1.27}$$

Now we can expand the metric $g_{\mu\nu}$ to a linear order in ϵ on both sides of the equation

$$\tilde{f}_{\mu\nu}(y) + \epsilon\tilde{h}_{\mu\nu}(y) = \frac{\partial x^\rho}{\partial y^\mu}\frac{\partial x^\sigma}{\partial y^\nu}(f_{\mu\nu}(x(y)) + \epsilon h_{\mu\nu}(x(y))). \tag{1.28}$$

Using (1.26) and the fact that at the zeroth order the metric does not change ($\tilde{f} = f$), we get the following gauge rule

$$\tilde{h}_{\mu\nu}(y) = h_{\mu\nu}(x(y)) - \partial_\rho f_{\mu\nu}\xi^\rho(y) - f_{\mu\rho}\partial_\nu\xi^\rho(y) - f_{\nu\rho}\partial_\mu\xi^\rho(y). \tag{1.29}$$

Note that h does not transform as a tensor. This should not come as a surprise since defining it as the first order perturbation in particular coordinate system is not a covariant definition.

Let us derive gauge rule (1.29) again in more geometrical terms. The metric $g_{\mu\nu}$ is a tensor, and thus it is a notion that does not depend on any choice of coordinates. It is the same tensor whether it is expressed in terms of x^μ or y^μ coordinates. Hence

$$g_{\mu\nu}(y) = g_{\mu\nu}(x(y)) \tag{1.30}$$

Plugging the coordinate transformation (1.26) and expanding the metric $g_{\mu\nu}$ on both sides, we get

$$\begin{aligned} \tilde{g}_{\mu\nu}(y) &= f_{\mu\nu}(y - \epsilon\xi) + \epsilon h_{\mu\nu}(x(y)), \\ f_{\mu\nu}(y) + \epsilon\tilde{h}_{\mu\nu}(y) &= f_{\mu\nu}(y) - \epsilon\mathcal{L}_\xi f_{\mu\nu}(y) + \epsilon h_{\mu\nu}(x(y)), \\ \tilde{h}_{\mu\nu}(y) &= h_{\mu\nu}(x(y)) - \mathcal{L}_\xi f_{\mu\nu}(y), \end{aligned} \tag{1.31}$$

where we have again used the invariance of zeroth order with respect to the transformation (1.26). Expressing the lie derivative ($\mathcal{L}$) by the standard definition, one obtains the same gauge rule (1.29).

By the same line of reasoning, we learn that perturbations of scalars also transform. This sounds quite odd at first since scalars by definition should not transform under coordinate changes. The catch is hidden again in the fact that the notion of dividing the scalar into its background value and the perturbation is not a covariant notion. Let us derive the transformation rule for a scalar $\varphi \to \varphi + \epsilon\pi$.

$$\begin{aligned} \tilde{\varphi}(y) &= \varphi(x(y)), \\ \varphi(y) + \epsilon\tilde{\pi}(y) &= \varphi(y - \epsilon\xi) + \epsilon\pi(x(y)), \\ \tilde{\pi}(y) &= \pi(x(y)) - \mathcal{L}_\xi\varphi(y). \end{aligned} \tag{1.32}$$

This rule will hold in general for any perturbed tensor quantity $T_{\mu..} \to T_{\mu..} + \delta T_{\mu..}$

$$\delta\tilde{T}_{\mu..}(y) = \delta T_{\mu..}(x(y)) - \mathcal{L}_\xi T_{\mu..}(y). \tag{1.33}$$

One of our basic assumptions for construction of the FRW models is the existence of a privileged 3+1 decomposition of space-time. Such decomposition allows us to break down all 4-dimensional tensor quantities into 3-dimensional ones (scalar, vector and tensor). These quantities transform as 3-dimensional tensors and do not mix among themselves when we restrict ourselves to diffeomorphisms of the spatial slices alone. We will use this to decompose perturbations since on the level of equations of motion different types decouple, and thus can be studied separately, which simplifies the calculations significantly (we will discuss why that is later in this section). Before we decompose the metric perturbations in this manner, let us show how this works for vectors.

The first step is to separate the time component of the vector. This component acts as a scalar since the restrictions to the transformations of spatial slices effectively set all mixed time and space components of the Jacobian to zero. The rest of the components already look very much like a three vector; however, we can still separate an additional scalar from them. The assertion is that any 3-vector can be uniquely written as

$$v_i = \tilde{v}_i + \partial_i\varphi, \tag{1.34}$$

where $\tilde{v}$ is divergenceless. For this to be true, we have to restrict ourselves to quantities that vanish in spatial infinity. By doing so, we effectively eliminate the kernel of the Laplacian operator, and thus making it invertible. Such assumption is very common in physics since physical quantities should be localized. This allows us to uniquely calculate φ, which then defines the decomposition

$$\varphi = \frac{1}{\Delta}\partial_i v_i. \tag{1.35}$$

The geometrical way to see this is from the Hodge decomposition theorem. By the restriction above, we effectively compactify our manifold, and by doing so the theorem gives a unique decomposition

$$\begin{aligned} v &= d\varphi + \delta\alpha + \gamma, \\ &= d\varphi + \tilde{v}. \end{aligned} \tag{1.36}$$

Here v is a 1-form, α is a 2-form, γ is a harmonic 1-form and delta is the co-differential. Thus, we get for a 4-vector

$$\begin{aligned} v_0 &= v_0, \\ v_i &= \partial_i\varphi + \tilde{v}_i. \end{aligned} \tag{1.37}$$

Applying this to the 4-vector ξ that generates the infinitesimal coordinate transformation (1.26), we get

$$\begin{aligned} \xi^0 &= \xi^0, \\ \xi^i &= \xi^i_\perp + \partial^i\zeta. \end{aligned} \tag{1.38}$$

Here $\xi^i_\perp$ has zero divergence.

The decomposition of metric perturbations of the FRW metric in conformal form (1.4) proceeds in a similar manner (there is no Hodge theorem for symmetric tensors though)

$$\begin{aligned} \delta g_{00} &= 2a^2\phi, \\ \delta g_{0i} &= a^2(S_i - \partial_i B), \\ \delta g_{ij} &= a^2(2\delta_{ij}\psi + 2\partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}). \end{aligned} \tag{1.39}$$

The vectors S and F are divergenceless, and the tensor h is transverse traceless:

$$H^i_j = \partial_i H^{ij} = 0. \tag{1.40}$$

Now we consider the gauge transformation (1.29) to see how the perturbations change when they are decomposed. This will allow us to construct gauge invariants. For the coordinate change (1.26) in its decomposed form (1.38), scalar perturbations transform as follows (transformed quantities will be denoted here by tilde):

$$\tilde{\phi} = \phi - \frac{1}{a}\left(a\xi^0\right)', \tag{1.41}$$

$$\tilde{B} = B - \zeta' + \xi^0, \tag{1.42}$$

$$\tilde{\psi} = \psi + \mathcal{H}\xi^0, \tag{1.43}$$

$$\tilde{E} = E + \zeta. \tag{1.44}$$

$$\tag{1.45}$$

Here we have defined the conformal Hubble parameter $\mathcal{H} = a'/a$. There are two simple independent gauge invariants that can be constructed out of the 4 scalar perturbations

$$\Phi = \phi + \frac{1}{a}\big[a\big(B+E\big)'\big]', \tag{1.46}$$

$$\Psi = \psi - \mathcal{H}\big(B+E'\big). \tag{1.47}$$

For future reference let us consider a case where there is a 4-scalar function $\varphi = t$. Its perturbation π follows the gauge rule (1.32)

$$\tilde{\pi} = \pi - a\xi^0. \tag{1.48}$$

Since we have another scalar in play, we can construct additional gauge invariant

$$\mathcal{R} = \psi + \frac{\mathcal{H}}{a}\pi. \tag{1.49}$$

Scalar perturbations play a crucial role in cosmology as they are responsible for large structure formation and gravitational instability. The gauge invariants correspond to physical inhomogenities and they can be used to distinguish from fictitious perturbations.

For vector perturbations we get the transformations

$$\tilde{S}_i = S_i + \xi'_{\perp i}, \tag{1.50}$$

$$\tilde{F}_i = F_i + \xi_{\perp i}. \tag{1.51}$$

The obvious gauge invariant here is

$$V_i = S_i - F_i'. \tag{1.52}$$

Vector perturbations describe the rotational motion of the cosmic fluid; however, as they decay very quickly, they are not of much interest to cosmology.

Tensor perturbations are surprisingly the simplest among the others. Since there is no tensor part to the transformation (1.26), they are already gauge invariant. Furthermore, they describe the gravitational waves which carry gravity's own degrees of freedom.

1.2.1 Examples of gauge choices

As we mentioned earlier, the gauge invariance of gravity allows us choose a gauge that is advantageous for a particular problem. There are several gauges that are widely used in literature. We will focus only on two which are most important to our work.

The first example is the Newtonian gauge, sometimes also referred to as longitudinal gauge. It is defined by two conditions

$$B = E = 0. \tag{1.53}$$

One of the advantages of this choice is that it fixes the gauge freedom completely. One can see that from the transformation rules (1.45). Any non-zero ζ violates the condition for E, and any non-zero $\xi_\perp$ violates the condition for B. We do not have to worry about $\xi^i_\perp$ since it is a vector quantity. Furthermore, the gauge invariant quantities (1.47) become very simple, and the metric is easily expressible in their terms. If we consider only scalar perturbations, the metric in this gauge has the following diagonal form

$$ds^2 = a^2(t)\Big[(1+2\Phi)d\eta^2 - (1-2\Psi)\delta_{ij}dx^i dx^j\Big]. \tag{1.54}$$

The gauge invariant function Φ has a somewhat clear physical interpretation. In Newtonian limit, it becomes the gravitational potential. So Φ generalizes the Newtonian potential.

Similarly, using time t we can get the form

$$ds^2 = (1+2\Phi)dt^2 - a^2(t)(1-2\Psi)\delta_{ij}dx^i dx^j. \tag{1.55}$$

Another useful gauge is the spatially flat gauge. In the case with an additional scalar field, one can encode all the information about spatial inhomogenities into the field perturbation π [20] by setting the perturbations of spatial part of the metric to zero ($\psi = E = 0$). This is particularly useful when expanding the Einstein-Hilbert action in the ADM formalism, because one does not have to expand the spatial scalar curvature. Again this choice fixes the gauge completely. The metric takes the form

$$ds^2 = a^2(\eta)\Big[(1+2\phi)d\eta^2 - 2\partial_i B dt dx^i - \delta_{ij}dx^i dx^j\Big]. \tag{1.56}$$

1.2.2 Equations for perturbations

Dynamics of the scalar perturbations are encoded within Einstein's equations. By plugging the perturbed metric inside of them and expanding to the first order, one obtains dynamical equations for perturbations. Note that since we are expanding around a solution of these equations, the zeroth order vanishes. The perturbed Einstein's equations are

$$\delta G^\mu_\nu = \delta T^\mu_\nu. \tag{1.57}$$

Note that neither δT nor δG are gauge invariant; however, their transformations can be compensated by adding a suitable combination of metric perturbations. The corresponding gauge invariants are

$$\delta\overline{G}^0_0 = \delta G^0_0 - (G^0_0)'(B - E'), \tag{1.58}$$

$$\delta\overline{G}^0_i = \delta G^0_i - (G^0_0 - G^j_j/3)'\partial_i(B - E'), \tag{1.59}$$

$$\delta\overline{G}^i_j = \delta G^i_j - (G^i_j)'(B - E'). \tag{1.60}$$

Gauge invariant for the stress-energy tensor perturbations has the same form. The perturbed Einstein's equation can be rewritten in gauge invariant manner as

$$\delta\overline{G}^\mu_\nu = \delta\overline{T}^\mu_\nu. \tag{1.61}$$

Notice that in Newtonian gauge these quantities are equal to the original perturbations. The possibility of inspecting each type of perturbation (scalar, vector, tensor) alone becomes very fruitful while solving these equations. The perturbations of the stress-energy tensor on the right hand side of the equation must be categorized in the same manner.

Expansion of the Einstein's tensor for scalar perturbations yields

$$\Delta\Psi - 3\mathcal{H}(\Psi' + \mathcal{H}\Phi) = \frac{1}{2}a^2\delta\overline{T}^0_{\ 0}, \tag{1.62}$$

$$\partial_i(\Psi' + \mathcal{H}\Phi) = \frac{1}{2}a^2\delta\overline{T}^0_{\ i}, \tag{1.63}$$

$$\begin{aligned}\big[\Psi'' + \mathcal{H}(2\Psi + \Phi)' + (2\mathcal{H}' + \mathcal{H}^2)\Phi + \frac{1}{2}\Delta(\Phi - \Psi)\big]\delta_{ij} - \\ - \frac{1}{2}\partial_i\partial_j(\Phi - \Psi) = -\frac{1}{2}a^2\delta\overline{T}^i_{\ j}.\end{aligned} \tag{1.64}$$

Note that this is an equality of components not a tensor equation. We us this convention [6] since it simplifies the resulting form of the equations. This will be done often in the rest of the work for spatial tensors since the metric there is almost Minkowski.

For vector perturbations we obtain

$$\Delta V_i = 2a^2\delta\overline{T}^0_{\ i} \tag{1.65}$$

$$\Big(a^2(\partial_i V_j + \partial_j V_i)\Big)' = -2a^4\delta\overline{T}^i_{\ j}, \tag{1.66}$$

and finally for tensor perturbations we get

$$h''_{ij} + 2\mathcal{H}h_{ij} - \Delta h_{ij} = 2a^2\delta\overline{T}^i_{\ j}. \tag{1.67}$$

Let us revisit our reasoning of the decoupling of various types of perturbations on the level of equations of motion. There is a simple reason how to see this. We can derive equations (1.57) by first expanding the action about the classical solution to the second order in perturbations and then varying with respect to them. Any possible quadratic term containing two different kinds of perturbations will have at least one index contracted with a derivative. Since the zeroth order quantities do not depend on spatial coordinates, such derivative can be always integrated by parts to hit a vector or tensor term, and thus eliminating it by definition (vectors are divergenceless and tensors are transverse). Therefore, only nonzero quadratic terms contain perturbations of the same type.

1.2.3 Dust energy perturbations

Let us demonstrate how can one use the above equations to study the evolution of density inhomogenities. Consider a pressureless dust characterized by the equation of state $p = 0$. From the equations (1.64) for $i \neq j$, we obtain

$$\partial_i\partial_j(\Phi - \Psi) = 0 \qquad \text{for} \qquad i \neq j, \tag{1.68}$$

since pressureless dust has vanishing anisotropic stress ($\delta\overline{T}^i_{\ j} = 0$ for $i \neq j$). The only solution consistent with Φ and Ψ being perturbations is $\Phi = \Psi$. The diagonal terms then give us

$$\Phi'' + 3\mathcal{H}\Phi' + (2\mathcal{H}' + \mathcal{H}^2) = 0. \tag{1.69}$$

From (1.14) we can calculate that in dust dominated FRW universe $a \propto \eta^2$ and $\mathcal{H} = 2/\eta$. Moreover, Friedmann equations (1.8) give us $2\mathcal{H}' + \mathcal{H}^2 = p = 0$. Therefore the equation above simplifies to

$$\Phi'' + \frac{6}{\eta}\Phi' = 0. \tag{1.70}$$

The general solution to this equation is

$$\Phi = C_1(x) + \frac{C_2}{\eta^5}, \tag{1.71}$$

where C_1 and C_2 are arbitrary functions of comoving coordinates consistent with Φ being a perturbation. We now take this solution and plug it into (1.62) to obtain

$$\frac{\overline{\delta\rho}}{\rho} = \frac{1}{6}\Big[\big(\Delta C_1\eta^2 - 12C_1\big) + \big(\Delta C_2\eta^2 + 18C_2\big)\frac{1}{\eta^5}\Big], \tag{1.72}$$

where ρ is the unperturbed energy density. We see that the perturbations have a non-decaying part that is given by C_1 and a decaying part given by C_2. The behavior of solutions (1.72) depends strongly on the Hubble scale. To see this consider a plane wave expansion of the functions $C_{1,2} \propto \exp(ikx)$. We will first analyze the long wavelength solutions. Note that the wavenumber k is related to the comoving coordinates, and therefore the physical wavelength is $\lambda_k = a/k$. Long wavelength corresponds to $\lambda = a/k \gg H^{-1} \propto a\eta$. Equation (1.72) becomes

$$\frac{\overline{\delta\rho}}{\rho} \simeq -2C_1 + 3\frac{C_2}{\eta^5}. \tag{1.73}$$

So we see that (neglecting the decaying mode) inhomogenities on super-Hubble scales remain in proportion to the average energy density. The short wavelength solutions are characterized by $\lambda_k \ll H^{-1}$ and yield the solution

$$\frac{\overline{\delta\rho}}{\rho} \simeq -\frac{k^2}{6}\big(C_1\eta^2 + C_2\eta^{-3}\big) = \tilde{C}_1 t^{\frac{2}{3}} + \tilde{C}_2 t^{-1}. \tag{1.74}$$

Again there is a decaying mode; however, now the second mode is actually growing. Note that this growth is not very fast.

1.3 Dark Matter model

Dark matter was introduced into physics in order to explain the motion of spiral galaxies [29],[30], [32]. Their rotational motion was way too fast to be held together by the gravity of visible matter content. In particular, the velocity curves (velocity of stars as a function of radial distance from the center of galaxy) were expected to behave as $V(r) \propto r^{-1/2}$. This behavior follows from the Newtonian gravity for a galaxy whose mass is concentrated in its center as it is the case for ordinary matter. These curves; however, have been measured to have a different, more flat shape on the outskirts of galaxies (see 1.1).

The shape of velocity curves is tightly connected to the matter distribution inside the galaxy, and simply put, there is not enough luminous matter in the

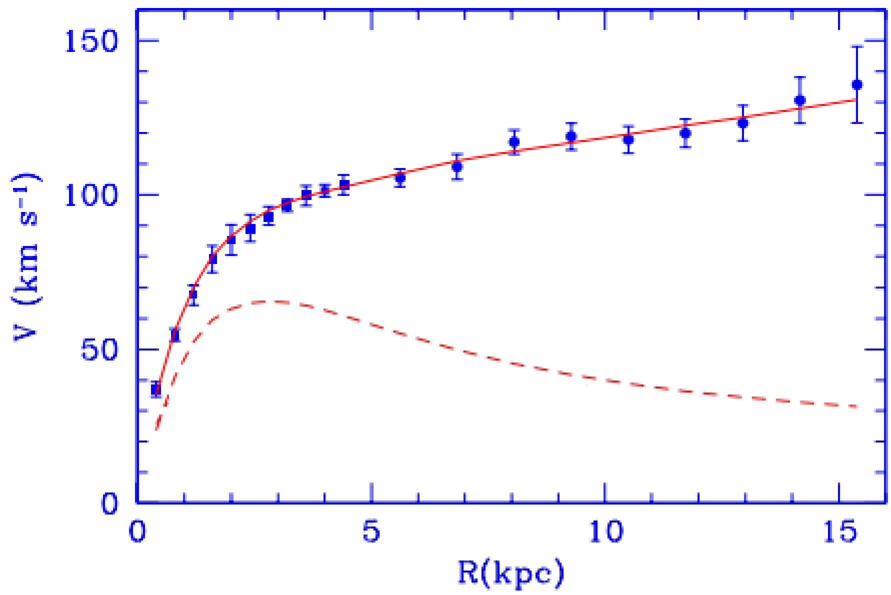

Figure 1.1: Rotational curves of galaxy M33. Classical prediction (dashed line). Adapted from [36]

galaxies to account for the measured shape of the curves. In order to explain the mass deficit, it was hypothesized that there is additional matter present that does not interact electromagnetically and therefore is invisible to our telescopes. Further evidence for dark matter comes from the weak gravitational lensing effects, which allow us to map the distribution of non-luminous matter in the universe.

Another piece of information is the formation of large scale structure of the Universe. In a universe where only baryonic matter is present structure formation can begin after recombination since photons keep the matter from clustering prior to recombination. Because of this delay, baryonic matter would not form structures that we observe simply because it did not have enough time to do so. Dark matter remedies this since it does not interact with photons, its structure formation could have started much earlier and grow into what we see today. Luminous matter to follow then follows its gravitational pull. Observations of the large scale structure showed that dark matter is distributed in a web like structures composed of filaments voids and walls with characteristic sizes of 100 Mpc. Around galaxies, dark matter is distributed in form of halos without which galaxies would be unstable [37].

Dark matter can be categorized into two distinct groups based on their equation of state: cold DM and hot DM corresponding to non-relativistic and relativistic matter respectively. From the models of structure formation, one can infer that most of dark matter is cold since hot dark matter moves too fast and is capable of escaping overdense regions and thus dissolving them.

One of the most obvious candidates for dark matter are ordinary neutrinos, who have just the right characteristics. Due to their tiny rest mass, they would fall to the category of hot DM and thus cannot be responsible for the majority of dark matter. Another possible candidates are primordial black holes (black holes that formed in the earliest stages of the universe). They obviously have a lot of the right properties; however, such black holes would have to be quite small otherwise their gravitational lensing would be too apparent. But they could have been produced in numbers large enough to explain cold dark matter. Unfortunately, searches for any evidence of such black holes have been so far unsuccessful [38]. For the lack of other candidates, the prevailing opinion nowadays is that dark matter is constituted by some exotic particles. Various supersymmetric theories and other extended particle models provide us with a plethora of possible candidates including the most favored weakly interacting massive particles. Unfortunately, such particles have not been observed. However, by definition dark matter has to be very difficult to observe, therefore the lack of detection is not yet a reason to dismiss it. However, as it has been foreshadowed in the introduction, another possibility is that gravity itself is responsible for dark matter. Since this is one of the upshots of mimetic gravity, we will analyze this possibility in the third chapter.

2. Beyond standard cosmology

Even though the standard model have performed amazingly well in the description of the cosmos, it has its problems and limitations. Apart from the issue with very high energies at the earliest stages ($t < 10^{-43}$s) of the universe that call for a theory of quantum gravity, there are questions that seem to be within our reach. In this chapter, we will discuss some of these questions and their possible solutions that go beyond the standard model and have not been verified by experiments. Specifically, we will lay out the basic ideas of inflation (following the introduction [35]) and quintessence, which share a similar idea.

2.1 Cosmological inflation

Let us first quickly review the problems that originally inflation sought to solve.

2.1.1 Initial condition problems

Flatness problem - Let us recall the first Friedmann equation (1.13) and rewrite it in the following form:

$$|\Omega - 1| = \frac{k^2}{a^2H^2}. \tag{2.1}$$

During the matter or radiation dominated epoch the combination $(aH)^2$ decreases as follows

$$\text{matter domination} : \propto t^{-2/3},$$
$$\text{radiation domination} : \propto t^{-1}.$$

So after the Big Bang (for most of the time) the expansion has driven Ω away from 1 (given that $k \neq 0$). Given that Ω is now within an order of 1 (which it is according to current measurements [1]), it is possible to extrapolate how much it had to be early in the universe. By doing so, we obtain values like $|\Omega - 1| < 10^{-27}$. This is an immense precision that requires some serious fine tuning of the initial conditions with which the universe began.

The horizon problem - One of the premises of the FRW models is homogeneity and isotropy at large scales. This has since been verified by direct observation of the distribution of galaxies and of the CMB. Variations in temperature in the CMB are of order 10^{-4}, which suggests that matter in the universe was in thermal equilibrium before recombination. This, however, is impossible, since given the age of the universe as predicted by the Big Bang theory, various regions of space could not have been in causal contact. Or in more quantitative language, the size of the particle horizon during the decoupling was much smaller then the optical horizon today.

$$d_p(t_r) \ll d_o(t). \tag{2.2}$$

The inequality is so large that any two regions separated by as little as 2 degrees were causally separated during recombination.

Relics of the Big Bang - Particle physics predicts that at very high temperatures (such as were present in the early universe) a variety of exotic objects such as magnetic monopoles, domain walls and cosmic strings is produced in abundance. Such objects dilute slower than ordinary matter and would soon dominate the universe. However, we do not observe anything like that in the universe.

2.1.2 Inflation as a solution

The main idea of inflation is that there was another phase early in the universe during which the universe expanded in a manner such that

$$\frac{d}{dt}\frac{1}{Ha} < 0. \tag{2.3}$$

This condition is equivalent with

$$\ddot{a} > 0 \qquad \Longleftrightarrow \qquad 3p < -\rho. \tag{2.4}$$

Here we assumed $a > 0$ and Friedmann equations. Notice that the first definition does exactly what we want to solve the flatness problem. It drives Ω to one. The only thing we need to ensure is that inflation lasts long enough to obtain high enough precision. Typically, this is not a problem in various inflationary scenarios.

The other important thing is that the condition (2.3) actually describes the comoving Hubble distance, and it is getting smaller. Even though the universe is getting larger, the amount of matter that one could observe after inflation is getting smaller in comparison to the era prior to inflation. In other words, when pictured in comoving coordinates, the patch of universe that was in causal contact before inflation was much larger than it is today. So if this patch came to thermal equilibrium before inflation, then it would have had the same temperature in regions that are causally disconnected today.

How inflation solves the problem of relics is less clear. Since the universe gets stretched very rapidly, the density of these relics decreases; however, so does the density of ordinary matter, and it does so even faster. Fortunately, there is a natural solution to this problem in the inflationary scheme. As we will see later, the driving force for inflation is usually somewhat constant energy density (similar behavior as the cosmological constant). In order for inflation to end, this energy has to dissipate into some other form of energy, possibly ordinary matter and radiation, in a process known as reheating. If, during reheating, the temperature of the universe does not exceed the values needed for creation of the aforementioned exotic objects, then their density remains low as it was diluted by the inflation. On the other hand, the density of ordinary matter and radiation gets a significant boost. Thus, we are able to recover the hot big bang with all its successes. The mechanics of reheating vary from model to model and are beyond the scope of this thesis.

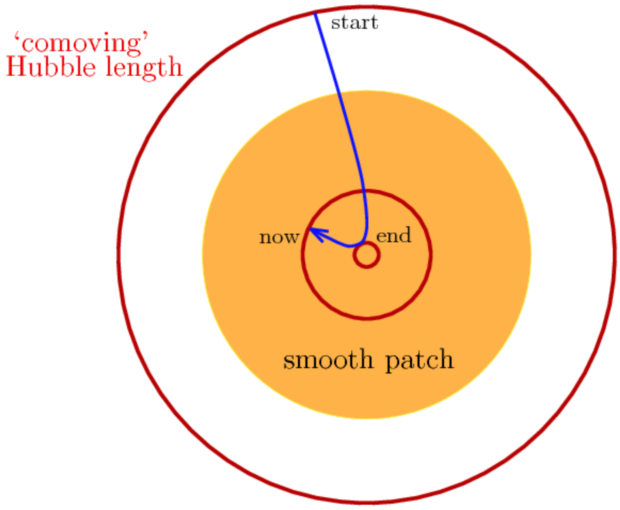

Figure 2.1: Solution to the horizon problem. At the start of inflation, the horizon is large. A patch inside comes to a thermal equilibrium and smooths out. Then the Hubble distance shrinks, and the consequent non-inflationary expansion is not strong enough to enlarge it beyond the smooth patch. [39]

2.1.3 Basics of inflation

One of the most beautiful things about inflation is that it can be realized by very simple object: the scalar field. Consider a scalar field ϕ (usually referred to as inflaton) in FRW model universe given by the action

$$S = \int d^4x\sqrt{-g}\Big(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\Big). \tag{2.5}$$

Since we restrict ourselves only to solutions that have the symmetries of the FRW model, the stress-energy tensor becomes

$$T^\mu_\nu = u^\mu u_\nu\dot{\phi}^2 + \delta^\mu_\nu(V(\phi) - \frac{1}{2}\dot{\phi}^2), \tag{2.6}$$

or equivalently

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{2.7}$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{2.8}$$

The Friedmann equations take the form

$$3H^2 = V(\phi) + \frac{1}{2}\dot{\phi}^2, \tag{2.9}$$

$$\ddot{\phi} + 3H\dot{\phi} = -\partial_\phi V(\phi). \tag{2.10}$$

The second equation exactly coincides with the equation of motion for ϕ. Conditions for inflation (2.3) give us the following constraint

$$\dot{\phi}^2 < V(\phi). \tag{2.11}$$

So we see that inflation takes place whenever the potential energy of the field dominates over its kinetic energy. In order for inflation to end, we also require that there is a stable point where such condition is violated. Furthermore, the model must provide us with a "graceful" exit to a radiation dominated epoch.

To solve these equations, one usually employs the so called slow roll approximation given by

$$\dot{\phi}^2 \ll V(\phi). \tag{2.12}$$

In this approximation the Friedmann equations become

$$3H^2 \simeq V(\phi), \tag{2.13}$$

$$3H\dot{\phi} \simeq -\partial_\phi V(\phi). \tag{2.14}$$

Notice that the order of the equations is lower then the order of the original set (2.10). Therefore, we need one less initial condition to give a unique solution. This sounds like we are doing something wrong, but it works thanks to the existence of an attractor solution of the original set (2.10). For more details on this, please refer to [40]. If the scalar field satisfies the slow roll conditions, then inflation is guaranteed. The shape of the potential is often characterized by slow roll parameters that are defined bellow [41]

$$\epsilon(\phi) = \frac{1}{2}\Big(\frac{\partial_\phi V}{V}\Big), \qquad\qquad \eta(\phi) = \frac{\partial_\phi^2 V}{V}. \tag{2.15}$$

ϵ measures the slope of the potential, and η measures the curvature. Necessary conditions for the slow roll approximation are

$$\epsilon \ll 1, \qquad\qquad |\eta| \ll 1. \tag{2.16}$$

As we mentioned, inflation has to last long enough in order to bring Ω sufficiently close to 1. A standard measure of the amount of expansion during inflation is the number of e-foldings, which is given as

$$N = \log \frac{a(t_{end})}{a(t_{ini})}. \tag{2.17}$$

Using the slow roll approximation, this number can be expressed without the need to solve the equations of motion

$$N = -\int_{\phi_{ini}}^{\phi_{end}} \frac{V}{\partial_\phi V} d\phi. \tag{2.18}$$

The minimum amount of inflation needed to obtain the desired result is about 70 e-foldings.

While inflation solves the original problems, which it has set out to solve, its even more profound feature is that it can produce cosmological perturbations from quantum fluctuations [42]. Therefore it is able to give rise to the structure formation in universe.

2.2 Quintessence

As we have seen in the above section, a scalar field can naturally provide us with a cosmic fluid with negative pressure that can drive accelerated expansion. It is quite obvious that one could consider applying the same mechanisms to give account of the observed dark energy. Such models are usually called quintessence. In this section, we will explore only a very basic model, in which the scalar field has only a standard kinetic term. However, we should stress that the variety of quintessence model is much richer. For example, there are models that derive the nontrivial behavior from various nontrivial kinetic terms. Such models are usually referred to as k-essence. Our discussion follows the review [41].

Let us consider a standard scalar field Q described by the following action

$$S = \int \mathrm{d}^4x \sqrt{-g}\big(\frac{1}{2}\partial_\mu Q \partial^\mu Q - V(Q)\big). \tag{2.19}$$

Similarly to the inflation the non-trivial behavior stems from the choice of the shape of the potential. In a flat FRW model universe ($k = 0$) the equations of motion are

$$\ddot{Q} + 3H\dot{Q} = -\partial_Q V(Q), \tag{2.20}$$

where the spatial derivatives are eliminated since Q has to respect the symmetries of the model. The energy momentum tensor is of the form (1.7) with

$$\rho = \frac{1}{2}\dot{Q}^2 + V(Q), \tag{2.21}$$

$$p = \frac{1}{2}\dot{Q}^2 - V(Q), \tag{2.22}$$

which yields

$$\frac{p}{\rho} = w_Q = \frac{\dot{Q}^2 - 2V(Q)}{\dot{Q}^2 + 2V(Q)}. \tag{2.23}$$

It is clear that $w \in [-1, 1]$. The lower bound corresponds to the slow roll approximation. The fluid equation (1.9) can be expressed in an integrated form as

$$\rho = \rho_0 \exp\Big(-\int 3(1 + w_Q)\frac{da}{a}\Big), \tag{2.24}$$

where ρ_0 is a constant of integration. Since we are interested in modeling accelerated expansion, it is of interest to derive what kind of shape should a potential have to satisfy this condition. The border between accelerated and decelerated expansion is

$$a(t) \propto t. \tag{2.25}$$

It is no extra work to derive the potential in a more general case where the scale factor obeys a power law

$$a(t) \propto t^p. \tag{2.26}$$

From Friedmann equations (1.8) we obtain:

$$\dot{H} = -\frac{1}{2}\dot{Q}^2, \tag{2.27}$$

which allows us to express the field Q and the potential V in terms of H:

$$V = 3H^2\Big(1 + \frac{\dot{H}}{3H^2}\Big), \tag{2.28}$$

$$Q = \int dt\Big(-2\dot{H}\Big)^{\frac{1}{2}}. \tag{2.29}$$

Substituting into these formulas from (2.26), we obtain

$$V = \frac{p}{t^2}\Big(3p - 1\Big), \tag{2.30}$$

$$Q = \pm\sqrt{2p}\log(t). \tag{2.31}$$

Expressing t from the positive branch of the second equation and plugging it in the first, we get

$$V(Q) = p(3p-1)\exp\Big(-\frac{Q}{\sqrt{2p}}\Big). \tag{2.32}$$

For $p = 2$ this represents a border between potentials that can and cannot lead to accelerated expansion.

The major difference between inflaton and quintessence is that the potential is chosen to achieve accelerated expansion in late times, while inflaton needs to enter such phase and exit it in the early universe.

One of major features of many quintessence models is so called tracking behavior that gives a possible explanation to the minuscule value of density of dark energy. For details about this please refer to [41].

3. Introduction to mimetic gravity

General relativity enjoys a very strong symmetry, the diffeomorphism invariance, which among other things, allow us to parametrize the physical metric $g_{\mu\nu}$ by means of an auxiliary metric $\tilde{g}_{\mu\nu}$ and a scalar field φ in the following way [19]:

$$g_{\mu\nu} = C(\varphi, X)\tilde{g}_{\mu\nu} + D(\varphi, X)\partial_\mu\varphi\partial_\nu\varphi, \tag{3.1}$$

where $X = \tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi/2$ is the standard kinetic term for scalar field defined through the auxiliary metric. C and D are free functions. Varying the Einstein-Hilbert action for the physical metric with respect to φ and $\tilde{g}_{\mu\nu}$ gives the same equations of motion for $g_{\mu\nu}$ as long as the reparametrization is not singular. In that case there are no new degrees of freedom in such theory. In the opposite case, new degrees of freedom are introduced, and the theory does not have an equivalent physical content. Singular transformations satisfy the following condition introduced in [43]:

$$D(\varphi, X) = f(\varphi) - \frac{C(\varphi, X)}{2X}, \tag{3.2}$$

where f is an arbitrary function. Reparametrizations of the type (3.1) (introduced by Bekenstein in 1993 [44]) are called disformal transformations, and they lie in the heart of the original mimetic dark matter as introduced in [3]. More specifically, mimetic dark matter is obtained by a particular singular disformal transformation of the physical metric $g_{\mu\nu}$. This transformation has the following form:

$$g_{\mu\nu} = \tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi\tilde{g}_{\mu\nu}. \tag{3.3}$$

Note that $\tilde{g}^{\mu\nu}$ is defined as the inverse auxiliary metric while the remaining indeces are raised by the physical metric. One can easily check that transformation (3.3) satisfies the singularity condition with $f(\varphi) = 1$, $C(\varphi, X) = 2X$ and $D(\varphi, X) = 0$. The two metrics are related by a conformal rescaling

$$g_{\mu\nu} = P\tilde{g}_{\mu\nu}, \tag{3.4}$$

and therefore the inverse metrics are related by the reciprocal value of $P = \tilde{g}^{\alpha\beta}\partial_\alpha\varphi\partial_\beta\varphi$

$$g^{\mu\nu} = \frac{1}{P}\tilde{g}^{\mu\nu}. \tag{3.5}$$

Contracting this equation with $\partial_\mu\varphi\partial_\nu\varphi$ yields the mimetic constraint

$$g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi = 1. \tag{3.6}$$

Dynamics are given by the standard Einstein-Hilbert action

$$S[\varphi, \tilde{g}_{\mu\nu}] = -\frac{1}{2}\int d^4x\sqrt{-g(X, \tilde{g}_{\mu\nu})}R(g_{\mu\nu}(X, \tilde{g}_{\mu\nu})) \tag{3.7}$$

with the exception that we now treat φ and $\tilde{g}_{\mu\nu}$ as the independent variables. If we consider an addition of some matter fields in the form of Lagrangian L_m, the variation of the auxiliary metric gives us the following equations of motion:

$$G^{\mu\nu} - T^{\mu\nu} - (G - T)\partial^\mu\varphi\partial^\nu\varphi = 0, \tag{3.8}$$

where G and T are the traces of the Einstein tensor and the stress-energy tensor respectively. Tensors $G^{\mu\nu}$ and $T^{\mu\nu}$ depend on φ and $\tilde{g}_{\mu\nu}$ only through the physical metric $g_{\mu\nu}$. Trace of this equation yields

$$(G-T)(1-\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi) = 0. \tag{3.9}$$

Thanks to the mimetic constraint, this equation is satisfied automatically. The equations (3.8) are therefore equivalent to the traceless Einstein's equations. The Euler-Lagrange equation for φ is

$$\nabla_\mu\big((G-T)\partial^\mu\varphi)\big) = 0, \tag{3.10}$$

which becomes equation relating G with T (after one solves for φ from the mimetic constraint (3.6)). These equations hold non-trivial solutions for φ even when there is no matter $T_{\mu\nu} = 0$, and thus gravity obtains an additional degree of freedom [3].

3.1 Mimetic dark matter

To better understand this model let us treat (3.8) as the standard Einstein's equation by grouping the novel terms together with matter contributions. We do this by introducing an energy tensor for mimetic matter $\tilde{T}^{\mu\nu}$ as

$$\tilde{T}^{\mu\nu} = (G-T)\partial^\mu\varphi\partial^\nu\varphi. \tag{3.11}$$

Note that this stress-energy tensor (3.11) has the same form as pressureless fluid:

$$\tilde{T}^{\mu\nu} = \rho u^\mu u^\nu \tag{3.12}$$

with energy density $\rho = G - T$ and velocity $u_\mu = \partial_\mu\varphi$. When there is no external matter, the energy density becomes $-R$, which does not vanish for generic solutions [3]. Let us further analyze the similarity between the two cases. First of all, the mimetic constraint (3.6) takes the form of normalization condition for a four-velocity u_μ

$$g^{\mu\nu}u_\mu u_\nu = 1. \tag{3.13}$$

Secondly, note that the tensor $\tilde{T}^{\mu\nu}$ is conserved as a consequence of the equations of motion for φ (3.10) and the constraint (3.6)

$$\nabla_\mu\tilde{T}^\mu_{\ \nu} = \partial_\nu\varphi\nabla_\mu\big((G-T)\partial^\mu\varphi)\big) + \frac{1}{2}(G-T)\nabla_\nu(\partial^\mu\varphi\partial_\mu\varphi) = 0. \tag{3.14}$$

Here we have used the fact that covariant derivatives commute when they act upon a scalar. Now consider the metric (1.1). Taking into account its symmetries, ones finds a unique solution of the constraint (3.6) (up to shifts in time)

$$\varphi(t) = t. \tag{3.15}$$

Plugging this into (3.10), we find

$$(\partial_t + 3H)(G-T) = 0 \qquad \Longrightarrow \qquad \rho = G - T = \frac{C}{a^3}, \tag{3.16}$$

where C is an integration constant and plays the role of a reference energy density.

To make the parallel even more obvious, let us rework the action (3.7) into a more familiar form. From (3.3) it is clear that a theory of gravity parametrized by φ and $\tilde{g}_{\mu\nu}$ is manifestly invariant with respect to the transformation (Weyl transformation)

$$\tilde{g}_{\mu\nu} \to \Omega^2 \tilde{g}_{\mu\nu} \tag{3.17}$$

of the auxiliary metric, where Ω is an arbitrary function of space-time. This is an additional gauge invariance that is not present in the standard theory of gravity. We fix this freedom by choosing a gauge where

$$g_{\mu\nu} = \tilde{g}_{\mu\nu}, \tag{3.18}$$

which can be realized by

$$\Omega^{-2} = \tilde{g}^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi. \tag{3.19}$$

This gauge eliminates the φ dependence of the Lagrangian (3.7); however, the field φ is still an independent variable of the action that is subjected to the constraint (3.6). We can include this external information to the action by introducing a Lagrange multiplier λ. The resulting action has the form [18]

$$S[\varphi, g_{\mu\nu}, \lambda] = \int d^4x \sqrt{-g(X, g_{\mu\nu})} \Big(-\frac{1}{2} R(X, g_{\mu\nu}) + \lambda (g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - 1) \Big). \tag{3.20}$$

The equations of motion for $g_{\mu\nu}$ are now

$$G^{\mu\nu} - T^{\mu\nu} + 2\lambda \partial^\mu \varphi \partial^\nu \varphi = 0. \tag{3.21}$$

Taking the trace of this equation yields an expression for λ:

$$\lambda = -\frac{1}{2}(G - T), \tag{3.22}$$

and thus we recover the previous case. The procedure introduced in the beginning of this chapter is a little exotic to a reader who is not acquainted with the subject; however, the resulting action (3.20) is actually quite familiar. Usually, it is also more convenient to work with and therefore we will prefer it over (3.7) in the rest of this work.

3.2 Potential for mimetic matter

It is important to note that while the procedure involving disformal transformation gives us a neat starting point and a possible explanation where a theory like (3.20) could originate, the following analysis is much more phenomenologically motivated. As we will soon see, we can achieve very interesting behaviors of this model by extending it through addition of further terms in the action. These terms; however, do not follow from the underlying scenario of mimetic dark matter and are added by hand.

The following sections follow the works [18], [19]. So far this new "mimetic matter", which we have plucked by a clever trick from the gravity itself, has all the right properties to play the role of cold dark matter. In what follows we will see that we can do even better.

Consider adding a potential term $V(\varphi)$ to the action (3.20):

$$S[\varphi, g_{\mu\nu}, \lambda] = \int d^4x\sqrt{-g}\Big(-\frac{1}{2}R + \lambda(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1) - V(\varphi)\Big). \tag{3.23}$$

The equation (3.22) becomes

$$\lambda = -\frac{1}{2}(G - T - 4V)., \tag{3.24}$$

and the equation of motion for φ is

$$\nabla_\mu\big((G - T - 4V)\partial^\mu\big) = 0. \tag{3.25}$$

Varying the action with respect to $g_{\mu\nu}$ provides us with the stress-energy tensor for mimetic matter:

$$\tilde{T}_{\mu\nu} = (G - T - 4V)\partial_\mu\varphi\partial_\nu\varphi + g_{\mu\nu}V(\varphi). \tag{3.26}$$

By a similar inspection as in previous section, we find that corresponding energy density and pressure are

$$\rho = G - T - 3V, \tag{3.27}$$
$$p = -V. \tag{3.28}$$

Continuing the parallel, we consider flat FRW metric (1.1), and we solve the mimetic constraint (3.6) to obtain

$$\varphi(t) = t + t_0. \tag{3.29}$$

We will, unless specified otherwise, use the choice $t_0 = 0$. This result allows us to integrate (3.25) to find

$$\rho = V - \frac{1}{a^3}\int a^3\dot{V}dt = \frac{3}{a^3}\int a^2V da. \tag{3.30}$$

So we see that we can control the behavior of the energy density by cleverly choosing the potential $V(\varphi) = V(t)$. Note that in action (3.20) the choice of $V = \text{const}$ is equivalent to inclusion of cosmological constant. In agreement with this note, the above equation yields for $V = \text{const.}$,

$$\rho = \text{const.} \tag{3.31}$$

Before we move to other examples, let us simplify the above equation. From the first Friedmann equation (1.8), we obtain

$$H^2 = \frac{1}{a^3}\int a^2V(t(a))da \qquad \Longleftrightarrow \qquad 2\dot{H} + 3H^2 = V. \tag{3.32}$$

Using a substitution

$$y = a^{\frac{2}{3}}, \tag{3.33}$$

one can simplify this equation to a linear second order equation

$$\ddot{y} - \frac{3}{4}V(t)y = 0, \tag{3.34}$$

which is easily solvable.

3.2.1 Cosmological solutions

Let us analyze an interesting potential given by the following expression:

$$V(\varphi) = \frac{\alpha}{\varphi^2}. \tag{3.35}$$

The mimetic field satisfies (3.29), and therefore (3.34) becomes

$$\ddot{y} - \frac{3\alpha}{4t^2} y = 0. \tag{3.36}$$

Fundamental solutions for this equation can be obtained by substituting a power law $y = t^p$, which in turn gives us a quadratic equation for p. For $\alpha \geq -1/3$ there are two real solutions for p, while for $\alpha < -1/3$ the solutions are complex. In the second case scale factor oscillates with ever increasing amplitude [18] and the resulting fluid is unlike anything we know. For that reason we will not discuss it any further. The solution for y in the first case $\alpha \geq -1/3$ is

$$y = C_1 t^{\left(\frac{1}{2} + \sqrt{1+3\alpha}\right)} + C_2 t^{\left(\frac{1}{2} + \sqrt{1+3\alpha}\right)}, \tag{3.37}$$

where C_1 and C_2 are constants of integration. Since the scale factor a is defined up to an overall normalization, we can get rid of one of these constants and write

$$a(t) = t^{\frac{1}{3}(1+\sqrt{1+3\alpha})} \left(1 + At^{-\sqrt{1+3\alpha}}\right), \tag{3.38}$$

where we defined $A = C_2/C1$. Using equations (3.30) and (3.28), we find

$$\frac{p}{\rho} = w(t) = -3\alpha \left(1 + \sqrt{1+3\alpha} \frac{1 - At^{-\sqrt{1+3\alpha}}}{1 + At^{-\sqrt{1+3\alpha}}}\right). \tag{3.39}$$

We see that the equation of state depends on time; however if one expands for large or small times $w(t)$ becomes constant. By inspection of this formula, we find that $\alpha = 0$ corresponds to pressureless dust for all times, which is consistent with the fact that nonexistent potential corresponds to the original mimetic dark matter scenario. The case $\alpha = -1/4$ corresponds to radiation in late times, but at early times it describes matter with $w = 3$. One other interesting case is $\alpha \gg 1$ for which $w = -1$, and thus describes cosmological constant.

3.2.2 Mimetic matter as quintessence

Let us consider a similar case as above (we keep the same potential), but this time we drop the condition that mimetic matter dominates. Instead, let the universe be dominated by some other type of matter characterized by $w =$ constant. In such case, the scale factor behaves as

$$a(t) \propto t^{\frac{2}{3(1+w)}}. \tag{3.40}$$

Using equation (3.30), we find that

$$\rho_{\text{mim}} = -\frac{\alpha}{wt^2}. \tag{3.41}$$

Since

$$p_{\text{mim}} = -V = -\frac{\alpha}{t^2}, \tag{3.42}$$

the mimetic matter mimics the dominant matter and acquires $w_{\text{mim}} = w$. However, this applies only when the mimetic matter is subdominant. The energy density of the external matter is given by

$$\rho_{\text{ext}} = 3H^2 = \frac{4}{3(1+w)^2 t^2}, \tag{3.43}$$

and therefore the subdominance condition is satisfied only when $\alpha/w \ll 1$.

We can consider more general solutions for φ, namely

$$\varphi = t + t_0. \tag{3.44}$$

In such case, the subdominant mimetic matter behaves as a cosmological constant for $t < t_0$ and then mimics the external matter for $t_0 < t$.

3.3 Cosmological perturbations

Let us analyze the behavior of cosmological perturbations in Newton gauge in a flat FRW universe dominated by mimetic matter. The stress tensor (3.26) has vanishing anisotropic stress, and therefore by means of the off-diagonal terms of equation (1.64) we can write the perturbed metric in the following form:

$$ds^2 = (1 + 2\Phi)\mathrm{d}t^2 - a^2(1 - 2\Phi)\mathrm{d}x^i \mathrm{d}x^j \delta_{ij}. \tag{3.45}$$

We consider perturbations of the mimetic matter about the solution (3.29) as follows:

$$\varphi = t + \pi(t, x). \tag{3.46}$$

The perturbed mimetic constraint (3.6) expanded to first order in perturbations fixes

$$\Phi = \dot{\pi}, \tag{3.47}$$

So we are left with only one independent scalar field, which we will take to be π. To derive an equation for this mode we will only need the 0-ith Einstein's perturbed equation (1.63), which in these coordinates take the form

$$\partial_i(\dot{\Phi} + H\Phi) = \delta T^0_{\ i}. \tag{3.48}$$

Perturbing the tensor (3.26) gives

$$\delta T^0_{\ i} = \frac{1}{2}(\rho + p)\partial_i \pi \tag{3.49}$$

$$= -\dot{H}\partial_i \pi, \tag{3.50}$$

where the second line was obtained by using the Friedmann equation (1.8). Considering (3.48) and that π is a perturbation, we obtain

$$\ddot{\pi} + H\dot{\pi} + \dot{H}\pi = 0. \tag{3.51}$$

A general solution to this equation is

$$\pi = \frac{A}{a} \int a dt, \tag{3.52}$$

where A is a constant of integration depending only on comoving spatial coordinates x. Solving for Φ, we get

$$\Phi = A\Big(1 - \frac{H}{a} \int a dt\Big). \tag{3.53}$$

The above solution is normally obtained for long wavelength perturbations, where the terms containing the speed of sound are negligible. However, since there are no spatial derivatives in (3.51), this solution is valid for all wavelengths. So the mimetic matter behaves as a pressureless dust in this respect while it still retains pressure. This introduces problems for defining quantum perturbations and thus makes the mimetic inflationary scenario fail since it would not be able to produce initial inhomogenities that would seed the large scale structure. This; however, can be remedied by yet another modification of the mimetic action (3.20).

3.3.1 Speed of sound for mimetic matter

We add a higher derivative term in the action (3.20) and show that the mimetic matter obtains a non-trivial speed of sound [18]. Consider a modified action

$$S[\varphi, g_{\mu\nu}, \lambda] = \int d^4x\sqrt{-g}\Big(-\frac{1}{2}R + \lambda(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - 1) - V(\varphi) + \frac{\gamma}{2}(\Box\varphi)^2\Big), \tag{3.54}$$

where γ is an arbitrary parameter. The equation of motion for $g_{\mu\nu}$ has the standard form

$$G_{\mu\nu} = \tilde{T}_{\mu\nu}, \tag{3.55}$$

where the stress-energy tensor is now

$$\tilde{T}^\mu_{\ \nu} = \Bigg(V + \gamma\Big(\partial_\alpha\varphi\partial^\alpha\Box\varphi + \frac{1}{2}(\Box\varphi)^2\Big)\Bigg)\delta^\mu_{\ \nu} + \\ + 2\lambda\partial^\mu\varphi\partial_\nu\varphi - \gamma\Big(\partial_\nu\varphi\partial^\mu\Box\varphi + \partial^\mu\varphi\partial_\nu\Box\varphi\Big). \tag{3.56}$$

The solution to (3.6) in FRW universe is as always $\varphi = t$. Evaluating the energy tensor for this solution and plugging into Friedmann equations (1.8), we obtain

$$3H^2 = V + \frac{3}{2}(3H^2 - 2\dot{H}) + 2\lambda, \tag{3.57}$$

$$V = \frac{2}{2 - 3\gamma}(2\dot{H} + 3H^2), \tag{3.58}$$

which allows us to solve for λ

$$\lambda = \dot{H}(3\gamma - 1). \tag{3.59}$$

Note that the equation (3.58) differs from the previous case only by a numerical factor multiplying V. This means that inclusion of the new term does not spoil the cosmological solutions we have described above. Let as analyze the behavior of scalar perturbations to see how it gets modified. Our approach is the same as in the modified case. The tensor (3.56) still has vanishing anisotropic stress, and thus we have again

$$g = (1+2\Phi)\mathrm{d}t^2 - a^2(1-2\Phi)\mathrm{d}x^i\mathrm{d}x^j\delta_{ij}, \tag{3.60}$$

$$\varphi = t + \pi, \tag{3.61}$$

$$\Phi = \dot{\pi}. \tag{3.62}$$

By perturbing the stress-energy tensor, we obtain

$$\delta T^0_{\ i} = 2\lambda\partial_i\pi - 3\gamma\dot{H}\partial_i\pi - \gamma\partial_i\delta(\Box\varphi), \tag{3.63}$$

where the perturbation of $\Box\varphi$ is

$$\delta\Box\varphi = -4\dot{\Phi} - 6H\Phi + \ddot{\pi} + 3H\dot{\pi} - \frac{\Delta}{a^2}\pi. \tag{3.64}$$

Substitution of the perturbed energy tensor into (3.48) yields the following equation for π:

$$\ddot{\pi} + \dot{H}\pi + H\dot{\pi} - c_s^2\frac{\Delta}{a^2}\pi = 0, \tag{3.65}$$

with

$$c_s^2 = \frac{\gamma}{2-3\gamma}. \tag{3.66}$$

We see that the equation for scalar perturbations developed a nontrivial speed of propagation. To solve this equation we switch to conformal time and make a plane wave expansion $\pi = \pi_k(\eta)\exp(ikx)$. Plugging this ansatz to (4.46), we obtain

$$\pi_k'' + \left(c_s^2k^2 + \mathcal{H}' - \mathcal{H}^2\right) = 0. \tag{3.67}$$

For short wavelengths, $a/k \ll c_s H^{-1}$, the solution is

$$\pi_k \propto e^{\pm i c_s k\eta}, \tag{3.68}$$

while for the long wavelengths, $a/k \gg c_s H^{-1}$, we obtain

$$\pi_k \propto \frac{1}{a}\int a^2 d\eta, \tag{3.69}$$

which is the same as (3.52).

As we have foreshadowed earlier, addition of higher time derivatives is not always safe as one might introduce new degrees of freedom that suffer from Ostrogradski instability. A simple reason to see why new degrees of freedom appear is that introducing higher time derivatives increases the order of resulting equations of motion. Such equations then require more initial conditions to define a unique solution. In our case, the solution for φ is determined by the mimetic constraint, which is unaffected by the addition of higher derivatives, and thus no new information is needed, and no additional degrees of freedom are introduced.

It is of interest to calculate the action for these perturbations by expanding the action (3.54) to second order in linear perturbations. Such action allows us to analyze the stability of the system. Moreover, it provides a check to the argument above about the number of propagating degrees of freedom. This was done in [20] in spatially flat gauge and in [24] without gauge fixing. Here we will only state the result of [20] since the calculation is a little cumbersome, and we will work out a similar analysis for a more general case in the following chapter. The action for scalar modes in spatially flat gauge is

$$\delta_2 S = \int a^3 \Bigg(-\frac{1}{c_s^2}\Big(\partial_t(H\pi)\Big)^2 + \Big(\partial_i(H\pi)\Big)^2\Bigg). \tag{3.70}$$

We see that the system suffers from a ghost instability for $c_s^2 > 0$ and from a gradient instability when $c_s^2 < 0$. It is important to mention that this behavior is already present in the original Einstein-Hilbert action; however, in the basic case when $\gamma = 0$ (such action cannot be obtained by simply evaluating this action for $\gamma = 0$) this instability is constrained and cannot grow [20]. In our case, this is no longer true.

Let us briefly discuss the problems that arise with ghosts. Ghosts are particles with a wrong sign of kinetic energy. If such particles are coupled to any ordinary matter, a spontaneous production of particle-ghost pairs occurs. This destabilizes any physical state. Since gravity couples to everything, this is bound to happen. As a result physical states decay divergently [45]. This problem can be in principle evaded by sacrificing Lorentz invariance. Consider a ghost- particle nucleation event characterized by four momentum of the ghost k_μ. The particle then has momentum $p_\mu = -k_\mu$. The decay rate of vacuum per unit volume must depend on p_μ only through the invariant $s = p_\mu p^\mu$. The full rate is given by an integral over all s and over all p_μ constrained by $s = p_\mu p^\mu$ [45]:

$$\Gamma = \int ds F(s) \int \frac{d^3p}{\sqrt{|\vec{p}|^2 + s}}, \tag{3.71}$$

where the weight $F(s)$ characterizes the given model. The latter integral is clearly divergent. However, it was argued that both integration in (3.71) should be subjected to an effective cut-off due to modifications of physical laws above certain energy scale μ [46]. Such cut-off inevitably breaks Lorentz invariance of the model. In cosmology Lorentz invariance is already broken by the presence of fundamental observers, and therefore such solution becomes more plausible.

Furthermore, in [24] it is suggested that the instability in our case is mild. Following [47] we can even find new canonical variables that are ghost free.

We find that there is really only one degree of freedom.

3.4 Ideal tracking

Let us now analyze the model (3.23) when the potential vanishes ($V = 0$) and there is external matter present characterized by some energy density and pressure [19]. As usual, we consider the flat Friedmann universe. Thanks to the vanishing potential, the theory is invariant to the shifts of the mimetic field

$$\varphi \to \varphi + c, \tag{3.72}$$

and therefore there is a conserved current

$$J^\mu = 2\lambda\partial^\mu\varphi - \gamma\partial^\mu\Box\varphi. \tag{3.73}$$

For $\varphi = t$ such current has only one nonvanishing component

$$J^0 = n = 2\lambda - 3\gamma\dot{H}, \tag{3.74}$$

which allows us to express λ in terms of the conserved charge density n. Due to the symmetries of the model, this charge density is constant on any spatial slice. Since the charge does not flow, we find that

$$\frac{d}{dt}(na^3) = 0, \tag{3.75}$$

and therefore

$$n \propto a^{-3}. \tag{3.76}$$

We see that this density describes CDM-like fluid. The energy density and pressure of mimetic matter based on (3.56), with λ eliminated using (3.73), is

$$\rho = n + \frac{9}{2}\gamma H^2, \tag{3.77}$$

$$p = -\frac{3\gamma}{2-3\gamma}\left(2\dot{H} + 3H^2\right). \tag{3.78}$$

The Friedmann equations (1.8) are therefore

$$3H^2 = \rho + \rho_{\text{ext}}, \tag{3.79}$$

$$2\dot{H} + 3H^2 = -p - p_{\text{ext}}, \tag{3.80}$$

which gives us

$$\rho = \frac{2}{2-3\gamma}n + \frac{3\gamma}{2-3\gamma}\rho_{\text{ext}}, \tag{3.81}$$

$$p = \frac{3\gamma}{2-3\gamma}p_{\text{ext}}. \tag{3.82}$$

We see that the mimetic matter in this case has two parts. One is a CDM-like (pressureless) component characterized by the shift charge density n, and the second part that tracks the external matter. To understand the effects of the mimetic matter on the cosmological dynamics, we restore the dependence on Newton's gravitational constant. The general Friedmann equations are

$$3H^2 = 8\pi G_{\text{N}}\rho, \tag{3.83}$$

$$2\dot{H} + 3H^2 = -8\pi G_{\text{N}}p. \tag{3.84}$$

By substituting from (3.82), these become

$$3H^2 = 8\pi G_{\text{N}}\frac{3\gamma}{2-3\gamma}\rho_{\text{ext}} + 8\pi G_{\text{N}}\frac{2}{2-3\gamma}n, \tag{3.85}$$

$$2\dot{H} + 3H^2 = -8\pi G_{\text{N}}\frac{3\gamma}{2-3\gamma}p_{\text{ext}}. \tag{3.86}$$

We see that the effects are twofold: mimetic matter provides us with a possible candidate for dark matter, and it effectively re-scales the Newton's gravitational constant:

$$G_{\text{eff}} = \left(1 + 3c_s^2\right)G_N. \tag{3.87}$$

4. Extended mimetic matter

Our research is motivated by the modification in action (3.54). If we allow ourselves to include higher derivative terms, then there is nothing to hold us from including terms of similar nature. Namely, we consider adding the following term to the Lagrangian (3.54) as was suggested in [19]:

$$\frac{\sigma}{2}\nabla_\mu\nabla_\nu\varphi\nabla^\mu\nabla^\nu\varphi. \tag{4.1}$$

By integrating this term by parts, we can get the original $(\Box\varphi)^2$ term with an addition of a direct coupling of the mimetic matter to curvature:

$$\int d^4x\sqrt{-g}\nabla_\mu\nabla_\nu\varphi\nabla^\mu\nabla^\nu\varphi = \int d^4x\sqrt{-g}\Big((\Box\varphi)^2 - R_{\mu\nu}\nabla^\mu\varphi\nabla^\nu\varphi\Big). \tag{4.2}$$

Keeping this in mind, we add another term with similar nature

$$\frac{\zeta}{2}R\nabla^\mu\varphi\nabla_\mu\varphi. \tag{4.3}$$

Our complete mimetic Lagrangian density has the following form:

$$\begin{aligned} L = -\frac{1}{2}R + \lambda((\partial\varphi)^2 - 1) - V(\varphi) + \gamma\frac{1}{2}(\Box\varphi)^2 + \\ + \sigma\frac{1}{2}\nabla_\mu\nabla_\nu\varphi\nabla^\mu\nabla^\nu\varphi + \zeta\frac{1}{2}R(\partial\varphi)^2. \end{aligned} \tag{4.4}$$

Building upon the ideas of [19], we can analyze an equivalent Lagrangian

$$\begin{aligned} L = -\frac{1}{2}R + \lambda((\partial\varphi)^2 - 1) - V(\varphi) - \\ - C_{\mu\nu\alpha\beta}\Big(\nabla^\nu\varphi\nabla^\mu\theta^{\alpha\beta} + \frac{1}{2}\theta^{\mu\nu}\theta^{\alpha\beta}\Big) + \zeta\frac{1}{2}R(\partial\varphi)^2, \end{aligned} \tag{4.5}$$

where $\theta^{\mu\nu}$ is an auxiliary tensor field and

$$C_{\mu\nu\alpha\beta} = \gamma g_{\mu\nu}g_{\alpha\beta} + \sigma g_{\mu\alpha}g_{\nu\beta}. \tag{4.6}$$

The equation of motion for $\theta^{\mu\nu}$ gives

$$C_{\mu\nu\alpha\beta}(\nabla^\mu\nabla^\nu\varphi - \theta^{\mu\nu}) = 0. \tag{4.7}$$

Taking the trace and then plugging the result back to this equation gives us

$$\nabla^\mu\nabla^\nu\varphi = \theta^{\mu\nu}. \tag{4.8}$$

Since equation (4.8) fully determines the field $\theta^{\mu\nu}$, it can be plugged back into the action, and thus we recover can recover the Lagrangian (4.4).

4.1 Energy-momentum tensor and Friedmann equations

In order to derive the energy-momentum tensor, we will use the Lagrangian (4.5). To simplify our calculation (more detailed calculation can be found in Appendix A), we will break up the whole energy momentum tensor into 5 parts:

$$T^{\mu}_{\ \nu} = \lambda T_{(\lambda)}{}^{\mu}{}_{\nu} + \gamma T_{(\gamma)}{}^{\mu}{}_{\nu} + \sigma T_{(\sigma)}{}^{\mu}{}_{\nu} + \zeta T_{(\zeta)}{}^{\mu}{}_{\nu} + V\delta^{\mu}_{\nu}. \tag{4.9}$$

These parts are given by

$$T_{(X)}{}_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta}{\delta g^{\mu\nu}}\frac{\partial S}{\partial X}, \tag{4.10}$$

where S is an action corresponding to (4.4) and $X \in \{\lambda, \gamma, \sigma, \zeta\}$. By varying the corresponding parts of the Lagrangian (4.5), we obtain

$$T_{(\lambda)}{}^{\mu}{}_{\nu} = \partial^{\mu}\varphi\partial_{\nu}\varphi, \tag{4.11}$$

$$T_{(\gamma)}{}^{\mu}{}_{\nu} = -\partial^{\mu}\theta\partial_{\nu}\varphi - \partial^{\mu}\varphi\partial_{\nu}\theta + \delta^{\mu}{}_{\nu}\big(\partial_{\alpha}\theta\partial^{\alpha}\varphi + \frac{\theta^2}{2}\big), \tag{4.12}$$

$$T_{(\sigma)}{}^{\mu}{}_{\nu} = \theta\theta^{\mu}{}_{\nu} + \nabla_{\alpha}\theta^{\mu}{}_{\nu}\partial^{\alpha}\varphi - \nabla_{\alpha}\theta^{\mu\alpha}\partial_{\nu}\varphi - \nabla_{\alpha}\theta_{\nu}{}^{\alpha}\partial^{\mu}\varphi - \delta^{\mu}{}_{\nu}\frac{1}{2}\theta^{\alpha\beta}\theta_{\alpha\beta}, \tag{4.13}$$

$$T_{(\zeta)}{}^{\mu}{}_{\nu} = G^{\mu}{}_{\nu} + R\partial^{\mu}\varphi\partial_{\nu}\varphi, \tag{4.14}$$

where we introduced $\theta = \theta^{\mu}_{\mu}$. In flat Friedmann universe ($k = 0$) the mimetic constraint (3.6) gives again the solution

$$\varphi(t) = t. \tag{4.15}$$

For the field θ^{μ}_{ν} we get

$$\theta^{i}_{j} = H\delta^{i}_{j}, \tag{4.16}$$

with all the other components vanishing. The parts of the total stress-energy tensor evaluated on this solution are

$$\begin{aligned}
T_{(\lambda)}{}^{0}_{0} &= 2, & T_{(\lambda)}{}^{i}_{j} &= 0,\\
T_{(\gamma)}{}^{0}_{0} &= \frac{9}{2}H^2 - 3\dot{H}, & T_{(\gamma)}{}^{i}_{j} &= \big(\frac{9}{2}H^2 + 3\dot{H}\big)\delta^{i}_{j},\\
T_{(\sigma)}{}^{0}_{0} &= \frac{9}{2}H^2, & T_{(\sigma)}{}^{i}_{j} &= \big(\frac{3}{2}H^2 + \dot{H}\big)\delta^{i}_{j},\\
T_{(\zeta)}{}^{0}_{0} &= -9H^2 - 6\dot{H}, & T_{(\zeta)}{}^{i}_{j} &= \big(3H^2 + 2\dot{H}\big)\delta^{i}_{j}.
\end{aligned} \tag{4.17}$$

The mixed space and time components are all zero.

Plugging these expressions into the formula (4.9) and the result into the Friedmann equations, we obtain a set of two equations

$$2V = (2 - 3\gamma - 2\zeta - \sigma)(2\dot{H} + 3H^2), \tag{4.18}$$

$$2\lambda = 3H^2(4\zeta - \sigma) + \dot{H}(6\gamma + 8\zeta + \sigma - 2). \tag{4.19}$$

We see that the inclusion of the new terms does not change the nature of the first equation (the potential just gets re-scaled by a numerical factor), and thus in this scenario we can again mimic cosmological expansions as we desire by choosing proper potential V.

4.2 Equations of motion for mimetic matter

Apart from the potential term, the Lagrangian (4.5) is invariant with respect to constant shifts in φ, thus the equations of motion for φ can be written as a continuity equation with the right hand side being proportional to the derivative of V instead of 0.

$$\nabla_\mu J^\mu = -V' \tag{4.20}$$

$$J^\mu = 2\lambda\partial^\mu\varphi + \zeta R\partial^\mu\varphi - \gamma\partial^\mu\theta - \sigma\nabla_\nu\theta^{\mu\nu} \tag{4.21}$$

We can plug in the solution $\varphi = t$ to obtain

$$\nabla_\mu J^\mu = \ddot{H} + 3H\dot{H}(3\gamma + 2\zeta + \sigma - 2), \tag{4.22}$$

where we used (4.19) to eliminate λ. By differentiating (4.18), we can verify that the right hand side vanishes, and therefore we see that $\varphi = t$ is indeed a solution to its equation of motion.

4.3 Fluid picture

Any solution to the mimetic constraint (3.6) provides us with a non-vanishing time-like vector $u_\mu = \partial_\mu\varphi$. Such vector defines a privileged local rest frame, which in turn defines a splitting of tensor quantities. In this section, we will decompose the stress-energy tensor (4.9) and the four current (4.21) as was done in [19]. Note that we perform this analysis while the EoM are satisfied; however, we do not assume any specific solution. The four current (4.21) in this rest frame can be written as

$$J_\mu = nu_\mu + \perp_\mu^\nu J_\nu, \tag{4.23}$$

where

$$\perp_\mu^\nu = \delta_\mu^\nu - u_\mu u^\nu. \tag{4.24}$$

It follows that

$$n = 2\lambda - \gamma\dot{\theta} + \zeta R + \sigma\theta_{\mu\nu}\theta^{\mu\nu} \tag{4.25}$$

and

$$\perp_\mu^\nu J_\nu = -\gamma(\partial_\mu\theta - u_\mu\dot{\theta}) - \sigma\big(u_\mu\theta_{\alpha\beta}\theta^{\alpha\beta} + \nabla_\alpha\theta_\mu{}^\alpha\big). \tag{4.26}$$

The dot over a letter now signifies covariant derivative along u^μ. The energy momentum tensor can be decomposed in a similar manner as the four current:

$$T_{\mu\nu} = u_\mu u_\nu\rho + q_\mu u_\nu + q_\nu u_\mu + \perp_\mu^\alpha\perp_\nu^\beta T_{\alpha\beta}, \tag{4.27}$$

where

$$q_\mu = \perp_\mu^\alpha u^\beta T_{\alpha\beta}, \tag{4.28}$$

$$\rho = u^\mu u^\nu T_{\mu\nu}. \tag{4.29}$$

Before we move on, let us take a closer look on the contribution (4.14). The term that is directly proportional to the Einstein tensor does not behave very nicely under the decomposition (4.27); however, its effect on the level of equations of motion is simple. Let us consider EFE for our system

$$\begin{aligned} G_{\mu\nu} &= \zeta G_{\mu\nu} + T^{\text{rest}}_{\mu\nu} \\ &= \frac{1}{1-\zeta} T^{\text{rest}}_{\mu\nu}. \end{aligned} \tag{4.30}$$

We see that it effectively re-scales the remaining terms in the energy tensor by a factor of $1/(1-\zeta)$. For that reason, we shall not decompose it but rather re-scale everything else. With this in mind, we obtain

$$\rho = \frac{1}{1-\zeta}\Big(2\lambda - \gamma\big(\dot{\theta} - \frac{1}{2}\theta^2\big) + \frac{3}{2}\sigma\theta_{\alpha\beta}\theta^{\alpha\beta} + \zeta R\Big), \tag{4.31}$$

$$q_\mu = \frac{1}{1-\zeta} \perp_\mu^\nu J_\nu, \tag{4.32}$$

$$\perp_\mu^\alpha \perp_\nu^\beta T_{\alpha\beta} = \frac{1}{1-\zeta}\Big(\sigma\big(\theta\theta_{\mu\nu} + \dot{\theta}_{\mu\nu} - \perp_{\mu\nu} \frac{1}{2}\theta_{\alpha\beta}\theta^{\alpha\beta}\big) + \gamma \perp_{\mu\nu} \big(\dot{\theta} + \frac{1}{2}\theta^2\big)\Big). \tag{4.33}$$

Important quantities we can calculate are the pressure, defined as

$$\begin{aligned} p &= -\frac{1}{3} \perp_{\mu\nu} T^{\mu\nu} \\ &= \frac{1}{1-\zeta}\Big(- \gamma\big(\dot{\theta} + \frac{1}{2}\theta^2\big) + \sigma\big(\frac{1}{2}\theta_{\alpha\beta}\theta^{\alpha\beta} - \frac{1}{3}(\theta^2 + \dot{\theta})\big)\Big), \end{aligned} \tag{4.34}$$

and anisotropic stress

$$\begin{aligned} \Pi_{\mu\nu} &= \perp_{\mu\alpha}\perp_{\nu\beta} T^{\alpha\beta} + \perp_{\mu\nu} p \\ &= \frac{\sigma}{1-\zeta}\Big(\theta\big(\theta_{\mu\nu} - \frac{1}{3} \perp_{\mu\nu} \theta\big) + u^\alpha\nabla_\alpha\big(\theta_{\mu\nu} - \frac{1}{3} \perp_{\mu\nu} \theta\big)\Big). \end{aligned} \tag{4.35}$$

While we introduced the field $\theta_{\mu\nu}$ as an auxiliary field to help us with calculations, it turns out that it has a rather nice kinematical interpretation. Namely $\theta_{\mu\nu}$ is exactly the expansion tensor for the congruence defined by u_μ. The trace θ is the expansion. Tthe shear tensor $\sigma_{\mu\nu}$,as we can notice, enters the anisotropic stress

$$\Pi_{\mu\nu} = \frac{\sigma}{1-\zeta}\Big(\theta\sigma_{\mu\nu} + \dot{\sigma}_{\mu\nu}\Big). \tag{4.36}$$

As we shall see in the next section, this anisotropic stress does not vanish at first order of perturbations around flat FRW universe. This signals a modification of propagation of gravitational waves [48].

An extensive survey of various of various imperfect fluid models of dark matter was carried out in [49].

4.4 Scalar perturbations

The most exciting prospect of extending the mimetic matter is the effect of new terms on the behavior of perturbations. Let us first analyze them on the level of perturbed Einstein equations of motion in the Newtonian gauge

$$ds^2 = (1+2\Phi)d\eta^2 - a^2(t)\Big[(1-2\Psi)\delta_{ij}dx^i dx^j\Big]. \tag{4.37}$$

To calculate equations for the perturbations we will only need the 0-ith (1.63) and off-diagonal i-jth (1.64) Einstein's equations, which in these coordinates take the form

$$\partial_i(\dot{\Psi} + H\Phi) = \delta T^0_{\ i}, \tag{4.38}$$
$$\partial_i\partial_j(\Phi - \Psi) = a^2\delta T^i_{\ j} \qquad \text{for } i \neq j. \tag{4.39}$$

The mimetic matter field perturbation is as in the previous chapter

$$\varphi = t + \pi(t, x). \tag{4.40}$$

The perturbed mimetic constraint (3.6) again fixes

$$\Phi = \dot{\pi}, \tag{4.41}$$

and the second equation gives us

$$\partial_i\partial_j(\Phi - \Psi) = \frac{\sigma}{\zeta - 1}\partial_i\partial_j(\dot{\pi} + H\pi), \tag{4.42}$$
$$\Psi = \frac{\sigma}{1 - \zeta}(\dot{\pi} + H\pi) + \Phi, \tag{4.43}$$

where we have integrated out the derivatives. This can be done free of integration constants since such terms would not be consistent with Φ and Ψ being perturbations. Note that the auxiliary field $\theta^{\mu\nu}$ has to be perturbed as well; however, such perturbations can be reduced to perturbations of φ by means of the equation of motion (4.8). For the details of these calculations please refer to the Appendix B. Using the above expression, we can eliminate Ψ from the first equation (4.38), which is

$$\partial_i(\dot{\Psi} + H\Phi) = \partial_i\Big(- \dot{H}\pi + \frac{\gamma + \sigma}{2 - 3\gamma - 2\zeta - \sigma}\frac{\Delta}{a^2}\pi\Big), \tag{4.44}$$
$$\dot{\Psi} + H\Phi = -\dot{H}\pi + \frac{\gamma + \sigma}{2 - 3\gamma - 2\zeta - \sigma}\frac{\Delta}{a^2}\pi, \tag{4.45}$$

where we again used the consistency conditions for perturbations. Combining equations (4.41), (4.43) and (4.45) and using π as an independent variable, we obtain

$$\ddot{\pi} + \dot{H}\pi + H\dot{\pi} - c_s^2\frac{\Delta}{a^2}\pi = 0, \tag{4.46}$$

where

$$c_s^2 = \frac{\gamma + \sigma}{(2 - 3\gamma - 2\zeta - \sigma)}\frac{1 - \zeta}{(1 + \sigma - \zeta)}. \tag{4.47}$$

Current constraints of such speed for cold dark matter in late times is provided by the CMB lensing that gives an upper bound [50]

$$c_s^2 < 10^{-5.9}. \tag{4.48}$$

In order to study instabilities of the system, we have to calculate the action for scalar perturbations (for a detailed calculation please refer to Appendix C). We do this in spatially flat gauge (1.56) following the example of [20]. For simplicity, we set the potential $V = 0$. This introduces a certain freedom to the result because we can freely use (4.18):

$$2\mathcal{H}' + \mathcal{H}^2 = 0. \tag{4.49}$$

This freedom can be fixed using the result of [20] for the unextended case $\sigma = \zeta = 0$. We will again break the calculation to the respective contributions of terms that are directly multiplied by our three parameters γ, σ, ζ. Note that the perturbation of Lagrange multiplier $\delta\lambda$ is again a Lagrange multiplier that fixes

$$\Phi = \frac{\pi'}{a}, \tag{4.50}$$

which can be used to eliminate Φ from the expansion. The second order of the expanded action yields

$$\frac{2a^2}{\gamma}\delta_2 L_\gamma = \Big[\Delta\big(B + \frac{\pi}{a}\big)\Big]^2 + 3\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + \frac{15}{2}\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 9\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2, \tag{4.51}$$

$$\frac{2a^2}{\sigma}\delta_2 L_\sigma = \Big[\Delta\big(B + \frac{\pi}{a}\big)\Big]^2 + \Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + \frac{5}{2}\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 3\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2, \tag{4.52}$$

$$\frac{2a^2}{\zeta}\delta_2 L_\zeta = 2\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + 3\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 6\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2 , \tag{4.53}$$

$$2a^2\delta_2 L_{EH} = 2\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + 3\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 6\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2. \tag{4.54}$$

The last term corresponds to terms that are not multiplied by any of the parameters (for example the Einstein-Hilbert term). The variation with respect to B yields

$$2\Delta\big(B + \frac{\pi}{a}\big) = \frac{2 - 3\gamma - 2\sigma - \zeta}{\gamma + \sigma}\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big). \tag{4.55}$$

This equation is not dynamical, and therefore it acts as a constraint of the field B and can be substituted back into the action given by the above Lagrangians [51]. By doing so, we obtain

$$\delta_2 S = (1 - \zeta)\int d^4x a^2\Bigg[-\frac{1}{c_s^2}\Big(\partial_\eta\big(\pi\mathcal{H}a^{-1}\big)\Big)^2 + \Big(\partial_i\big(\pi\mathcal{H}a^{-1}\big)\Big)^2\Bigg], \tag{4.56}$$

where c_s^2 is given by (4.47). Note that this action will not simply reproduce equation (4.46) because we are working in a different gauge.

The above analysis also gives us a solid check that there is indeed only one scalar degree of freedom propagating in our model, despite the inclusion of higher order time derivatives in the Lagrangian (4.4).

A noteworthy feature of this model is that depending on the sign of $1-\zeta$ this action possesses a ghost instability. This seems to be encouraging since the action derived in [20] has ghost instabilities for any non-trivial choice of the parameter γ. In this extension we obtained control over this problem. Unfortunately, as we eliminate ghosts here, a new instability pops out elsewhere. We will show this in the following section.

4.5 Tensor perturbations

The new terms have had effect on the speed of scalar perturbations; however, this effect did not bring anything crucially new to the model apart from having more free parameters to play with. However, there is a much more significant novel effect of these terms on the behavior of gravitational waves. For a detailed calculation please refer to Appendix D.

Considering only the tensor perturbations, one finds that

$$\delta T_{(\gamma)}{}^{\mu}_{\nu} = 0, \tag{4.57}$$

and thus we see that in the original model such behavior is nonexistent.

Transforming the perturbed equations of motion (1.67) to normal time t yields

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta}{a^2}h_{ij} = 2\delta T^i{}_j. \tag{4.58}$$

After we calculate the perturbed energy tensor, we obtain the following equation for tensor modes

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - c_T^2\frac{\Delta}{a^2}h_{ij} = 0, \tag{4.59}$$

where

$$c_T^2 = \frac{1-\zeta}{1-\zeta+\sigma}. \tag{4.60}$$

We see that the novel terms in the mimetic action allow us to model different speeds of propagation of gravitational waves. Thus the imperfect DM can have a nontrivial refractive index for the gravitational waves. This speed is related to the speed of scalar mode by a simple relation:

$$c_s^2 = \frac{\gamma+\sigma}{2-3\gamma-2\zeta-\sigma}c_T^2. \tag{4.61}$$

Current observational bounds on the speed of gravitational waves are very mild, but we expect them to improve as soon as the first electromagnetic signals accompanying gravity waves are detected. The current upper bound from the first detections [52], [53] are

$$c_T < 1.7. \tag{4.62}$$

A lower bound is provided from the pulsar timing [54]

$$1 - c_T < 10^{-2}. \tag{4.63}$$

In order to check whether or not the perturbations suffer from some sort of instability, we derive the quadratic action for such perturbations. This is done by expanding the action corresponding to the Lagrangian (4.4) to second order in linear perturbations. Since we are expanding about the solution of the classical equations of motion, the first order perturbations vanish. Including the second order only and making a similar decomposition as (4.9) for respective Lagrangian densities we obtain

$$\delta_2 S_\gamma = -\int d^4x \frac{3}{2}\gamma a^3 H\Big(\frac{3}{4}Hh_{ij}h_{ij} + h_{ij}\dot{h}_{ij}\Big), \tag{4.64}$$

$$\delta_2 S_\sigma = \int d^4x \frac{\sigma}{2} a^3\Big(\frac{1}{4}\dot{h}_{ij}\dot{h}_{ij} - Hh_{ij}\dot{h}_{ij} - \frac{3}{4}H^2 h_{ij}h_{ij}\Big), \tag{4.65}$$

$$\delta_2 S_\zeta = -\int d^4x \frac{\zeta}{8} a^3\Big(\dot{h_{ij}}\dot{h_{ij}} - \partial_k h_{ij}\partial^k h_{ij} - 4Hh_{ij}\dot{h}_{ij} - 6(\dot{H} + 2H^2)h_{ij}h_{ij}\Big), \tag{4.66}$$

$$\delta_2 S_\lambda = 0, \tag{4.67}$$

$$\delta_2 S_V = \int d^4x \frac{V}{4} h_{ij}h_{ij}, \tag{4.68}$$

$$\delta_2 S_{EH} = \int d^4x \frac{1}{8} a^3\Big(\dot{h_{ij}}\dot{h_{ij}} - \partial_k h_{ij}\partial^k h_{ij} - 4Hh_{ij}\dot{h}_{ij} - 6(\dot{H} + 2H^2)h_{ij}h_{ij}\Big), \tag{4.69}$$

where $\delta_2 S_{EH}$ is the perturbation of the Einstein-Hilbert action. By integrating by parts and using the first Friedmann equation (4.18), we obtain

$$\delta_2 S_T = \frac{1}{8}\int d^3x dt\ a^3\Big((1-\zeta+\sigma)\dot{h_{ij}}\dot{h_{ij}} - (1-\zeta)\partial_k h_{ij}\partial^k h_{ij}\Big). \tag{4.70}$$

The above action reproduces the correct equations of motion (4.59). We see that, while both $(1-\zeta+\sigma)$ and $(1-\zeta)$ are positive, gravitational waves do not suffer from ghost instabilities neither from gradient instability. Therefore, if we eliminate ghosts in the action (4.56) by setting $1-\zeta < 0$, we create at least a gradient instability for tensors.

4.6 Tracking behavior

Following the example of [19], we want to confirm whether the mimetic matter has the same tracking properties as it had before the modifications we introduced. We also derive how the effective Newton's constant changes. Therefore, we consider our model with no potential in the presence of external matter, in flat Friedmann universe. From the current (4.21) we obtain

$$J^0 = n = 2\lambda - 6\zeta(\dot{H} + 2H^2) - 3\gamma\dot{H} + 3\sigma H^2. \tag{4.71}$$

Since this is the only nonzero component of the four-current, it follows that this charge density dissipates as

$$n \propto a^{-3}. \tag{4.72}$$

The stress-energy tensor (4.17) gives

$$\rho = n + \frac{3}{2}H^2\big(3\gamma + 2\zeta + \sigma\big), \tag{4.73}$$

$$p = \big(3H^2 + 2\dot{H}\big)\frac{3\gamma + 2\zeta + \sigma}{2}. \tag{4.74}$$

From Friedmann equations (1.8) we then obtain

$$\rho = \frac{2}{2-3\gamma-2\zeta-\sigma} n + \frac{3\gamma+2\zeta+\sigma}{2-3\gamma-2\zeta-\sigma} \rho_{\rm ext}, \tag{4.75}$$

$$p = \frac{3\gamma+2\zeta+\sigma}{2-3\gamma-2\zeta-\sigma} p_{\rm ext}. \tag{4.76}$$

We see that qualitatively the tracking behavior does not change; however, we do see a difference in the modification of Newton's constant. In the previous case, it was (apart from a factor of 3) the speed of sound of scalar mode. In our model this is no longer true. The effective Newton's constant is

$$G_{\rm eff} = \Big(1 + \frac{3\gamma+2\zeta+\sigma}{2-3\gamma-2\zeta-\sigma}\Big) G_{\rm N}. \tag{4.77}$$

Conclusion

In order to set the stage for our research, we first laid down some basic concepts from cosmology and tools from general relativity. In particular, we reviewed the Friedmann-Robertson-Walker models and their elementary solutions. We discussed several noteworthy physical horizons that are present in these models and are relevant for understanding the cosmos. After that we continued with an introduction to the gauge problem of cosmological perturbations and their dynamics as given by standard general theory of relativity. We also briefly mentioned dark matter, quintessence and inflation to provide the reader with a surface knowledge of related ideas in contemporary cosmology that go beyond the standard model of cosmology.

Our research focused on mimetic matter, a recently proposed model that introduces a minimal modification of general relativity [3]. This modification provides us with a pressureless fluid (described by a scalar field). This fluid can mimic cold dark matter on cosmological scales. We showed how such modification works, and how mimetic matter arises from it. We continued to review research of extensions of this scenario prior to this thesis [19], [18]. Firstly, we discussed the possibilities of adding a potential for mimetic matter, which allows us to mimic other types of matter (apart from pressureless dust) like various cosmic fluids and quintessence. Secondly, we showed how addition of higher derivatives introduced in [18] of mimetic matter allows us to deviate from perfect fluid to an imperfect fluid with non-vanishing speed of sound for cosmological perturbations.

In this thesis we introduced a novel higher derivative terms of more general structure and terms that couple directly to curvature. We found that such extensions provide us with further changes of the speed of sound for cosmological perturbations as well as changes of the speed of propagation of gravity waves. Thus imperfect dark matter can have a nontrivial refractive index for gravity waves. The current bounds on this refractive index are mild [53]. But now after the discovery of the gravitational waves [52] by the LIGO collaboration one can expect that the constraints will drastically improve. This will happen when the first electromagnetic signal accompanying GW is observed. Furthermore, the additional terms renormalize Newton's constant in Friedmann equations. Lastly, we investigated the stability of this system on the level of linear perturbations. In particular, we derived quadratic action for scalar and tensor perturbations to analyze a potential ghost instability. On the level of linear perturbations in the usual variables the system either exhibits ghost instabilities in the scalar or in the tensor sector. However, following the previous discussion in [24] these instabilities can be rather mild and phenomenologically acceptable. Moreover, there correct canonical variables where there no ghost instabilities around cosmological solutions at all [47]. Finally we analyse the observational bounds on the parameters of our model arising from the constraint on the speed of gravitational waves etc. We think the models of imperfect DM with anisotropic stress are very interesting because their parameters can be constrained in near future by the GW observations. Thus from GW observations one can infer mechanical properties of DM. This opens up a new exciting opportunity to learn about the origins of the dark sector.

A. Energy-momentum tensor for flat FRW universe

In this appendix we derive the energy momentum tensors (4.9), and then we evaluate them for the flat FRW solution of EFE. We consider the Lagrangian density with the auxiliary field $\theta_{\mu\nu}$ (4.5) and break it up to contributions from different parameters:

$$L_\gamma = -\gamma\Big(\partial_\mu\varphi\partial^\mu\theta + \frac{1}{2}\theta^2\Big), \tag{A.1}$$

$$L_\zeta = \zeta\Big(R\partial_\mu\varphi\partial^\mu\varphi\Big), \tag{A.2}$$

$$L_\sigma = \sigma\Big(\nabla_\mu\nabla_\nu\varphi\theta^{\mu\nu} - \frac{1}{2}\theta_{\mu\nu}\theta^{\mu\nu}\Big). \tag{A.3}$$

Note that the σ contribution is obtained from (4.5) by integrating by parts. The rest of the terms in (4.5) are trivial to vary so we will not discuss them here. The variation of (A.3) is

$$\delta L_\gamma = \gamma\partial^\mu\varphi\partial^\nu\theta\delta g_{\mu\nu}, \tag{A.4}$$

$$\delta L_\sigma = \sigma\Big(\partial^\nu\nabla_\rho\theta^{\mu\rho} - \frac{1}{2}\theta\theta^{\mu\nu} - \frac{1}{2}\partial^\rho\nabla_\rho\theta^{\mu\nu}\Big)\delta g_{\mu\nu}, \tag{A.5}$$

$$\delta L_\zeta = -\zeta\Big(R\partial^\mu\varphi\partial^\nu\varphi + R^{\mu\nu}\Big). \tag{A.6}$$

Note that we omitted total derivatives and we used the equations of motion (4.8) and the mimetic constraint (3.6) after the variation. We also used the standard formulas for variation of the Christoffel symbol and of the Ricci tensor

$$\delta\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\iota}\Big(\nabla_\mu\delta g_{\iota\nu} + \nabla_\nu\delta g_{\iota\mu} - \nabla_\iota\delta g_{\mu\nu}\Big), \tag{A.7}$$

$$\delta R_{\mu\nu} = \nabla_\rho\delta\Gamma^\rho_{\mu\nu} - \nabla_\mu\delta\Gamma^\rho_{\rho\nu}. \tag{A.8}$$

Our sign convention for the Einstein-Hilbert term gives us the following expression for the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}}. \tag{A.9}$$

Combining the above, we the energy momentum tensor

$$T^\mu_{\ \nu} = \lambda T_{(\lambda)}{}^\mu{}_\nu + \gamma T_{(\gamma)}{}^\mu{}_\nu + \sigma T_{(\sigma)}{}^\mu{}_\nu + \zeta T_{(\zeta)}{}^\mu{}_\nu + V\delta^\mu_\nu., \tag{A.10}$$

$$T_{(\lambda)}{}^\mu{}_\nu = \partial^\mu\varphi\partial_\nu\varphi, \tag{A.11}$$

$$T_{(\gamma)}{}^\mu{}_\nu = -\partial^\mu\theta\partial_\nu\varphi - \partial^\mu\varphi\partial_\nu\theta + \delta^\mu{}_\nu\big(\partial_\alpha\theta\partial^\alpha\varphi + \frac{\theta^2}{2}\big), \tag{A.12}$$

$$T_{(\sigma)}{}^\mu{}_\nu = \theta\theta^\mu{}_\nu + \nabla_\alpha\theta^\mu{}_\nu\partial^\alpha\varphi - \nabla_\alpha\theta^{\mu\alpha}\partial_\nu\varphi - \nabla_\alpha\theta_\nu{}^\alpha\partial^\mu\varphi - \delta^\mu{}_\nu\frac{1}{2}\theta^{\alpha\beta}\theta_{\alpha\beta}, \tag{A.13}$$

$$T_{(\zeta)}{}^\mu{}_\nu = G^\mu{}_\nu + R\partial^\mu\varphi\partial_\nu\varphi. \tag{A.14}$$

In flat FRW universe the solution for mimetic field is$\varphi = t$ from (3.6)and the Christoffel symbols are

$$\Gamma^0_{ij} = \delta_{ij} a^2 H, \qquad \Gamma^i_{0j} = H\delta^i_j, \tag{A.15}$$

with all the others components vanishing. For the θ fields we obtain

$$\theta = 3H, \qquad \theta^i{}_j = \delta^i_j H. \tag{A.16}$$
$$\tag{A.17}$$

The other components of $\theta^\mu{}_\nu$ are 0. The derivatives of $\theta^\mu{}_\nu$ are:

$$\nabla_\mu \theta_0{}^\mu = -3H^2, \tag{A.18}$$
$$\nabla_\mu \theta_i{}^\mu = 0, \tag{A.19}$$
$$\nabla_0 \theta^i{}_j = \delta^i{}_j \dot{H}, \tag{A.20}$$
$$\nabla_0 \theta^0{}_\mu = 0. \tag{A.21}$$
$$\tag{A.22}$$

The Einstein tensor and scalar curvature are:

$$R = -6\big(\dot{H} + 2H^2\big), \tag{A.23}$$
$$G^0_0 = 3H^2, \tag{A.24}$$
$$G^i{}_j = 3H^2 + 2\dot{H}. \tag{A.25}$$

Combining the above results we obtain

$$\begin{aligned} T_{(\lambda)}{}^0_0 &= 2 & T_{(\lambda)}{}^i_j &= 0 \\ T_{(\gamma)}{}^0_0 &= \frac{9}{2}H^2 - 3\dot{H} & T_{(\gamma)}{}^i_j &= \big(\frac{9}{2}H^2 + 3\dot{H}\big)\delta^i_j \\ T_{(\sigma)}{}^0_0 &= \frac{9}{2}H^2 & T_{(\sigma)}{}^i_j &= \big(\frac{3}{2}H^2 + \dot{H}\big)\delta^i_j \\ T_{(\zeta)}{}^0_0 &= -9H^2 - 6\dot{H} & T_{(\zeta)}{}^i_j &= \big(3H^2 + 2\dot{H}\big)\delta^i_j. \end{aligned} \tag{A.26}$$

Hence the equations

$$2V = (2 - 3\gamma - 2\zeta - \sigma)(2\dot{H} + 3H^2), \tag{A.27}$$
$$2\lambda = 3H^2(4\zeta - \sigma) + \dot{H}(6\gamma + 8\zeta + \sigma - 2). \tag{A.28}$$

B. First order scalar perturbations

We perturb the system around flat FRW universe in the Newtonian gauge to the first order in linear perturbations:

$$ds^2 = (1 + 2\Phi)d\eta^2 - a^2(t)\Big[(1 - 2\Psi)\delta_{ij}dx^i dx^j\Big]. \tag{B.1}$$

The variation of the mimetic field is $\varphi \to t + \pi$. From the mimetic constraint we obtain $\dot{\pi} = \Phi$. The zero order quantities are calculated in the Appendix A. The relevant perturbed Christoffel symbols are

$$\delta\Gamma^{\mu}_{0\nu} = \begin{pmatrix} \dot{\Phi} & \vec{\nabla}\Phi \\ a^{-2}\vec{\nabla}\Phi & -\dot{\Psi}\mathbf{I} \end{pmatrix}, \qquad \delta\Gamma^{0}_{\mu\nu} = \begin{pmatrix} \dot{\Phi} & \vec{\nabla}\Phi \\ \vec{\nabla}\Phi & -a^2\big(2H\Psi + \dot{\Psi} + 2H\Phi\big)\mathbf{I} \end{pmatrix}, \tag{B.2}$$

$$\delta\Gamma^{\nu}_{\mu\nu} = \partial_\mu\big(\Phi - 3\Psi\big), \qquad \delta\Gamma^{i}_{\mu i} = -3\partial_\mu\Psi, \tag{B.3}$$

where we have expanded the indexes μ and ν into matrices. $\vec{\nabla}$ is the standard gradient operator from 3-dimensional vector analysis and $\mathbf{I}$ is a 3x3 identity matrix. Using the above results it follows that for $i \neq j$

$$\begin{aligned} \delta(\theta\theta^i{}_j) &= -3Ha^{-2}\partial_i\partial_j\pi, \\ \delta(\nabla_\alpha\theta^i{}_j\partial^\alpha\varphi) &= -a^{-2}\partial_i\partial_j(\dot{\pi} - 2H\pi), \\ \delta(\nabla_\alpha\theta^{i\alpha}\partial_j\varphi) &= 0, \\ \delta(\nabla_\alpha\theta_j{}^\alpha\partial^i\varphi) &= 0, \\ \delta(R\partial^i\varphi\partial_j\varphi) &= 0, \end{aligned}$$

and for $0, i$

$$\begin{aligned} \delta(\theta\theta^0{}_i) &= -3H^2\partial_i\pi, \\ \delta(\nabla_\alpha\theta^0{}_i\partial^\alpha\varphi) &= -\dot{H}\partial_i\pi, \\ \delta(\nabla_\alpha\theta^{0\alpha}\partial_i\varphi) &= -3H^2\partial_i\pi, \\ \delta(\nabla_\alpha\theta_i{}^\alpha\partial^0\varphi) &= -\dot{H}\partial_i\pi - 3H^2\partial_i, \pi - \partial_i(\dot{\Psi} + H\Phi) - \partial_i\frac{\Delta}{a^2}\pi, \\ \delta(R\partial^0\varphi\partial_i\varphi) &= -6(\dot{H} + 2H^2)\partial_i\pi. \end{aligned}$$

The variation of the Einstein tensor can be inferred from equations (1.63), (1.64)

$$\begin{aligned} \delta G^i{}_j &= a^{-2}\partial_i\partial_j(\Phi - \Psi) \qquad \text{for } i \neq j, \\ \delta G^0{}_i &= 2\partial_i(\dot{\Psi} + H\Phi). \end{aligned}$$

Substituting these into the perturbation of (A.10) we obtain for $i \neq j$

$$\delta T_{(\gamma)\ j}^{\ i} = 0, \tag{B.4}$$
$$\delta T_{(\sigma)\ j}^{\ i} = -a^{-2}\partial_i\partial_j(\dot{\pi} + H\pi), \tag{B.5}$$
$$\delta T_{\zeta\ j}^{\ i} = a^{-2}\partial_i\partial_j(\Phi - \Psi), \tag{B.6}$$
$$\tag{B.7}$$

and for $0, i$

$$\delta T_{(\gamma)\ i}^{\ 0} = \partial_i\Big(- 3\dot{H}\pi + 3(\dot{\Psi} + H\Phi) + \frac{\Delta}{a^2}\pi\Big), \tag{B.8}$$
$$\delta T_{(\sigma)\ i}^{\ 0} = \partial_i\Big(3H^2\pi + (\dot{\Psi} + H\Phi) + \frac{\Delta}{a^2}\pi\Big), \tag{B.9}$$
$$\delta T_{(\zeta)\ i}^{\ 0} = \partial_i\Big(2(\dot{\Psi} + H\Phi) - 6(\dot{H} + 2H^2)\pi\Big). \tag{B.10}$$
$$\tag{B.11}$$

Substituting the above results into (1.63) and (1.64) we obtain

$$\Psi = \frac{\sigma}{1-\rho}(\dot{\pi} + H\pi) + \Phi, \tag{B.12}$$
$$(\dot{\Psi} + H\Phi) = -\dot{H}\pi + \frac{\gamma + \sigma}{(2 - 3\gamma - 2\rho - \sigma)}\frac{\Delta}{a^2}\pi. \tag{B.13}$$
$$\tag{B.14}$$

Note that a direct substitution yields only derivatives of the above formulas. Our result follows from the fact that Φ, Ψ and π are perturbations (any other solution does not decay properly in spatial infinity). Furthermore we have $\dot{\pi} = \Phi$ and therefore we can eliminate Ψ and Φ to obtain

$$\ddot{\pi} + \dot{H}\pi + H\dot{\pi} - c_s^2\frac{\Delta}{a^2} = 0, \tag{B.15}$$

where

$$c_s^2 = \frac{\gamma + \sigma}{(2 - 3\gamma - 2\zeta - \sigma)}\frac{1 - \zeta}{(1 + \sigma - \zeta)}. \tag{B.16}$$

C. Second order perturbations for scalars

We perturb the system (4.4) around a flat FRW universe in spatially flat gauge to second order in linear perturbations with vanishing potential. Let us start by defining the basic perturbations:

$$ds^2 = a^2(\eta)\Big[(1+2\phi)d\eta^2 - 2\partial_i B dt dx^i - \delta_{ij}dx^i dx^j\Big], \tag{C.1}$$

$$\varphi = t + \pi, \tag{C.2}$$

$$\lambda = \frac{1}{2}\big(\mathcal{H}^2(4\zeta - 4\sigma - 6\gamma + 2) + \dot{H}(6\gamma + 8\zeta + \sigma - 2)\big) + \delta\lambda. \tag{C.3}$$

The only term in the action that contains $\delta\lambda$ is

$$2\delta\lambda\Big(\frac{\pi}{a} - \phi\Big), \tag{C.4}$$

and therefore it is a Lagrange multiplier and enforces $\pi = a\phi$. For the time being, we will not eliminate ϕ from our calculations just because of the inconvenient factor of a; however, keep in mind that we can always switch to π. The zero order quantities are the same as in Appendix A except that now we work in conformal time. Relevant objects defined by the perturbed metric are: the inverse metric

$$g^{00} = a^{-2}\big((1 - 2\phi - \partial_i B\partial_i B + 4\phi^2)\big), \tag{C.5}$$

$$g^{0i} = a^{-2}\big(\partial_i B(1-2\phi)\big), \tag{C.6}$$

$$g^{ij} = a^{-2}\big(\partial_i B\partial_j B - \delta_{ij}\big), \tag{C.7}$$

the volume element

$$\sqrt{-g} = a^4\Big(1 + \phi + \frac{1}{2}(\partial_i B\partial_i B - \phi^2)\Big), \tag{C.8}$$

and the Christoffel symbols of the first kind

$$\Gamma_{i\mu\nu} = -a^2\begin{pmatrix} 2\mathcal{H}\partial_i B + \partial_i B' - \partial_i\phi & \mathcal{H}\vec{\delta}_i \\ \mathcal{H}\vec{\delta}_i & 0\end{pmatrix}, \tag{C.9}$$

$$\Gamma_{0\mu\nu} = a^2\begin{pmatrix} \mathcal{H}(1+2\phi) + \phi' & \vec{\nabla}\phi \\ \vec{\nabla}\phi & \mathcal{H}\mathbf{I} - \vec{\nabla}\vec{\nabla}B\end{pmatrix}, \tag{C.10}$$

where we have again expanded the indeces μ and ν into matrices.$\vec{\delta}_i$ is a 3-vector whose ith entry is equal to 1 and the rest to 0. For the second covariant derivatives of φ we have

$$\nabla_\mu\nabla_\nu\varphi = \partial_\mu\partial_\nu\varphi - \Gamma_{\rho\mu\nu}\partial^\rho\varphi. \tag{C.11}$$

The covariant and contravariant gradient of φ is

$$\partial_\mu\varphi = \begin{pmatrix} a(1+\phi) \\ \vec{\nabla}\pi\end{pmatrix}, \tag{C.12}$$

$$\partial^\mu\varphi = a^{-1}\begin{pmatrix} 1 - \phi - \partial_i B\partial_i(B + \pi/a) + 2\phi^2 \\ -\vec{\nabla}(B + \pi/a) + \vec{\nabla}B\phi\end{pmatrix}. \tag{C.13}$$

Combining the above results, we obtain

$$\nabla_0\nabla_0\varphi = a\Big((B' + \mathcal{H}B + \phi)\Delta(B + \pi/a) + \phi\phi'\Big), \tag{C.14}$$

$$\nabla_0\nabla_i\varphi = a\big(\mathcal{H}\phi\partial_i B + \phi\partial_i\phi - \mathcal{H}\partial_i(B + \pi/a)\big), \tag{C.15}$$

$$\nabla_i\nabla_j\varphi = \Big(- \mathcal{H}(1 + B\Delta(B + \pi/a) - \phi + 2\phi^2)\Big)\delta_{ij} + \partial_i\partial_j(B + \pi/a) - \phi\partial_i\partial_j B. \tag{C.16}$$

The second order perturbation of some of the terms in the action corresponding to (4.4) can be obtained from the above results by contraction with the inverse metric.

$$\begin{aligned}\delta_2\sqrt{-g}\nabla_\mu\nabla_\nu\varphi\nabla^\mu\nabla^\nu\varphi = a^2\Big(&\big[\Delta(B + \pi/a)\big]^2 + \frac{3}{2}\mathcal{H}^2\partial_i B\partial_i B +\\ &+6B\Delta(B + \pi/a) - 2\frac{\mathcal{H}}{a^2}\partial_i\pi\partial_i\pi + 2\Delta B\mathcal{H}\phi\Big),\end{aligned} \tag{C.17}$$

$$\begin{aligned}\delta_2\sqrt{-g}(\Box\varphi)^2 = a^2\Big(&\big[\Delta(B + \pi/a)\big]^2 + 6B'\Delta(B + \pi/a) + \frac{21}{2}\partial_i B\partial_i B\mathcal{H}^2 +\\ &+24B\Delta(B + \pi/a)\mathcal{H} + 12\mathcal{H}^2\partial_i B\partial_i\pi a^{-1} + 6\Delta B\mathcal{H}\phi +\\ &+6\mathcal{H}\phi\Delta(B + \pi/a) + \frac{45}{2}\mathcal{H}^2\phi^2 + 6\mathcal{H}\phi\phi'\Big),\end{aligned} \tag{C.18}$$

$$\begin{aligned}\delta_2\sqrt{-g}\lambda(\partial_\mu\varphi\partial^\mu\varphi - 1) = \frac{a^2}{2}&\big(\mathcal{H}^2(4\zeta - 4\sigma - 6\gamma + 2) + \dot{H}(6\gamma + 8\zeta + \sigma - 2)\big)\\ &\Big(\phi^2 - \partial_i\big(B + \pi/a\big)\partial_i\big(B + \pi/a\big)\Big).\end{aligned} \tag{C.19}$$

Note that we have omitted several total derivatives. The second order perturbation of the Einstein-Hilbert term was calculated in [20]:

$$\begin{aligned}\delta_2\sqrt{-g}R = a^2\Big(&2\Delta B\big(2\mathcal{H}\phi - 3\mathcal{H}^2\pi/a\big) - 3\mathcal{H}^2\partial_i\pi\partial_i\pi a^{-2} - 6\mathcal{H}^2\phi^2 +\\ &+2(\mathcal{H}^2 - \mathcal{H}')\Big(\phi^2 - \partial_i\big(B + \pi/a\big)\partial_i\big(B + \pi/a\big)\Big)\Big).\end{aligned} \tag{C.20}$$

This result allows us to simply calculate the perturbation of the last term in our action since the first order perturbation of $\partial_\mu\varphi\partial^\mu\varphi$ vanishes:

$$\begin{aligned}\delta_2\big(\sqrt{-g}R\partial_\mu\varphi\partial^\mu\varphi\big) &= \delta_2\big(\sqrt{-g}R\big) + \sqrt{-g}R\delta_2\partial_\mu\varphi\partial^\mu\varphi\\ &= \delta_2\big(\sqrt{-g}R\big) - 6a^2(\mathcal{H}^2 + \mathcal{H}')\Big(\phi^2 - \partial_i\big(B + \pi/a\big)\partial_i\big(B + \pi/a\big)\Big).\end{aligned} \tag{C.21}$$

Substituting the above expressions into the perturbation of (4.4) and decomposing the result with respect to the various parameters we obtain

$$\frac{2a^2}{\gamma}\delta_2\mathcal{L}_\gamma = \Big[\Delta\big(B+\frac{\pi}{a}\big)\Big]^2 + 3\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + \frac{15}{2}\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 9\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2 \tag{C.22}$$

$$\frac{2a^2}{\sigma}\delta_2\mathcal{L}_\sigma = \Big[\Delta\big(B+\frac{\pi}{a}\big)\Big]^2 + \Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + \frac{5}{2}\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 3\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2 \tag{C.23}$$

$$\frac{2a^2}{\zeta}\delta_2\mathcal{L}_\zeta = 2\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + 3\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 6\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2 \tag{C.24}$$

$$2a^2\delta_2\mathcal{L}_{EH} = 2\Delta B\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big) + 3\mathcal{H}^2\Big(\frac{\partial_i\pi}{a}\Big)^2 + 6\mathcal{H}^2\Big(\frac{\pi'}{a}\Big)^2, \tag{C.25}$$

where we have again omitted total derivatives. The last Lagrangian density corresponds to terms that have no parameter dependence. Variation of resulting action with respect to B yields

$$2\Delta^2\big(B+\frac{\pi}{a}\big) = \frac{2-3\gamma-2\sigma-\zeta}{\gamma+\sigma}\Delta\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big). \tag{C.26}$$

We again invoke the consistency of perturbations (kernel of Δ does not decay properly in spatial infinity, and therefore is inconsistent with perturbations) to find a unique solution

$$2\Delta\big(B+\frac{\pi}{a}\big) = \frac{2-3\gamma-2\sigma-\zeta}{\gamma+\sigma}\Big(2\mathcal{H}\frac{\pi'}{a} - 3\mathcal{H}^2\frac{\pi}{a}\Big). \tag{C.27}$$

Now we substitute back into the above Lagrangians to eliminate any B dependence. One final intermediate result before conclusion is

$$\begin{aligned}\int d\eta d^3x\big(3\mathcal{H}^2\pi - 2\mathcal{H}\pi'\big)^2 &= \int d\eta d^3x 4\mathcal{H}^2(\pi')^2 \\ &= \int d\eta d^3x a^2\Big(\partial_\eta\big(\mathcal{H}\pi/a\big)\Big)^2,\end{aligned} \tag{C.28}$$

which follows from (4.18) for vanishing potential. For the final perturbed action at second order we obtain

$$\delta_2 S = (1-\zeta)\int d^4x a^2\Big[-\frac{1}{c_s^2}\Big(\partial_\eta\big(\pi\mathcal{H}a^{-1}\big)\Big)^2 + \Big(\partial_i\big(\pi\mathcal{H}a^{-1}\big)\Big)^2\Big], \tag{C.29}$$

where

$$c_s^2 = \frac{\gamma+\sigma}{(2-3\gamma-2\zeta-\sigma)}\frac{1-\zeta}{(1-\zeta+\sigma)}. \tag{C.30}$$

D. Tensor perturbations at 1st and 2nd order

The analysis of perturbations is much easier for tensors since there is only one gauge invariant field that enters the problem. Namely the 3-tensor h_{ij}. The basic setup is

$$ds^2 = dt^2 - a^2(\delta_{ij} - h_{ij})dx^i dx^j, \tag{D.1}$$

where h_{ij} is transverse and traceless. Such metric defines the following composite objects: the inverse metric

$$g^{00} = 1, \tag{D.2}$$

$$g^{0i} = 0, \tag{D.3}$$

$$g^{ij} = -a^{-2}(\delta_{ij} + h_{ij} + h_{ik}h_{kj}), \tag{D.4}$$

the volume element

$$\sqrt{-g} = a^3\Big(1 - \frac{1}{4}h_{ij}h_{ij}\Big), \tag{D.5}$$

and the Christoffel symbols

$$\Gamma^0_{ij} = a^2\Big(H(\delta_{ij} - h_{ij}) - \frac{1}{2}\dot{h}_{ij}\Big), \tag{D.6}$$

$$\Gamma^i_{0j} = H\delta_{ij} - \frac{1}{2}\big(\dot{h}_{ij} + h_{ik}\dot{h}_{kj}\big). \tag{D.7}$$

The components Γ^0_{0i} and Γ^0_{00} vanish and Γ^i_{jk} does not appear anywhere in our calculations. For the auxiliary field $\theta_{\mu\nu} = \nabla_\mu\nabla_\nu t$ we obtain

$$\theta^i{}_j = H\delta_{ij} - \frac{1}{2}\big(\dot{h}_{ij} + h_{ik}\dot{h}_{kj}\big), \tag{D.8}$$

$$\theta = -\frac{1}{2}h_{ij}\dot{h}_{ij}, \tag{D.9}$$

$$\theta_{00} = \theta_{0i} = 0. \tag{D.10}$$

Now we will perturb the i-jth components of energy-momentum tensor (4.9) to first order. The perturbation of relevant terms is

$$\delta(\theta\theta^i_j) = -\frac{3}{2}H\dot{h}_{ij}, \tag{D.11}$$

$$\delta(\nabla_\alpha\theta^i_j\partial^\alpha\varphi) = -\frac{1}{2}\ddot{h}_{ij}. \tag{D.12}$$

$$\tag{D.13}$$

From (1.67) we have

$$\delta G^i{}_j = \frac{1}{2}\Big(\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta}{a^2}h_{ij}\Big). \tag{D.14}$$

Substituting the above results into the perturbation of (4.9) we obtain

$$\delta T_{(\gamma)}{}^{i}{}_{j} = 0, \tag{D.15}$$

$$\delta T_{(\sigma)}{}^{i}{}_{j} = -\frac{\sigma}{2}\Big(\ddot{h}_{ij} + 3H\dot{h}_{ij}\Big), \tag{D.16}$$

$$\delta T_{(\zeta)}{}^{i}{}_{j} = \frac{\zeta}{2}\Big(\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta}{a^2}h_{ij}\Big). \tag{D.17}$$

$$\tag{D.18}$$

Substituting these results into (1.67) we find

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - c_T^2\frac{\Delta}{a^2}h_{ij} = 0, \tag{D.19}$$

where

$$c_T^2 = \frac{1-\zeta}{1+\sigma-\zeta}. \tag{D.20}$$

Now we keep perturbations to second order. Perturbed terms from action corresponding to (4.4) are

$$\delta_2\big(\sqrt{-g}\theta^i{}_j\theta^j{}_i\big) = a^3\Big(\frac{1}{4}\dot{h}_{ij}^2 - h_{ij}\dot{h}_{ij} - \frac{3}{4}H^2h_{ij}h_{ij}\Big), \tag{D.21}$$

$$\delta_2\big(\sqrt{-g}\theta^2\big) = -3a^3\Big(h_{ij}\dot{h}_{ij} + \frac{3}{4}H^2h_{ij}h_{ij}\Big), \tag{D.22}$$

$$\delta_2\big(V\sqrt{-g}\big) = \frac{2-3\gamma-2\zeta-\sigma}{2}(2\dot{H}+3H^2)\frac{1}{4}h_{ij}h_{ij}. \tag{D.23}$$

The second order perturbation of the Einstein-Hilbert term has been calculated in [6]:

$$\delta_2\sqrt{-g}R = -\frac{1}{4}a^3\Big(\dot{h_{ij}}\dot{h_{ij}} - \partial_k h_{ij}\partial^k h_{ij} - 4Hh_{ij}\dot{h}_{ij} - 6(\dot{H}+2H^2)h_{ij}h_{ij}\Big). \tag{D.24}$$

From which it follows:

$$\delta_2\sqrt{-g}R(\partial_\mu\varphi\partial^\mu\varphi) = -\frac{1}{4}a^3\Big(\dot{h_{ij}}\dot{h_{ij}} - \partial_k h_{ij}\partial^k h_{ij} - 4Hh_{ij}\dot{h}_{ij} - 6(\dot{H}+2H^2)h_{ij}h_{ij}\Big). \tag{D.25}$$

Now we substitute into perturbation of the action (4.4). Terms with no derivatives and with one derivative all vanish as they combine with contribution from the potential V. The resulting action is

$$\delta_2 S_T = \frac{1}{8}\int d^3x dt\ a^3\Big((1-\zeta+\sigma)\dot{h_{ij}}\dot{h_{ij}} - (1-\zeta)\partial_k h_{ij}\partial^k h_{ij}\Big). \tag{D.26}$$

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List of Abbreviations

HD - higher derivatives
FLRW - Friedman-Lemaître-Robertson-Walker
FRM - Friedman-Robertson-Walker
EoM - equation of motion
EFE - Einstein field equation
DM - dark matter
TeS - Tensor-Scalar
CDM - cold dark matter
ADM - Arnowitt-Deser-Misner
CMB - cosmic microwave background
GW - gravitational wave