

# Posudek práce

předložené na Matematicko-fyzikální fakultě  
Univerzity Karlovy

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Název práce: Modelování anizotropních viskoelastických tekutin

Studijní program a obor: Fyzika – Matematické a počítačové modelování

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Jméno a tituly vedoucího/opponenta: Mgr. Vít Průša, PhD.

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## Odborná úroveň práce:

- vynikající  velmi dobrá  průměrná  podprůměrná  nevyhovující

## Věcné chyby:

- téměř žádné  vzhledem k rozsahu přiměřený počet  méně podstatné četné  závažné

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- téměř žádné  vzhledem k rozsahu a tématu přiměřený počet  četné

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- vynikající  velmi dobrá  průměrná  podprůměrná  nevyhovující

## Slovní vyjádření, komentáře a připomínky vedoucího/opponenta:

Viz příložený dokument.

**Případné otázky při obhajobě a náměty do diskuze:**

Viz příložený dokument.

**Práci**

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**Navrhuji hodnocení stupněm:**

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Místo, datum a podpis vedoucího/oponenta:

Praha, 29. ledna 2020

Vít Průša

# MODELING OF ANISOTROPIC VISCOELASTIC FLUIDS

BY MARTIN ŠÍPKA

## 1. OVERVIEW

The author proposes a framework for the derivation of thermodynamically consistent models for description of anisotropic viscoelastic fluids. The framework developed by the author exploits the ideas introduced by Rajagopal and Srinivasa (2000, 2001) and Málek et al. (2015), and using the framework the author derives two models for anisotropic viscoelastic fluids – anisotropic variants of Oldroyd-B and Giesekus model. Finally, the author shows a few results obtained by numerical solution of the corresponding governing equations.

## 2. MAJOR REMARKS/QUESTIONS

- The prevailing approach to the modelling of anisotropic fluids is based on the concept of directors, see for example Leslie (1966) and follow up works. (In the context of liquid crystals see especially Leslie (1968) and Leslie (1983), for fiber suspensions see for example Folgar and Tucker (1984) and related works.) *The author should explain how his approach, or more precisely the approach by Rajagopal and Srinivasa (2001), differs from the established approach based on the theory of directors.*
- The author frequently mentions the (nematic) liquid crystals as a material for which the theory might be applied. However, most of the studies focused on nematic liquid crystals work with the Oseen–Frank long-range elastic energy that contains gradients of  $\mathbf{n}$ . The presence of the gradient would however require substantial changes in the proposed approach. *Could the author indicate what changes would be necessary provided that one wants to work with the Oseen–Frank long-range elastic energy?*

In this context, I would like to note that the issue of (additional) boundary conditions does not seem to be as critical as Rajagopal and Srinivasa (2001) point out. In fact, the issue of the specification of boundary conditions seems to be well understood in the classical Leslie–Ericksen theory of nematic liquid crystals, see for example Stewart (2004) or Rey and Denn (2002).

- Given the complexity of the thesis objective, I would expect that the author would first develop models for anisotropic elastic solids (in the Eulerian formulation), then for anisotropic viscous fluids, and *after* that he would finally proceed with anisotropic viscoelastic fluids. Instead of this, the author goes directly to anisotropic viscoelastic fluids. *Would it be possible to show the governing equations for an anisotropic elastic solid in the Eulerian formulation? (Including the orientation vector  $\mathbf{n}$  that satisfies the constraint  $|\mathbf{n}| = 1$ ?)* This should not be too difficult, since formulae for the time derivatives of the rotation and stretch tensors are known, see for example Carroll (2004).

Next, the author should probably discuss whether his constitutive relations reduce well to the governing equations for an anisotropic solid provided that the corresponding terms in the governing equations vanish. The rate-type models by Oldroyd and Giesekus

for (isotropic) viscoelastic fluids do have this property. *Does the same hold true for the proposed anisotropic variants of the models?*

- The author works with a variable length vector  $\mathbf{n}$ . If  $\mathbf{n}$  is interpreted as a vectorial variable that keeps track of the *orientation*, one would expect that the length of the vector  $\mathbf{n}$  is constant. *Is there a specific reason for working with variable length  $\mathbf{n}$ ? Is there a specific reason for the choice of Helmholtz free energy in the form (2.11)?*
- The last section (Section 4: Simulations) is written in a very confusing manner. Let me give some examples.

In Figure 4.15 and Figure 4.14 the author compares two plots of the apparent viscosity. However, Figure 4.14 shows apparent viscosity versus *time*, while Figure 4.15 shows apparent viscosity versus *strain* – different quantities are plotted on the horizontal axis! Furthermore, Figure 4.15 is definitely not taken from Rajagopal and Srinivasa (2004), in fact, one can find it in Ternet et al. (1999).

Furthermore, although the quantity  $\mathbf{n}$  evolves in time, the author does not specify the initial condition for the quantity  $\mathbf{n}$ . The reader is left to infer the initial condition from the figures.

The geometry of the problem studied in Section 4.6 entitled “3D Simulation” is not specified at all (radius of the plates, gap width and so forth).

### 3. DECISION

The author has chosen a very interesting topic, but his work on the subject matter leaves much to be desired. The thesis opens many interesting questions, and I fully agree with the author in his claim that “there is a lot of potential for further work”, but the thesis itself does not provide specific clues for the answers. (However, the same can be probably said about the original work by Rajagopal and Srinivasa (2001).) Furthermore, some parts of the thesis, especially the final chapter, are poorly written. For these reasons the thesis can hardly aspire for better than average grade. I propose that the student can defend the thesis.

Praha, 29th January 2020

Vít Průša

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