Charles University in Prague

Faculty of Mathematics and Physics

DOCTORAL THESIS

Kay Condie Erb

Algebraic Approaches to Elementary Excitations in Media with Broken Spatial or Time-reversal Symmetry

Institute of Physics of the Czech Academy of Sciences

Supervisor of the doctoral thesis: Ing. Jiří Hlinka, Ph.D.

Study programme: Physics

Study branch: Physics of Condensed Matter and Materials Research

Prague 2019

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Název práce: Algebraické přístupy k elementárním excitacím v prostředích s narušenou invariancí vůči prostorové nebo časové inverzi

Autor: Kay Condie Erb

Ústav: Fyzikální ústav Akademie věd České republiky

Školitel: Ing. Jiří Hlinka, Ph.D., Fyzikální ústav Akademie věd České republiky

Abstrakt:

Strukturní fázové přechody vykazující narušení makroskopické symetrie lze rozdělit do 212 nemagnetických druhů (species) podle vzájemné prostorové orientace bodových grup obou fází. Zařazení do daného druhu implikuje řadu univerzálních vlastností přechodu jako např. počet makroskopických doménových stavů nízkosymetrické fáze a jejich rozlišitelnost pomocí parametrů uspořádání.

V této práci byla studována rozlišitelnost makroskopických doménových stavů pomocí všech parametrů uspořádání, které se transformují jako vektory nebo jako vektorům podobné veličiny, tzv. bidirectory. Pro řešení úlohy byl navržen počítačový algoritmus, který umožnil explicitní výpis výskytu všech vektorových a vektorům podobných parametrů uspořádání a to nejen pro 212 nemagnetických species, ale i pro všech 1602 magnetických species, která zahrnují také přechody mezi krystalografickými bodovými grupami šedými a dvoubarevnými. Ireducibilní reprezentace 122 magnetických krystalografických bodových grup, které se transformují jako vektory nebo vektorům podobné veličiny, byly navíc přímo vepsány do tabulek charakterů. Provedená systematická analýza ireducibilních reprezentací magnetických grup má za cíl usnadnit klasifikaci dlouhovlnných elementárních excitací. V této práci byla použita pro identifikaci fázových přechodů připouštějících existenci tzv. skyrmionových fází.

Klíčová slova: symetrie krystalů, fázové přechody, časová inverze, chiralita

Title: Algebraic Approaches to Elementary Excitations in Media with Broken Spatial or Time-reversal Symmetry

Author: Kay Condie Erb

Institute: Institute of Physics of the Czech Academy of Sciences

Supervisor: Ing. Jiří Hlinka, Ph.D., Institute of Physics of the Czech Academy of Sciences

Abstract: Structural phase transitions with macroscopic symmetry breaking can be divided into 212 non-magnetic *species* according to the mutual spatial orientation of the point groups of both phases. Classification into the given species implies a set of universal transition properties such as the number of macroscopic domain states of the low-symmetry phase and their distinguishability by order parameter.

In this work, the distinguishability of macroscopic domain states by all order parameters which transform as vectors or vectorlike quantities (called bidirectors) was studied. For solving this task, a computer algorithm was designed which enabled an explicit listing of all vector and vectorlike order parameters, not only for the 212 non-magnetic species, but even for all 1602 magnetic species which includes transitions between crystallographic gray and bicolor point groups. In addition, irreducible representations of the 122 magnetic crystallographic point groups which transform as vectors or vectorlike quantities are given in character tables. The aim of this systematic analysis of irreducible representations of magnetic groups is to facilitate classification of long-wave elementary excitations. This work was used to identify phase transitions that allow the existence of so-called skyrmion phases.

Keywords: crystal symmetry, phase transitions, time-reversal symmetry, chirality

Dedication

I am deeply indebted to my wife Shelyse for her immense contributions to this thesis. She cared for our home and children; she provided delicious meals and comfort; and she did it all cheerfully with no recognition expected. This simply wouldn't have been possible without her.

My advisor Jirka Hlinka has also contributed a good deal more of his mental and temporal resources than I can repay. From help with mundane paperwork, to private lectures and tutoring and training, he also made this work possible and sacrificed much to help me get where I am today. Děkuji!

Contents

Introduction

In 1937 Lev Landau described solid-state phase transitions in terms of the symmetry of the crystal lattice. Of interest in many phase transitions are those properties of the phase which are describable as a 'vectorlike' quantity, meaning the property has an axis, a sign, and a magnitude. For example, spontaneous magnetization can be parametrized by the vector order parameter of magnetic dipole moment. This means that a symmetry-lowering phase transition must remove any symmetry elements of the high-temperature phase which are incompatible with the symmetry of a magnetic dipole moment. This is the first topic of the thesis: symmetry rules for phase transitions involving a vectorlike order parameter.

The other aspect of this line of inquiry is elementary excitations. Due to the complexity of the dynamics of solid-state phenomena, it is often useful to characterize a macroscopic property in terms of particle-like excitations (e.g. magnons and phonons). In experimental physics, we can probe a solid with various measurement techniques and understand spectra in terms of these excitations and their irreducible character. Thus the second topic of this thesis is using symmetry to understand elementary excitations via the irreducible character of vectorlike properties.

This thesis is divided into two halves. The first half, Chapters 1, 2, 3, 4, and 5, introduces the basic theory and conceptual framework needed to understand the second half. The second half, Chapters 6, 7, 8, and 9, presents the published and soon-to-be published research of the author on this topic.

Following those chapters, there is a glossary and a rather lengthy appendix of attachments. These attachments are the primary accomplishment of this thesis work. They tabulate data which is essential to taking an algebraic approach to understanding elementary excitations in media with broken spatial or time-reversal symmetry. What follows should make clear the meaning of that sentence as well as the usefulness of such data in condensed matter research.

1. Symmetry

The intuitive concept of visual symmetry tells us that our right and left hands are reflections of each other, or that a blank piece of paper doesn't really have a top or bottom until you start writing on it. We can experience temporal symmetry as the regular ticking of a clock or beating of a drum. The two can even be combined as we watch a windmill lazily rotating in the breeze. If you close your eyes, the steady whoosh of its blades will never tell you how many blades it has or which one has passed.

1.1 Spatiotemporal Symmetry

1.1.1 Spatial Symmetry

In the solid state, it is often the case that atoms tend towards an energy minimum when in a symmetric arrangement called a **crystal lattice**. The only requirement we have of this arrangement, so that it may qualify as a crystal lattice, is that there is some fixed distance and direction by which the lattice can be translated and it will remain unchanged (see Figure 1.1). This can only be true if the lattice has no beginning or end which means it's not physical. Luckily for us, in most cases, the lattice size is so much smaller than the size of the material, that it is inconsequential from a macroscopic standpoint. We can focus on the atomic level and treat the lattice as infinite or we can focus on the macroscopic level and disregard **translational symmetry**.

The set of symmetries which we'll not disregard at either level are the **point isometries**:

- rotation

- reflection

- improper rotation

- and inversion.

So named because they leave invariant at least one point in the object (the center point in Figure 1.2) and they are **isometries** meaning they do not change the lengths and angles that describe the object i.e. they do not distort the object.
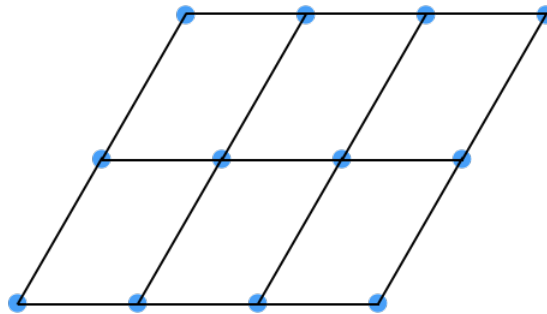
Figure 1.1: Illustration of a 2D lattice.
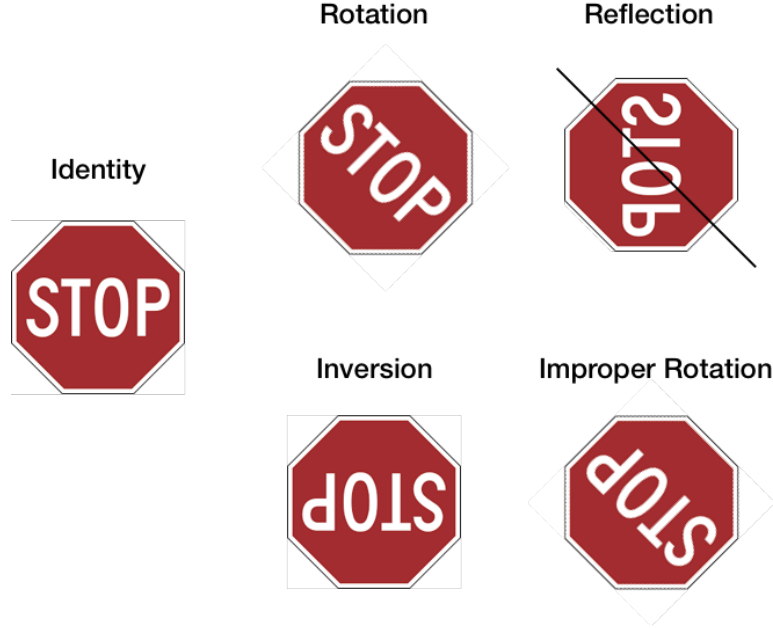
Figure 1.2: The five types of point isometry. The red octagon is invariant with respect to these operations, the word "STOP" is not. Hence both are used to illustrate the effect of the isometries. The two rotational isometries rotate about an axis normal to the figure with the same angle and direction; the improper rotation is inverted after rotating.

Due to the translational requirement, there are only 4 kinds of rotation available to our crystal lattice: 2-fold, 3-fold, 4-fold, and 6-fold; this fact is commonly known as the "crystallographic restriction theorem" and has many simple proofs [1].

Inversion symmetry involves inverting the coordinates of each atom about a point, i.e. for the origin, the transformation would be $(x, y, z) \rightarrow (-x, -y, -z)$. This symmetry can be combined with rotational symmetry to create a new operation, the **improper rotation**. An improper rotation is an operation which both rotates and inverts. The **reflection** operation is reflection with respect to a plane. To calculate the reflection of a point through a plane, one needs only to find a line normal to the plane which passes through the point and then move the point to the unique position on that line which is the same distance from the plane as the original point but on the opposite side of the plane.

With pictures and words, we can describe these symmetry operations in the abstract. One way to reify them is as matrices operating on a Cartesian coordinate system. If our object is a **3-tuple** (ordered list of 3 elements) corresponding to x, y, z coordinates, then we can "rotate" each tuple by matrix multiplication with

$$R = \begin{bmatrix} \xi_c + u_x^2 (1 - \xi_c) & u_x u_y (1 - \xi_c) - u_z \xi_s & u_x u_z (1 - \xi_c) + u_y \xi_s \\ u_y u_x (1 - \xi_c) + u_z \xi_s & \xi_c + u_y^2 (1 - \xi_c) & u_y u_z (1 - \xi_c) - u_x \xi_s \\ u_z u_x (1 - \xi_c) - u_y \xi_s & u_z u_y (1 - \xi_c) + u_x \xi_s & \xi_c + u_z^2 (1 - \xi_c) \end{bmatrix}. \quad (1.1)$$

To make this fit on one page: ξ_c is $\cos \theta$; ξ_s is $\sin \theta$; u_i are the components of the axis about which the rotation will take place; and θ is the angle of the rotation

about that axis. This matrix only describes *active* rotations (meaning we rotate the object not the coordinates) that are counterclockwise in a right-handed sense defined with respect to a vector pointing from the origin to point $\{u_x, u_y, u_z\}$.

Matrices can be defined for each of the operations this way, but let's dispense with them because they are complicated. In fact, this thesis includes this example precisely to point out that though we often can write things out explicitly, doing so brings with it a lot of detail in order to make the concrete idea match the abstract one. Sometimes the abstract idea, though necessarily more vague, has sufficient detail to solve the problem at hand. This is a key reason why "symmetry considerations" are so important to physicists. They often allow one to leave aside many difficult details in favor of simply expressed and understood truths.

1.1.2 Temporal Symmetry

Time can be modeled as a one-dimensional quantity and, if so modelled, the only operators it has are translation and inversion. Temporal translational symmetry comes up in the examples given before as the passage of time. Temporal inversion symmetry, i.e. $t \rightarrow -t$ is useful for simply deciding whether a system is time-dependent or not. For example, a wrench sitting on a table possesses time-reversal symmetry because the system at time t is indistinguishable from the system at time $-t$. A river flowing by would not possess this symmetry because its state (easily observable with the help of a stick or a bubble floating on the surface for example) is unequal at times t and $-t$.

If the system is non-classical, temporal inversion is not as simple. In quantum mechanics, we work with richer mathematical objects like spinors and their behavior under temporal inversion is more complex (literally). We'll come back to this point briefly in a moment but won't need it for the work in this thesis.

1.2 Group Theory

The mathematical structure designed for nicely organizing the concept of isometries (also called symmetry operations) is a **group**. It's beyond the scope of this thesis to give a proper primer on group theory (the bibliography contains three of many possible introductions: [1, 2, 3]), so remarks will be restricted to the core idea: we abstractly consider a **set** of elements and an operation which we can use to take two of these elements as input and produce a third as output

$$G_1 G_2 = G_3. \tag{1.2}$$

In order for such a set and operation to qualify as a group (and thus inherit the rich toolset of group theory) there are four axioms which must be satisfied:

1. Identity: There must exist some element of the group which only returns the *other* argument to the operation. We'll denote this element as E.

$$EG = GE = G$$

2. Inverse: For every element in the group, there must exist some element which under the group operation produces the identity element. We'll denote the inverse of an element G as G^{-1}

$$GG^{-1} = G^{-1}G = E.$$

3. Closure: The combination of any two elements under the group operation must result in another element of the group. By counterexample, if our candidate group is spectral colors (i.e. rainbow colors) we can define an addition operator as the act of shining two wavelengths of light on a sheet of white paper. If we choose as input red and blue light, the resultant color is called magenta and it is a non-spectral color (i.e. there is no wavelength of light which produces the color magenta. Other non-spectral colors include white, black, and metallic colors like gold). Thus spectral colors under addition do not form a group.

4. Associativity: The group operation cannot vary based on choice of pairs. Thus for any three group elements

$$(G_1 G_2)G_3 = G_1(G_2 G_3).$$

A simple counter example can be given by division for integers

$$(12 \div 6) \div 3 \neq 12 \div (6 \div 3)$$
$$2 \div 3 \neq 12 \div 2.$$

The set need not be finite. Let's take for example the integers, then our operation could be addition. The inverse elements are the negative integers and the identity element is 0. These constitute an infinite group. The number of elements in the set is the **order** of the group.

With isometries, the elements of the set are themselves operations: rotations, translations, and so on. Then the 'operation' for combining two symmetry elements is simply to perform the symmetry operations sequentially. For example, if I take a rectangle and act on it with a 2-fold (180 degrees) and a 4-fold (90 degrees) rotation, the result is the same as if I had acted on it with a 2-fold[3] (270 degrees) rotation.

$$R_2 R_4 = R_2^3 \tag{1.3}$$

A **normal subgroup** is a set of elements N in a group G which in addition to forming a subgroup also obey the rule that for every element G_i in G and for every element N_j in N

$$G_i N_j G_i^{-1} \in N. \tag{1.4}$$

A normal subgroup is just one kind of **factor group** which is a special kind of subgroup that preserves some of the group structure via an equivalence relation. Because the group structure is preserved, such a subgroup will be of an order that is an even divider of the order of the group. Thus a group of order 12, for example, may have a subgroup which preserves some element of the structure

and has order three. Then, necessarily, there will be four such subgroups since the equivalence relation partitions the group.

A **group action** is defined as the action of a group on some set. For example, a symmetry group can have multiple actions associating it to different sets. If the set were physical objects, then the group action connecting an abstract rotation operation to this concrete set may be to pick the object up and rotate it (in one's hands for example). If the set were points in a linear coordinate system, the group action would be coordinate transformations and we would associate to each symmetry element a matrix (see Section 3.1). In each case, some set was chosen and an association between group elements and transformations on the set was identified to establish the group action.

1.3 Point Groups

Just as the lattice requirement restricted possible rotations to a finite number of elements, it also restricts us to a finite number of combinations of elements. If we take a microscopic view and include translations (which implies other composite operators such as a "glide plane" which is the composition of translation with reflection) we obtain the 230 well-known **space groups**. If we take the macroscopic view and consider only the point-preserving operations, we obtain 32 groups called the **point groups**. These are often classified according to six crystal families based on the possible lattices of crystal symmetry. The families, corresponding point groups, and a brief description are given in Table 1.1.

Family	Point Groups	Description
Triclinic	1, $\bar{1}$	The three axes have no special relationship among them.
Monoclinic	$2/m$, 2, m	Two axes are orthogonal; side lengths are unrelated.
Orthorhombic	mmm, 222, $2mm$	All three axes are orthogonal; side lengths are unrelated.
Tetragonal	$4/mmm$, $4/m$, 422, $4mm$, $\bar{4}2m$, $\bar{4}$, 4	All three axes are orthogonal; two side lengths are the same.
Hexagonal	$6/mmm$, $6/m$, 622, $6mm$, $\bar{6}2m$, $\bar{6}$, 6, $\bar{3}m$, $3m$, 32, $\bar{3}$, 3	Like tetragonal, except lattice variables are chosen so as to produce 3 and 6-fold rotational symmetry.
Cubic	$m\bar{3}m$, $m\bar{3}$, 432, $\bar{4}3m$, 23	All three axes are orthogonal; side lengths are equal.

Table 1.1: The six crystal families and associated point groups.

1.3.1 Magnetic Point Groups

In 1930 Heesch introduced the **antisymmetry** operation to describe the symmetry of a system where the elements could carry a binary property in addition to their location [4]. This idea was later independently rediscovered and developed in terms of an abstract binary "color" and can thus also be called "bicolor symmetry" [1]. Adding this operation to the other point isometries increases the total number of groups to 122. These abstract mathematical groups have found application in condensed matter physics where the antisymmetry operation acts as a time-reversal operation and the binary property is the time-reversible magnetic moment. Hence, a common name for these 122 groups is the "magnetic point groups".

Magnetic point groups come in two varieties. The gray groups and the black-white groups. These names derive from the original bicolor formulation of the idea.

The gray groups essentially possess two copies of each of the symmetries of their non-magnetic counterparts: the original (which we may call black for example), and a time-inverted version (which we may call white). This is actually best understood in terms of the group product which is defined simply as the combination (by group operation) of every member of one group with every member of the other. For example, the time-inversion group has two operations $\{1, 1'\}$ (identity, and time inversion). The group product of this group and $2mm$ (which has elements $\{1, 2, m_1, m_2\}$) would be $\{1, 2, m_1, m_2, 1', 2', m_1', m_2'\}$. Thus a gray group has a white and a black copy of every operation.

Things get a little trickier with black-white groups. In that case, we find the valid subgroups of each gray group which will generally have some operators black and others white. We say valid subgroups because the nature of the symmetry operations does not allow all combinations of colors among subgroups to be consistent with the symmetry. For example, group $42'2'$ colors all of the 2-fold axes in the plane since the 4-fold axis requires they be indistinguishable and thus we could not have a group $422'$. By contrast, group $4'22'$ distinguishes two sets of planar 2-fold axes because the 4-fold rotation inverts color on rotation.

This is only a very brief overview of a few kinds of symmetry groups and their operations. For a more thorough introduction to these groups and their elements and behaviors, I can recommend references [1] and [5], as well as the comprehensive International Tables for Crystallography [6].

There exists another formalism for working with time reversal in quantum mechanics. Instead of the groups above, another set is defined, the **double groups** [7]. These are useful because in that setting, temporal inversion is **anti-linear**:

$$T(ax) = a^*T(x) \tag{1.5}$$

where the * operation indicates complex conjugation [3]. This generally comes up when we want to expand the group of Schrödinger's equation to include spin-orbit coupling in the Hamiltonian and anti-linear time reversal in the group. Hence temporal inversion may take very different forms depending on the application.

1.4 Examples of Symmetry Groups in Physics

1.4.1 Group of Schrödinger's Equation

An important group action in condensed matter physics is the action of symmetry operations on the (Hermitian) operators of quantum mechanics and their eigenfunctions ψ_n.

$$\left(\mathcal{H} - E\hat{I}\right)\psi = 0 \tag{1.6}$$

$$\mathcal{H}\psi_n = E_n\psi_n. \tag{1.7}$$

If we have a group G and we specify an action $\hat{G}$ on wave functions, then we can define the group of Schrödinger's equation [3] as the group of symmetry elements which commute with the Hamiltonian:

$$\left[\mathcal{H}, \hat{G}\right] = 0. \tag{1.8}$$

It is therefore possible to simultaneously diagonalize the Hamiltonian and any operator $\hat{G}$ in the group (as long as $\hat{G}$ is Hermitian). Thus the **invariant subspaces** of the Hamiltonian can be identified with the invariant subspaces of the symmetry operators (such as an axis of rotation or a mirror plane).

Furthermore, if ψ_n is an eigenfunction with eigenvalue E_n then

$$\hat{G}\psi_n = E_n\psi_n. \tag{1.9}$$

Thus for all elements G_i in G we find that each of the $\hat{G}_i\psi_n$ (for fixed n) are eigenfunctions of the Hamiltonian. Because they all have the eigenvalue E_n it is degenerate to the extent that the $\hat{G}_i\psi_n$ are distinct functions. The result is that when the symmetry is broken, the degeneracy is lifted and a direct connection between structure and observable energy levels can be made.

1.4.2 Group of Force Matrix

In a different but analogous problem space, we may consider a collection of N classical point masses with 3 degrees of freedom each. If we let $q_n = x_n\sqrt{m_n}$ represent the generalized coordinate of a degree of freedom for a point mass m_n displaced from equilibrium by an amount x_n (the point where potential V is 0). Then we may describe the force of one mass on another under the influence of this potential as the generalized force matrix **F** with elements:

$$F_{mn} = \frac{\partial^2 V}{\partial q_m \partial q_n}. \tag{1.10}$$

Lagrange's equations for this system can be written in matrix form as

$$\ddot{\mathbf{q}} + \mathbf{F}\mathbf{q} = 0 \tag{1.11}$$

where **q** is the 3N x 1 column matrix with q_m as its mth component [8, 9]. To compare with Equation 1.6, we may write the solution as

$$\left(\mathbf{F} - \omega^2\hat{I}\right)\mathbf{c} = 0, \tag{1.12}$$

where **c** is the 3N x 1 column matrix of constants of integration. Again a group of symmetry elements can be identified (now written in matrix form **G**) which commute with the force matrix [9]:

$$\mathbf{FG} = \mathbf{GF}. \qquad (1.13)$$

Therefore we can again simultaneously diagonalize both matrices **F** and **G**. The result is that we can identify **invariant subspaces** of the force matrix with invariant subspaces of the symmetries. Similar commutation relations are valid for the Fourier-transformed force constant matrix, and this matrix can be made real for $q = 0$ (the Brillouin zone center, long-wavelength phonon limit).

The difference between these two groups ($\hat{G}$ and **G**) is that the group action defined here corresponds to real matrices $\mathbf{G_i}$ since **F** is a real matrix of real coordinates. By contrast, the group action in a quantum mechanical setting is a Hermitian operator acting on wave functions. Thus not only do we need to keep in mind that it is in a function space over the field of complex numbers, but we also need to keep in mind that the symmetries of wave functions are different as well. For example, a time-reversal symmetry operator will be anti-linear in the group of Schrödinger's equation but linear in the group of the force matrix.

1.4.3 Conclusion

We briefly covered some basics of symmetry operators and group theory, and gave some examples of groups in quantum mechanical and classical systems. In this thesis, emphasis will be placed on classical systems specifically in the form of the force matrix as the intended application of the theory and results presented here is to lattice vibrations (i.e. phonons) [10, 11]. These can be measured spectroscopically by a variety of methods [12, 13, 14], but the focus here is on the underlying theory which connects emission and absorption of energy quanta to symmetry changes in the crystal lattice. Therefore it is sufficient to focus on real numbers and linear time inversion, dispensing with double groups and group actions on complex-valued vector spaces.

The connection between changed symmetry and changed material properties is the theme of the next chapter and is applied more concretely in the following chapter (Section 3.4.2).

2. Phase Transitions

2.1 Introduction

A **phase** of a substance is a spatially distinct region of matter with identical physical properties. A familiar demonstration is found in an ordinary glass of ice water. Water exists in three phases: ice, liquid water, and water vapor. In this example, the word phase is being used to describe that water is coexisting with itself in distinguishable forms. In isolation, ice isn't so much a *phase* of water as a state of matter, hence we use *phase* to make a point of distinguishing two things which are together in some sense (in the same system, or same space) and share some attributes but not others.

As another example, if we mix water and oil, the mixture will spontaneously separate into two phases: the water phase and oil phase. In this case, the "same" is that they are liquid and in the same container and the "different" is that they don't mix.

Of particular interest in solid state physics is when we have the same material throughout in terms of chemical composition *and* state of matter (all solid) and still distinguishable properties are present in different regions (sometimes called domains). A good example of such a property is ferromagnetism. A ferromagnetic material may have different regions with magnetic dipole moment pointing in unique directions.

If the crystal lattice requires a 2-fold axis of symmetry in the x, y, and z directions, then no magnetic dipole moment could form because the symmetry requirements make both directions on each axis the same and a magnetic dipole moment would create a special direction. If the symmetry were somehow lowered (i.e. if some symmetry elements were removed from the group), we may have a **phase transition** where magnetic dipole moment is now allowed. In some regions, it would point one way and in others another way. Such regions are called magnetic domains to distinguish them in terms of this parameter.

Let's now turn our attention to phase transitions that lower symmetry in such a way that they allow domains of some new property to spontaneously arise, or in other words, some domains become distinguishable in terms of that property.

2.2 Species

In addition to magnetization, there are other macroscopic order parameters which can be introduced by symmetry-lowering phase transitions. To study these generally, the term **species** has been introduced. In this thesis, we use the definition described in Reference [15] which refers to *Aizu* species [16, 17]. Since the bulk of the body of research this thesis is based on makes extensive use of this particular definition of species, it's worth taking some time to fully explain the notation and meaning of the various types of magnetic and non-magnetic species.

The name of each species is given in the pattern $G > F$ where the group order of G is greater than the order of F *and* F is a **proper subgroup** of G. If we only cared about group-subgroup relations, the task would be a simple matter of determining set inclusion. Species describe *how* a subgroup corresponds

orientationally to its super group. For example, in point group 222 there are three 2-fold axes orthogonal to one another. There is no way to distinguish these 2-fold axes and so there is only one species associating these two groups: $222 > 2$. If we instead consider point group 422, there are two distinct kinds of 2-fold rotation, one that is parallel to the 4-fold rotation and four that are perpendicular. Therefore we may recognize two distinguishable subgroups and give them species labels $422 < 2_|$ and $422 < 2_-$. The meaning is that the high symmetry phase is point group 422 and the low symmetry phase is point group 2 where the 2-fold axis is respectively parallel to the 4-fold axis (using symbol |) or in the plane perpendicular to that axis (using symbol $-$).

Triclinic, monoclinic, and orthorhombic groups are not able to distinguish their axes and do not use a label. Tetragonal and hexagonal groups have one special axis and use the subscripts just given (see Figure 2.1).
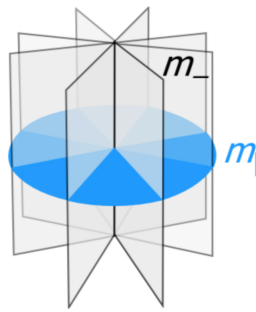
Figure 2.1: Tetragonal and hexagonal groups possess a special axis to which all rotation axes or mirror-plane normals are either parallel or perpendicular. The subscript indicates which type of axis the symmetry element corresponds to and in the figure, the planes of mirror symmetries are highlighted for clarity. This figure is reused with permission from [15].

For the cubic groups, it turns out that it is sufficient to distinguish again two types of axes (or planes as the figures highlight) because it is only the 2-fold and mirror operations which are distinguishable. See Figure 2.2 for a depiction of the planes and corresponding symbols.
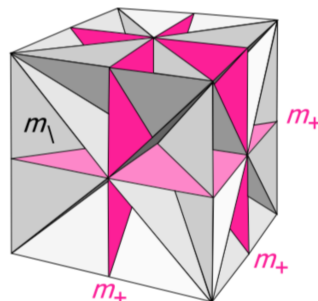
Figure 2.2: Cubic groups also posses two kinds of axis to which all rotation axes or mirror-plane normals are either parallel or perpendicular. The subscript indicates to which type of axis the symmetry element corresponds and in the figure, the planes of mirror symmetries are highlighted for clarity. This figure is reused with permission from [15].

By counting all of the possible group-subgroup relationships between point groups, and counting distinctly the possible unique orientations of one subgroup within its parent, we arrive at 212 distinct species of macroscopic symmetry-breaking phase transitions. These are said to break macroscopic symmetry because point groups exclude the microscopic translation symmetry elements and thus explicitly deal with macroscopic properties only. This is closely related to the important **Neumann's principle** which states that physical properties of a crystal must possess at least the symmetry of the point group of that crystal [5]. Hence, to determine the macroscopic properties of a crystal, it is sufficient to consider only its point group symmetry. If we add time-inversion symmetry, the work is expanded to include magnetic point groups. In that case, we obtain even more species (see Chapter 8 for more details).

Ultimately the goal of such a classification system is to understand, by argument from symmetry, which macroscopic properties can possibly be introduced by a symmetry lowering process and in how many distinct domains can we expect the lower symmetry phase to exhibit the property. This latter point is important and bears further explanation.

2.3   Orientational Domains

If the symmetry of an object is lowered from point group G of order m to point group F of order n then the subgroup F is a **factor group** and there are m/n unique ways to preserve the chosen elements of F in G while breaking the others. A nice way to remember this idea (which will play a large role later in this thesis) is to consider an arrangement of atoms where each atom corresponds to one of the symmetry elements.
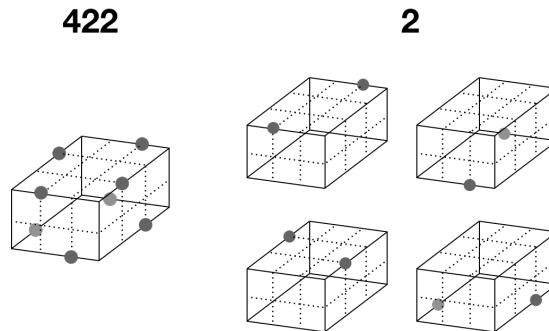
Figure 2.3: A symmetry-breaking transition from group 422 to group 2 will have four distinct ways for the symmetry to be broken given a specific choice of two-fold axis. In a physical system, for energetic reasons, regions will form in the material where one of the four arrangements on the right dominates. These regions are called orientational domains because they differ only in their orientation of the symmetry reduction with respect to the parent group.

For example, consider Figure 2.3. We may select any atom to represent the identity element. Then we can produce each of the other seven atoms (on the left, representing group 422) by operating on that first atom with the action of one of the remaining seven symmetry elements. Group G (422) has eight elements and group F (2) has two elements, thus there are four **orientational domains**

states. The figure shows one possible way those domains could be realized which is consistent with preserving the 2-fold axis parallel to the 4-fold axis. Whether or not in a physical system these different orientations could be distinguished macroscopically will be discussed more fully in Chapter 7.

2.4   Landau Theory Introduction

Unlike state-of-matter phase transitions, such as the transition from solid to liquid, symmetry-lowering phase transitions (like the kind discussed here) do not coexist in an equilibrium between the two phases. Either the crystal possesses one symmetry or another and in each phase the symmetry rules determine the possible physical phenomena. We may characterize such a phase transition by some order parameter, for example a ferromagnetic phase transition by magnetic dipole moment. By definition, this order parameter must be zero above the critical temperature and non-zero below the critical temperature (note that temperature is sometimes used when discussing symmetry since the high temperature phase often corresponds to higher symmetry and the low symmetry phase to lower symmetry) [18].

In 1937, Lev Landau first described such phase transitions giving qualitative and quantitative tools for reasoning about them [19]. Put simply, continuous change can introduce discontinuous symmetry breaking. A full account of Landau theory is beyond the scope of this paper, but some parts of the argument bear repeating here.

We consider some distribution function $\rho(x, y, z)$ which gives us the distribution of atoms in a system. We also consider that there is some group of symmetry operations under which this function is invariant. Next we consider a continuous transition where our function gains a small term with lower symmetry $\delta\rho$:

$$\rho = \rho_0 + \delta\rho. \tag{2.1}$$

Using a one-dimensional drawing for the general three-dimensional distribution function, ρ_0 corresponds to Figure 2.4a and $\delta\rho$ corresponds to a continuous change in distribution which leads to a continuous change in thermodynamic potential while discontinuously breaking the symmetry to the lower group of $\delta\rho$ (Figure 2.4b).

The insight is that the whole function only has the symmetry of the lowest symmetry component, so no matter how small that new component is, no matter how continuously it comes on, the symmetry is suddenly broken. The result is that when we consider the phase transition in terms of thermodynamic potential and its derivatives, we have an explanation for so called "second-order" phase transitions which are continuous from zero in the first derivative (ρ changes continuously) and discontinuous in the second derivative. The system may spontaneously gain an order parameter as a result of the symmetry breaking as a function ρ. This order parameter is a constant zero beneath the critical point and thus has zero curvature. At the moment $\delta\rho$ is introduced this parameter can grow continuously with $\delta\rho$ and thus discontinuously have non-zero curvature.

From this we find that symmetry-lowering phase transitions can spontaneously introduce macroscopic physical properties as a second-order phase transition ac-
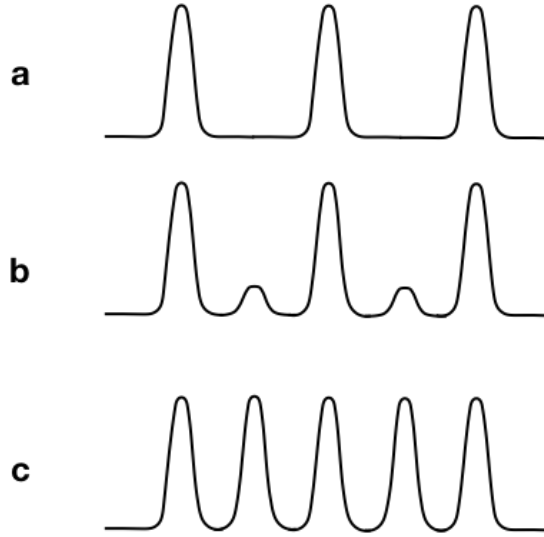
Figure 2.4: One-dimensional demonstration of how discontinuous symmetry breaking can occur as a result of continuous changes in crystal density. a. Some high symmetry distribution corresponding to ρ_0. b. A small change in the distribution $\delta\rho$ instantly breaks the symmetry. c. The new distribution is more clearly seen to have the symmetry of $\delta\rho$.

cording to the Landau theory of phase transitions. The rest of that theory and its use in the modern-day theory of phase transitions is not discussed here due to its sheer breadth, but one powerful aspect of it rests on the topic of the next chapter: representation theory. So we'll come back to this topic in Section 4.3.

3. Irreducible Representations

3.1 Representation Theory

A mathematical group in the abstract is purely a set with operations that obeys the group axioms (see Section 1.2). When we assign to each element of the set a square matrix such that matrix multiplication preserves the group structure, we call this a **representation**. In fact, the action of a group on any vector space is called a representation of the group.

Let's consider a few representations of a simple group: $AB = BA = B$, $BC = CB = A$, $AC = CA = C$. Given some function $M^i(G)$ that maps group elements to square matrices:

$$M^1(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$M^1(B) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.1}$$
$$M^1(C) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$M^2(A) = 1$$
$$M^2(B) = 5$$
$$M^2(C) = \frac{1}{5} \tag{3.2}$$

$$M^3(A) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$M^3(B) = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \tag{3.3}$$
$$M^3(C) = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

We can verify that these are indeed representations of the group by checking that for each product in the multiplication table $M^i(G_j)M^i(G_k) = M^i(G_jG_k)$.

These examples illustrate some important points:

- A group has many representations.

- The function $M^i(G)$ does not have to be injective (i.e. the representation does not have to be faithful).

- Scalars may be included as 1x1 matrices.

3.2   Irreps

It should be clear that the set of matrices M^i constitute transformations on a vector space V and that the symmetry of an object corresponds to some subspace V_i of V which is invariant with respect to the group action. It is then possible that there exists some subspace of V_i which is also invariant in G. In particular, we may find by **similarity transform** that a given M^i can be put into "block-diagonal" form showing that it is the direct sum of two other representations:

$$M^4(B) = \begin{pmatrix} 5 & 0 & 0 \\ 0 & 2 & 3 \\ 0 & 1 & 2 \end{pmatrix}. \tag{3.4}$$

The terminology for a representation which is not **similar** to any block-diagonal matrix (and thus does not possess a lower-dimensional invariant subspace) is an "irreducible representation" or **irrep** for short.

3.3   Character Tables

For a finite group, there are as many distinct (up to similarity) irreps as conjugacy classes [3, 20]. Because trace is invariant with respect to similarity transform, a tabulation of irreps for a group uses only the traces of the matrices. The trace of the representation of a group element takes on the special name **character** in representation theory and thus tables tabulating the irreps of a finite group are called character tables.

Table 3.1 gives a character table for point group 23. The columns give the four conjugacy classes and the rows give the four irreps.

23	1	3	3^2	2
Γ_1	1	1	1	1
Γ_2	1	$e^{2\pi i/3}$	$e^{-2\pi i/3}$	1
Γ_3	1	$e^{-2\pi i/3}$	$e^{2\pi i/3}$	1
Γ_4	3	0	0	-1

Table 3.1: Character table for point group 23 with imaginary character. Contrast with Table 3.2 which gives a real-valued character.

Each row gives the characters (traces) of the matrices in one irreducible representation of the group. Since the representation for the identity element is always an identity matrix, its trace tells us the dimensionality of the **invariant subspace** of the irrep. We may also note that the trivial representation of mapping every symmetry element to an identity matrix produces the *fully symmetric* representation here labeled as Γ_1. Table 3.1 is for a complex vector space as we can see from the exponential terms. This is a commonly used vector space, useful in contexts where imaginary character is needed and is easily converted to another common choice of space: real vector space (see Section 1.4.2). One can determine the real character of a point group from the complex character by noting that multiplication in the complex plane by a unit complex number is equivalent to

rotation. In the real plane, a rotation operation is represented by a 2x2 matrix. Thus, we may convert the Γ_2 and Γ_3 one-dimensional irreps to a two-dimensional irrep with real character. To do so we will use the trace of a two-dimensional rotation matrix representing an angle of rotation equivalent to the argument of the complex number ($2\pi/3$ in this case). Table 3.2 gives the result of this conversion.

23	1	3	3^2	2
A	1	1	1	1
E	2	-1	-1	2
T	3	0	0	-1

Table 3.2: Character table for point group 23 using real numbers.

Here the **Mulliken symbols** were also introduced in the first column as they are well-known in the physical sciences (and preferred to the generic Γ_n or χ_n labels more common in a pure mathematics setting).

3.4 Excitations

So why should anyone get excited about character tables? For mathematicians, this is merely an entry point to many exciting depths to explore in the realm of representation theory. For the physical sciences, there is a concrete connection between the data in these tables and measurable phenomena in the laboratory. That connection lies in the concept of **basis functions** of an irrep.

3.4.1 Basis Functions

We can obtain a representation of a group by defining a group action on a vector space. We can also define a group action on a set of functions. Inasmuch as the functions can be written as vectors in some space (for example coefficients of polynomial terms of increasing power) this group action will also be a representation. If that representation is irreducible, then the functions must form a basis of one of the irreps of the group. We commonly say that those functions "transform as" the irrep to which they correspond.

Let's take for example cubic point group 23. We may define an action of the group on the vector space of functions in terms of spatial coordinates (x, y, z). Then the effect of the symmetry operators on a point at (1,0,0) for example (we can call this point x or can just think of the x-directed unit vector, it's all the same) will be to rotate it to point along either the $\pm x$, $\pm y$, or $\pm z$ axes as in Table 3.3.

E	2	3_1	3_1^2	3_2	3_2^2	3_3	3_3^2	3_4	3_4^2
x	-x	z	y	-y	-z	-z	y	-y	z

Table 3.3: Action of symmetry operators of point group 23 on a unit vector x and hence any function of x.

If function x were to have character A, Table 3.3 would instead just read x in every column. In order for it to have character E, there would have to exist

some 2x2 matrix which could, acting on a 2D basis, transform x into both y and z, this is also impossible. Therefore, it must have character T which corresponds to 3D rotational matrices.

The pattern is that we identify the desired group action on some object, determine matrices that accomplish that action, and if they form an irreducible representation we say that that object transforms as the given irreducible representation.

A more rigorous definition involves the notion of basis vector in general (of which functions form just one possible vector space) and ties the explicit matrix elements of each representation to the basis vectors like so:

$$\hat{G}v_j = \sum_i M_{ij}^n(G)v_i \tag{3.5}$$

Where abstract group element G acts on vector v according to a group action denoted by $\hat{G}$ and $M_{ij}^n(G)$ is the matrix element i, j of the nth irrep of G which acts on v_i by matrix multiplication. Hence $\hat{G}$ is the group action on v and M is a specific matrix.

It is common when listing character tables to include some basis functions as in Table 3.4. Notice that in the two-dimensional representation, there is a basis of two functions separated by a comma in parentheses; and in the three-dimensional representation, we have a three-dimensional basis of unit vectors x, y, and z.

23	1	3	3^2	2	Basis
A	1	1	1	1	$x^2 + y^2 + z^2$
E	2	-1	-1	2	$(x^2 - y^2, 2z^2 - x^2 - y^2)$
T	3	0	0	-1	(x, y, z)

Table 3.4: Character table for point group 23 including a few example basis functions.

3.4.2 Selection Rules

In Section 1.4.1, the idea of the group of Schrödinger's equation was introduced. It was shown that for a given eigenfunction ψ_n, the eigenvalue could be degenerate insofar as the group action on the eigenfunction produced new eigenfunctions. This degeneracy is linked directly to the dimensionality of the irrep since

$$\hat{G}\psi_{nj} = \sum_i \psi_{ni} M_{ij}^n(G). \tag{3.6}$$

Where n labels the energy level and i (or j) labels a degenerate eigenfunction in that level. Thus the irreps of the group label each of the eigenvalues of the Hamiltonian as well as determining their degeneracy.

Given the above, group theory can instruct us on what kinds of perturbations $\mathcal{H}'$ are forbidden or allowed to induce transitions between the eigenstates of $\mathcal{H}_0$:

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}'. \tag{3.7}$$

Reasoning about which transitions are allowed or forbidden results in what are known as selection rules. A full derivation and discussion of the logic behind

such rules is beyond the scope of this thesis, but an important result is presented here and the interested reader may read a fuller treatment in Reference [3].

It can be shown that a transition between states ψ_{ni} and ψ_{mj} can arise by means of a Hamiltonian $\mathcal{H}'$ and have probability given by matrix element $\langle \psi_{ni} | \mathcal{H}' \psi_{mj} \rangle$. If $\mathcal{H}'$ transforms as some irrep in some group (let's call it M_k), then the following holds true: the matrix element is zero if $M_i \otimes M_j$ is orthogonal to M_k where $\otimes$ indicates the **direct product of representations**. The details of this calculation and proof are withheld here to focus on the important content: it is possible, even if one knows only the irreducible character of a Hamiltonian and perturbation, to determine whether the matrix element joining two states of interest is nonzero. Thus an important motivation for including basis functions in a list of character tables is that by knowing the irrep of the perturbing function, we can determine which transitions are allowed and which are forbidden.

4. Vectorlike Quantities

In physics, we are often interested in phenomena which can be parameterized by 3 properties: magnitude, axis, and binary sign. The most familiar example of this abstract idea is the physicist's "vector" which is represented by an arrow and is used to describe velocity, force, and other physical properties requiring *magnitude and direction*. Here, we will call this kind of vector a **polar vector** and note that it is the *combination* of a binary sign and axis which constitutes direction.

Another familiar example to physicists is what we may call an **axial vector** which is used to describe rotating phenomena like angular momentum. The axis in this case is normal to the plane of rotation, and the binary sign indicates the direction of rotation with respect to a convention (usually the right-hand rule).

A generalization of such **vectorlike quantities** in terms of their symmetry requirements and broad applicability was published in 2014 by Jiří Hlinka [21]. In that work, eight vectorlike symmetry groups were identified as 1D irreps of the limiting group $\infty/mm1'$ which (as argued in that paper) is the highest symmetry group which can be constructed consistent with the requirement of axis, sign, and magnitude.

4.1 Limiting Groups

The point groups we've dealt with so far have often included the symmetry operation of an "n-fold" rotation. If we take the limit as n goes to infinity the symmetry of the rotation axis become continuous (i.e. cylindrical symmetry). The presence of an n-fold rotation axis in a point group creates n copies of symmetry elements which are oriented perpendicular to that axis. Thus in the limit as n goes to infinity, so too will the order of the group in such circumstances. Such a group may be called a **limiting group**.

We also seek symmetry requirements consistent with the magnitude and sign properties. We can maximally introduce mirror planes and a 2-fold rotation to relate the ends of the axis to each other (function of z in cylindrical coordinates). The resultant highest symmetry limiting group which may be used as a group theoretical foundation for establishing these vectorlike quantities is a parent group with ∞-fold rotation as an infinite element, and 2-fold and mirror symmetries as in Figure 4.1b.

When we add also time inversion symmetry by taking the group product with the time inversion group $\{1, 1'\}$, we obtain a group of eight elements as in Figure 4.1a.

Group ∞/mm is a subgroup of $\infty/mm1'$ and it in turn has three subgroups which gives four time-odd vectorlike quantities as in Figure 4.2. Taking the group product between each of these and the time inversion group gives four **time-even vectorlike quantities** and four **time-odd vectorlike quantities**. Hence there are eight in total which are named as in Figure 4.3.

We may divide these eight into two groups of four by the type of vectorlike quantity they are. Four of these elements are not properly vectors, but rather are **bidirectors** because their binary sign does not make one end distinct from the

$\infty/mm1'$

$E(\infty)$	$m_\parallel$	$m_\perp$	2
$E(\infty)'$	$m'_\parallel$	$m'_\perp$	$2'$

$\infty/mm1$

$E(\infty)$	$m_\parallel$	$m_\perp$	2
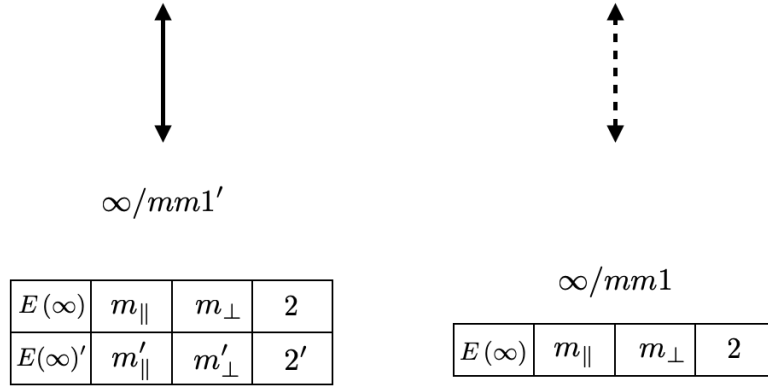
Figure 4.1: The table of symmetry elements beneath each pictogram marks the identity element E by an ∞ symbol to indicate that the continuous rotational symmetry provides an identity element in this group. The m symbols indicate mirror planes where the subscript $\parallel$ is an infinite set of mirror planes parallel to the axis and $\perp$ indicates a single mirror plane perpendicular to the axis.

∞m

$E(\infty)$	$m_\parallel$

∞/mm

∞/m

$E(\infty)$	$m_\perp$

$E(\infty)$	$m_\parallel$	$m_\perp$	2

$\infty 2$

$E(\infty)$	2
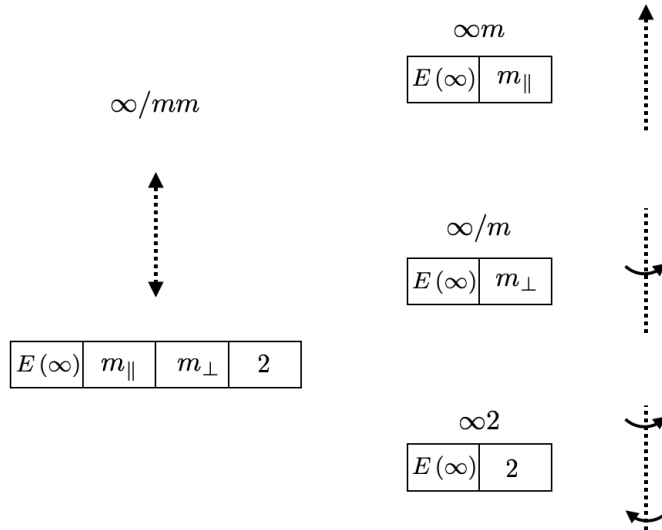
Figure 4.2: The group-subgroup relationship among the four time-odd vectorlike quantities. A dashed line is used in these pictographs to indicate the quantity is antisymmetric under time reversal.

other (i.e. they possess a two-fold symmetry axis perpendicular to the main axis). We'll discuss further in Chapter 9 the consequences of the bidirectorial nature of the objects described in Figure 4.3 as a **neutral bidirector** and a **chiroaxial bidirector**.

4.2 Properties

Let's consider a few examples of these vectors and bidirectors and physical quantities which possess the same symmetry.

- Time-even polar vector: electric dipole moment.

- Time-odd polar vector: velocity or force.
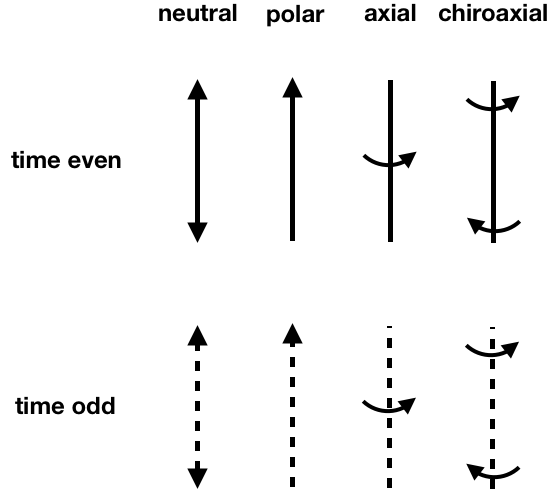
Figure 4.3: The eight vectorlike quantities from Reference [21].

- Time-odd axial vector: angular momentum or magnetic dipole moment.

- Time-even neutral bidirector: liquid crystal nematic disclination angle [22].

- Time-odd neutral bidirector: linear source or sink.

- Time-even chiroaxial bidirector: helical symmetry as in DNA or Bloch-type skyrmions [23].

Using this analytical scheme, many physical quantities can be identified with some limiting group and can be more simply treated in terms of group symmetry. Instead of writing out an explicit function for a property, like the Hamiltonian for the electromagnetic interaction which gives electric dipole transitions:

$$\mathcal{H}'_{\text{em}} = -\frac{e}{2mc}\vec{p} \cdot \vec{A},\tag{4.1}$$

we may note that the Hamiltonian here is for the interaction of a polar vector $\vec{p}$ with the external vector potential field $\vec{A}$. If we treat the potential field as constant while $\vec{p}$ changes, then this perturbation will transform as a time-even polar vector. With that bit of information, selection rules can be calculated on symmetry grounds via the methods described in Section 3.4.2.

4.3   Structural Phase Transitions

Consider a crystal which has macroscopic symmetry of point group G. Let's say that by symmetry, G forbids a macroscopic property of interest such as magnetic dipole moment. Precisely, this means G is not a subgroup of the limiting group corresponding to that property. G may for example require inversion symmetry and the vectorlike property may not possess that symmetry. This is in essence a restatement of **Neumann's principle** (see Section 2.2).

If the crystal undergoes a symmetry-lowering phase transition to point group F (necessarily a subgroup of G) we can quickly determine by inspection whether F is a also a subgroup of any limiting group of a vectorlike quantity. If so, then

we can expect the appearance of that quantity in the low-symmetry phase purely in terms of symmetry and group theory.

Quantitatively we can expand the thermodynamic potential of the system near the phase transition temperature in powers of the order parameter η since it has arbitrarily small values as it grows continuously from zero in the high-symmetry phase. For example, in the case of a scalar or one-component order parameter:

$$\Phi(\eta) = \Phi_0 + \alpha\eta + A\eta^2 + B\eta^3 + C\eta^4. \tag{4.2}$$

It can be shown, that the requirement that this function model a second-order phase transition restricts the expression to even powers of η ($\alpha = B = 0$), requires that C is always greater than 0 and that A is greater than 0 in the high-symmetry phase and less than 0 in the low-symmetry phase [19]. From these requirements, more precise equations for other thermodynamic quantities like entropy and specific heat can be formulated, all in terms of the order parameter.

With representation theory, we can take this a step further and make a connection between the symmetry of the order parameter and the irreducible representations of the high-symmetry phase of the crystal and thus thermodynamic quantities of interest can be further understood quantitatively in terms of irreps and symmetry.

This connection is made first by writing the density function of the crystal (introduced in Section 2.4) as a linear combination of irreps ψ (since they form a complete basis)

$$\rho = \sum_n \sum_i c_{in}\psi_{in}. \tag{4.3}$$

n is the irrep index and i indexes the basis function in that irrep. Next we note that the fully symmetric irrep ψ_0 is invariant with respect to all symmetry elements and thus has the symmetry of the high-temperature phase. That means we can separate our density function into a high-symmetry component and a small perturbation to it:

$$\rho = \rho_0 + \delta\rho \tag{4.4}$$

$$\delta\rho = \sum_{n>0} \sum_i c_{ni}\psi_{ni}. \tag{4.5}$$

Where we've assigned $n = 0$ to the fully-symmetric irrep. Now our expression for $\delta\rho$ is guaranteed to have lower-symmetry than the high-temperature phase and we require that the coefficients c_{ni} are all 0 at and above the transition temperature and thus they, like the order parameter η, should be arbitrarily small at the transition and the thermodynamic potential can be written similar to before

$$\Phi = \Phi_0 + \sum_{n>0} A_n \sum_i c_{in}^2. \tag{4.6}$$

where the A_n are the (functional) coefficients of each irrep. Note that here, the reason we drop linear terms is that we require the thermodynamic potential to be invariant with respect to all transformations in the high-symmetry group. It's not possible to write such a function as a linear combination of irreps when the

fully-symmetric irrep is missing but the next highest order (quadratic) does allow it (for more details see [18] or [19]).

Next, we require all of the c_{in} to be zero at the transition point as before and recognize that the A_n must be greater than zero so that Φ can be a minimum. However, one of them must change sign at the transition point if a new lower-symmetry equilibrium is to be found. Which one changes sign is determined by the symmetry lowering. Hence, we can identify with a specific symmetry lowering an irrep n such that A_n changes sign at the transition point. And thus $\delta\rho$ can be written as a linear combination of the basis functions of that irrep.

$$\delta\rho = \sum_i c_{in}\psi_{in} \tag{4.7}$$

with n fixed according to the symmetry-lowering at hand. If we omit the n as a fundamental aspect of the description of the problem, we can see that the expansion of Φ in terms of $\sum c_i$ has identical requirements to its expansion in terms of η and hence the order parameter is written in terms of irreps

$$\eta = \sum_i c_i. \tag{4.8}$$

Thus, knowing the irrep of a physical quantity can lead to a quantitative description of the thermodynamic potential at a phase transition which introduces that quantity.

4.4   Vectorlike Irreps

In Section 3.4, we covered some background on how the symmetry of initial and final states combined with the symmetry of a perturbing Hamiltonian can be used to determine if that Hamiltonian can join the two states or not. In practice, this is often done by writing out functions in terms of polar vector components x, y, and z or axial vector components R_x, R_y, and R_z [3]. These two sets of coordinates were treated as special bases but now in this more general framework, we take interest in all vectorlike quantities. In order to make this practical for experimentalists to use, ideally we would have character tables which give the irreducible character of all eight vectorlike quantities. Creating such tables is the topic of Chapter 9.

5. Software Methodology

5.1 Introduction

The International Union of Crystallography lists hundreds of software libraries in the field of crystallography [24]. Many of these libraries are created for the purpose of performing calculations such as expected X-ray diffraction patterns or effective one-particle potentials. A search through all listed libraries revealed that only a few work with group theory for its own sake (for example the `Space Group Explorer` software which tabulates space group data [25], or the `SUBGROUP GRAPH` tool which gives group-subgroup relations between point groups [26]) and only one tool was found that mentions both the application of group-theoretical methods and irreducible representations—the `ISOTROPY` software suite [27].

For this reason, it is important to describe in greater detail and call attention to the set of libraries which were created for this thesis. This library is written in the relatively new, Ruby-inspired, object-oriented programming language coincidentally named *Crystal*. Though the libraries introduced here are designed to be small, modular, and broadly applicable, in this chapter, special attention is called to their application in answering the questions raised in this thesis.

Libraries in the Crystal programming language are called *shards*. Collectively, the body of shards described here is called `CrystalSymm` [28]. All of the shards belonging to the `CrystalSymm` collection make use of software engineering best practices which are included directly in the Crystal compiler such as automated tests and automatically-generated documentation (from code comments). In References [29], [30], and [31] a URL is provided to the GitLab repository of each shard. This repository hosts the code, tests, and documentation and it is highly recommended that the interested reader consult the documentation for implementation details as well as the tests for operational definitions of the expected functionality of each library.

5.2 Shards Overview

Due to the way scope and namespace are handled in the Ruby family of languages, the intention of publishing a shard is so that another programmer can access the objects and logic of the shard independent of their specific application. The final shard in this section `SymmMagnetic` is perhaps the most useful generally because it imports the other shards mentioned here as a foundation for its mechanics, but a brief overview of each shard will enable a researcher to import only those objects and tools needed for a specific task rather than the whole library. Furthermore, as we expect some researchers will be interested in Space Groups, they will want to build on more basic shards rather than a top-level shard like `SymmMagnetic`.

5.2.1 SymmBase

The `SymmBase` shard is intended to provide abstract and base classes shared by all other shards in this collection. It provides the abstract class `SymmGroup` which

includes the `isometries` property which is a set of objects implementing the `Isometry` interface (via Crystal's `module` construct).

In addition to these foundations for groups of isometries, this base class also provides some basic math tools which can be used concretely by implementing libraries such as a `Vector3` struct, generalized `RotationMatrix` class, and the stateful `Point` object which combines a three-dimensional location (`Vector3`) with an arbitrary internal (i.e. not position-dependent) binary state for applications like Bicolor symmetry where an isometry should effect the point in an abstract way.

5.2.2 Symm32

The `Symm32` shard demonstrates the implementation of `SymmBase` to the modelling of the 32 crystallographic point groups. It provides the mirror, rotation, inversion, identity, and improper rotation point isometries. Except for inversion and identity, it defines these with respect to an `Axis` which for the sake of computational clarity has a fixed orientation in space and a name. For example, the library provides a point group $2/m$ instantiated like so (see Documentation for details) `PointGroup.parse("monoclinic", "2/m", ["e", "2z", "i", "mz"])` which is interpreted as providing a point group with name $2/m$, class "monoclinic", and four isometries: identity, 2-fold parallel to z, inversion, and mirror plane normal to z. The short-hand code devised for describing isometries (seen above) is simple but not appropriate for discussion here as it is carefully documented in the shard.

For creation of isometries and groups, a method `parse` is designed to produce the same instance (Singleton class) of a given object for use with the `Set` struct in Crystal. This struct requires all of its contents to be unique and does not order them just like the corresponding concept of a mathematical set. By using this struct throughout the design of these shards, it is much easier to use set theory and by extension some theorems of group theory when working with these objects.

This shard provides to code which imports it (via the `require` keyword in Crystal) an array of 32 point groups built according to some self-consistent conventions. An example program could make use of this library like so

```
require 'symm32'

# Array of all 32 point groups for iteration
Symm32::POINT_GROUPS.each do |point_group|
  puts point_group.name  # => [1, 1b, 2, m, 2/m, etc.]
end

# Get a specific one by name
pg222 = Symm32.point_group("222")
pg2 = Symm32.point_group("2")
pg2.isometries.subset? pg222.isometries # => true
```

That `subset?` method is provided by Crystal core since the `isometries` method's return type is `Set`.

A more unique feature of this library is that each group organizes its isome-

tries according to an object called a `Direction`. This object associates a set of isometries to an `Axis` and associates to that "direction" in the group a classification from the crystal family according to the scheme in [15]. These two features are motivated by application to species of symmetry-breaking phase transitions and are helpful in determining the orientational relationships that a subgroup may have in a parent group.

Much of the work to which we have applied this library, has not involved any actual calculations with vectors and matrices. This is perhaps apparent from the description of the above design, but there is a final feature worth mentioning here which makes this possible: `Axes` as an `Enum`.

If an arbitrary orientation of isometries is chosen for each point group, it is possible to minimize the total number of axes needed to just 17. With a finite set like this, it suffices to list them as an `Enum` (i.e. the internal representation of the object in the program is as an integer). Each `Enum` has associated with it a `Vector3` object for use in calculations when that is necessary, but for many of our applications we have found that it is sufficient to take advantage of the fact that the axes are unique.

5.2.3 SymmMagnetic

The last shard which we'll describe in this chapter demonstrates the power of building on the foundation of simple, modular, generalized components in software design. Excluding the listing of 122 point groups, this shard uses less than 100 lines of code to accommodate the extra logic required to include time inversion symmetry. Most of that code is responsible for adding a `parse` method so that new symbols like `i'` will be interpreted as spatial + temporal inversion. One of the reasons this task is so simple is that much of it is done automatically by judicious use of another powerful feature of the Crystal programming language: macros. From the online documentation "Macros are methods that receive AST nodes at compile-time and produce code that is pasted into a program" [32], so it is possible to programmatically create source code which is automatically written and compiled without the user ever interacting with it in full.

`SymmBase` provides a `CompoundIsometry` module which allows isometry `transformation` operations to be stacked arbitrarily. In `Symm32` this is used to create an `ImproperRotation` as the combination of rotation and inversion, and here it is used (via macros) to recreate all of the previous isometries while adding a time inversion operation to their transformation stack.

The only truly new bit of logic in this library is the concept of an `AntiSymmetry` operation, and even that was moved to a separate shard `SymmBicolor` to neatly separate the concern of an abstract isometry like a "color change" operation and the purpose of this shard which is to produce the 122 magnetic point groups.

5.3 Applications

Chapters 7, 8, and 9 cover the applications of these libraries but a demonstration of how the above tools were used to solve problems which had not been thought of during the design phase of their development will be briefly given here. It is
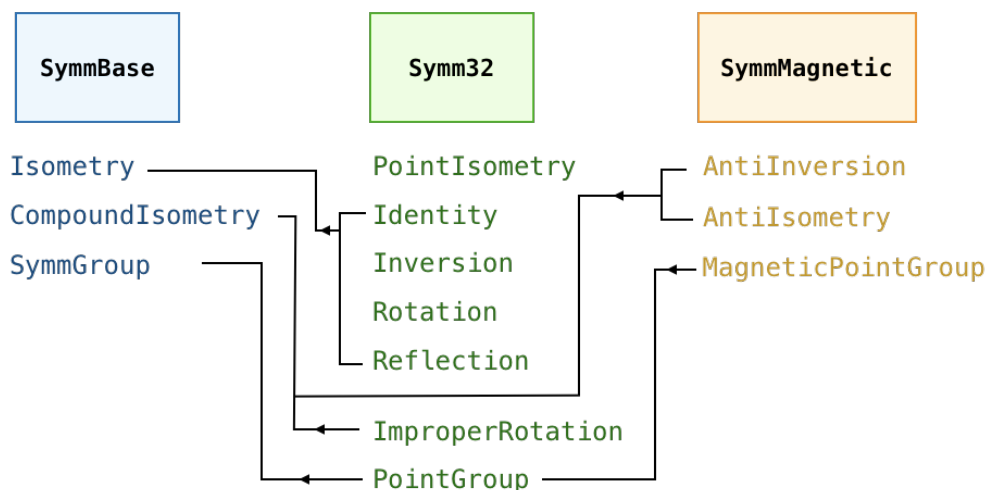
Figure 5.1: The inheritance pattern among a few important objects in the shards described here. The arrow head points from a child to its parent. For a full list of the modules, classes, constants, methods, and so on, please see the complete documentation at the respective repository for each library.

the opinion of the author that this is a good test of the usefulness of general purpose software.

This software was written to solve problems related to **species** (Chapter 7). When that project was finished, it seemed reasonable to try to use the same objects for representing groups to generate the character tables of the 122 magnetic point groups.

To do this, one can proceed with a number of different algorithms and we chose the simple-to-implement Burnside algorithm [33] which relies on complete information of the multiplication rules of the group. As described above, our tools did not have this information. We decided to add two methods to the abstract class `SymmBase::SymmGroup`: a `product` method and an `inverse` method. Then in the base class `Symm32::PointGroup` we added about 40 lines of code for determining inverses and products for any given group. The raw information about the group relationships was really already in there, it just wasn't exposed. With just those two changes, we were able to implement Burnside's algorithm and compute the character table for any group in the system.

Another application which demonstrates the general-purpose design of these libraries was the task of computing the irreducible character of so-called 'vector-like' functions (Chapter 9). In order to accomplish this, we would need a way for a symmetry operator to act on not only a polynomial function but also a vector-valued polynomial, meaning that like the `Point` object described above, there would have to be some way of tracking an abstract operation such as inverting chirality or color.

The first step was to expand on the `SymmBase` shard by creating the abstract notions of `Numberlike` and `Vectorlike` like so:

```
abstract struct Numberlike
  alias Num = (Int32 | Float64)

  abstract def —
```

```
  abstract def +(other : self)
  abstract def +(other : Num)
  abstract def *(other : self)
  abstract def *(other : Num)
end

alias Num = Float64 | Int32 | Numberlike
alias Vectorlike = Tuple(Num, Num, Num)
```

This code basically defines something numberlike as anything that responds to `+`, `-`, or `*` for inclusion in a tuple of numbers constituting a vector. (Since our goal is polynomials, no division requirement is given.)

With that, our five isometries were updated to include logic for how to operate on an arbitrary vectorlike quantity (mostly sharing the same matrix operation logic since the matrix operations only require +, -, and *). And from there all further inherited and compound properties automatically gain the ability to work on this more generalized object. In our research app (the software making use of these libraries), we added a `Numberlike` object called `Term` for representing the product of a scalar and variables such as $3xy^2$ and another object for representing the sum of these called `TermSum`. `TermSum` supported the `Numberlike` operations as well as adding simple division operation for simplifying terms. The result is the ability to handle rudimentary polynomial algebra such as $(2x+3y)/2x = 1+3y/x$.

Because these designs stay close to the fundamental mathematical definitions of these objects, when we discovered a surprising 2D character of bidirector quantities in cubic groups, we were able to explicitly convert the action of each isometry on our vector-valued functions into a 2x2 matrix and check if the matrices obeyed the multiplication rules of the group. A quick script of about 50 lines pulled in the above-mentioned shards and was able to interact with the objects described here in such a way as to construct the matrices and iteratively check each element in the multiplication table to be sure it was completely homomorphic to the group's.

5.4   Conclusion

We've very briefly covered a few of the design principles and decisions involved in creating a collection of software libraries written in the Crystal programming language. These libraries provide objects, data, and base classes intended to be useful as a general-purpose programming tool in approaching crystallographic problem solving from a symmetry-focused and group-theoretical point of view.

Because the crystallographic point groups are implemented in an abstract way using an object-oriented language, the raw contents of the symmetry relationships are available programmatically. It is too common in the field of scientific software engineering to see problems which are solved with elegant symmetry arguments on paper but with brute force number crunching *in silico*. If properly designed, we don't have to crunch numbers to get the answers, we can just politely ask.

6. Chiroaxial Transitions

Let's now apply the foregoing principles to modern-day research problems. This chapter builds on work published in 2016 which analyzed non-magnetic **species** in terms of **polar vectors** and **axial vectors** [15]. The additional analysis here is considering a third type of **vectorlike quantity** from the possible eight: the time-even **chiroaxial bidirector** and was published in 2018 [23].

Due to the absence of inversion and reflection operations in the chiral bidirector symmetry group, only the 9 point groups given in Table 6.1 can host such a property.

Triclinic	1
Monoclinic	2
Orthorhombic	222
Tetragonal	4, 422
Hexagonal	3, 32, 6, 622
Cubic	-

Table 6.1: List of 9 point groups which allow macroscopic chiral bidirector properties.

In recent years, a quasiparticle has been predicted and observed which is only present in crystal structures belonging to the 3, 4, 6, 32, 422, and 622 crystal classes or their subclasses. This is the Bloch-type magnetic skyrmion and it arises from the chiroaxial Dzyaloshinskii–Moriya interaction [34, 35]. One can immediately see the chiroaxial symmetry by inspection of Figure 6.1.
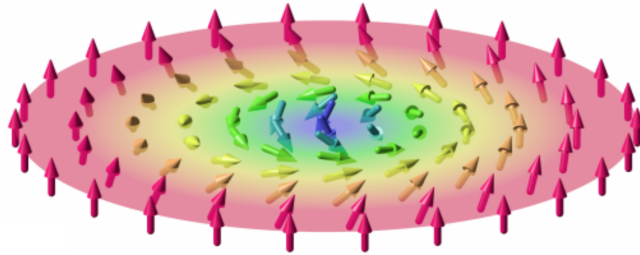
Figure 6.1: Illustration of Bloch-type skyrmion from Wikimedia [36].

A related elementary excitation, the Néel-type skyrmion, may arise when a symmetry-lowering structural phase transition creates a paramagnetic phase with uniaxial polar anisotropy [37, 38]. In the magnetic phase, the process of switching the structural domains can be used to switch skyrmion texture. It's therefore interesting to explore the possible structural transitions that could introduce Bloch-skyrmion texture. This can be accomplished by inspecting all symmetry-lowering transitions that allow **spontaneous components** of chiral bidirector macroscopic properties. We shall call these transitions chiroaxial phase transitions.

6.1 Methods

Each point group in a species will either permit or forbid an axis with chiral bidirector symmetry. If both groups forbid it, this species of transition is not a chiroaxial transition. If the high symmetry group forbids but the low symmetry group permits, then this is in all cases a chiroaxial transition. If both groups permit, then the classification depends on whether the transition increases the freedom of the chiral bidirector axis.

There are four kinds of restrictions that can be placed on the orientation of the axis. (1) Some classes have a special axis which has a high-order (3-, 4-, or 6-fold) rotation: tetragonal and hexagonal classes. In these classes, the chiral bidirector is restricted to that axis. (2) Members of the orthorhombic class possess three axes which are unrelated by symmetry but perpendicular to each other. In the case of point group 222, the chiral bidirector axis may be parallel to any of these three. (3) A chiral bidirector may be oriented anywhere in the plane perpendicular to the 2-fold axis as well as parallel to it. (4) Point group 1 gives no restrictions to the orientation of the axis.

Species 8 (222 > 2) is an example of a species where the freedom of orientation of the axis increases. In the high-symmetry phase (222) the bidirector axis can be imagined as restricted to one axis or to a subset of axes which constitute the plane orthogonal to that axis (i.e. just the x or y axis). In this sense, the lower symmetry phase (2) increases the freedom of the axis by allowing it to be oriented anywhere in the plane and thus species 8 is a chiroaxial transition. A transition which is not chiroaxial is found in species 100 (622 > 2) where the low-symmetry phase (6) permits the chiral bidirector axis only parallel to the 6-fold axis and so too does the parent phase (point group 622). In both of these species, a chiroaxial property is found in the high and low phases, but in only one case is it considered a chiroaxial transition.

There is one further distinction that we should make based on the **orientational domains**. The value of this distinction will be more completely demonstrated in Chapter 7 where it is naturally illustrated in the larger context of all **time-even vectorlike quantities**, but here we will focus only on the concept specifically as it applies to chiroaxial transitions.

If we turn our attention to species 22 ($\bar{4} > 2_|$), we'll observe that the parent phase has four isometries. They each will act on an object with chiroaxial symmetry differently. If we place that object in a suitable location in the child phase (i.e. parallel to the 2-fold axis or perpendicular to it) then the parent isometries would have the following effect:

- The identity element leaves the object in its original orientation. Since it is chiral, we may call this initial chirality "right handed" for convenience.

- Next, the $\bar{4}$ rotation inverts the object and rotates it $\pi/2$ radians. This object is not invariant with respect to inversion, so let's say that it is now left-handed in addition to having a new orientation.

- The 2-fold rotation (which is shared between parent and child) acts as identity on this object: it does nothing.

- Lastly the $\bar{4}^3$ rotation produces the same object as the $\bar{4}$ rotation both in terms of orientation and handedness.

In the above, we did not specify what makes a location suitable. If we chose only the 2-fold axis then none of the isometries would reorient the bidirector axis and only the chirality change would have effect. For this analysis, we need to consider all possibilities. We therefore chose to work with a composite object with two chiral bidirector axes. One axis is parallel to the 2-fold axis and the other is at a "general" angle in the plane. By avoiding a special orientation in the plane (with respect to the parent) we guarantee that the parent isometries will reorient this planar part of the composite. With such an object in mind, we can count each of the unique orientations of the object with confidence that we haven't missed one due to the object being in some special configuration. If the number of unique objects produced by these isometries is equal to the number of orientational states, then it is a fully chiroaxial transition. If it is less, it is only a partial chiroaxial transition.

Another illuminating example into this process is provided by species 32 (422 > 222). In the low symmetry phase, we must be careful to not place three identical objects on the three axes. If the vectorlike quantities were of equal magnitude, our object would posses cubic symmetry by mistake! Instead we must be sure to make the magnitudes unequal.

With this setup, we see that the 4-fold rotation causes the y and x axes to reverse position which we may count as a new orientation. The remaining 2-fold axes in the plane have the same effect and the remaining 2-fold axis parallel to the 4-fold axis is equivalent to identity. Thus we count two orientations and because this species has two domains we again have a fully chiroaxial transition.

6.2   Results and Discussion

Attachment A includes a table which lists the 212 non-magnetic species alongside full, half-full, and empty circles indicating full, partial, and non-chiroaxial transitions respectively. That table gives 93 full chiroaxial transitions, 12 partial chiroaxial transitions, and 107 non-chiroaxial transitions.

The results in the attachment can be applied to the study of any interesting phenomena of chiroaxial symmetry. For example, we may imagine a material which undergoes a ferromagnetic phase transition which may be classified as a chiroaxial species. The magnetic phase would then support a Bloch skyrmion texture by reason of symmetry alone. Because this is a chiral property, we may expect structural domains with opposite-sense skyrmion axes. Of particular interest would be a fully chiroaxial species (so that we obtain all possible senses through all possible domains) where the symmetry of the parent phase forbids a chiroaxial axis and the symmetry of the child phase guarantees only one chiroaxial axis so that the effect of a dual-sense Block skyrmion material is maximized.

The above example of a search query returns just 17 species which could have such properties. They are given in Table 6.2 by crystal class. If we include previous work in this vein (see Reference [15]) the possibility for a query expands to include ferroelectric and ferroelastic transitions.

Crystal Class	Species
Tetragonal	25, 36, 50
Hexagonal	73, 77, 88, 92, 107, 116, 125
Cubic	146, 152, 162, 164, 173, 187, 194

Table 6.2: A table of species which are full chiroaxial transitions; where the parent phase forbids a chiroaxial property; and where the child permits it only along one axis. Such species could give new skyrmion material functionalities such as flipping the sense of the skyrmion according to structural (orientational) domain. In the cubic case, this would include flipping the crystallographic orientation of the skyrmion.

6.3 Conclusion

In this work, we analyzed the 212 macroscopic symmetry-breaking species of phase transitions. Special attention was paid to physical properties with chiroaxial symmetry, i.e. those that could be described by a chiral bidirector. We found 105 chiroaxial species and also determined the number of orientational states which become distinguishable by measurement of a chiroaxial property. Comparison with the number of macroscopic orientational domain states enables us to classify the species as either full chiroaxial or partial chiroaxial. We selected one salient example among many to demonstrate the usefulness of such a dataset by discussing the symmetry requirements of materials with switchable Bloch-skyrmion textures.

In the next chapter, we will expand on this work by moving focus from specific applications to the generalization of all time-invariant vectorlike phenomena. This will greatly simplify queries like the one above where multiple vectorlike quantities may be of interest in the search for new material functionalities.

7. Non-magnetic Vectorlike Transitions

In the previously mentioned studies, each of the 212 species was analyzed by experts case-by-case. The high and low symmetry phases had to be oriented in the mind or on paper and the rules pertaining to various axes were applied to determine the compatibility and incompatibility of each vectorlike quantity with the symmetry setting of the parent and child groups as described in detail in Chapter 6 and Reference [23].

The obvious next step, having considered three of the four **time-even vectorlike quantities**, is to finish out the non-magnetic analysis and work through the 212 cases by hand once again, this time for the **neutral bidirector**. We opted instead to create software which could perform the analysis algorithmically. This chapter introduces that algorithm and presents results which confirm previous work and add to it the missing neutral bidirector property [39].

7.1 Overview

The task is to fully classify the 212 non-magnetic **species** as either partial or full transitions based on the number of orientational domains which can be distinguished by vectorlike quantities (Table 7.1). In the previous chapter, this analysis was described strictly in terms of the chiroaxial property; here we'll explain it more generally.

Symbol	Limiting Group	Characteristic Property
$\updownarrow$	∞/mm	Neutral Bidirector
$\uparrow$	∞m	Polar Vector
$\leftbarbtip$	∞/m	Axial Vector
$\rightleftharpoons$	$\infty 2$	Chiroaxial Bidirector

Table 7.1: Table summarizing multiple notations for the same four vectorlike quantities.

In Section 2.2, the example of species 33 and 34 was given to explain why two species exist for the same group-subgroup pair. In Chapter 6 and Section 2.3, the notion of orientational domain states was summarized, but here we will describe it in full detail.

Let's again consider species 33 and do so in the context of a quantity with polar vector symmetry (Figure 7.1 top). From the high-symmetry phase, we may choose some way to break the symmetry which gives the symmetry of the low-symmetry phase. By comparing the **order** of the two phases (eight and two respectively) we can see that there must be four symmetry-equivalent orientations of the child with respect to the parent. These are called orientational states and when the same orientational state is found throughout a region of a crystal it can be called an **orientational domain**. We see in the top row of the figure that

there are only two unique polar vectors available among these four states. In the bottom row, because the polar vector has four orientations, there is a unique polar vector for each one.

Species 33 (422 > 2₎)
partial ferroelectric transition
4,2

Species 34 (422 > 2₋)
full ferroelectric transition
2
2
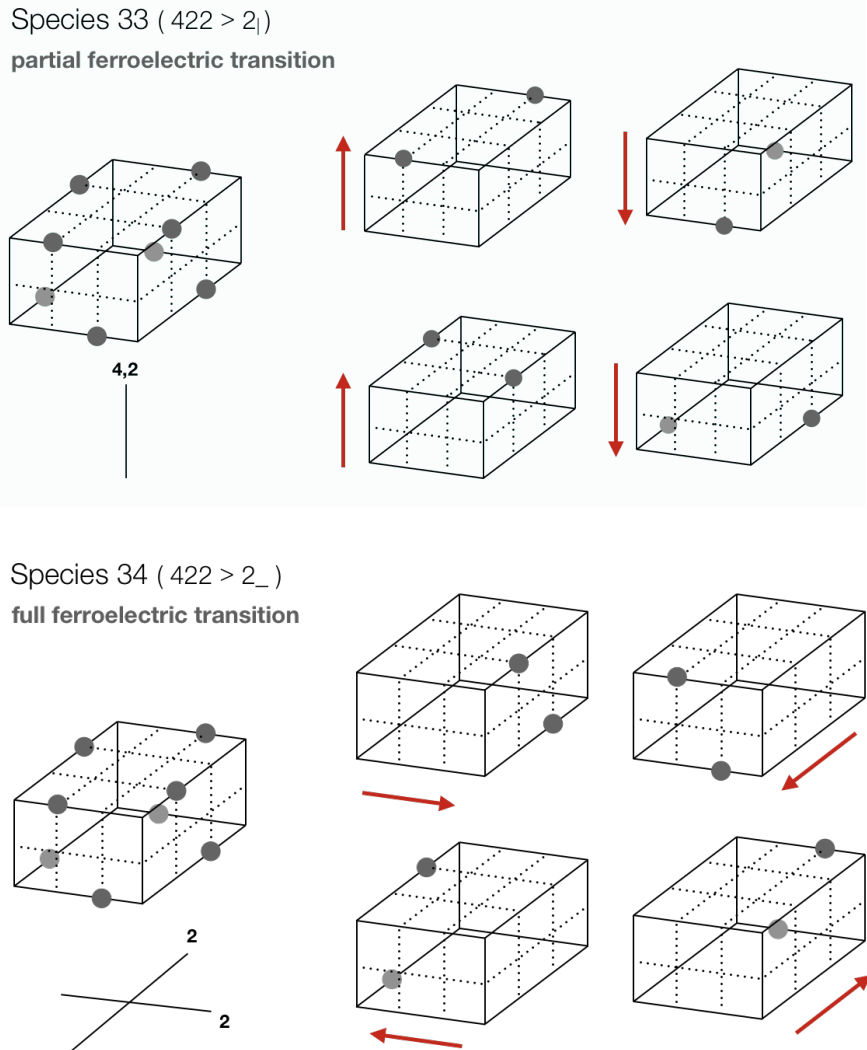
Figure 7.1: An illustration of how in species 33 and 34 a polar vector may or may not fully distinguish each of the possible, energetically equivalent, domain states. By this reasoning, species 33 is a "partial" polar transition and species 34 is a "full" polar transition.

As noted previously, depending on the symmetry of the vectorlike property of interest, the low-symmetry phase may or may not restrict the property to a single axis only. For example, with polar vectors, the symmetry of point group m confines the axis to a 2D plane of orientations.

7.2   Algorithm

Let's now go through the conceptual details of the algorithm which was written to solve this problem in a general and automated way. We first created a software package which contained the raw data and object structure to encapsulate the symmetry rules of the 32 non-magnetic point groups in such a way that the 212

species could be determined automatically including metadata about specific orientations between high and low symmetry phases. This library is called Symm32 and is publicly available with documentation, examples, and a full-coverage test suite [30].

With the 212 species in hand, the next step is to represent the four **limiting groups** under study. Our algorithm uses a pair of `Point` objects to represent the four limiting groups. A point pair defines the axis of the quantity, and the coordinates of each point are chosen so that their midpoint is the shared special point of the point group (i.e. the origin). The final quality, binary sign, is stored in an *internal binary state* of the point. For the neutral bidirector and polar vector, the binary "sign" (plus or minus) is used which is unaffected by symmetry operations. For the chiroaxial bidirector and axial vector, a point is used which has a "handedness" property for keeping track of the effect of the inversion operations.
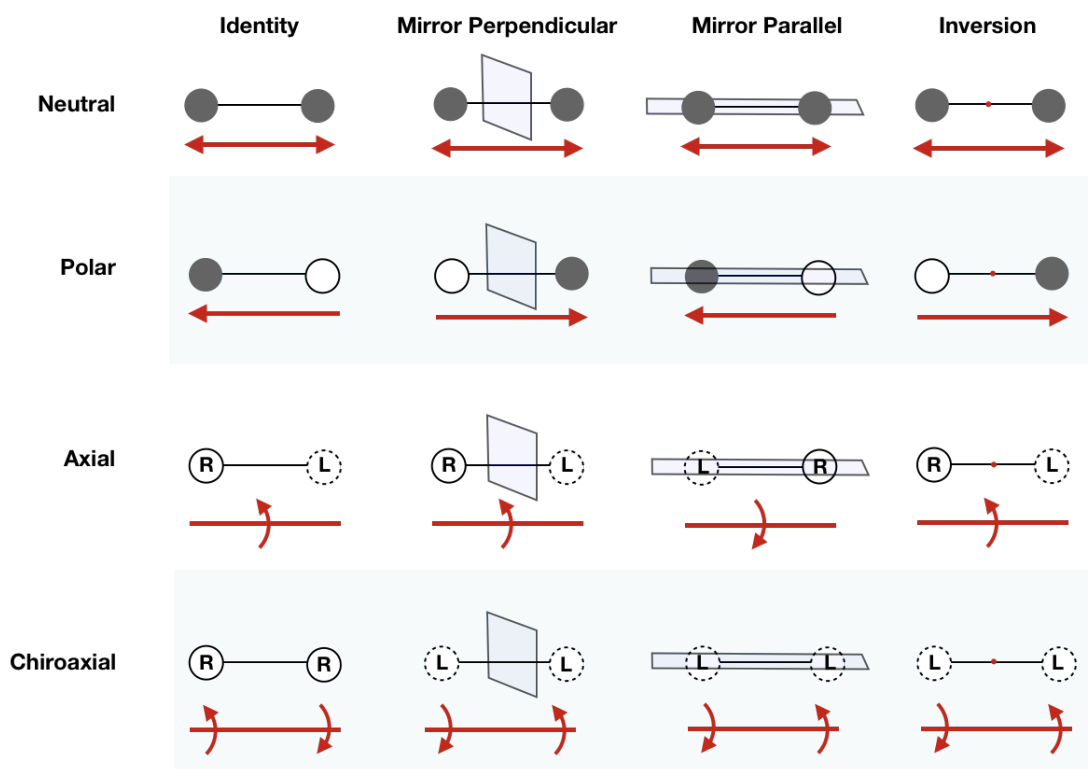
Figure 7.2: Chart explaining how plus and minus points (gray and white) as well as "chiral" points (R and L) can be used to algorithmically represent the four vector and bidirector quantities of interest. Each row gives a vectorlike quantity and each column gives a symmetry operation. The diagrams show the final state of the object after the operation, hence the identity operation is included to show the original orientation of the object. The diagram for mirror symmetries contains a plane about which the reflection should take place. Thus, for example, in the second column third row, the object appears unchanged because the mirror plane has both swapped position of the points and their chirality.

Figure 7.2 shows the scheme: because the vector quantities (polar and axial) have opposite state at their ends, either an operation that switches the locations or an operation that switches the binary state will have the effect of reversing

the vector ('sign' and 'handedness' for polar and axial vectors respectively). The same holds for the bidirector properties which are unaffected by a 2-fold rotation because they have the same binary state at both ends. In the case of chiral points, inversion and mirror operations will change the sense from right-handed to left-handed.

With the help of Figure 7.2, we can demonstrate the utility of this approach. Let's take for example a polar vector perpendicular to a mirror plane (second row second column). The mirror plane swaps the position coordinates of the two points but does not effect their internal state (plus or minus) the result is that the vector is considered as 'pointing' the other way. By contrast, the case where an axial vector is parallel to the plane of the mirror is given in row three column three of the figure. The position coordinates of the points are unaffected but the internal state (left or right handed) is affected and thus the sense or sign of the axial vector is reversed.

The next task is to create a composite object of points which will represent the possible restrictions of the axis to a plane, an axis, or some combination (see Section 6.1). Species 8 provides a good pair of point groups for this demonstration and the placement of each of the four quantities in parent and child is given in Figure 7.3. In point group 2, the bidirectors (top and bottom row, neutral and chiroaxial respectively) are free to be oriented anywhere in the plane (red in the figure) while in the parent they are restricted to one of the three two-fold axes. As described in the aforementioned section, the representation will then contain four points (symbolized here with two arrows), two form a line vertical in the figure and parallel to the 2-fold axis and two form a line at a general angle in the plane.

The logic from the perspective of groups is quite simple. The chiroaxial bidirector has symmetry $\infty 2$, thus it possesses two axes which bear a 2-fold rotation making it a supergroup of point group 2.

7.2.1 Species Classification

The final step is to determine the number of possible unique orientations of the child group within its parent. As in the previous chapter, we let the symmetry operations of the parent group act on the composite object representing the child group and count the number of unique results. If this number is equal to the number of orientational domain states, then each domain would be distinguishable by this property and it is a *full* transition. If it is less (often half), then it is classified as a partial transition.

As mentioned before, in the case where both the parent and child groups are subgroups of the limiting group of the vector or bidirector, we must determine if the child group increases the freedom of the orientation of the axis. Algorithmically, we implemented this by adding a score for each of the types of constraints. The highest score is given when no constraint is present (100), the next highest score when constrained to a plane (10), and the lowest score for each axis of constraint (1). Thus when constrained to a single axis (1) or all three axes (3) the score is always less than if free in the plane (10). Likewise, freedom in a plane plus an axis (11) has a higher score than only in the plane (10).
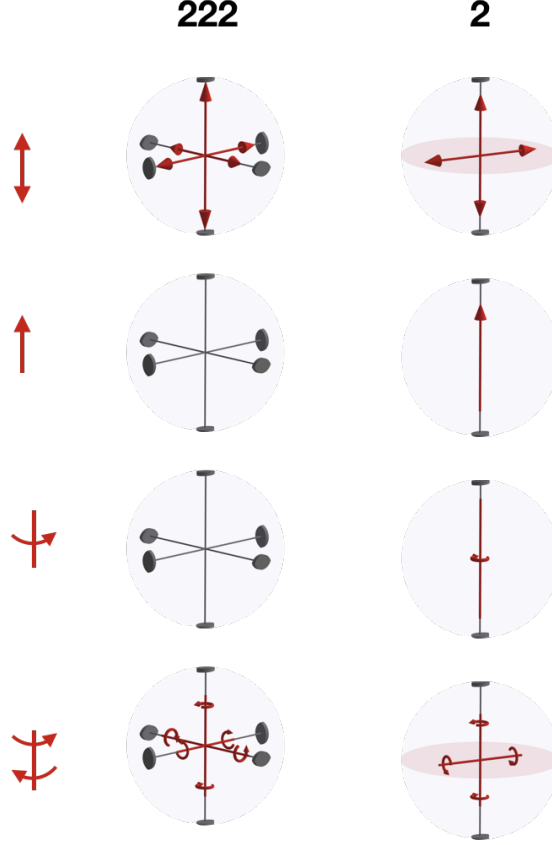
Figure 7.3: Diagram illustrating placement of representative points for each vectorlike quantity (rows) in the high and low symmetry phases of species 8 (columns). The bidirector quantities in the 222 setting have a different magnitude parallel to each axis because the three axes are unrelated by symmetry operations of the group. The vectors (second and third rows) are forbidden by point group 222 thus these diagrams are intentionally empty.

7.3 Results and Discussion

The table in Attachment B gives a complete classification of the 212 species in terms of neutral transitions, polar transitions, axial transitions, and chiroaxial transitions. The species' labels are consistent with Chapter 6 and are explained in greater detail there.

These tables can be used in several ways. First, it can help to identify the species of the phase transition in cases where macroscopic observations of the domain structure allow to determine the number of distinct orientational domain states and their distinguishability by spontaneous vectorlike properties. For example, polar character known from measurements under an applied field, axial or chiral bidirector orientation known from polarized Raman scattering, second harmonic generation microscopy [40, 41], etc. More frequently, the species is known for a given material and the tables allow one to tell immediately whether macroscopic domains can be labeled and experimentally distinguished by the directional quantities considered. Finally, one could also inspect various combinations that can or cannot occur. For example, by simple inspection of the table, one can see that no nonferroelastic species can be simultanously fully ferroelectric, fully

ferroaxial and fully chiroaxial. Some species are very rare in the list, for example a row with four partial marks occurs only twice (species 189 and 197), or there is only one species which is purely chiroaxial and it is only partially chiroaxial (species 131), implying that the set of available vectorlike properties is not enough to distinguish all macroscopic domains uniquely; some other tensorial quantity is needed for a full description of domain states there.

7.4 Conclusion

In this chapter, we described generally an algorithm for fully classifying species of phase transitions in terms of the vectorlike properties which they do and do not spontaneously gain in the low-symmetry phase. This algorithm was implemented as described in Chapter 5. The output of this program was used to generate a complete table of partial and full transitions with respect to the four **time-even vectorlike quantities**. The data in this table confirm the work of previous researchers and complete the study of time-even vectorlike quantities with respect to structural phase transitions. This format should be more useful for researchers as a single point of reference with respect to any possible vectorlike physical property.

This research also lays the foundation for expanding this type of analysis to include **time-odd vectorlike quantities** in the context of magnetic species.

8. Magnetic Vectorlike Transitions

The last step in the logical progression set forth by Chapters 6 and 7 is to include all eight **vectorlike quantities** rather than only focusing on the four time-even ones. In order to do that, we need to consider all magnetic point groups and magnetic species. While the magnetic point groups are well known, the full set of magnetic species was introduced in 2014 as the logical extension of the original Aizu species [42]. In that work, the authors reported 1601 species. To check all 1601 species for each of the eight possible properties would involve checking by hand 12,808 cases. Thus, the creation of the software described in Chapter 7 is a necessary first step to this kind of analysis.

As will be shown in this chapter, not only can an extra species be identified, bringing the total to 1602, but it is useful to organize these species into 212 **genera** of symmetry-related time-even transition properties called the "signature" of the species.

8.1 Methods

Algorithmically speaking, the addition of time-inversion symmetry does not add any fundamentally new difficulties to the methods described in Chapter 7. We do however need to expand the list of point groups to include *magnetic* point groups and add an additional isometry for each time-odd antisymmetric operation. For example, the symbol 4 represents a 4-fold rotation about some axis and the symbol $4'$ represents a 4-fold rotation combined with time reversal. Aside from these considerations though, everything else is identical to the non-magnetic situation implying there is little to add in terms of methods for this research; however, the output of this algorithm is 1602 species which is one more than previously reported. Additionally, for convenience, some automatic procedure must be determined for consistently and algorithmically labelling the species. For these two reasons, though nothing fundamentally new is taking place, it is worth reintroducing and contextualizing previously introduced concepts in this more complex setting.

We'll start with the algorithm which determines species. This algorithm works with point groups in terms of an origin and a set of axes. **Origin**: the identity (1), inversion ($\bar{1}$), and antisymmetry ($1'$) operations act with respect to a point and thus constitute the complete set of symmetries which the origin of the group can bear. **Axes**: the rotation and improper-rotation isometries are naturally associated with some axis of the point group. The mirror operation, though perhaps more naturally described by a plane, can for algorithmic simplicity and consistency also be defined by its normal to that plane.

For example, point group m has two elements, an identity element and a mirror plane, so we model it as an origin with an isometry (1) and an axis with an isometry (m). A higher symmetry group like $2/m$ is an origin with two isometries (1 and $\bar{1}$) and an axis with two isometries (2 and m).

Using this origin + axes model, we compute the possible orientations in three

steps:

1. Compare the origins of each group.

2. Select and compare one axis of the child to all axes of the parent.

3. Select and compare a second axis of the child to the remaining axes of the parent.

For step one, the origin isometries of the candidate child group must form a subset of the origin isometries of the parent group (including of course set equality). For step two, we select some axis of the child and create a list of axes in the parent which could be supersets of this axis. In our algorithm, the Hermann–Mauguin naming scheme was used to give unambiguous correspondence between group name and isometry identification and then the first axis given by that Hermann–Mauguin name was selected in the child. For the third step, we select the second axis specified by the name and note its orientational relation to the first. With the first axis aligned along some member of the list of candidate axes in the parent, the complete set of child axes is rotated leaving this axis invariant until all are parallel to some axis in the parent. Inasmuch as these alignments are such that parallel axes form a group-subgroup relationship, each is counted as a possible species.

Also, it should be noted that all three steps are only required in the case where the child possess two axes. If it possesses only one, then step two is sufficient to identify the orientations with each of the members of the list. If it has only an origin, then no alignment is needed; it is either a simple subgroup or not.

This method counts all possible species but it overcounts if the symmetry of the group, which makes some axes indistinguishable, is not taken into account. A simple method for determining which axes are redundant with respect to the symmetry of the group, is to use the labelling scheme described in Chapter 6. That scheme shows the identity of the planar axes in the tetragonal and hexagonal families as well as the identity of the axes normal to the faces of the cube in the cubic family. Other families cannot distinguish their axes at all.

To illustrate, let's take point group 222 as the low-symmetry phase and a cubic parent as the high-symmetry phase. The first step is to verify that the origin of the cubic parent contains at least the elements of the origin of group 222, it does, the identity element. The second step is to take a 2-fold axis from the child and identify the candidate alignments in the parent. In a cubic parent, there are many, but there are at least two in terms of labelling: normal to a face ($+$) and not normal to a face ($\backslash$). Now we move to step three using a 2-fold axis of the child parallel to a 2-fold axis in the parent normal to a face. In one orientation, all three 2-fold axes are normal to the faces of the cube so we could mark this orientation as $2_+2_+2_+$ (though putting a subscript on every element in the name is redundant). If we hold that first alignment fixed, and rotate to find another place for the second 2-fold axis (repeating step three) then we find two 2-fold axes which are not normal to the faces $2_+2_\backslash2_\backslash$. This exhausts the list of candidates where the first axis is parallel to a 2_+ axis. So we go back to step two and fix as the parent axis a $\backslash$ axis. Now the remaining 2-fold axes in the child are always found to have one normal to a face and one not, so the candidate third

orientation is $2_\backslash 2_+ 2_\backslash$. Due to the symmetry of 222, this is identical to $2_+ 2_\backslash 2_\backslash$ hence we have only two unique orientations.

As a second illustration of this algorithm, and to explain the additional species in our list of 1602, let's now consider the magnetic transition from point group $4321'$ to point group $4'22'$ (species numbers 446 and 447 in Attachment C) in Figure 8.1.
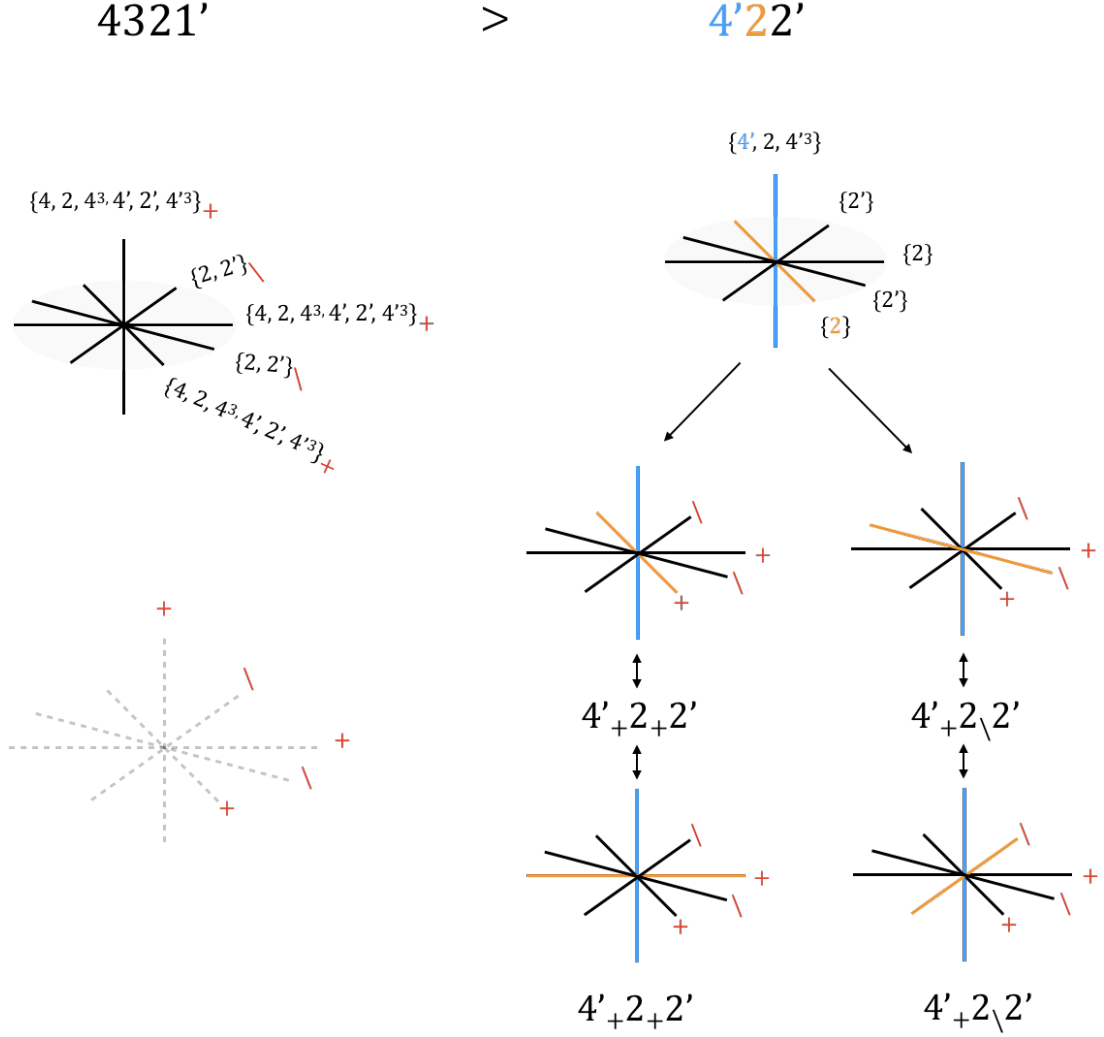
Figure 8.1: Diagram for determining species 446 and 447. Left: A 4-fold axis and plane orthogonal to it of the cubic point group $4321'$ are used as a reference coordinate system. The dashed axes give the template on which $4'22'$ will be oriented. Right: point group $4'22'$ is oriented within 4321' by rotating first one axis (marked in blue) and then a second axis (marked in orange). The bottom-right quadrant of the diagram gives the four possible alignments and the labels in red indicate why only two are considered unique.

As before, the origins are checked first and we find that $\{1\} \subseteq \{1, 1'\}$. Next, the first axis in $4'22'$ is a $4'$ operation and point group $4321'$ has three axes which have symmetries that form a superset of the symmetries on this axis in the child

$$\{4', 2, 4'^3\} \subseteq \{4, 2, 4^3, 4', 2', 4'^3\}. \tag{8.1}$$

These three axes are normal to the faces of the cube and thus under the symmetry of the group count as one.

In Figure 8.1 both groups are represented by the same five axes. The cubic parent has other axes, but they do not need to be considered for this step in the algorithm since the child group has no other axes that would need to align with them.

Next, we choose a second axis, and by convention we select the second symmetry element in the Hermann–Mauguin name which is a 2-fold axis. There are four axes in the parent which possess high enough symmetry to form supersets of this axis (x, y, xy, and $-xy$) and by symmetry two of these are unique (marked $+$ and $\backslash$ in the figure). Both choices of second axis fulfill the requirements mentioned above and thus both satisfy the condition of being unique orientations and could use the labels: $4'_+2_+2'$ and $4'_+2\backslash2'$. (Note that in Attachment C the subscript on the $4'$ element is ignored because it is not needed to differentiate the labels).

8.1.1 Species Classification

With the complete list of 1602 magnetic species of structural phase transitions, we may determine which transitions introduce new orientational degrees of freedom for the eight **vectorlike quantities** according to the process outlined in Chapter 7. We only add the antisymmetry version of the point symmetry operators as potential members of the set of isometries of the origin or axes of a magnetic point group. There are many possible methods for ordering and labelling both the point groups and species, hence we'll next discuss the conventions used here.

There are 122 magnetic point groups [43]. In our context, it is logical to order them from high to low symmetry. Ties are settled by an arbitrary priority of rotation symmetries to mirror symmetries. For black-white groups (see Section 1.3.1) the **halving groups** can settle ties. As an example, the black-white mmm groups ($mm'm'$, $m'm'm'$, and $m'mm$) can be ordered according to their halving groups (respectively $2/m$, 222, and $2mm$). Table 8.1 gives in order the names of the magnetic point groups.

To order the 1602 species, they are divided into 6 groups:

- Section 1: gray > gray

- Section 2: gray > black-white

- Section 3: gray > black

- Section 4: black-white > black-white

- Section 5: black-white > black

- Section 6: black > black

This puts them in high to low symmetry order same as the magnetic groups (see Table 8.2).

It is also necessary to determine a consistent naming scheme for the species which can be implemented automatically by software. Unfortunately this has the disadvantage of varying from previously published systems [16, 42] but the

Table 8.1 (The 122 magnetic point groups):

#	gray	#		#		#		#	
	gray	25	$6mm1'$	49	$4'/m'$	74	$62'2'$	98	mmm
1	$11'$	26	$\bar{6}m21'$	50	$42'2'$	75	$6'mm'$	99	4
2	$\bar{1}1'$	27	$6/mmm1'$	51	$4'22'$	76	$6m'm'$	100	$\bar{4}$
3	$21'$	28	$231'$	52	$4'mm'$	77	$\bar{6}'m'2$	101	$4/m$
4	$m1'$	29	$m\bar{3}1'$	53	$4m'm'$	78	$\bar{6}'m2'$	102	422
5	$2/m1'$	30	$4321'$	54	$\bar{4}'2m'$	79	$\bar{6}m'2'$	103	$4mm$
6	$2221'$	31	$\bar{4}3m1'$	55	$\bar{4}'2m$	80	$6'/m'mm'$	104	$\bar{4}2m$
7	$2mm1'$	32	$m\bar{3}m1'$	56	$\bar{4}2'm'$	81	$6/mm'm'$	105	$4/mmm$
8	$mmm1'$		black-white	57	$4'/mmm'$	82	$6/m'm'm'$	106	3
9	$41'$	33	$\bar{1}'$	58	$4/mm'm'$	83	$6/m'mm$	107	$\bar{3}$
10	$\bar{4}1'$	34	$2'$	59	$4/m'm'm'$	84	$6'/mmm'$	108	32
11	$4/m1'$	35	m'	60	$4/m'mm$	85	$m'\bar{3}'$	109	$3m$
12	$4221'$	36	$2'/m'$	61	$4'/m'm'm$	86	$4'32'$	110	$\bar{3}m$
13	$4mm1'$	37	$2/m'$	62	$\bar{3}'$	87	$\bar{4}'3m'$	111	6
14	$\bar{4}2m1'$	38	$2'/m$	63	$32'$	88	$m\bar{3}m'$	112	$\bar{6}$
15	$4/mmm1'$	39	$22'2'$	64	$3m'$	89	$m'\bar{3}'m'$	113	$6/m$
16	$31'$	40	$2m'm'$	65	$\bar{3}m'$	90	$m'\bar{3}m$	114	622
17	$\bar{3}1'$	41	$2'mm'$	66	$\bar{3}'m'$		black	115	$6mm$
18	$321'$	42	$mm'm'$	67	$\bar{3}'m$	91	1	116	$\bar{6}m2$
19	$3m1'$	43	$m'm'm'$	68	$6'$	92	$\bar{1}$	117	$6/mmm$
20	$\bar{3}m1'$	44	$m'mm$	69	$\bar{6}'$	93	2	118	23
21	$61'$	45	$4'$	70	$6'/m'$	94	m	119	$m\bar{3}$
22	$\bar{6}1'$	46	$\bar{4}'$	71	$6/m'$	95	$2/m$	120	432
23	$6/m1'$	47	$4'/m$	72	$6'/m$	96	222	121	$\bar{4}3m$
24	$6221'$	48	$4/m'$	73	$6'22'$	97	$2mm$	122	$m\bar{3}m$

Table 8.1: The 122 magnetic point groups ordered according to the scheme described in the text.

	gray	black-white	black
gray	1	2	3
black-white	-	4	5
black	-	-	6

Table 8.2: Demonstration of reasoning behind magnetic group ordering used in this thesis. The numbers in this table give a secondary order relation from high to low symmetry.

advantage of being simpler in terms of implementation logic. In orthorhombic groups, we must choose either "xyz" or "zxy" order. Since "zxy" order is used for uniaxial groups (tetragonal and hexagonal) we chose to use it for the orthorhombic groups as well. Of course in the present context, a label like z has no special meaning. We would do better to call them the "first, second, and third axes" since no coordinate system is implied. Still, some choice must be made between labels $2mm$ and $mm2$ or labels $22'2'$ and $2'2'2$. In each case we decided on the former as it places the unique axis in the first position, similar to uniaxial groups.

In cases where two species would share the same name if only parent and child names were given, the orientation of the child's axes within the parent

is indicated with subscripts from one or more of the four symbols previously explained. Technically, all elements in the name could be labelled in this way, so the convention is to label the first two elements of the name and then remove all redundant labels.

8.2 Results and Discussion

Attachment C gives the 1602 magnetic species in a style consistent with the other tables in the attachments. The species are presented alongside a row of circles indicating transition classification as in the previous chapters. Because each slot can hold three values (full, partial, and non), we can view this row as a ternary **species signature**.

In the attached table, we see that the time-invariant portion of this signature is identical among species which share time-invariant symmetry elements, i.e. for each group bearing time-inversion symmetry (a gray group or a black-white group) we can identify a group which has no time-inversion symmetries (a black group) as being identical with respect to time-even vectorlike quantities. This sort of correspondence is really as simple as dropping the ′ symbols from the names of the groups (the 1′ symbol in the case of a gray group). This kind of grouping allows us to identify 212 magnetic **genera** of species which share their time-independent signature and Attachment D gives the 1602 species arranged according to genus. The result is that we may, by investigation, determine the time-even signature of a transition and then find the genus of that transition. Other species in the same genus may have the same time-even signature but vary with respect to time-odd vectorlike properties and thereby open the door to new avenues of research.

8.3 Conclusion

With this final installment, we have completed our classification of the 1602 species of macroscopic symmetry-breaking transitions in terms of vectorlike quantities. The possible applications of this work to discovering and predicting novel material functionalities are derived from the generalized symmetry-based approach presented here.

One such application has been highlighted with respect to elementary excitations. One way that such excitations will be observed, will be through absorption and emission spectra. These kinds of experiments produce data which can be interpreted in terms of symmetry as well as with the aid of suitable character tables. Unfortunately, until now, character tables do not exist tabulating the irreducible character of all vectorlike quantities. In the next chapter, we'll address this issue and present the final data for this thesis which is all 122 magnetic point group character tables including the irreducible characters of vectorlike quantities.

9. Magnetic Irrep Tables with Vectorlike Bases

In Chapter 3, we introduced the basic principles that make **irrep** tables an important tool for material science research. In Chapter 8, we created a symmetry guide to species of macroscopic symmetry-breaking phase transitions in terms of the eight vectorlike quantities consistent with the properties of magnitude, axis, and sign. The concluding research of this thesis is to determine the irreducible character of these quantities in each of the 122 magnetic point groups. For reference and clarity, Table 9.1 gives these eight quantities along with their characteristic property and symbol.

Name	Symbol	Characteristic Property
Time-even Neutral	$\updownarrow$	Bidirector
Time-even Polar	$\uparrow$	Vector
Time-even Axial	$\leftarrow\!\!\!\rightarrow$	Vector
Time-even Chiroaxial	$\leftarrow\!\!\!\rightarrow$	Bidirector
Time-odd Neutral	$\updownarrow$	Bidirector
Time-odd Polar	$\uparrow$	Vector
Time-odd Axial	$\leftarrow\!\!\!\rightarrow$	Vector
Time-odd Chiroaxial	$\leftarrow\!\!\!\rightarrow$	Bidirector

Table 9.1: The eight vectorlike quantities with names, symbols, and characteristic properties which are used to identify them in this thesis.

9.1   Methods

The complex character table of a group can be computed from its regular representation (i.e. knowing only its multiplication table) and over the years many algorithms have been devised to compute the character table using only this information (for example: [44, 45, 46]). Modern sophisticated methods are orders of magnitude faster than their older counterparts, but for simple finite groups like these, we chose to use the older and simpler method of Burnside [33]. This method starts by computing a set of coefficient matrices from the multiplication table of the group and then seeks the eigenvectors and values which simultaneously diagonalize that set. It turns out that the solution can be obtained by solving the eigenvalue problem for a randomized linear combination of these matrices. The resultant eigenvalues were shown by Burnside to be scaled versions of the row data for the character table of that group.

Once a complete complex-valued character table for each group was determined, it was converted to a real-valued character table as described in Section 3.3. Then two methods were used for determining the characters of vectors and bidirectors. For vectors, one may compute a row of the table by determining the matrices which act on components of the vector in the same way the operators should act.

For example, considering a unit vector $\hat{x}$, a 4-fold rotation about the z-axis will rotate it onto the y-axis

$$G_4\hat{x} = \hat{y}. \tag{9.1}$$

Likewise, the unit vector $\hat{y}$ will be rotated into the negative x direction.

$$G_4\hat{y} = -\hat{x} \tag{9.2}$$

Thus a planar vector with components (x, y) can be transformed into $(y,-x)$ by some suitable matrix. Once a matrix like this has been determined for all operators, the character of the representation is immediately calculated from the traces. If this object (a planar vector) transforms as an irreducible representation of the group, then the character generated will be in the irrep table of that group. With our software, we automatically generated all such matrices and found that the corresponding character was always present in the resultant tables thus verifying the results of the previous step as well as calculating the characters of each component of each vector quantity.

Bidirectors, on the other hand, do not form a vector basis. This is illustrated in Figure 9.1 where the addition of the horizontal and vertical bidirectors could be taken to result in either of the diagonal bidirectors in the figure.
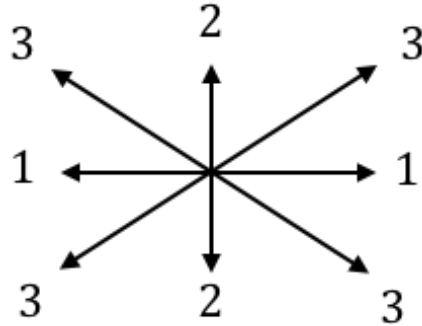
Figure 9.1: In the Cartesian plane, both bidirectors labelled 3 could be the result of adding bidirectors 2 and 1.

To overcome this difficulty, we may work with a function instead of the abstract bidirector object itself. If this function possesses the symmetry of the bidirector property in question, as well as capturing the physical property of interest, then it will be useful to determine its irreducible character. In this work, we selected a simple and general function for demonstration. It's important to note that because bidirectors prevent us from using the concept of 'components' analogous to vector components, we have to calculate the character of the function as its bidirector axis oriented along some special axes. The choice of axes used here were (1) in monoclinic and triclinic groups: parallel to the only axis in the group; (2) in orthorhombic groups: parallel to one of the axes (all are equivalent with respect to bidirector symmetry); (3) in all other groups the "high-symmetry" axis is labelled as z and an axis perpendicular to it is labelled p.

9.2 Results and Discussion

In Attachment E, we give the 122 magnetic point group irrep character tables alongside vector components and bidirector functions which act as bases of the irreps. Because bidirector functions *can* form a vector space, we ignored results which rely on a linear combination (irreps with dimension greater than one). In practice, only the z-parallel bidirectors appear in the tables. In uniaxial and cubic groups, a bidirector perpendicular to the high-order axis (subscript p) would imply that a symmetry-related image exists which is rotated about the z-axis in a two-dimensional plane by angles other than π.

Standard tables often include the vector components as functions written $\{x, y\}$ for example instead of $\{\uparrow_x, \uparrow_y\}$. It is even common for those tables to include what are sometimes called the "angular momentum components" with a capital R as in $\{R_x, R_y\}$ instead of $\{\curlywedge_x, \curlywedge_y\}$ [3]. These tables add to that same conceptual framework another six possible vector components $\uparrow_x$, $\uparrow_y$, $\uparrow_z$, $\curlywedge_x$, $\curlywedge_y$, and $\curlywedge_z$ as well as four new bidirector quantities along as many as two axes giving eight more characters: $\updownarrow_z$, $\updownarrow_p$, Φ_z, Φ_p, $\updownarrow_z$, $\updownarrow_p$, Φ_z, and Φ_p.

As mentioned previously, because bidirectors do not form a vector basis, their results require a little more careful discussion. Table 9.2 gives the one-dimensional irreducible characters of the bidirectors and is taken directly from Reference [21]. Any function which we may want to use to represent a bidirector quantity must have the one-dimensional characters in this table.

	$E(\infty)$	$m_\perp$	$m_\parallel$	2	E'	$m'_\perp$	$m'_\parallel$	$2'$
Time-even Neutral ($\updownarrow$)	1	1	1	1	1	1	1	1
Time-even Chiroaxial (Φ)	1	-1	-1	1	1	-1	-1	1
Time-odd Neutral ($\updownarrow$)	1	1	1	1	-1	-1	-1	-1
Time-odd Chiroaxial (Φ)	1	-1	-1	1	-1	1	1	-1

Table 9.2: Table of one-dimensional irreducible characters of the four bidirector quantities according to Reference [21].

For these tables, we used the vector-valued function $\vec{v}(x, y, z) = z(x\hat{j} - y\hat{i})$ which gives vector components $\{\hat{i}, \hat{j}, \hat{k}\}$ for a vector at coordinates (x, y, z). Now we may check the character of this function by reflecting it through the plane perpendicular to z.

$$\begin{aligned}
\vec{g}(x, y, z) &= m_\perp \vec{f}(x, y, z) \\
&= \vec{f}(x, y, -z) \\
&= -z(x\hat{j} - y\hat{i}) \\
\vec{g} &= -\vec{f}
\end{aligned}$$

(9.3)

and we see that the character is -1. To check that the z-axis has continuous rotational symmetry we can rotate by an arbitrary angle θ like so:

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

$$z = z' \tag{9.4}$$

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \hat{\imath} \\ \hat{\jmath} \end{pmatrix} = \begin{pmatrix} \hat{\imath}' \\ \hat{\jmath}' \end{pmatrix}.$$

Using this transformation we obtain a new function $\vec{g}$:

$$\begin{aligned}
\vec{g}(x,y,z) &= z'(x'\hat{\jmath}' - y'\hat{\imath}') \\
&= z[(x\cos\theta - y\sin\theta)(-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) \\
&\quad - (-x\sin\theta + y\cos\theta)(\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath})] \\
&= z(x\hat{\jmath} - y\hat{\imath}) \\
\vec{g} &= \vec{f}
\end{aligned} \tag{9.5}$$

which proves the desired invariance.

This same kind of reasoning can be applied for all symmetry operations to prove that the vector-valued function indeed possesses the required symmetry of the abstract vectorlike quantity (the **chiroaxial bidirector** in this case).

To obtain the **neutral bidirector**, we may instead consider what might be termed an "axial-vector"-valued function $\breve{v}(x,y,z)$. Such a function would assign an orientation, magnitude, and sign for an *axial* vector at every coordinate triplet (x,y,z). Because these vectors obey different symmetry rules than polar vectors, we will get different results. Let's rework the previous example. As before we may write the mirror operation on the function as

$$\begin{aligned}
\breve{g}(x,y,z) &= m_\perp \breve{f}(x,y,z) \\
&= \breve{f}(x,y,-z).
\end{aligned} \tag{9.6}$$

Only this time, the full picture is not captured by considering only the positional coordinates. The effect of a mirror on an axial vector is to reverse the unit vectors parallel to the mirror and leave unchanged the unit vector perpendicular to the mirror:

$$\begin{aligned}
\breve{g}(x,y,z,\hat{\imath},\hat{\jmath},\hat{k}) &= m_\perp \breve{f}(x,y,z,\hat{\imath},\hat{\jmath},\hat{k}) \\
&= \breve{f}(x,y,-z,-\hat{\imath},-\hat{\jmath},\hat{k})
\end{aligned} \tag{9.7}$$

Thus the rest follows differently:

$$\begin{aligned}
&= -z(-x\hat{\jmath} + y\hat{\imath}) \\
&= z(x\hat{\jmath} - y\hat{\imath}) \\
\breve{g} &= \breve{f}
\end{aligned} \tag{9.8}$$

So the character is $+1$ just as the table requires for neutral vectors. If we continue this kind of reasoning for all operations we'll prove that this function now behaves according to the character of the neutral bidirector.

9.2.1 Application to Elementary Excitations

The standard inclusion of vector components as basis functions of irreducible representations in character tables can and should be expanded to include all eight possible vectorlike quantities for all magnetic point groups. This is valuable in its own right as a generalization of already useful principles but it also provides a systematic approach to identifying host materials for physical properties of interest.

To further elucidate this point, we'll now turn our attention to Néel and Bloch-type ferromagnetic skyrmion host materials as identified by Bogdanov and Yablonskii in 1989. In their paper (Reference [47]), we read the conditions, for certain axial crystal families, under which stable magnetic vortices can form (now commonly known as Néel and Bloch type skyrmions). In Reference [21], the symmetry requirements in terms of vectorlike quantities were given for Bloch-type skyrmion hosts. The algorithm provided here allows us to systematically find all magnetic point groups in which these excitations could be found in a symmetry-lowing phase transition. Let's start with Bloch-type magnetic skyrmions.

A Bloch-type magnetic skyrmion material must allow simultaneously for time-odd polar symmetry (**T** in Reference [21]), time-odd axial symmetry (**M** in the same reference) and time-even chiral symmetry (**C**). Therefore, a host material will have a point group symmetry which possesses one or more (but not all) of those symmetries as bases of the fully symmetric representation and the remaining (one or two) as a basis of one of its other irreps indicating that as a result of symmetry reduction, the necessary vectorlike symmetry is introduced.

A Néel-type magnetic skyrmion likewise requires a triplet of symmetries: **PMF** (again from the notation of Reference [21] where **P** is time-even polar and **F** is time-odd chiroaxial).

Knowing this, a search can be carried out automatically using the character library we've developed in just a few lines of code:

```
require "symm_character"

quantities = {
  :bloch => Set{:chiroaxial_z, :antiaxial_z, :antipolar_z},
  :neel  => Set{:polar_z, :antiaxial_z, :antichiroaxial_z},
}

quantities.each do |quantity_name, basis_set|
  puts quantity_name
  SymmCharacter::TABLES.each do |table|
    fully_symmetric = table.rows.first
    test_set = basis_set.dup - bases_set2id(fully_symmetric.
   bases)
    # next if all are in fully symmetric irrep
    next if test_set.empty?

    # for remaining rows, duplicate test_set and try to empty
    # it out with the ids in each basis.
    table.rows.skip(1).each do |row|
      test_set_dup = test_set.dup
      id_set = bases_set2id(row.bases)
```

```
      test_set_dup -= id_set
      if test_set_dup.empty?
        # success, this is a winner
        puts "#{table.group.name} - #{row.name}"
      end
    end
  end
end

# Takes an array of sets of bases
# converts each basis set to an array of ids
# and flattens it into a set of ids
def bases_set2id(set_arr)
  set_arr.flat_map(&.map(&.id)).to_set
end
```

The above code will print to the console all of the point groups which can act as a host for each kind of skyrmion. We may compare the output of this program (in Tables 9.3 and 9.4) to the published results in Reference [47]. For uniaxial gray groups, and excluding anti-skyrmions, the results exactly match, i.e. we find that classes C_n and D_n host Bloch-type skyrmions and classes C_n and C_{nv} host Néel-type skyrmions.

gray	$11'$, $21'$, $2221'$, $41'$, $4221'$, $31'$, $321'$, $61'$, $6221'$
black-white	$\bar{1}'$, $2'$, m', $2/m'$, $22'2'$, $2m'm'$, $4'$, $\bar{4}'$, $4/m'$, $4'22'$, $4m'm'$, $\bar{4}'2'm$, $\bar{4}2'm'$, $4/mm'm'$, $4/m'mm$, $\bar{3}'$, $3m'$, $\bar{3}m'$, $\bar{3}'m$, $6'$, $\bar{6}'$, $6/m'$, $6'22'$, $6m'm'$, $\bar{6}'m2'$, $\bar{6}m'2'$, $6/mm'm'$, $6/m'mm$
black	$\bar{1}$, m, $2/m$, 222, $2mm$, $\bar{4}$, $4/m$, 422, $4mm$, $\bar{3}$, 32, $3m$, $\bar{6}$, $6/m$, 622, $6mm$

Table 9.3: Bloch-type magnetic skyrmion host point groups. The point groups in the second column are those which introduce one or more of the three required vectorlike quantities (**CMT**) as a basis of an irrep other than the fully-symmetric irrep.

We can turn our attention in a new direction with almost no additional effort. A footnote in Reference [21] mentions the possibility of such a vortex in a polar field i.e. in a ferroelectric material. To systematically identify potential host materials, we again only need to identify the symmetry of the quantity and it is even provided in [21] as **PGC** (time-even polar, time-even axial, and time-even chiral). Thus to the code above we can simply add an entry to the `quantities` hash:

```
:bloch_polar => Set{:polar_z, :axial_z, :chiroaxial_z},
```

The result is provided in Table 9.5.

Lastly, let us mention that a similar analysis could be performed for a description of skyrmion bubble domain structures such as those discussed recently in [48, 49].

gray	$11'$, $21'$, $2mm1'$, $41'$, $4mm1'$, $31'$, $3m1'$, $61'$, $6mm1'$
black-white	$\bar{1}'$, $2'$, m', $2/m'$, $2'mm'$, $m'm'm'$, $4'$, $\bar{4}'$, $4/m'$, $42'2'$, $4'mm'$, $\bar{4}'2m'$, $\bar{4}2'm'$, $4/mm'm'$, $4/m'm'm'$, $\bar{3}'$, $32'$, $\bar{3}m'$, $\bar{3}'m'$, $6'$, $\bar{6}'$, $6/m'$, $62'2'$, $6'mm'$, $\bar{6}'m'2$, $\bar{6}m'2'$, $6/mm'm'$, $6/m'm'm'$
black	$\bar{1}$, m, $2/m$, 222, $2mm$, $\bar{4}$, $4/m$, 422, $4mm$, $\bar{3}$, 32, $3m$, $\bar{6}$, $6/m$, 622, $6mm$

Table 9.4: Néel-type magnetic skyrmion host point groups. The point groups in the second column are those which introduce one or more of the three required vectorlike quantities (**PMF**) as a basis of an irrep other than the fully-symmetric irrep.

gray	$\bar{1}1'$, $m1'$, $2/m1'$, $2221'$, $2mm1'$, $\bar{4}1'$, $4/m1'$, $4221'$, $4mm1'$, $\bar{3}1'$, $321'$, $3m1'$, $\bar{6}1'$, $6/m1'$, $6221'$, $6mm1'$
black-white	$\bar{1}'$, m', $2'/m'$, $2/m'$, $2'/m$, $22'2'$, $2m'm'$, $2'mm'$, $\bar{4}'$, $4'/m$, $4/m'$, $4'/m'$, $42'2'$, $4'22'$, $4'mm'$, $4m'm'$, $\bar{3}'$, $32'$, $3m'$, $\bar{6}'$, $6'/m'$, $6/m'$, $6'/m$, $6'22'$, $62'2'$, $6'mm'$, $6m'm'$
black	$\bar{1}$, m, $2/m$, 222, $2mm$, $\bar{4}$, $4/m$, 422, $4mm$, $\bar{3}$, 32, $3m$, $\bar{6}$, $6/m$, 622, $6mm$

Table 9.5: Bloch-type ferroelectric skyrmion host point groups. The point groups in the second column are those which introduce one or more of the three required vectorlike quantities (**PGC**) as a basis of an irrep other than the fully-symmetric irrep.

Conclusion

Polar vectors and axial vectors already play a prominent role in mathematical and experimental physics due to their mathematical robustness and their usefulness in visualization and reasoning. They are not often recognized according to the group theoretical classification used throughout this thesis and introduced in Reference [21] and so they may mistakenly be thought of as the only two such important quantities. For some time, only these two special cases of the eight vectorlike quantities have been used to determine selection rules and label the spectroscopists' peaks in previous decades via their irreducible character.

Given such broad applicability and utility, it's little wonder that we might try to understand these objects for their own sake, in terms of their own group symmetries. The eight vectorlike quantities discussed herein constitute a complete body of related tools for understanding physical phenomena in terms of their vectorlike properties.

In this thesis, a relatively new line of symmetry-based reasoning—species and genera of macroscopic symmetry-breaking structural phase transitions—was expanded upon and more thoroughly fleshed out in terms of all possible spontaneous components of vectorlike properties. Additionally, the standard point group character tables were expanded on by both (1) adding all vectorlike quantities instead of only the polar and axial ones and (2) including the magnetic point groups and time-inversion-symmetry breaking systems.

This work was limited in scope to the consideration of vector spaces over the field of real numbers. While this has broad applicability in the study of classical systems (e.g. lattice vibrations via the force matrix), future work will be needed for application to quantum mechanical systems over Hilbert space which include anti-linear time reversal and specialized symmetry groups like the crystallographic double groups.

One avenue of future work which we've highlighted here is in the search for potential host materials of various elementary excitations. At a second-order phase transition boundary, we can expect that in the direction of spatial and temporal symmetry breaking, some new properties can be introduced. According to the symmetry of the excitation in question and the vectorlike quantity bases of the irreps of the point group, suitable media can be systematically identified thanks to the algebraic approaches pursued here.

It is the author's hope that the results and tables included here will prove valuable in the search for useful physical characteristics of novel materials via an understanding of the underlying symmetries of elementary excitations both presently in use and as yet undiscovered.

Bibliography

[1] M. Glazer. *Space Groups for Solid State Scientists.* Academic Press, 2013.

[2] D. Harris. *Symmetry and Spectroscopy: An Introduction to Vibrational and Electronic Spectroscopy.* Dover Publications, 1989.

[3] M. Dresselhaus. *Group Theory: Application to the Physics of Condensed Matter.* Springer, 2008.

[4] H. Heesch. XIX. zur systematischen strukturtheorie. III. *Zeitschrift für Kristallographie - Crystalline Materials*, 73:325, 1930.

[5] R. Newnham. *Properties of Materials: Anisotropy, Symmetry, Structure.* Oxford University Press, 2005.

[6] T. Hahn, editor. *International Tables for Crystallography.* International Union of Crystallography, 2006.

[7] T. Damhus. Double groups as symmetry groups for spin-orbit coupling hamiltonians. *MATCH Communications in Mathematical and in Computer Chemistry*, 16:21–82, 1984.

[8] P. Brüesch. *Phonons: Theory and Experiments I: Lattice Dynamics and Models of Interatomic Forces.* Springer, 2012.

[9] J. Cornwell. *Group Theory in Physics (Techniques in Physics, Vol 1).* Academic Press, 1986.

[10] G. Mahan. *Condensed Matter in a Nutshell.* Princeton University Press, 2010.

[11] C. Kittel. *Introduction to Solid State Physics.* Wiley, 2004.

[12] W. Hayes. *Scattering of Light by Crystals.* Dover Publications, 1978.

[13] P. Rostron and D. Gerber. Raman spectroscopy, a review. *International Journal of Engineering and Technical Research*, 6:50–64, 2016.

[14] E. Parrott and J. Zeitler. Terahertz time-domain and low-frequency Raman spectroscopy of organic materials. *Applied Spectroscopy*, 69:1–25, 2015.

[15] J. Hlinka, J. Privratska, P. Ondrejkovic, and V. Janovec. Symmetry guide to ferroaxial transitions. *Physical Review Letters*, 116:177602, 2016.

[16] K. Aizu. Possible species of ferromagnetic, ferroelectric, and ferroelastic crystals. *Physical Review B*, 2:754–772, 1970.

[17] K. Aizu. Comprehensive tabulation of the four categories of ferroic point groups derived from each of the 31 prototype point groups. *Journal of the Physical Society of Japan*, 46:1716–1725, 1979.

[18] L. Landau. *Statistical Physics, Third Edition, Part 1: Volume 5.* Butterworth-Heinemann, 1980.

[19] L. Landau. On the theory of phase transitions. *Journal of Experimental and Theoretical Physics*, 7:19–32, 1937.

[20] B. Souvignier. Representations of crystallographic groups. In *International Union of Crystallography MaThCryst: Summer School on Irreducible of Space Groups*, June 2010.

[21] J. Hlinka. Eight types of symmetrically distinct vectorlike physical quantities. *Physical Review Letters*, 113:165502, 2014.

[22] M. Hird. *Introduction To Liquid Crystals: Chemistry and Physics.* CRC Press, 2007.

[23] K. Erb and J. Hlinka. Symmetry guide to chiroaxial transitions. *Phase Transitions*, 91:953–958, 2018.

[24] International Union of Crystallography. Crystallographic software list. `https://www.iucr.org/resources/other-directories/software`, 2019. [Online; accessed 2-July-2019].

[25] Space Group Explorer. `https://www.iucr.org/resources/other-directories/software/space-group-explorer`, 2019. [Online; accessed 2-July-2019].

[26] S. Ivantchev, E. Kroumova, G. Madariaga, J. Pérez-Mato, and M. Aroyo. *SUBGROUPGRAPH*: a computer program for analysis of group–subgroup relations between space groups. *Journal of Applied Crystallography*, 33:1190–1191, 2000.

[27] H. Stokes, D. Hatch, and B. Campbell. ISOTROPY Software Suite. `iso.byu.edu`, 2019. [Online; accessed 2-July-2019].

[28] CrystalSymm - collection of software packages for modelling group-theoretical approaches to problems in crystallography. `https://gitlab.com/crystal-symmetry`.

[29] SymmBase - software package written in the crystal programming language. `https://gitlab.com/crystal-symmetry/symm_base`.

[30] Symm32 - software package written in the crystal programming language. `https://gitlab.com/crystal-symmetry/symm32`.

[31] SymmMagnetic - software package in the crystal programming language. `https://gitlab.com/crystal-symmetry/symm_magnetic`.

[32] Crystal Language Documentation. `https://crystal-lang.org/reference/syntax_and_semantics/macros.html`, 2019. [Online; accessed 2-July-2019].

[33] W. Burnside. *Theory Of Groups Of Finite Order.* Cambridge University Press, 1911.

[34] A. Bogdanov and A. Hubert. Thermodynamically stable magnetic vortex states in magnetic crystals. *Journal of Magnetism and Magnetic Materials*, 138:255–269, 1994.

[35] A. Leonov, T. Monchesky, N. Romming, A. Kubetzka, A. Bogdanov, and R. Wiesendanger. The properties of isolated chiral skyrmions in thin magnetic films. *New Journal of Physics*, 18:065003, 2016.

[36] K. Everschor-Sitte and M. Sitte. 2skyrmions.png. https://commons.wikimedia.org/w/index.php?curid=37682157, 2015. [Online; accessed 19-June-2019].

[37] I. Kézsmárki, S. Bordács, P. Milde, E. Neuber, L. Eng, J. White, H. Rønnow, C. Dewhurst, M. Mochizuki, K. Yanai, H. Nakamura, D. Ehlers, V. Tsurkan, and A. Loidl. Néel-type skyrmion lattice with confined orientation in the polar magnetic semiconductor GaV_4S_8. *Nature Materials*, 14:1116, 2015.

[38] J. Hlinka, F. Borodavka, I. Rafalovskyi, Z. Docekalova, J. Pokorny, I. Gregora, V. Tsurkan, H. Nakamura, F. Mayr, C. Kuntscher, A. Loidl, S. Bordács, D. Szaller, H. Lee, J. Lee, and I. Kézsmárki. Lattice modes and the Jahn-Teller ferroelectric transition of GaV_4S_8. *Physical Review B*, 94:060104, 2016.

[39] K. Erb and J. Hlinka. Vector, bidirector and bloch skyrmion phases induced by crystallographic symmetry breaking. *submitted*, 2019.

[40] M. Fiebig, V. Pavlov, and R. Pisarev. Second-harmonic generation as a tool for studying electronic and magnetic structures of crystals: review. *The Journal of the Optical Society of America B*, 22:96–118, 2005.

[41] S. Gallego, J. Etxebarria, L. Elcoro, E. Tasci, and J. Perez-Mato. Automatic calculation of symmetry-adapted tensors in magnetic and non-magnetic materials: a new tool of the Bilbao Crystallographic Server. *Acta Crystallographica Section A: Foundations and Advances*, 75:438–447, 2019.

[42] D. Litvin and V. Janovec. Spontaneous tensor properties for multiferroic phases. *Ferroelectrics*, 461:10–15, 2014.

[43] G. Rado and H. Suhl. *Magnetism: A Treatise on Modern Theory and Materials, Volume II Part A: Statistical Models, Magnetic Symmetry, Hyperfine Interactions, and Metals.* Academic Press, 1965.

[44] G. Schneider. Dixon's character table algorithm revisited. *Journal of Symbolic Computation*, 9:601–606, 1990.

[45] J. Dixon. High speed computation of group characters. *Numerische Mathematik*, 10:446–450, 1967.

[46] W. Unger. Computing the character table of a finite group. *Journal of Symbolic Computation*, 41:847–862, 2006.

[47] A. Bogdanov and D. Yablonskii. Thermodynamically stable "vortices" in magnetically ordered crystals. the mixed state of magnets. *Journal of Experimental and Theoretical Physics*, 68:178, 1989.

[48] Z. Hong, A. Damodaran, F. Xue, S. Hsu, J. Britson, A. Yadav, C. Nelson, J. Wang, J. Scott, L. Martin, R. Ramesh, and L. Chen. Stability of polar vortex lattice in ferroelectric superlattices. *Nano Letters*, 17:2246–2252, 2017.

[49] S. Das, Y. Tang, Z. Hong, M. Gonçalves, M. McCarter, C. Klewe, K. Nguyen, F. Gómez-Ortiz, P. Shafer, E. Arenholz, V. Stoica, S. Hsu, B. Wang, C. Ophus, J. Liu, C. Nelson, S. Saremi, B. Prasad, A. Mei, D. Schlom, J. Íñiguez, P. García-Fernández, D. Muller, L. Chen, J. Junquera, L. Martin, and R. Ramesh. Observation of room-temperature polar skyrmions. *Nature*, 568:368–372, 2019.

[50] R. Horn. *Matrix Analysis*. Cambridge University Press, 1990.

[51] T. Rowland. "Group Action." From MathWorld—A Wolfram Web Resource. [Online; accessed 26-July-2019].

List of Figures

List of Tables

Glossary

anti-linear Also called conjugate-linear or semilinear [50], an anti-linear map is a mapping T between vector spaces (say, from V) which instead of being homogeneous is conjugate homogeneous

$$T(ax) = a^*T(x)$$

antisymmetry A symmetry operation which does not affect the spatial positions of points, but instead affects some other abstract property. For example, if the points were colored black and white, then the antisymmetry operation could be designated to change black to white and vice versa. The antisymmetry formalism is commonly used to handle

axial vector A vector which is symmetric under inversion but antisymmetric under reflection through any plane not normal to its axis. Such a vector is also called a pseudovector, especially in the case where it is the result of a

basis function Any function which is an element of a basis in function space. Any continuous function can be represented as a linear combination of basis functions and so in representation theory, special attention is paid to

bidirector A vectorlike quantity which is symmetric under a 2-fold rotation

character (group theory) The traces of the representation of a group

chiroaxial bidirector A bidirector which is antisymmetric under inversion

crystal lattice A three-dimensional arrangement of atoms which possesses

direct product (of groups) The direct product of two groups G and F is defined as the set of all possible ordered pairs of elements from each $\{(G_1, H_1), (G_2, H_1), ...(G_i, H_j)\}$ combined with an operation defined component-wise as $(G_1, H_1)(G_1, H_1) = (G_1 G_2, H_1 H_2)$ using the group

double group A group of symmetry elements which commute with a Hamiltonian that includes spin-orbit coupling. The group is doubled in order because each element in the original group is paired with a time-reversal operator, like the gray magnetic groups, except that the

factor group The set of right cosets of an invariant subgroup I of a group G [9]. A factor group is also called a quotient group and denoted G/I.

genus (of phase transitions) Continuing the taxonomic theme of *species*, a genus of phase transitions is a collection of species. In particular, each genus contains species which are related by transforming the time-odd elements of their point groups into the time-even equivalents. . .

group A set of elements and an operation for mapping pairs of those elements to members of the set such that the mapping obeys the group axioms—identity, associativity, inversion, and closure as described in Section 1.2. . . . .

group action A group G is said to act on a set X when there is a map $\phi : G \times X \to X$ such that the following conditions hold for all elements x in X [51]:

1. $\phi(e, x) = x$ where e is the identity element of G.
2. $\phi(g, \phi(h, x)) = \phi(gh, x)$ for all g,h in G.

Hence it associates to each group element a transformation on a set. .

halving group A subgroup of a group which has exactly half of the elements of its parent. . . . .

invariant subspace The space spanned by vectors which are invariant with respect to a linear transformation. . . . .

inversion One of the axioms of group theory is that every element in a group must have an inverse which when combined with the element, under the group operation, produces the identity element of the group. Geometrically, inverting a point (x, y, z) through the origin gives the inverse point $(-x, -y, -z)$. . . . .

irrep Common abbreviation of irreducible representation. . . . .

isometry An operation which leaves invariant the distance metric(s) of an object. For the metric spaces used in crystallography, this simply means preserving the lengths and angles involved in describing the relative positions of atoms in a crystal lattice to one another. . . . .

limiting group A group with order n as the limit n approaches infinity is called a limiting group and is used to analyze continuous symmetry such as the continuous rotational symmetry of a cylinder about its axis.

Mulliken symbol A short string of characters used to label irreps in a character table based on the character of various symmetry elements in the group. . . . .

Neumann's principle "The symmetry of any physical property of a crystal must include the symmetry elements of the point group of the crystal" from Reference [5]. 15, 27

neutral bidirector The vectorlike quantity which is symmetric under all ∞/mm symmetry operations. 26, 41, 56

order (group theory) The number of elements in the group. 8, 41

orientational domain In the low-symmetry phase of a macroscopic symmetry-breaking phase transition, there are two or more energetically equivalent ways for the low symmetry structure to be oriented with respect to the high symmetry structure. A region in a crystal lattice where these orientations are uniform is an orientational domain. See Chapter 7 for full discussion. 15, 38, 41

phase A spatially distinct region of matter with identical physical properties. 13

phase transition The transition of a substance from one phase to another, usually as a function of some thermodynamic quantity like temperature. 13

point group A group of point isometries. 9

point isometry An isometry which leaves at least one point in an object fixed with respect to transformation. 5

polar vector An ordinary vector, in the physicist's vocabulary; a quantity with both magnitude and direction. It can be represented by an arrow and is called polar because it is simply an axis with two ends that are distinguished (polarized). 25, 37

proper subgroup A subgroup which has fewer elements than its parent. This term is used to contrast with the more general term "subgroup" which, by the standard definition used in set theory, may have the same order (i.e. be identical to) its parent. 13

representation (group theory) In group theory, the term "representation" is strictly used to describe a group action defined on a vector space i.e. a representation of a group is a group action which assigns to each element of the group a square matrix such that matrix multiplication for that set of matrices has the same multiplication table as the abstract elements of the group. 19

set In the context of set theory, a set is an abstract, unordered collection of distinguishable objects. The objects may be any distinct things that can be described, including other sets. 7

Attachments

A. Species of Chiroaxial Transitions

The following page contains the complete table of all 212 Aizu species of non-magnetic macroscopic symmetry-breaking structural phase transitions. Each is marked as either a full chiroaxial, partial chiroaxial, or non-chiroaxial transition using the symbols given in Table A.1.

Transition Type	Symbol
Full Chiroaxial	●
Partial Chiroaxial	◖
Non-chiroaxial	○

Table A.1: Guide to symbols used in this attachment.

Chapter 6 describes the motivation and methods for creating this table as well as discussing the results and presenting some conclusions about its contents, limitations, and possible applications.

No.	G	F	C
1	$\bar{1}$	1	●
2	2	1	●
3	m	1	●
4	$2/m$	m	○
5	$2/m$	2	●
6	$2/m$	$\bar{1}$	○
7	$2/m$	1	●
8	222	2	●
9	222	1	●
10	$2mm$	m	○
11	$2mm$	2	●
12	$2mm$	1	●
13	mmm	$2mm$	○
14	mmm	222	●
15	mmm	$2/m$	○
16	mmm	m	○
17	mmm	2	●
18	mmm	$\bar{1}$	○
19	mmm	1	●
20	4	$2_|$	●
21	4	1	●
22	$\bar{4}$	$2_|$	●
23	$\bar{4}$	1	●
24	$4/m$	$\bar{4}$	○
25	$4/m$	4	●
26	$4/m$	$2/m_|$	○
27	$4/m$	$m_|$	○
28	$4/m$	$2_|$	●
29	$4/m$	$\bar{1}$	○
30	$4/m$	1	●
31	422	4	○
32	422	222	●
33	422	$2_|$	●
34	422	2_-	●
35	422	1	●
36	$4mm$	4	●
37	$4mm$	$2_|mm$	○
38	$4mm$	m_-	○
39	$4mm$	$2_|$	●
40	$4mm$	1	●
41	$\bar{4}2m$	$\bar{4}$	○
42	$\bar{4}2m$	$2_|mm$	○
43	$\bar{4}2m$	222	●
44	$\bar{4}2m$	m_-	○
45	$\bar{4}2m$	$2_|$	●
46	$\bar{4}2m$	2_-	●
47	$\bar{4}2m$	1	●
48	$4/mmm$	$\bar{4}2m$	○
49	$4/mmm$	$4mm$	○
50	$4/mmm$	422	●
51	$4/mmm$	$4/m$	○
52	$4/mmm$	$\bar{4}$	○
53	$4/mmm$	4	◐

No.	G	F	C
54	$4/mmm$	mmm	○
55	$4/mmm$	$2_|mm$	○
56	$4/mmm$	2_-mm	○
57	$4/mmm$	222	●
58	$4/mmm$	$2/m_|$	○
59	$4/mmm$	$2/m_-$	○
60	$4/mmm$	$m_|$	○
61	$4/mmm$	m_-	○
62	$4/mmm$	$2_|$	●
63	$4/mmm$	2_-	●
64	$4/mmm$	$\bar{1}$	○
65	$4/mmm$	1	●
66	3	1	●
67	$\bar{3}$	3	●
68	$\bar{3}$	$\bar{1}$	○
69	$\bar{3}$	1	●
70	32	3	○
71	32	2_-	●
72	32	1	●
73	$3m$	3	●
74	$3m$	m_-	○
75	$3m$	1	●
76	$\bar{3}m$	$3m$	○
77	$\bar{3}m$	32	●
78	$\bar{3}m$	$\bar{3}$	○
79	$\bar{3}m$	3	◐
80	$\bar{3}m$	$2/m_-$	○
81	$\bar{3}m$	m_-	○
82	$\bar{3}m$	2_-	●
83	$\bar{3}m$	$\bar{1}$	○
84	$\bar{3}m$	1	●
85	6	3	○
86	6	$2_|$	●
87	6	1	●
88	$\bar{6}$	3	●
89	$\bar{6}$	$m_|$	○
90	$\bar{6}$	1	●
91	$6/m$	$\bar{6}$	○
92	$6/m$	6	●
93	$6/m$	$\bar{3}$	○
94	$6/m$	3	◐
95	$6/m$	$2/m_|$	○
96	$6/m$	$m_|$	○
97	$6/m$	$2_|$	●
98	$6/m$	$\bar{1}$	○
99	$6/m$	1	●
100	622	6	○
101	622	32	○
102	622	3	○
103	622	222	●
104	622	$2_|$	●
105	622	2_-	●
106	622	1	●

No.	G	F	C
107	$6mm$	6	●
108	$6mm$	$3m$	○
109	$6mm$	3	◐
110	$6mm$	$2_|mm$	○
111	$6mm$	m_-	○
112	$6mm$	$2_|$	●
113	$6mm$	1	●
114	$\bar{6}m2$	$\bar{6}$	○
115	$\bar{6}m2$	$3m$	○
116	$\bar{6}m2$	32	●
117	$\bar{6}m2$	3	◐
118	$\bar{6}m2$	2_-mm	○
119	$\bar{6}m2$	$m_|$	○
120	$\bar{6}m2$	m_-	○
121	$\bar{6}m2$	2_-	●
122	$\bar{6}m2$	1	●
123	$6/mmm$	$\bar{6}m2$	○
124	$6/mmm$	$6mm$	○
125	$6/mmm$	622	●
126	$6/mmm$	$6/m$	○
127	$6/mmm$	$\bar{6}$	○
128	$6/mmm$	6	◐
129	$6/mmm$	$\bar{3}m$	○
130	$6/mmm$	$3m$	○
131	$6/mmm$	32	◐
132	$6/mmm$	$\bar{3}$	○
133	$6/mmm$	3	◐
134	$6/mmm$	mmm	○
135	$6/mmm$	$2_|mm$	○
136	$6/mmm$	2_-mm	○
137	$6/mmm$	222	●
138	$6/mmm$	$2/m_|$	○
139	$6/mmm$	$2/m_-$	○
140	$6/mmm$	$m_|$	○
141	$6/mmm$	m_-	○
142	$6/mmm$	$2_|$	●
143	$6/mmm$	2_-	●
144	$6/mmm$	$\bar{1}$	○
145	$6/mmm$	1	●
146	23	3	●
147	23	22_+2	●
148	23	2_+	●
149	23	1	●
150	$m\bar{3}$	23	○
151	$m\bar{3}$	$\bar{3}$	○
152	$m\bar{3}$	3	●
153	$m\bar{3}$	mm_+m	○
154	$m\bar{3}$	2_+m_+m	○
155	$m\bar{3}$	22_+2	●
156	$m\bar{3}$	$2/m_+$	○
157	$m\bar{3}$	m_+	○
158	$m\bar{3}$	2_+	●
159	$m\bar{3}$	$\bar{1}$	○

No.	G	F	C
160	$m\bar{3}$	1	●
161	432	23	○
162	432	32	●
163	432	3	◐
164	432	422	●
165	432	4	◐
166	432	22_+2	●
167	432	$22_\backslash 2$	●
168	432	2_+	●
169	432	$2_\backslash$	●
170	432	1	●
171	$\bar{4}3m$	23	○
172	$\bar{4}3m$	$3m$	○
173	$\bar{4}3m$	3	●
174	$\bar{4}3m$	$\bar{4}2_+m$	○
175	$\bar{4}3m$	$\bar{4}$	○
176	$\bar{4}3m$	$2_+m_\backslash m$	○
177	$\bar{4}3m$	22_+2	●
178	$\bar{4}3m$	$m_\backslash$	○
179	$\bar{4}3m$	2_+	●
180	$\bar{4}3m$	1	●
181	$m\bar{3}m$	$\bar{4}3m$	○
182	$m\bar{3}m$	432	○
183	$m\bar{3}m$	$m\bar{3}$	○
184	$m\bar{3}m$	23	○
185	$m\bar{3}m$	$\bar{3}m$	○
186	$m\bar{3}m$	$3m$	○
187	$m\bar{3}m$	32	●
188	$m\bar{3}m$	$\bar{3}$	○
189	$m\bar{3}m$	3	◐
190	$m\bar{3}m$	$4/mmm$	○
191	$m\bar{3}m$	$\bar{4}2_+m$	○
192	$m\bar{3}m$	$\bar{4}2_\backslash m$	○
193	$m\bar{3}m$	$4mm$	○
194	$m\bar{3}m$	422	●
195	$m\bar{3}m$	$4/m$	○
196	$m\bar{3}m$	$\bar{4}$	○
197	$m\bar{3}m$	4	◐
198	$m\bar{3}m$	mm_+m	○
199	$m\bar{3}m$	$mm_\backslash m$	○
200	$m\bar{3}m$	2_+m_+m	○
201	$m\bar{3}m$	$2_+m_\backslash m$	○
202	$m\bar{3}m$	$2_\backslash mm$	○
203	$m\bar{3}m$	22_+2	●
204	$m\bar{3}m$	$22_\backslash 2$	●
205	$m\bar{3}m$	$2/m_+$	○
206	$m\bar{3}m$	$2/m_\backslash$	○
207	$m\bar{3}m$	m_+	○
208	$m\bar{3}m$	$m_\backslash$	○
209	$m\bar{3}m$	2_+	●
210	$m\bar{3}m$	$2_\backslash$	●
211	$m\bar{3}m$	$\bar{1}$	○
212	$m\bar{3}m$	1	●

B. Species of Time-even Vectorlike Transitions

The following page contains the complete table of all 212 Aizu species of non-magnetic macroscopic symmetry-breaking structural phase transitions. Each is listed with four symbols indicating the classification of the transition with respect to a vectorlike quantity. The vectorlike quantities label the columns and are given by the symbols in Table B.1.

Symbol	Limiting Group	Characteristic Property
$\updownarrow$	∞/mm	Neutral Bidirector
$\uparrow$	∞m	Polar Vector
$\curlyvee$	∞/m	Axial Vector
$\skew{-2}\curlyvee$	$\infty 2$	Chiroaxial Bidirector

Table B.1: Reproduction of Table 7.1. Table summarizing multiple notations for the same four vectorlike quantities.

The transition classification is either full (e.g. full chiroaxial, full polar, etc.), partial (e.g. partial axial, partial neutral, etc.), or non (e.g. non-chiroaxial, non-polar, etc.) according to the symbols in Table B.2.

Transition Type	Symbol
Full	●
Partial	◑
Non	○

Table B.2: Guide to symbols used in this attachment.

Chapter 7 describes the motivation and methods for creating this table as well as discussing the results and presenting some conclusions about its contents, limitations, and possible applications.

No.	G	F	n	↕	↑	⊬	⊭
1	$\bar{1}$	1	2	○	●	○	●
2	2	1	2	●	●	○	●
3	m	1	2	●	●	●	●
4	$2/m$	m	2	○	●	○	○
5	$2/m$	2	2	○	●	●	○
6	$2/m$	$\bar{1}$	2	●	○	●	○
7	$2/m$	1	4	◐	●	●	●
8	222	2	2	●	●	●	●
9	222	1	4	●	●	●	●
10	$2mm$	m	2	●	●	●	○
11	$2mm$	2	2	●	○	●	○
12	$2mm$	1	4	●	●	●	●
13	mmm	$2mm$	2	○	●	○	○
14	mmm	222	2	○	○	○	●
15	mmm	$2/m$	2	●	○	●	○
16	mmm	m	4	◐	●	○	●
17	mmm	2	4	◐	◐	●	●
18	mmm	$\bar{1}$	4	●	○	●	○
19	mmm	1	8	◐	●	○	●
20	4	$2_|$	2	○	○	●	○
21	4	1	4	●	●	●	●
22	$\bar{4}$	$2_|$	2	●	●	○	●
23	$\bar{4}$	1	4	●	●	●	●
24	$4/m$	$\bar{4}$	2	○	○	○	○
25	$4/m$	4	2	●	○	●	●
26	$4/m$	$2/m_|$	2	○	○	○	○
27	$4/m$	$m_|$	4	◐	●	○	○
28	$4/m$	$2_|$	4	◐	◐	○	●
29	$4/m$	$\bar{1}$	4	●	○	●	○
30	$4/m$	1	8	◐	●	○	●
31	422	4	2	●	○	●	●
32	422	222	2	●	○	○	●
33	422	$2_|$	4	●	◐	◐	●
34	422	2_-	4	●	○	●	●
35	422	1	8	●	●	●	●
36	$4mm$	4	2	○	○	●	●
37	$4mm$	$2_|mm$	2	●	○	○	○
38	$4mm$	m_-	4	●	●	○	●
39	$4mm$	$2_|$	4	●	◐	◐	●
40	$4mm$	1	8	●	●	○	●
41	$\bar{4}2m$	$\bar{4}$	2	○	○	●	○
42	$\bar{4}2m$	$2_|mm$	2	●	●	○	○
43	$\bar{4}2m$	222	2	●	●	●	○
44	$\bar{4}2m$	m_-	4	●	●	●	○
45	$\bar{4}2m$	$2_|$	4	●	◐	●	●
46	$\bar{4}2m$	2_-	4	●	●	●	●
47	$\bar{4}2m$	1	8	●	●	●	●
48	$4/mmm$	$\bar{4}2m$	2	○	○	○	○
49	$4/mmm$	$4mm$	2	○	●	○	○
50	$4/mmm$	422	2	○	○	○	●
51	$4/mmm$	$4/m$	2	○	○	●	○
52	$4/mmm$	$\bar{4}$	4	○	○	◐	○
53	$4/mmm$	4	4	○	◐	○	◐

No.	G	F	n	↕	↑	⊬	⊭
54	$4/mmm$	mmm	2	●	○	○	○
55	$4/mmm$	$2_|mm$	4	○	◐	○	○
56	$4/mmm$	2_-mm	4	◐	●	○	○
57	$4/mmm$	222	4	◐	○	○	●
58	$4/mmm$	$2/m_|$	4	●	○	◐	○
59	$4/mmm$	$2/m_-$	4	●	○	●	○
60	$4/mmm$	$m_|$	8	◐	●	◐	○
61	$4/mmm$	m_-	8	◐	●	○	○
62	$4/mmm$	$2_|$	8	◐	◐	◐	●
63	$4/mmm$	2_-	8	●	○	○	●
64	$4/mmm$	$\bar{1}$	8	●	○	●	○
65	$4/mmm$	1	16	◐	●	◐	●
66	3	1	3	●	●	●	●
67	$\bar{3}$	3	2	○	●	○	●
68	$\bar{3}$	$\bar{1}$	3	●	○	●	○
69	$\bar{3}$	3	6	●	○	◐	●
70	32	3	2	○	●	●	●
71	32	2_-	3	●	●	●	●
72	32	1	6	●	●	●	●
73	$3m$	3	2	○	●	●	●
74	$3m$	m_-	3	●	●	●	○
75	$3m$	1	6	●	●	●	●
76	$\bar{3}m$	$3m$	2	○	●	○	○
77	$\bar{3}m$	32	2	○	○	○	●
78	$\bar{3}m$	$\bar{3}$	2	●	○	●	○
79	$\bar{3}m$	3	4	○	◐	○	◐
80	$\bar{3}m$	$2/m_-$	3	●	●	●	○
81	$\bar{3}m$	m_-	6	◐	●	◐	○
82	$\bar{3}m$	2_-	6	◐	●	◐	●
83	$\bar{3}m$	$\bar{1}$	6	●	○	●	○
84	$\bar{3}m$	1	12	◐	●	◑	●
85	6	3	2	○	○	○	○
86	6	$2_|$	3	○	○	●	○
87	6	1	6	●	●	●	●
88	$\bar{6}$	3	2	○	●	●	●
89	$\bar{6}$	$m_|$	3	●	●	○	○
90	$\bar{6}$	1	6	●	●	●	●
91	$6/m$	$\bar{6}$	2	○	○	○	○
92	$6/m$	6	2	○	●	○	●
93	$6/m$	$\bar{3}$	2	○	○	○	○
94	$6/m$	3	4	○	◐	○	◐
95	$6/m$	$2/m_|$	3	●	○	○	○
96	$6/m$	$m_|$	6	◐	●	○	○
97	$6/m$	$2_|$	6	◐	◐	○	●
98	$6/m$	$\bar{1}$	6	●	○	●	○
99	$6/m$	1	12	◐	●	◐	●
100	622	6	2	○	●	●	○
101	622	32	2	○	○	○	○
102	622	3	4	○	◐	◐	○
103	622	222	3	●	○	○	○
104	622	$2_|$	6	●	◐	◐	●
105	622	2_-	6	●	●	●	●
106	622	1	12	●	●	●	●

No.	G	F	n	↕	↑	⊬	⊭
107	$6mm$	6	2	○	○	●	●
108	$6mm$	$3m$	2	○	○	○	○
109	$6mm$	3	4	○	○	◐	◐
110	$6mm$	$2_|mm$	3	●	○	○	○
111	$6mm$	m_-	6	●	●	●	○
112	$6mm$	$2_|$	6	●	◐	○	●
113	$6mm$	1	12	●	●	●	●
114	$\bar{6}m2$	$\bar{6}$	2	○	○	●	○
115	$\bar{6}m2$	$3m$	2	●	●	○	○
116	$\bar{6}m2$	32	2	○	○	○	○
117	$\bar{6}m2$	3	4	○	◐	◐	◐
118	$\bar{6}m2$	2_-mm	3	●	●	○	○
119	$\bar{6}m2$	$m_|$	6	●	●	◐	○
120	$\bar{6}m2$	m_-	6	●	●	●	○
121	$\bar{6}m2$	2_-	6	●	◐	●	●
122	$\bar{6}m2$	1	12	●	●	●	●
123	$6/mmm$	$\bar{6}m2$	2	○	○	●	●
124	$6/mmm$	$6mm$	2	○	●	○	○
125	$6/mmm$	622	2	○	○	○	●
126	$6/mmm$	$6/m$	2	○	○	●	○
127	$6/mmm$	$\bar{6}$	4	○	○	◐	◐
128	$6/mmm$	6	4	○	○	◐	◐
129	$6/mmm$	$\bar{3}m$	2	○	○	○	○
130	$6/mmm$	$3m$	4	○	◐	○	◐
131	$6/mmm$	32	4	○	○	○	◐
132	$6/mmm$	$\bar{3}$	4	○	○	◐	○
133	$6/mmm$	3	8	○	◐	○	◐
134	$6/mmm$	mmm	3	●	○	○	○
135	$6/mmm$	$2_|mm$	6	○	◐	○	○
136	$6/mmm$	2_-mm	6	◐	●	○	○
137	$6/mmm$	222	6	◐	○	○	●
138	$6/mmm$	$2/m_|$	6	●	○	◐	○
139	$6/mmm$	$2/m_-$	6	●	○	●	○
140	$6/mmm$	$m_|$	12	◐	●	◐	○
141	$6/mmm$	m_-	12	◐	●	○	○
142	$6/mmm$	$2_|$	12	◐	◐	○	●
143	$6/mmm$	2_-	12	●	○	○	●
144	$6/mmm$	$\bar{1}$	12	●	○	●	○
145	$6/mmm$	1	24	◐	●	◐	●
146	23	3	4	●	●	●	●
147	23	22_+2	3	●	○	○	○
148	23	2_+	6	●	●	●	●
149	23	1	12	●	●	●	●
150	$m\bar{3}$	23	2	○	○	○	○
151	$m\bar{3}$	$\bar{3}$	4	●	○	●	○
152	$m\bar{3}$	3	8	◐	●	◐	●
153	$m\bar{3}$	mm_+m	3	●	○	○	○
154	$m\bar{3}$	$2_+m_|m$	6	◐	●	○	○
155	$m\bar{3}$	22_+2	6	●	○	○	●
156	$m\bar{3}$	$2/m_+$	6	●	○	◐	○
157	$m\bar{3}$	m_+	12	◐	●	○	○
158	$m\bar{3}$	2_+	12	◐	◐	○	●
159	$m\bar{3}$	$\bar{1}$	12	●	○	●	○

No.	G	F	n	↕	↑	⊬	⊭
160	$m\bar{3}$	1	24	◐	●	◐	●
161	432	23	2	○	○	○	○
162	432	32	4	●	○	○	●
163	432	3	8	◐	●	●	◐
164	432	422	3	●	○	○	●
165	432	4	6	◐	●	●	◐
166	432	22_+2	6	●	○	○	●
167	432	$22_|2$	6	●	○	○	●
168	432	2_+	12	●	◐	◐	●
169	432	$2_|$	12	●	●	●	●
170	432	1	24	●	●	●	●
171	$\bar{4}3m$	23	2	○	○	○	○
172	$\bar{4}3m$	$3m$	4	●	●	○	○
173	$\bar{4}3m$	3	8	◐	◐	●	●
174	$\bar{4}3m$	$\bar{4}2_+m$	3	●	○	○	○
175	$\bar{4}3m$	$\bar{4}$	6	◐	○	●	○
176	$\bar{4}3m$	$2_+m_|m$	6	●	●	○	○
177	$\bar{4}3m$	22_+2	6	●	○	○	○
178	$\bar{4}3m$	$m_|$	12	●	●	●	○
179	$\bar{4}3m$	2_+	12	●	◐	◐	●
180	$\bar{4}3m$	1	24	●	●	●	●
181	$m\bar{3}m$	$\bar{4}3m$	2	○	○	○	○
182	$m\bar{3}m$	432	2	○	○	○	○
183	$m\bar{3}m$	$m\bar{3}$	2	○	○	○	○
184	$m\bar{3}m$	23	4	○	○	○	○
185	$m\bar{3}m$	$\bar{3}m$	4	○	○	○	○
186	$m\bar{3}m$	$3m$	8	◐	●	○	○
187	$m\bar{3}m$	32	8	◐	○	○	●
188	$m\bar{3}m$	$\bar{3}$	8	○	○	●	○
189	$m\bar{3}m$	3	16	◐	◐	◐	◐
190	$m\bar{3}m$	$4/mmm$	3	●	○	○	○
191	$m\bar{3}m$	$\bar{4}2_+m$	6	○	○	○	○
192	$m\bar{3}m$	$\bar{4}2_|m$	6	○	○	○	○
193	$m\bar{3}m$	$4mm$	6	○	●	○	○
194	$m\bar{3}m$	422	6	○	○	○	●
195	$m\bar{3}m$	$4/m$	6	○	○	●	○
196	$m\bar{3}m$	$\bar{4}$	12	○	○	○	○
197	$m\bar{3}m$	4	12	○	◐	○	◐
198	$m\bar{3}m$	mm_+m	6	●	○	○	○
199	$m\bar{3}m$	$mm_|m$	6	●	○	○	○
200	$m\bar{3}m$	$2_+m_|m$	12	◐	◐	○	○
201	$m\bar{3}m$	$2_+m_|m$	12	◐	◐	○	○
202	$m\bar{3}m$	$2_|mm$	12	◐	●	○	○
203	$m\bar{3}m$	22_+2	12	◐	○	○	●
204	$m\bar{3}m$	$22_|2$	12	◐	○	○	●
205	$m\bar{3}m$	$2/m_+$	12	●	○	◐	○
206	$m\bar{3}m$	$2/m_|$	12	●	○	●	○
207	$m\bar{3}m$	m_+	24	◐	●	○	○
208	$m\bar{3}m$	$m_|$	24	◐	●	○	○
209	$m\bar{3}m$	2_+	24	◐	◐	○	●
210	$m\bar{3}m$	$2_|$	24	◐	◐	○	●
211	$m\bar{3}m$	$\bar{1}$	24	●	○	●	○
212	$m\bar{3}m$	1	48	◐	●	●	●

C. Species of All Vectorlike Transitions

The following pages contain the complete table of all 1602 species of macroscopic symmetry-breaking structural phase transitions between magnetic point groups. Each is listed with eight symbols indicating the classification of the transition with respect to a vectorlike quantity. The vectorlike quantities label the columns and are given by the symbols in Table C.1.

Name	Symbol	Characteristic Property
Time-even Neutral	$\updownarrow$	Bidirector
Time-even Polar	$\uparrow$	Vector
Time-even Axial	$\nleftrightarrow$	Vector
Time-even Chiroaxial	$\nleftrightarrow$	Bidirector
Time-odd Neutral	$\updownarrow$	Bidirector
Time-odd Polar	$\uparrow$	Vector
Time-odd Axial	$\nleftrightarrow$	Vector
Time-odd Chiroaxial	$\nleftrightarrow$	Bidirector

Table C.1: Reproduction of Table 9.1. The eight vectorlike quantities with names, symbols, and characteristic properties which are used to identify them in this thesis.

The transition classification is either full (e.g. full time-even chiroaxial, full time-odd polar, etc.), partial (e.g. partial time-odd axial, partial time-even neutral, etc.), or non (e.g. non-time-odd chiroaxial, non-time-even polar, etc.) according to the symbols in Table C.2.

Transition Type	Symbol
Full	●
Partial	◑
Non	○

Table C.2: Guide to symbols used in this attachment.

Chapter 8 describes the motivation and methods for creating this table as well as discussing the results and presenting some conclusions about its contents, limitations, and possible applications.

No.	G	F	n	↕	↑	⊹	⊞	↕	↑	⊹	⊞
		gray > gray									
1	$\bar{1}1'$	$11'$	2	○	●	○	●	○	○	○	○
2	$21'$	$11'$	2	●	●	●	●	○	○	○	○
3	$m1'$	$11'$	2	●	●	●	●	○	○	○	○
4	$2/m1'$	$m1'$	2	○	●	○	○	○	○	○	○
5	$2/m1'$	$21'$	2	○	●	○	●	○	○	○	○
6	$2/m1'$	$\bar{1}1'$	2	●	○	○	○	○	○	○	○
7	$2/m1'$	$11'$	4	◐	●	◐	●	○	○	○	○
8	$2221'$	$21'$	2	●	●	●	●	○	○	○	○
9	$2221'$	$11'$	4	●	●	●	●	○	○	○	○
10	$2mm1'$	$m1'$	2	●	●	●	●	○	○	○	○
11	$2mm1'$	$21'$	2	●	○	●	●	○	○	○	○
12	$2mm1'$	$11'$	4	●	●	●	●	○	○	○	○
13	$mmm1'$	$2mm1'$	2	○	●	○	○	○	○	○	○
14	$mmm1'$	$2221'$	2	○	○	○	●	○	○	○	○
15	$mmm1'$	$2/m1'$	2	●	○	●	○	○	○	○	○
16	$mmm1'$	$m1'$	4	◐	●	●	○	○	○	○	○
17	$mmm1'$	$21'$	4	◐	◐	◐	●	○	○	○	○
18	$mmm1'$	$\bar{1}1'$	4	●	○	●	○	○	○	○	○
19	$mmm1'$	$11'$	8	◐	●	●	◐	○	○	○	○
20	$41'$	$2_|1'$	2	○	○	○	●	○	○	○	○
21	$41'$	$11'$	4	●	●	●	●	○	○	○	○
22	$\bar{4}1'$	$2_|1'$	2	●	●	○	●	○	○	○	○
23	$\bar{4}1'$	$11'$	4	●	●	●	●	○	○	○	○
24	$4/m1'$	$\bar{4}1'$	2	○	○	○	○	○	○	○	○
25	$4/m1'$	$41'$	2	○	●	○	●	○	○	○	○
26	$4/m1'$	$2/m_|1'$	2	●	○	○	○	○	○	○	○
27	$4/m1'$	$m_|1'$	4	◐	●	○	○	○	○	○	○
28	$4/m1'$	$2_|1'$	4	◐	◐	○	●	○	○	○	○
29	$4/m1'$	$\bar{1}1'$	4	●	○	●	○	○	○	○	○
30	$4/m1'$	$11'$	8	◐	●	◐	●	○	○	○	○
31	$4221'$	$41'$	2	○	●	●	○	○	○	○	○
32	$4221'$	$2221'$	2	●	○	○	●	○	○	○	○
33	$4221'$	$2_|1'$	4	●	◐	◐	●	○	○	○	○
34	$4221'$	$2\_1'$	4	●	●	●	●	○	○	○	○
35	$4221'$	$11'$	8	●	●	●	●	○	○	○	○
36	$4mm1'$	$41'$	2	○	○	●	●	○	○	○	○
37	$4mm1'$	$2_|mm1'$	2	●	○	○	●	○	○	○	○
38	$4mm1'$	m_1'	4	●	●	●	○	○	○	○	○
39	$4mm1'$	$2_|1'$	4	○	●	◐	●	○	○	○	○
40	$4mm1'$	$11'$	8	●	●	●	●	○	○	○	○
41	$\bar{4}2m1'$	$\bar{4}1'$	2	○	○	●	○	○	○	○	○
42	$\bar{4}2m1'$	$2_|mm1'$	2	●	●	○	○	○	○	○	○
43	$\bar{4}2m1'$	$2221'$	2	●	○	○	●	○	○	○	○
44	$\bar{4}2m1'$	m_1'	4	●	●	●	○	○	○	○	○
45	$\bar{4}2m1'$	$2_|1'$	4	●	◐	◐	●	○	○	○	○
46	$\bar{4}2m1'$	$2\_1'$	4	●	●	●	●	○	○	○	○
47	$\bar{4}2m1'$	$11'$	8	●	●	●	●	○	○	○	○
48	$4/mmm1'$	$\bar{4}2m1'$	2	○	○	○	○	○	○	○	○
49	$4/mmm1'$	$4mm1'$	2	○	●	○	○	○	○	○	○
50	$4/mmm1'$	$4221'$	2	○	○	○	●	○	○	○	○
51	$4/mmm1'$	$4/m1'$	2	○	○	●	○	○	○	○	○
52	$4/mmm1'$	$\bar{4}1'$	4	○	○	○	●	○	○	○	○
53	$4/mmm1'$	$41'$	4	○	◐	◐	◐	○	○	○	○
54	$4/mmm1'$	$mmm1'$	2	●	○	○	○	○	○	○	○
55	$4/mmm1'$	$2_|mm1'$	4	◐	◐	○	○	○	○	○	○
56	$4/mmm1'$	$2\_mm1'$	4	◐	●	○	○	○	○	○	○
57	$4/mmm1'$	$2221'$	4	◐	○	○	●	○	○	○	○
58	$4/mmm1'$	$2/m_|1'$	4	●	○	◐	○	○	○	○	○
59	$4/mmm1'$	$2/m\_1'$	4	●	○	●	○	○	○	○	○
60	$4/mmm1'$	$m_|1'$	8	◐	●	◐	○	○	○	○	○
61	$4/mmm1'$	m_1'	8	◐	●	◐	○	○	○	○	○
62	$4/mmm1'$	$2_|1'$	8	◐	◐	◐	●	○	○	○	○
63	$4/mmm1'$	$2\_1'$	8	◐	◐	◐	●	○	○	○	○
64	$4/mmm1'$	$\bar{1}1'$	8	●	○	●	○	○	○	○	○
65	$4/mmm1'$	$11'$	16	◐	●	◐	●	○	○	○	○
66	$31'$	$11'$	3	●	●	●	●	○	○	○	○
67	$\bar{3}1'$	$31'$	2	○	●	○	●	○	○	○	○
68	$\bar{3}1'$	$\bar{1}1'$	3	●	○	●	○	○	○	○	○
69	$\bar{3}1'$	$11'$	6	◐	●	◐	●	○	○	○	○
70	$321'$	$31'$	2	○	●	●	○	○	○	○	○
71	$321'$	$2\_1'$	3	●	●	●	●	○	○	○	○
72	$321'$	$11'$	6	●	●	●	●	○	○	○	○
73	$3m1'$	$31'$	2	○	○	●	●	○	○	○	○
74	$3m1'$	m_1'	3	●	●	●	○	○	○	○	○
75	$3m1'$	$11'$	6	●	●	●	●	○	○	○	○
76	$\bar{3}m1'$	$3m1'$	2	○	●	○	○	○	○	○	○
77	$\bar{3}m1'$	$321'$	2	○	○	○	●	○	○	○	○
78	$\bar{3}m1'$	$\bar{3}1'$	2	○	○	●	○	○	○	○	○
79	$\bar{3}m1'$	$31'$	4	○	◐	◐	◐	○	○	○	○
80	$\bar{3}m1'$	$2/m\_1'$	3	●	○	●	○	○	○	○	○
81	$\bar{3}m1'$	m_1'	6	◐	●	●	○	○	○	○	○
82	$\bar{3}m1'$	$2\_1'$	6	◐	●	●	●	○	○	○	○
83	$\bar{3}m1'$	$\bar{1}1'$	6	●	○	●	○	○	○	○	○
84	$\bar{3}m1'$	$11'$	12	◐	●	◐	●	○	○	○	○
85	$61'$	$31'$	2	○	○	●	●	○	○	○	○
86	$61'$	$2_|1'$	3	●	○	○	●	○	○	○	○
87	$61'$	$11'$	6	●	●	●	●	○	○	○	○
88	$\bar{6}1'$	$31'$	2	○	●	○	●	○	○	○	○
89	$\bar{6}1'$	$m_|1'$	3	●	●	○	○	○	○	○	○
90	$\bar{6}1'$	$11'$	6	●	●	●	●	○	○	○	○
91	$6/m1'$	$\bar{6}1'$	2	○	○	○	○	○	○	○	○
92	$6/m1'$	$61'$	2	○	●	○	●	○	○	○	○
93	$6/m1'$	$\bar{3}1'$	2	○	○	●	○	○	○	○	○
94	$6/m1'$	$31'$	4	○	◐	◐	◐	○	○	○	○
95	$6/m1'$	$2/m_|1'$	3	●	○	○	○	○	○	○	○
96	$6/m1'$	$m_|1'$	6	◐	●	○	○	○	○	○	○
97	$6/m1'$	$2_|1'$	6	◐	◐	○	●	○	○	○	○
98	$6/m1'$	$\bar{1}1'$	6	●	○	●	○	○	○	○	○
99	$6/m1'$	$11'$	12	◐	●	◐	●	○	○	○	○
100	$6221'$	$61'$	2	○	●	●	○	○	○	○	○
101	$6221'$	$321'$	2	○	○	○	●	○	○	○	○
102	$6221'$	$31'$	4	○	◐	◐	○	○	○	○	○
103	$6221'$	$2221'$	3	●	○	○	●	○	○	○	○
104	$6221'$	$2_|1'$	6	●	◐	◐	●	○	○	○	○
105	$6221'$	$2\_1'$	6	●	●	●	●	○	○	○	○
106	$6221'$	$11'$	12	●	●	●	●	○	○	○	○
107	$6mm1'$	$61'$	2	○	○	●	●	○	○	○	○
108	$6mm1'$	$3m1'$	2	○	○	○	●	○	○	○	○
109	$6mm1'$	$31'$	4	○	○	◐	◐	○	○	○	○
110	$6mm1'$	$2_|mm1'$	3	●	○	○	○	○	○	○	○
111	$6mm1'$	m_1'	6	●	●	●	○	○	○	○	○
112	$6mm1'$	$2_|1'$	6	●	○	◐	●	○	○	○	○
113	$6mm1'$	$11'$	12	●	●	●	●	○	○	○	○
114	$\bar{6}m21'$	$\bar{6}1'$	2	○	○	●	○	○	○	○	○
115	$\bar{6}m21'$	$3m1'$	2	○	●	○	○	○	○	○	○
116	$\bar{6}m21'$	$321'$	2	○	○	○	●	○	○	○	○
117	$\bar{6}m21'$	$31'$	4	○	◐	◐	◐	○	○	○	○
118	$\bar{6}m21'$	$2\_mm1'$	3	●	●	○	○	○	○	○	○
119	$\bar{6}m21'$	$m_|1'$	6	●	●	◐	○	○	○	○	○
120	$\bar{6}m21'$	m_1'	6	●	●	●	○	○	○	○	○
121	$\bar{6}m21'$	$2\_1'$	6	●	◐	●	●	○	○	○	○
122	$\bar{6}m21'$	$11'$	12	●	●	●	●	○	○	○	○
123	$6/mmm1'$	$\bar{6}m21'$	2	○	○	○	○	○	○	○	○
124	$6/mmm1'$	$6mm1'$	2	○	●	○	○	○	○	○	○
125	$6/mmm1'$	$6221'$	2	○	○	○	●	○	○	○	○
126	$6/mmm1'$	$6/m1'$	2	○	○	●	○	○	○	○	○
127	$6/mmm1'$	$\bar{6}1'$	4	○	○	○	◐	○	○	○	○
128	$6/mmm1'$	$61'$	4	○	◐	◐	◐	○	○	○	○
129	$6/mmm1'$	$\bar{3}m1'$	2	○	○	○	○	○	○	○	○
130	$6/mmm1'$	$3m1'$	4	○	◐	○	○	○	○	○	○
131	$6/mmm1'$	$321'$	4	○	○	○	◐	○	○	○	○
132	$6/mmm1'$	$\bar{3}1'$	4	○	○	◐	○	○	○	○	○
133	$6/mmm1'$	$31'$	8	○	◐	◐	◐	○	○	○	○
134	$6/mmm1'$	$mmm1'$	3	●	○	○	○	○	○	○	○
135	$6/mmm1'$	$2_|mm1'$	6	◐	◐	○	○	○	○	○	○
136	$6/mmm1'$	$2\_mm1'$	6	◐	●	○	○	○	○	○	○
137	$6/mmm1'$	$2221'$	6	◐	○	○	●	○	○	○	○
138	$6/mmm1'$	$2/m_|1'$	6	●	○	◐	○	○	○	○	○
139	$6/mmm1'$	$2/m\_1'$	6	●	○	●	○	○	○	○	○
140	$6/mmm1'$	$m_|1'$	12	◐	●	◐	○	○	○	○	○
141	$6/mmm1'$	m_1'	12	◐	●	◐	○	○	○	○	○
142	$6/mmm1'$	$2_|1'$	12	◐	◐	◐	●	○	○	○	○
143	$6/mmm1'$	$2\_1'$	12	◐	◐	◐	●	○	○	○	○
144	$6/mmm1'$	$\bar{1}1'$	12	●	○	●	○	○	○	○	○
145	$6/mmm1'$	$11'$	24	◐	●	◐	●	○	○	○	○
146	$231'$	$31'$	4	●	●	●	●	○	○	○	○
147	$231'$	$22_+21'$	3	●	○	○	●	○	○	○	○
148	$231'$	$2_+1'$	6	●	●	●	●	○	○	○	○
149	$231'$	$11'$	12	●	●	●	●	○	○	○	○

No.	G	F	n	↕ ↑ ⊹ ⊱ ↕ ↑ ⊹ ⊱
150	$m\bar{3}1'$	$231'$	2	○○○○○○○○○
151	$m\bar{3}1'$	$\bar{3}1'$	4	●○●○○○○○○
152	$m\bar{3}1'$	$31'$	8	◐●●◐●○○○○
153	$m\bar{3}1'$	mm_+m1'	3	●○○○○○○○○
154	$m\bar{3}1'$	$2_+m_+m1'$	6	◐●○○○○○○○
155	$m\bar{3}1'$	$22_+21'$	6	◐○○○●○○○○
156	$m\bar{3}1'$	$2/m_+1'$	6	●○●○○○○○○
157	$m\bar{3}1'$	m_+1'	12	◐●●◐○○○○○
158	$m\bar{3}1'$	$2_+1'$	12	◐◐◐●○○○○○
159	$m\bar{3}1'$	$\bar{1}1'$	12	●○●○○○○○○
160	$m\bar{3}1'$	$11'$	24	◐●◐●●○○○○
161	$4321'$	$231'$	2	○○○○○○○○○
162	$4321'$	$321'$	4	●○○○●○○○○
163	$4321'$	$31'$	8	◐●●◐○○○○○
164	$4321'$	$4221'$	3	●○○○●○○○○
165	$4321'$	$41'$	6	◐●●◐○○○○○
166	$4321'$	$22_+21'$	6	●○○○●○○○○
167	$4321'$	$22_\searrow21'$	6	●○○○●○○○○
168	$4321'$	$2_+1'$	12	●◐◐●○○○○○
169	$4321'$	$2_\searrow1'$	12	●●●●○○○○○
170	$4321'$	$11'$	24	●●●●○○○○○
171	$\bar{4}3m1'$	$231'$	2	○○○○○○○○○
172	$\bar{4}3m1'$	$3m1'$	4	●●○○○○○○○
173	$\bar{4}3m1'$	$31'$	8	◐◐●●○○○○○
174	$\bar{4}3m1'$	$\bar{4}2_+m1'$	3	●○○○○○○○○
175	$\bar{4}3m1'$	$\bar{4}1'$	6	◐○○●○○○○○
176	$\bar{4}3m1'$	$2_+m_\searrow m1'$	6	●●○○○○○○○
177	$\bar{4}3m1'$	$22_+21'$	6	●○○○●○○○○
178	$\bar{4}3m1'$	$m_\searrow1'$	12	●●●◐○○○○○
179	$\bar{4}3m1'$	$2_+1'$	12	●◐◐●○○○○○
180	$\bar{4}3m1'$	$11'$	24	●●●●○○○○○
181	$m\bar{3}m1'$	$\bar{4}3m1'$	2	○○○○○○○○○
182	$m\bar{3}m1'$	$4321'$	2	○○○○○○○○○
183	$m\bar{3}m1'$	$m\bar{3}1'$	2	○○○○○○○○○
184	$m\bar{3}m1'$	$231'$	4	○○○○○○○○○
185	$m\bar{3}m1'$	$\bar{3}m1'$	4	●○○○○○○○○
186	$m\bar{3}m1'$	$3m1'$	8	◐●●○○○○○○
187	$m\bar{3}m1'$	$321'$	8	◐○○○●○○○○
188	$m\bar{3}m1'$	$\bar{3}1'$	8	◐○●○○○○○○
189	$m\bar{3}m1'$	$31'$	16	◐◐◐◐◐○○○○
190	$m\bar{3}m1'$	$4/mmm1'$	3	●○○○○○○○○
191	$m\bar{3}m1'$	$\bar{4}2_+m1'$	6	◐○○○○○○○○
192	$m\bar{3}m1'$	$\bar{4}2_\searrow m1'$	6	◐○○○○○○○○
193	$m\bar{3}m1'$	$4mm1'$	6	◐●○○○○○○○
194	$m\bar{3}m1'$	$4221'$	6	◐○○○●○○○○
195	$m\bar{3}m1'$	$4/m1'$	6	◐○●○○○○○○
196	$m\bar{3}m1'$	$\bar{4}1'$	12	◐○○●○○○○○
197	$m\bar{3}m1'$	$41'$	12	◐◐◐◐○○○○○
198	$m\bar{3}m1'$	mm_+m1'	6	●○○○○○○○○
199	$m\bar{3}m1'$	$mm_\searrow m1'$	6	●○○○○○○○○

No.	G	F	n	↕ ↑ ⊹ ⊱ ↕ ↑ ⊹ ⊱
200	$m\bar{3}m1'$	$2_+m_+m1'$	12	◐◐○○○○○○○
201	$m\bar{3}m1'$	$2_+m_\searrow m1'$	12	◐◐○○○○○○○
202	$m\bar{3}m1'$	$2_\searrow mm1'$	12	●○○○○○○○○
203	$m\bar{3}m1'$	$22_+21'$	12	◐○○●○○○○○
204	$m\bar{3}m1'$	$22_\searrow21'$	12	◐○○●○○○○○
205	$m\bar{3}m1'$	$2/m_+1'$	12	●○◐○○○○○○
206	$m\bar{3}m1'$	$2/m_\searrow1'$	12	●○●○○○○○○
207	$m\bar{3}m1'$	m_+1'	24	◐●◐○○○○○○
208	$m\bar{3}m1'$	$m_\searrow1'$	24	●●○○○○○○○
209	$m\bar{3}m1'$	$2_+1'$	24	◐◐◐●○○○○○
210	$m\bar{3}m1'$	$2_\searrow1'$	24	◐◐●●○○○○○
211	$m\bar{3}m1'$	$\bar{1}1'$	24	●○●○○○○○○
212	$m\bar{3}m1'$	$11'$	48	◐●◐●○○○○○
gray > black-white				
213	$\bar{1}1'$	$\bar{1}'$	2	○○○○○●○●
214	$21'$	$2'$	2	○○○○○○●●○
215	$m1'$	m'	2	○○○○○○●●●
216	$2/m1'$	$2'/m$	2	○○○○○○●○○
217	$2/m1'$	$2/m'$	2	○○○○○○○○●
218	$2/m1'$	$2'/m'$	2	○○○○○○○●○
219	$2/m1'$	m'	4	○◐○○○◐◐◐
220	$2/m1'$	$2'$	4	○◐○◐○●◐○
221	$2/m1'$	$\bar{1}'$	4	◐○◐○○●○●
222	$2221'$	$22'2'$	2	○○○○○○●●●
223	$2221'$	$2'$	4	◐◐◐◐○●●○
224	$2mm1'$	$2'mm'$	2	○○○○○○●●●
225	$2mm1'$	$2m'm'$	2	○○○○○○○●●
226	$2mm1'$	m'	4	◐◐◐○○○●●
227	$2mm1'$	$2'$	4	○◐○◐◐○●●
228	$mmm1'$	$m'mm$	2	○○○○○○●○○
229	$mmm1'$	$m'm'm'$	2	○○○○○○○○●
230	$mmm1'$	$mm'm'$	2	○○○○○○●○○
231	$mmm1'$	$2'mm'$	4	○◐○○○○◐○
232	$mmm1'$	$2m'm'$	4	○◐○○○○●◐
233	$mmm1'$	$22'2'$	4	○○○◐○○◐○
234	$mmm1'$	$2'/m$	4	◐○○◐○●○○
235	$mmm1'$	$2/m'$	4	◐○◐○○◐○●
236	$mmm1'$	$2'/m'$	4	◐○◐○○◐○●
237	$mmm1'$	m'	8	◐◐◐○○○○◐
238	$mmm1'$	$2'$	8	◐◐◐◐○○◐○
239	$mmm1'$	$\bar{1}'$	8	◐○◐○○●○●
240	$41'$	$4'$	2	○○○○○○●○○
241	$41'$	$2_\mid$	4	◐○○○◐○●●○
242	$\bar{4}1'$	$\bar{4}'$	2	○○○○○○●○●
243	$\bar{4}1'$	$2_\mid$	4	◐○○◐○○●●○
244	$4/m1'$	$4'/m'$	2	○○○○○○●○○
245	$4/m1'$	$4/m'$	2	○○○○○○●○●
246	$4/m1'$	$4'/m$	2	○○○○○○●○○
247	$4/m1'$	$\bar{4}'$	4	○○○○○◐○◐
248	$4/m1'$	$4'$	4	○◐○◐○○○○

No.	G	F	n	↕ ↑ ⊹ ⊱ ↕ ↑ ⊹ ⊱
249	$4/m1'$	$2'/m_\mid$	4	◐○○○○●○○
250	$4/m1'$	$2/m_\mid$	4	◐○○○○◐○●
251	$4/m1'$	$2'/m_\mid$	4	◐○○○○○●○
252	$4/m1'$	$m_\mid$	8	◐○◐○○○◐◐
253	$4/m1'$	$2_\mid$	8	◐○◐◐○◐○◐
254	$4/m1'$	$\bar{1}'$	8	◐○◐○○●○●
255	$4221'$	$4'22'$	2	○○○○○●○○
256	$4221'$	$42'2'$	2	○○○○○○●●
257	$4221'$	$4'$	4	○◐○◐○○●○
258	$4221'$	$2_\mid2'2'$	4	○○○◐○◐◐◐
259	$4221'$	$2_-2'2'$	4	○○○◐○○◐●
260	$4221'$	$2_\mid$	8	◐◐◐◐○●●○
261	$4221'$	$2'$	8	◐◐◐◐○●●○
262	$4mm1'$	$4m'm'$	2	○○○○○○●●
263	$4mm1'$	$4'mm'$	2	○○○○○○●●
264	$4mm1'$	$4'$	4	○○○◐◐○●○
265	$4mm1'$	$2_\mid'mm'$	4	◐○○○○●●○
266	$4mm1'$	$2_\mid m'm'$	4	◐○○○○○●●
267	$4mm1'$	m'_-	8	◐◐◐◐○○◐◐
268	$4mm1'$	$2_\mid$	8	◐○○◐◐○●○
269	$\bar{4}2m1'$	$\bar{4}2'm'$	2	○○○○○○●●
270	$\bar{4}2m1'$	$\bar{4}'2'm$	2	○○○○○●○○
271	$\bar{4}2m1'$	$\bar{4}'2m'$	2	○○○○○○●○
272	$\bar{4}2m1'$	$\bar{4}'$	4	○○○◐○●○◐
273	$\bar{4}2m1'$	$2_\mid'mm'$	4	◐○◐○○●●○
274	$\bar{4}2m1'$	$2_\mid m'm'$	4	◐◐○○○○●●
275	$\bar{4}2m1'$	$2_\mid2'2'$	4	○○○◐○◐○◐
276	$\bar{4}2m1'$	$2_-2'2'$	4	○◐○◐○○●●
277	$\bar{4}2m1'$	m'_-	8	◐◐◐◐○○●●
278	$\bar{4}2m1'$	$2_\mid$	8	◐◐◐◐○●●○
279	$\bar{4}2m1'$	2_-	8	◐◐◐◐○●●○
280	$4/mmm1'$	$4'/m'm'm$	2	○○○○○○●○
281	$4/mmm1'$	$4/m'mm$	2	○○○○○●○○
282	$4/mmm1'$	$4/m'm'm'$	2	○○○○○○○●
283	$4/mmm1'$	$4/mm'm'$	2	○○○○○○●○
284	$4/mmm1'$	$4'/mmm'$	2	○○○○○○●○
285	$4/mmm1'$	$\bar{4}2'm'$	4	○○○◐○○●○
286	$4/mmm1'$	$\bar{4}'2m$	4	○○○○○○◐○
287	$4/mmm1'$	$\bar{4}'2m'$	4	○○○○○○○◐
288	$4/mmm1'$	$4m'm'$	4	○◐○○○○◐◐
289	$4/mmm1'$	$4'mm'$	4	○◐○○○○●○
290	$4/mmm1'$	$4'22'$	4	○○○◐◐○○○
291	$4/mmm1'$	$42'2'$	4	○○○◐○○◐○
292	$4/mmm1'$	$4'/m'$	4	○○◐○◐○●○
293	$4/mmm1'$	$4/m'$	4	○○◐○○○○◐
294	$4/mmm1'$	$4'/m$	4	○○◐○◐○○◐
295	$4/mmm1'$	$\bar{4}'$	8	◐○◐◐○◐○◐
296	$4/mmm1'$	$4'$	8	◐◐◐◐○○○○
297	$4/mmm1'$	$m_\mid'mm$	4	◐○○○○●○○
298	$4/mmm1'$	m'_-mm	4	◐◐○○○●○○

83

No.	G	F	n									
299	4/mmm1′	m′m′m′	4	◐	○	○	○	○	○	○	○	●
300	4/mmm1′	m|m′m′	4	◐	○	○	○	○	○	○	◐	○
301	4/mmm1′	m_m′m′	4	◐	○	○	○	○	○	○	●	○
302	4/mmm1′	2′|mm′	8	◐	◐	○	○	○	○	◐	◐	○
303	4/mmm1′	2′_m_m′	8	◐	◐	○	○	○	○	◐	◐	○
304	4/mmm1′	2′_m|m′	8	◐	◐	○	○	○	○	◐	◐	○
305	4/mmm1′	2|m′m′	8	◐	◐	○	○	○	○	○	◐	◐
306	4/mmm1′	2_m′m′	8	◐	◐	○	○	○	○	○	◐	◐
307	4/mmm1′	2|2′2′	8	◐	○	○	○	◐	○	◐	◐	○
308	4/mmm1′	2_2′2′	8	◐	○	○	○	◐	○	◐	◐	○
309	4/mmm1′	2′/m|	8	◐	○	○	◐	○	○	●	○	○
310	4/mmm1′	2′/m_	8	◐	○	○	◐	○	○	●	○	○
311	4/mmm1′	2/m′|	8	◐	○	○	◐	○	○	◐	○	●
312	4/mmm1′	2/m′_	8	◐	○	○	◐	○	○	◐	○	●
313	4/mmm1′	2′/m′|	8	◐	○	○	◐	○	○	○	●	○
314	4/mmm1′	2′/m′_	8	◐	○	○	◐	○	○	○	●	○
315	4/mmm1′	m′|	16	◐	◐	◐	○	○	○	◐	◐	◐
316	4/mmm1′	m′_	16	◐	◐	◐	○	○	○	◐	◐	◐
317	4/mmm1′	2′|	16	◐	◐	◐	◐	○	○	◐	◐	○
318	4/mmm1′	2′_	16	◐	◐	◐	◐	○	○	◐	◐	○
319	4/mmm1′	1̄′	16	◐	○	◐	○	○	○	●	○	●
320	3̄1′	3̄′	2	○	○	○	○	○	○	●	○	●
321	3̄1′	1̄′	6	◐	○	○	○	○	○	●	○	●
322	321′	32′	2	○	○	○	○	○	○	●	●	○
323	321′	2′_	6	◐	◐	◐	◐	○	○	●	●	○
324	3m1′	3m′	2	○	○	○	○	○	○	○	●	●
325	3m1′	m′_	6	◐	◐	◐	○	○	○	●	●	●
326	3̄m1′	3̄′m	2	○	○	○	○	○	●	○	○	○
327	3̄m1′	3̄′m′	2	○	○	○	○	○	○	○	○	●
328	3̄m1′	3̄m′	2	○	○	○	○	○	○	●	○	○
329	3̄m1′	3m′	4	○	◐	○	○	○	○	○	◐	◐
330	3̄m1′	32′	4	○	○	○	◐	○	○	◐	◐	○
331	3̄m1′	3̄′	4	○	○	◐	○	○	○	◐	○	◐
332	3̄m1′	2′/m_	6	◐	○	○	○	○	●	○	○	○
333	3̄m1′	2/m′_	6	◐	○	◐	○	○	●	○	○	●
334	3̄m1′	2′/m′_	6	◐	○	○	○	○	○	●	○	○
335	3̄m1′	m′_	12	◐	◐	◐	○	○	○	◐	◐	◐
336	3̄m1′	2′_	12	◐	◐	◐	◐	○	○	◐	◐	○
337	3̄m1′	1̄′	12	◐	○	◐	○	○	○	●	○	●
338	61′	6′	2	○	○	○	○	○	○	●	○	○
339	61′	2′|	6	◐	○	○	◐	○	○	●	●	○
340	6̄1′	6̄′	2	○	○	○	○	○	○	○	○	●
341	6̄1′	m′|	6	◐	◐	◐	○	○	○	◐	●	●
342	6/m1′	6′/m	2	○	○	○	○	○	○	○	○	○
343	6/m1′	6/m′	2	○	○	○	○	○	●	○	●	○
344	6/m1′	6′/m′	2	○	○	○	○	○	●	○	●	○
345	6/m1′	6̄′	4	○	◐	○	○	○	◐	○	○	◐
346	6/m1′	6′	4	○	◐	○	◐	○	○	○	○	○
347	6/m1′	3̄′	4	○	○	○	○	○	◐	○	○	◐
348	6/m1′	2′/m|	6	◐	○	○	○	○	●	○	○	○

No.	G	F	n									
349	6/m1′	2/m′|	6	◐	○	○	○	○	○	◐	○	●
350	6/m1′	2′/m′|	6	◐	○	○	○	○	○	●	○	○
351	6/m1′	m′|	12	◐	◐	○	○	○	○	◐	◐	◐
352	6/m1′	2′|	12	◐	◐	○	◐	○	○	◐	◐	○
353	6/m1′	1̄′	12	◐	○	◐	○	○	○	●	○	●
354	6221′	62′2′	2	○	○	○	○	○	●	●	○	
355	6221′	6′22′	2	○	○	○	○	○	○	○	○	
356	6221′	6′	4	○	◐	◐	○	○	○	○	○	
357	6221′	32′	4	○	○	○	○	○	◐	◐	○	
358	6221′	2|2′2′	6	◐	○	○	◐	○	◐	◐	○	
359	6221′	2_2′2′	6	◐	○	○	◐	○	●	●	○	
360	6221′	2′|	12	◐	◐	◐	◐	○	●	●	○	
361	6221′	2′_	12	◐	◐	◐	◐	○	●	●	○	
362	6mm1′	6m′m′	2	○	○	○	○	○	○	●	●	
363	6mm1′	6′mm′	2	○	○	○	○	○	○	○	○	
364	6mm1′	6′	4	○	○	◐	◐	○	○	○	○	
365	6mm1′	3m′	4	○	○	○	○	○	○	◐	◐	
366	6mm1′	2′|mm′	6	◐	○	○	○	○	●	●	○	
367	6mm1′	2|m′m′	6	◐	○	○	○	○	○	◐	●	
368	6mm1′	m′_	12	◐	◐	◐	○	○	◐	●	●	
369	6mm1′	2′|	12	◐	○	○	◐	○	●	●	○	
370	6̄m21′	6̄m′2′	2	○	○	○	○	○	○	●	○	
371	6̄m21′	6̄′m2′	2	○	○	○	○	○	●	○	○	
372	6̄m21′	6̄′m′2	2	○	○	○	○	○	○	○	●	
373	6̄m21′	6̄′	4	○	○	◐	○	○	◐	○	◐	
374	6̄m21′	3m′	4	○	◐	○	○	○	○	◐	◐	
375	6̄m21′	32′	4	○	○	○	◐	○	◐	◐	○	
376	6̄m21′	2′_m_m′	6	◐	◐	○	○	○	○	◐	●	
377	6̄m21′	2′_m|m′	6	◐	◐	○	○	○	○	●	◐	
378	6̄m21′	2_m′m′	6	◐	◐	○	○	○	○	○	●	
379	6̄m21′	m′|	12	◐	◐	◐	○	○	○	◐	●	
380	6̄m21′	m′_	12	◐	◐	◐	○	○	○	◐	●	
381	6̄m21′	2′_	12	◐	◐	◐	◐	○	●	●	○	
382	6/mmm1′	6′/mmm′	2	○	○	○	○	○	○	○	○	
383	6/mmm1′	6/m′mm	2	○	○	○	○	○	●	○	○	
384	6/mmm1′	6/m′m′m′	2	○	○	○	○	○	○	○	●	
385	6/mmm1′	6/mm′m′	2	○	○	○	○	○	●	○	○	
386	6/mmm1′	6′/m′mm′	2	○	○	○	○	○	○	○	○	
387	6/mmm1′	6̄m′2′	4	○	○	○	○	○	○	◐	○	
388	6/mmm1′	6̄′m2′	4	○	○	○	○	○	◐	○	○	
389	6/mmm1′	6̄′m′2	4	○	○	○	○	○	○	○	◐	
390	6/mmm1′	6m′m′	4	○	◐	○	○	○	○	◐	◐	
391	6/mmm1′	6′mm′	4	○	◐	○	○	○	○	○	○	
392	6/mmm1′	62′2′	4	○	○	○	◐	○	◐	◐	○	
393	6/mmm1′	6′22′	4	○	○	○	◐	○	○	○	○	
394	6/mmm1′	6′/m	4	○	○	◐	○	○	◐	○	○	
395	6/mmm1′	6/m′	4	○	○	◐	○	○	◐	○	◐	
396	6/mmm1′	6′/m′	4	○	○	◐	○	○	◐	○	○	
397	6/mmm1′	6̄′	8	○	○	◐	○	○	◐	○	◐	
398	6/mmm1′	6′	8	○	◐	◐	◐	○	○	○	○	

No.	G	F	n									
399	6/mmm1′	3̄′m	4	○	○	○	○	○	◐	○	○	
400	6/mmm1′	3̄′m′	4	○	○	○	○	○	○	○	◐	
401	6/mmm1′	3̄m′	4	○	○	○	○	○	○	◐	○	
402	6/mmm1′	3m′	8	○	◐	○	○	○	○	◐	◐	
403	6/mmm1′	32′	8	○	○	○	◐	○	○	◐	◐	
404	6/mmm1′	3̄′	8	○	○	◐	○	○	○	◐	○	
405	6/mmm1′	m′|mm	6	◐	○	○	○	○	●	○	○	
406	6/mmm1′	m′_mm	6	◐	○	○	○	○	●	○	○	
407	6/mmm1′	m′m′m′	6	◐	○	○	○	○	○	○	●	
408	6/mmm1′	m|m′m′	6	◐	○	○	○	○	○	◐	○	
409	6/mmm1′	m_m′m′	6	◐	○	○	○	○	○	●	○	
410	6/mmm1′	2′|mm′	12	◐	◐	○	○	○	○	◐	◐	
411	6/mmm1′	2′_m_m′	12	◐	◐	○	○	○	○	◐	◐	
412	6/mmm1′	2′_m|m′	12	◐	◐	○	○	○	○	◐	◐	
413	6/mmm1′	2|m′m′	12	◐	◐	○	○	○	○	○	◐	
414	6/mmm1′	2_m′m′	12	◐	◐	○	○	○	○	○	◐	
415	6/mmm1′	2|2′2′	12	◐	○	○	◐	○	○	◐	◐	
416	6/mmm1′	2_2′2′	12	◐	○	○	◐	○	○	◐	◐	
417	6/mmm1′	2′/m|	12	◐	○	◐	○	○	●	○	○	
418	6/mmm1′	2′/m_	12	◐	○	◐	○	○	●	○	○	
419	6/mmm1′	2/m′|	12	◐	○	◐	○	○	◐	○	●	
420	6/mmm1′	2/m′_	12	◐	○	◐	○	○	◐	○	●	
421	6/mmm1′	2′/m′|	12	◐	○	○	○	○	○	●	○	
422	6/mmm1′	2′/m′_	12	◐	○	○	○	○	○	●	○	
423	6/mmm1′	m′|	24	◐	◐	◐	○	○	○	◐	◐	◐
424	6/mmm1′	m′_	24	◐	◐	◐	○	○	○	◐	◐	◐
425	6/mmm1′	2′|	24	◐	◐	◐	◐	○	○	◐	◐	○
426	6/mmm1′	2′_	24	◐	◐	◐	◐	○	○	◐	◐	○
427	6/mmm1′	1̄′	24	◐	○	◐	○	○	○	●	○	●
428	231′	2_+2′_+2	6	○	○	○	◐	○	●	●	○	
429	231′	2′_+	12	◐	◐	◐	◐	○	●	●	○	
430	m3̄1′	m′3̄′	2	○	○	○	○	○	○	○	○	
431	m3̄1′	3̄′	8	◐	○	◐	○	○	●	○	●	
432	m3̄1′	m′_+m_+m	6	◐	○	○	○	○	●	○	○	
433	m3̄1′	m′m′_+m′	6	◐	○	○	○	○	○	○	●	
434	m3̄1′	m_+m′_+m′	6	◐	○	○	○	○	○	●	○	
435	m3̄1′	2′_+m_+m′	12	◐	◐	○	○	○	○	◐	◐	
436	m3̄1′	2_+m′_+m′	12	◐	◐	○	○	○	○	◐	◐	
437	m3̄1′	2_+2′_+2	12	◐	○	○	◐	○	○	◐	◐	
438	m3̄1′	2′/m_+	12	◐	○	◐	○	○	●	○	○	
439	m3̄1′	2/m′_+	12	◐	○	◐	○	○	◐	○	●	
440	m3̄1′	2′/m′_+	12	◐	○	○	○	○	○	●	○	
441	m3̄1′	m′_+	24	◐	◐	◐	○	○	○	◐	◐	◐
442	m3̄1′	2′_+	24	◐	◐	◐	◐	○	○	◐	◐	○
443	m3̄1′	1̄′	24	◐	○	◐	○	○	●	○	●	
444	4321′	4′32′	2	○	○	○	○	○	○	○	○	
445	4321′	32′	8	◐	○	○	◐	○	●	●	○	
446	4321′	4′2_+2′	6	◐	○	○	◐	○	○	○	○	
447	4321′	4′2\2′	6	◐	○	○	◐	○	○	○	○	
448	4321′	42′2′	6	◐	○	○	◐	○	●	●	○	

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
449	$4321'$	$4'$	12	◐	●	◐	◐	◐	○	○	○
450	$4321'$	$2_+2'_+2'$	12	◐	○	○	◐	○	◐	●	○
451	$4321'$	$2_+2'_\backslash 2'$	12	◐	○	○	◐	○	◐	●	○
452	$4321'$	$2_\backslash 2'2'$	12	◐	○	○	◐	○	●	●	○
453	$4321'$	$2'_+$	24	◐	◐	◐	◐	◐	○	●	● ○
454	$4321'$	$2'_\backslash$	24	◐	◐	◐	◐	◐	○	●	● ○
455	$\bar43m1'$	$\bar4'3m'$	2	○	○	○	○	○	○	○	○
456	$\bar43m1'$	$3m'$	8	◐	◐	○	○	○	○	●	●
457	$\bar43m1'$	$\bar42'_+m'$	6	◐	○	○	○	○	○	●	○
458	$\bar43m1'$	$\bar4'2'_+m$	6	◐	○	○	○	●	○	○	○
459	$\bar43m1'$	$\bar4'2_+m'$	6	◐	○	○	○	○	○	○	●
460	$\bar43m1'$	$\bar4'$	12	◐	○	◐	○	○	○	◐	◐
461	$\bar43m1'$	$2'_+m_\backslash m'$	12	◐	◐	○	○	○	●	●	○
462	$\bar43m1'$	$2_+m'_\backslash m'$	12	◐	◐	○	○	○	○	◐	●
463	$\bar43m1'$	$2_+2'_+2'$	12	◐	○	○	◐	○	◐	◐	○
464	$\bar43m1'$	$m'_\backslash$	24	◐	◐	◐	○	○	◐	●	●
465	$\bar43m1'$	$2'_+$	24	◐	◐	◐	◐	○	●	●	○
466	$m\bar3m1'$	$m'\bar3'm$	2	○	○	○	○	○	○	○	○
467	$m\bar3m1'$	$m'\bar3'm'$	2	○	○	○	○	○	○	○	○
468	$m\bar3m1'$	$m\bar3m'$	2	○	○	○	○	○	○	○	○
469	$m\bar3m1'$	$\bar4'3m'$	4	○	○	○	○	○	○	○	○
470	$m\bar3m1'$	$4'32'$	4	○	○	○	○	○	○	○	○
471	$m\bar3m1'$	$m'\bar3'$	4	○	○	○	○	○	○	○	○
472	$m\bar3m1'$	$\bar3'm$	8	◐	○	○	○	○	●	○	○
473	$m\bar3m1'$	$\bar3'm'$	8	◐	○	○	○	○	○	○	●
474	$m\bar3m1'$	$\bar3m'$	8	◐	○	○	○	○	●	○	○
475	$m\bar3m1'$	$3m'$	16	◐	◐	○	○	○	○	◐	◐
476	$m\bar3m1'$	$32'$	16	◐	○	○	◐	○	◐	◐	○
477	$m\bar3m1'$	$\bar3'$	16	◐	○	◐	○	○	◐	○	◐
478	$m\bar3m1'$	$4'/m'm'_+m$	6	◐	○	○	○	○	○	○	○
479	$m\bar3m1'$	$4'/m'm'_\backslash m$	6	◐	○	○	○	○	○	○	○
480	$m\bar3m1'$	$4/m'mm$	6	◐	○	○	○	○	●	○	○
481	$m\bar3m1'$	$4/m'm'm'$	6	◐	○	○	○	○	○	○	●
482	$m\bar3m1'$	$4/mm'm'$	6	◐	○	○	○	○	○	●	○
483	$m\bar3m1'$	$4'/mm_+m'$	6	◐	○	○	○	○	○	○	○
484	$m\bar3m1'$	$4'/mm_\backslash m'$	6	◐	○	○	○	○	○	○	○
485	$m\bar3m1'$	$\bar42'_+m'$	12	◐	○	○	○	○	○	◐	◐
486	$m\bar3m1'$	$\bar42'_\backslash m'$	12	◐	○	○	○	○	○	◐	◐
487	$m\bar3m1'$	$\bar4'2'_+m$	12	◐	○	○	○	○	◐	○	◐
488	$m\bar3m1'$	$\bar4'2'_\backslash m$	12	◐	○	○	○	○	◐	○	◐
489	$m\bar3m1'$	$\bar4'2_+m'$	12	◐	○	○	○	○	○	○	◐
490	$m\bar3m1'$	$\bar4'2_\backslash m'$	12	◐	○	○	○	○	○	○	◐
491	$m\bar3m1'$	$4m'm'$	12	◐	◐	○	○	○	○	◐	◐
492	$m\bar3m1'$	$4'm_+m'$	12	◐	◐	○	○	○	○	○	○
493	$m\bar3m1'$	$4'm_\backslash m'$	12	◐	◐	○	○	○	○	○	○
494	$m\bar3m1'$	$4'2_+2'$	12	◐	○	○	◐	○	○	○	○
495	$m\bar3m1'$	$4'2_\backslash 2'$	12	◐	○	○	◐	○	○	○	○
496	$m\bar3m1'$	$42'2'$	12	◐	○	○	◐	○	◐	◐	○
497	$m\bar3m1'$	$4'/m'$	12	◐	○	◐	○	○	○	○	◐
498	$m\bar3m1'$	$4/m'$	12	◐	○	◐	○	○	○	◐	◐
499	$m\bar3m1'$	$4'/m$	12	◐	○	○	○	○	○	○	○
500	$m\bar3m1'$	$\bar4'$	24	◐	○	◐	○	○	○	◐	◐
501	$m\bar3m1'$	$4'$	24	◐	◐	◐	◐	○	○	○	○
502	$m\bar3m1'$	m'_+m_+m	12	◐	○	○	○	○	◐	○	○
503	$m\bar3m1'$	$m'_+m_\backslash m$	12	◐	○	○	○	○	◐	○	○
504	$m\bar3m1'$	$m'_\backslash mm$	12	◐	○	○	○	○	●	○	○
505	$m\bar3m1'$	$m'm'_+m'$	12	◐	○	○	○	○	○	○	●
506	$m\bar3m1'$	$m'm'_\backslash m'$	12	◐	○	○	○	○	○	○	●
507	$m\bar3m1'$	$m_+m'_+m'$	12	◐	○	○	○	○	○	◐	○
508	$m\bar3m1'$	$m_+m'_\backslash m'$	12	◐	○	○	○	○	○	◐	○
509	$m\bar3m1'$	$m_\backslash m'm'$	12	◐	○	○	○	○	●	○	○
510	$m\bar3m1'$	$2'_+m_+m'$	24	◐	◐	○	○	○	○	◐	◐
511	$m\bar3m1'$	$2'_+m_\backslash m'$	24	◐	◐	○	○	○	○	◐	◐
512	$m\bar3m1'$	$2'_\backslash m_\backslash m'$	24	◐	◐	○	○	○	○	◐	◐
513	$m\bar3m1'$	$2'_\backslash m_+m'$	24	◐	◐	○	○	○	○	◐	◐
514	$m\bar3m1'$	$2_+m'_+m'$	24	◐	◐	○	○	○	○	◐	◐
515	$m\bar3m1'$	$2_+m'_\backslash m'$	24	◐	◐	○	○	○	○	◐	◐
516	$m\bar3m1'$	$2_\backslash m'm'$	24	◐	◐	○	○	○	○	◐	◐
517	$m\bar3m1'$	$2_+2'_+2'$	24	◐	○	○	◐	○	◐	◐	○
518	$m\bar3m1'$	$2_+2'_\backslash 2'$	24	◐	○	○	◐	○	◐	◐	○
519	$m\bar3m1'$	$2_\backslash 2'2'$	24	◐	○	○	◐	○	◐	◐	○
520	$m\bar3m1'$	$2'/m_+$	24	◐	○	◐	○	○	●	○	○
521	$m\bar3m1'$	$2'/m_\backslash$	24	◐	○	◐	○	○	●	○	○
522	$m\bar3m1'$	$2/m'_+$	24	◐	○	◐	○	○	◐	○	●
523	$m\bar3m1'$	$2/m'_\backslash$	24	◐	○	◐	○	○	◐	○	●
524	$m\bar3m1'$	$2'/m'_+$	24	◐	○	◐	○	○	○	●	○
525	$m\bar3m1'$	$2'/m'_\backslash$	24	◐	○	◐	○	○	○	●	○
526	$m\bar3m1'$	m'_+	48	◐	◐	◐	○	○	◐	◐	◐
527	$m\bar3m1'$	$m'_\backslash$	48	◐	◐	◐	○	○	◐	◐	◐
528	$m\bar3m1'$	$2'_+$	48	◐	◐	◐	◐	○	◐	◐	○
529	$m\bar3m1'$	$2'_\backslash$	48	◐	◐	◐	◐	○	◐	◐	○
530	$m\bar3m1'$	$\bar1'$	48	◐	○	◐	○	○	●	○	●

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
531	$11'$	1	2	○	○	○	○	●	●	●	●
532	$\bar11'$	$\bar1$	2	○	○	○	○	●	○	●	○
533	$\bar11'$	1	4	○	◐	○	◐	◐	◐	◐	◐
534	$21'$	2	2	○	○	○	○	●	●	●	●
535	$21'$	1	4	◐	◐	◐	◐	●	●	●	●
536	$m1'$	m	2	○	○	○	○	●	●	●	○
537	$m1'$	1	4	◐	◐	◐	◐	●	●	●	●
538	$2/m1'$	$2/m$	2	○	○	○	○	●	○	●	○
539	$2/m1'$	m	4	○	◐	○	◐	◐	◐	◐	◐
540	$2/m1'$	2	4	○	◐	○	◐	◐	◐	◐	◐
541	$2/m1'$	$\bar1$	4	◐	○	◐	○	●	○	●	○
542	$2/m1'$	1	8	◐	◐	◐	◐	◐	◐	◐	◐
543	$2221'$	222	2	○	○	○	○	●	○	○	●
544	$2221'$	2	4	○	◐	○	◐	●	◐	◐	●
545	$2221'$	1	8	◐	◐	◐	◐	●	●	●	●
546	$2mm1'$	$2mm$	2	○	○	○	○	●	●	○	○
547	$2mm1'$	m	4	◐	◐	◐	○	●	●	◐	○
548	$2mm1'$	2	4	○	◐	○	◐	◐	●	◐	●
549	$2mm1'$	1	8	◐	◐	◐	◐	●	●	●	●
550	$mmm1'$	mmm	2	○	○	○	○	●	○	○	○
551	$mmm1'$	$2mm$	4	○	◐	○	○	◐	◐	○	◐
552	$mmm1'$	222	4	○	○	○	◐	◐	○	○	◐
553	$mmm1'$	$2/m$	4	◐	○	◐	○	●	○	◐	○
554	$mmm1'$	m	8	◐	◐	◐	○	◐	◐	◐	○
555	$mmm1'$	2	8	◐	◐	○	◐	◐	◐	◐	◐
556	$mmm1'$	$\bar1$	8	◐	○	◐	○	●	○	●	○
557	$mmm1'$	1	16	◐	◐	◐	◐	◐	◐	◐	◐
558	$41'$	4	2	○	○	○	○	●	●	●	●
559	$41'$	$2_\mid$	4	◐	○	○	◐	●	◐	◐	●
560	$41'$	1	8	◐	◐	◐	◐	●	●	●	●
561	$\bar41'$	$\bar4$	2	○	○	○	○	●	○	●	○
562	$\bar41'$	$2_\mid$	4	◐	○	○	◐	●	◐	◐	●
563	$\bar41'$	1	8	◐	◐	◐	◐	●	●	●	●
564	$4/m1'$	$4/m$	2	○	○	○	○	●	○	●	○
565	$4/m1'$	$\bar4$	4	○	○	○	◐	◐	○	◐	◐
566	$4/m1'$	4	4	○	◐	○	◐	◐	◐	◐	◐
567	$4/m1'$	$2/m_\mid$	4	◐	○	○	○	●	○	◐	○
568	$4/m1'$	$m_\mid$	8	◐	◐	○	○	◐	◐	◐	◐
569	$4/m1'$	$2_\mid$	8	◐	◐	○	◐	◐	◐	◐	◐
570	$4/m1'$	$\bar1$	8	◐	○	◐	○	●	○	●	○
571	$4/m1'$	1	16	◐	◐	◐	◐	◐	◐	◐	◐
572	$4221'$	422	2	○	○	○	○	●	○	○	●
573	$4221'$	4	4	○	◐	○	◐	◐	◐	◐	◐
574	$4221'$	222	4	◐	○	○	◐	●	○	○	●
575	$4221'$	$2_\mid$	8	◐	◐	◐	◐	●	◐	◐	●
576	$4221'$	2_-	8	◐	◐	◐	◐	●	◐	◐	●
577	$4221'$	1	16	◐	◐	◐	◐	●	●	●	●
578	$4mm1'$	$4mm$	2	○	○	○	○	●	●	○	○
579	$4mm1'$	4	4	○	◐	○	◐	◐	◐	◐	◐
580	$4mm1'$	$2_\mid mm$	4	◐	○	○	○	●	●	○	○
581	$4mm1'$	m_-	8	◐	◐	◐	○	●	●	◐	○
582	$4mm1'$	$2_\mid$	8	◐	◐	◐	◐	●	●	◐	●
583	$4mm1'$	1	16	◐	◐	◐	◐	●	●	●	●
584	$\bar42m1'$	$\bar42m$	2	○	○	○	○	●	○	○	○
585	$\bar42m1'$	$\bar4$	4	○	○	○	◐	◐	○	◐	◐
586	$\bar42m1'$	$2_\mid mm$	4	◐	◐	○	○	●	●	○	○
587	$\bar42m1'$	222	4	○	○	○	◐	●	○	○	◐
588	$\bar42m1'$	m_-	8	◐	◐	◐	○	●	●	◐	○
589	$\bar42m1'$	$2_\mid$	8	◐	◐	◐	◐	●	●	◐	●
590	$\bar42m1'$	2_-	8	◐	◐	◐	◐	●	◐	◐	●
591	$\bar42m1'$	1	16	◐	◐	◐	◐	●	●	●	●
592	$4/mmm1'$	$4/mmm$	2	○	○	○	○	●	○	○	○
593	$4/mmm1'$	$\bar42m$	4	○	○	○	◐	◐	○	○	○
594	$4/mmm1'$	$4mm$	4	○	◐	○	○	◐	◐	○	○
595	$4/mmm1'$	422	4	○	○	○	◐	◐	○	○	◐
596	$4/mmm1'$	$4/m$	4	○	○	◐	○	◐	○	◐	○
597	$4/mmm1'$	$\bar4$	8	○	○	◐	◐	○	○	◐	○

85

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
598	$4/mmm1'$	4	8	○	◐	●	◐	●	◐	●	◐
599	$4/mmm1'$	mmm	4	◐	○	○	○	●	○	○	○
600	$4/mmm1'$	$2_|mm$	8	◐	◐	○	○	◐	◐	○	○
601	$4/mmm1'$	2_-mm	8	◐	◐	○	○	◐	◐	○	○
602	$4/mmm1'$	222	8	◐	○	○	◐	◐	○	○	◐
603	$4/mmm1'$	$2/m_|$	8	○	◐	○	◐	●	○	◐	○
604	$4/mmm1'$	$2/m_-$	8	○	◐	○	◐	●	○	◐	○
605	$4/mmm1'$	$m_|$	16	◐	◐	◐	○	◐	◐	◐	○
606	$4/mmm1'$	m_-	16	◐	◐	◐	○	◐	◐	◐	○
607	$4/mmm1'$	$2_|$	16	◐	◐	◐	◐	◐	◐	◐	◐
608	$4/mmm1'$	2_-	16	◐	◐	◐	◐	◐	◐	◐	◐
609	$4/mmm1'$	$\bar{1}$	16	◐	○	◐	○	●	○	●	○
610	$4/mmm1'$	1	32	◐	◐	◐	◐	◐	◐	◐	◐
611	$31'$	3	2	○	○	○	○	●	●	●	●
612	$31'$	1	6	◐	◐	◐	◐	●	●	●	●
613	$\bar{3}1'$	$\bar{3}$	2	○	○	○	○	●	○	●	○
614	$\bar{3}1'$	3	4	○	◐	○	◐	●	◐	●	◐
615	$\bar{3}1'$	$\bar{1}$	6	◐	○	◐	○	●	○	●	○
616	$\bar{3}1'$	1	12	◐	◐	◐	◐	◐	◐	◐	◐
617	$321'$	32	2	○	○	○	○	●	○	○	●
618	$321'$	3	4	○	◐	◐	◐	◐	◐	○	○
619	$321'$	2_-	6	◐	◐	◐	◐	●	●	●	●
620	$321'$	1	12	◐	◐	◐	◐	●	●	●	●
621	$3m1'$	$3m$	2	○	○	○	○	●	●	○	○
622	$3m1'$	3	4	○	○	◐	◐	◐	◐	◐	◐
623	$3m1'$	m_-	6	◐	◐	◐	○	●	●	●	○
624	$3m1'$	1	12	◐	◐	◐	◐	●	●	●	●
625	$\bar{3}m1'$	$\bar{3}m$	2	○	○	○	○	●	○	○	○
626	$\bar{3}m1'$	$3m$	4	○	◐	○	○	◐	◐	○	○
627	$\bar{3}m1'$	32	4	○	○	○	◐	◐	○	○	◐
628	$\bar{3}m1'$	$\bar{3}$	4	○	○	◐	○	◐	○	◐	○
629	$\bar{3}m1'$	3	8	○	◐	◐	◐	◐	◐	◐	◐
630	$\bar{3}m1'$	$2/m_-$	6	◐	○	◐	○	●	○	●	○
631	$\bar{3}m1'$	m_-	12	◐	◐	◐	○	◐	◐	◐	○
632	$\bar{3}m1'$	2_-	12	◐	◐	◐	◐	◐	◐	◐	◐
633	$\bar{3}m1'$	$\bar{1}$	12	◐	○	◐	○	●	○	●	○
634	$\bar{3}m1'$	1	24	◐	◐	◐	◐	◐	◐	◐	◐
635	$61'$	6	2	○	○	○	○	●	●	●	●
636	$61'$	3	4	○	○	○	○	◐	◐	◐	◐
637	$61'$	$2_|$	6	◐	○	○	○	●	●	◐	●
638	$61'$	1	12	◐	◐	◐	◐	●	●	●	●
639	$\bar{6}1'$	$\bar{6}$	2	○	○	○	○	●	○	●	○
640	$\bar{6}1'$	3	4	○	◐	○	◐	◐	◐	◐	◐
641	$\bar{6}1'$	$m_|$	6	◐	◐	○	○	●	●	◐	○
642	$\bar{6}1'$	1	12	◐	◐	◐	◐	●	●	●	●
643	$6/m1'$	$6/m$	2	○	○	○	○	●	○	●	○
644	$6/m1'$	$\bar{6}$	4	○	○	○	○	◐	○	◐	○
645	$6/m1'$	6	4	○	◐	○	◐	◐	◐	◐	◐
646	$6/m1'$	$\bar{3}$	4	○	○	○	◐	○	◐	○	◐
647	$6/m1'$	3	8	○	◐	○	◐	◐	◐	◐	◐

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
648	$6/m1'$	$2/m_|$	6	◐	○	○	○	●	○	◐	○
649	$6/m1'$	$m_|$	12	◐	◐	○	○	◐	◐	◐	○
650	$6/m1'$	$2_|$	12	◐	◐	○	◐	◐	◐	◐	◐
651	$6/m1'$	$\bar{1}$	12	◐	○	◐	○	●	○	●	○
652	$6/m1'$	1	24	◐	◐	◐	◐	◐	◐	◐	◐
653	$6221'$	622	2	○	○	○	○	●	○	○	●
654	$6221'$	6	4	○	◐	◐	○	◐	◐	◐	◐
655	$6221'$	32	4	○	○	○	○	◐	○	○	◐
656	$6221'$	3	8	○	◐	◐	○	◐	◐	◐	◐
657	$6221'$	222	6	◐	○	○	◐	●	○	○	●
658	$6221'$	$2_|$	12	◐	◐	◐	◐	●	◐	◐	●
659	$6221'$	2_-	12	◐	◐	◐	◐	●	◐	◐	●
660	$6221'$	1	24	◐	◐	◐	◐	●	●	●	●
661	$6mm1'$	$6mm$	2	○	○	○	○	●	●	○	○
662	$6mm1'$	6	4	○	○	◐	◐	◐	◐	◐	◐
663	$6mm1'$	$3m$	4	○	○	○	○	◐	◐	○	○
664	$6mm1'$	3	8	○	○	◐	◐	◐	◐	◐	◐
665	$6mm1'$	$2_|mm$	6	◐	○	○	○	●	●	○	○
666	$6mm1'$	m_-	12	◐	◐	◐	○	●	●	◐	○
667	$6mm1'$	$2_|$	12	◐	○	◐	◐	●	●	◐	●
668	$6mm1'$	1	24	◐	◐	◐	◐	●	●	●	●
669	$\bar{6}m21'$	$\bar{6}m2$	2	○	○	○	◐	●	○	○	○
670	$\bar{6}m21'$	$\bar{6}$	4	○	○	◐	○	◐	○	◐	○
671	$\bar{6}m21'$	$3m$	4	○	◐	○	○	◐	◐	○	○
672	$\bar{6}m21'$	32	4	○	○	○	◐	◐	○	○	◐
673	$\bar{6}m21'$	3	8	○	◐	◐	◐	◐	◐	◐	◐
674	$\bar{6}m21'$	2_-mm	6	◐	◐	○	○	●	●	○	○
675	$\bar{6}m21'$	$m_|$	12	◐	◐	◐	○	●	●	◐	○
676	$\bar{6}m21'$	m_-	12	◐	◐	◐	○	●	●	◐	○
677	$\bar{6}m21'$	2_-	12	◐	◐	◐	◐	●	◐	◐	●
678	$\bar{6}m21'$	1	24	◐	◐	◐	◐	●	●	●	●
679	$6/mmm1'$	$6/mmm$	2	○	○	○	○	●	○	○	○
680	$6/mmm1'$	$\bar{6}m2$	4	○	○	○	○	◐	○	○	○
681	$6/mmm1'$	$6mm$	4	○	◐	○	○	◐	◐	○	○
682	$6/mmm1'$	622	4	○	○	○	◐	◐	○	○	◐
683	$6/mmm1'$	$6/m$	4	○	○	◐	○	◐	○	◐	○
684	$6/mmm1'$	$\bar{6}$	8	○	○	◐	○	◐	○	◐	○
685	$6/mmm1'$	6	8	○	◐	◐	◐	◐	◐	◐	◐
686	$6/mmm1'$	$\bar{3}m$	4	○	○	○	◐	○	◐	○	○
687	$6/mmm1'$	$3m$	8	○	◐	○	○	◐	◐	○	○
688	$6/mmm1'$	32	8	○	○	○	◐	◐	○	○	◐
689	$6/mmm1'$	$\bar{3}$	8	○	○	◐	○	◐	○	◐	○
690	$6/mmm1'$	3	16	○	◐	◐	◐	◐	◐	◐	◐
691	$6/mmm1'$	mmm	6	◐	○	○	○	●	○	○	○
692	$6/mmm1'$	$2_|mm$	12	◐	◐	○	○	◐	◐	○	○
693	$6/mmm1'$	2_-mm	12	◐	◐	○	○	◐	◐	○	○
694	$6/mmm1'$	222	12	◐	○	○	◐	◐	○	○	◐
695	$6/mmm1'$	$2/m_|$	12	◐	○	◐	○	●	○	◐	○
696	$6/mmm1'$	$2/m_-$	12	◐	○	◐	○	●	○	◐	○
697	$6/mmm1'$	$m_|$	24	◐	◐	◐	○	◐	◐	◐	○

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
698	$6/mmm1'$	m_-	24	◐	◐	◐	○	◐	◐	◐	○
699	$6/mmm1'$	$2_|$	24	◐	◐	◐	◐	◐	◐	◐	◐
700	$6/mmm1'$	2_-	24	◐	◐	◐	◐	◐	◐	◐	◐
701	$6/mmm1'$	$\bar{1}$	24	◐	○	◐	○	●	○	●	○
702	$6/mmm1'$	1	48	◐	◐	◐	◐	◐	◐	◐	◐
703	$231'$	23	2	○	○	○	○	○	○	○	○
704	$231'$	3	8	◐	◐	◐	◐	●	●	●	●
705	$231'$	22_+2	6	◐	◐	○	◐	●	○	○	●
706	$231'$	2_+	12	◐	◐	◐	◐	●	◐	◐	●
707	$231'$	1	24	◐	◐	◐	◐	●	●	●	●
708	$m\bar{3}1'$	$m\bar{3}$	2	○	○	○	○	○	○	○	○
709	$m\bar{3}1'$	23	4	○	○	○	○	○	○	○	○
710	$m\bar{3}1'$	$\bar{3}$	8	◐	○	◐	●	○	●	◐	○
711	$m\bar{3}1'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
712	$m\bar{3}1'$	mm_+m	6	◐	○	○	○	●	○	○	○
713	$m\bar{3}1'$	2_+m_+m	12	◐	◐	○	○	◐	◐	○	○
714	$m\bar{3}1'$	22_+2	12	◐	○	○	◐	◐	○	○	◐
715	$m\bar{3}1'$	$2/m_+$	12	◐	○	◐	○	●	○	◐	○
716	$m\bar{3}1'$	m_+	24	◐	◐	◐	○	◐	◐	◐	○
717	$m\bar{3}1'$	2_+	24	◐	◐	◐	◐	◐	◐	◐	◐
718	$m\bar{3}1'$	$\bar{1}$	24	◐	○	◐	○	●	○	●	○
719	$m\bar{3}1'$	1	48	◐	◐	◐	◐	◐	◐	◐	◐
720	$4321'$	432	2	○	○	○	○	◐	○	○	◐
721	$4321'$	23	4	○	○	○	○	○	○	○	○
722	$4321'$	32	8	◐	◐	○	◐	●	○	○	●
723	$4321'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
724	$4321'$	422	6	◐	○	○	◐	●	○	○	●
725	$4321'$	4	12	◐	◐	◐	◐	◐	◐	◐	◐
726	$4321'$	22_+2	12	◐	○	○	◐	◐	○	○	●
727	$4321'$	$22_\backslash 2$	12	◐	○	○	◐	◐	○	○	●
728	$4321'$	2_+	24	◐	◐	◐	◐	●	◐	◐	●
729	$4321'$	$2_\backslash$	24	◐	◐	◐	◐	●	◐	◐	●
730	$4321'$	1	48	◐	◐	◐	◐	●	●	●	●
731	$\bar{4}3m1'$	$\bar{4}3m$	2	○	○	○	○	○	○	○	○
732	$\bar{4}3m1'$	23	4	○	○	○	○	○	○	○	○
733	$\bar{4}3m1'$	$3m$	8	◐	◐	○	○	●	●	○	○
734	$\bar{4}3m1'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
735	$\bar{4}3m1'$	$\bar{4}2_+m$	6	◐	○	○	◐	●	○	○	●
736	$\bar{4}3m1'$	$\bar{4}$	12	◐	◐	○	◐	○	◐	○	◐
737	$\bar{4}3m1'$	$2_+m_\backslash m$	12	◐	◐	○	○	●	◐	○	○
738	$\bar{4}3m1'$	22_+2	12	◐	○	○	◐	◐	○	○	●
739	$\bar{4}3m1'$	$m_\backslash$	24	◐	◐	◐	○	●	●	◐	○
740	$\bar{4}3m1'$	2_+	24	◐	◐	◐	◐	●	◐	◐	●
741	$\bar{4}3m1'$	1	48	◐	◐	◐	◐	●	●	●	●
742	$m\bar{3}m1'$	$m\bar{3}m$	2	○	○	○	○	○	○	○	○
743	$m\bar{3}m1'$	$\bar{4}3m$	4	○	○	○	○	○	○	○	○
744	$m\bar{3}m1'$	432	4	○	○	○	○	○	○	○	○
745	$m\bar{3}m1'$	$m\bar{3}$	4	○	○	○	○	○	○	○	○
746	$m\bar{3}m1'$	23	8	○	○	○	○	○	○	○	○
747	$m\bar{3}m1'$	$\bar{3}m$	8	◐	○	○	○	●	○	○	○

No.	G	F	n	↕	↑	⊹	⌖	↕	↑	⊹	⌖
748	$m\bar{3}m1'$	$3m$	16	◐	◐	○	○	○	◐	◐	○
749	$m\bar{3}m1'$	32	16	◐	○	○	○	◐	◐	○	◐
750	$m\bar{3}m1'$	$\bar{3}$	16	◐	○	◐	○	○	◐	◐	○
751	$m\bar{3}m1'$	3	32	◐	◐	◐	◐	◐	◐	◐	◐
752	$m\bar{3}m1'$	$4/mmm$	6	○	○	○	○	●	○	○	○
753	$m\bar{3}m1'$	$\bar{4}2_+m$	12	◐	○	○	○	○	◐	○	○
754	$m\bar{3}m1'$	$\bar{4}2_\backslash m$	12	◐	○	○	○	○	◐	○	○
755	$m\bar{3}m1'$	$4mm$	12	◐	◐	○	○	○	◐	◐	○
756	$m\bar{3}m1'$	422	12	◐	○	○	○	◐	◐	○	◐
757	$m\bar{3}m1'$	$4/m$	12	◐	○	◐	○	○	◐	○	◐
758	$m\bar{3}m1'$	$\bar{4}$	24	◐	○	◐	○	○	◐	○	◐
759	$m\bar{3}m1'$	4	24	◐	◐	◐	◐	◐	◐	◐	◐
760	$m\bar{3}m1'$	mm_+m	12	◐	○	○	○	●	○	○	○
761	$m\bar{3}m1'$	$mm_\backslash m$	12	◐	○	○	○	●	○	○	○
762	$m\bar{3}m1'$	2_+m_+m	24	◐	◐	○	○	○	◐	◐	○
763	$m\bar{3}m1'$	$2_+m_\backslash m$	24	◐	◐	○	○	○	◐	◐	○
764	$m\bar{3}m1'$	$2_\backslash mm$	24	◐	◐	○	○	○	◐	◐	○
765	$m\bar{3}m1'$	22_+2	24	◐	○	○	◐	◐	○	○	◐
766	$m\bar{3}m1'$	$22_\backslash 2$	24	◐	○	○	◐	◐	○	○	◐
767	$m\bar{3}m1'$	$2/m_+$	24	◐	○	◐	○	●	○	◐	○
768	$m\bar{3}m1'$	$2/m_\backslash$	24	◐	○	◐	○	●	○	◐	○
769	$m\bar{3}m1'$	m_+	48	◐	◐	○	○	◐	◐	◐	○
770	$m\bar{3}m1'$	$m_\backslash$	48	◐	◐	○	○	◐	◐	◐	○
771	$m\bar{3}m1'$	2_+	48	◐	◐	◐	◐	◐	◐	◐	◐
772	$m\bar{3}m1'$	$2_\backslash$	48	◐	◐	◐	◐	◐	◐	◐	◐
773	$m\bar{3}m1'$	$\bar{1}$	48	◐	○	◐	○	●	○	●	○
774	$m\bar{3}m1'$	1	96	◐	◐	◐	◐	◐	◐	◐	◐
	black-white > black-white										
775	$2'/m'$	m'	2	○	●	○	○	○	●	●	●
776	$2'/m'$	$2'$	2	○	●	○	●	○	○	●	○
777	$2/m'$	m'	2	○	●	○	○	○	○	●	○
778	$2/m'$	$\bar{1}'$	2	●	○	●	○	○	○	●	●
779	$2'/m$	$2'$	2	○	●	○	●	○	○	●	○
780	$2'/m$	$\bar{1}'$	2	●	○	●	○	○	●	○	●
781	$22'2'$	$2'$	2	●	●	●	●	●	○	●	○
782	$2m'm'$	m'	2	●	●	●	○	○	○	●	●
783	$2'mm'$	m'	2	●	●	●	○	○	○	●	●
784	$2'mm'$	$2'$	2	●	○	●	○	●	○	●	○
785	$mm'm'$	$2'mm'$	2	○	●	○	○	○	○	●	○
786	$mm'm'$	$2m'm'$	2	○	●	○	○	○	○	●	●
787	$mm'm'$	$22'2'$	2	○	○	○	●	○	●	○	○
788	$mm'm'$	$2'/m'$	2	●	○	●	○	○	○	●	○
789	$mm'm'$	m'	4	◐	●	◐	○	○	◐	◐	●
790	$mm'm'$	$2'$	4	◐	◐	◐	●	○	●	●	○
791	$m'm'm'$	$2m'm'$	2	○	●	○	○	○	●	○	●
792	$m'm'm'$	$2/m'$	2	●	○	●	○	●	○	●	●
793	$m'm'm'$	m'	4	◐	●	◐	○	○	○	●	◐
794	$m'm'm'$	$\bar{1}'$	4	●	○	●	○	○	○	●	●
795	$m'mm$	$2'mm'$	2	○	●	○	○	○	○	●	○
796	$m'mm$	$22'2'$	2	○	○	○	●	○	○	●	○
797	$m'mm$	$2'/m$	2	●	○	●	○	○	●	○	○
798	$m'mm$	$2/m'$	2	●	○	●	○	○	○	○	●
799	$m'mm$	m'	4	◐	●	○	○	○	●	●	◐
800	$m'mm$	$2'$	4	◐	○	◐	●	○	◐	●	○
801	$m'mm$	$\bar{1}'$	4	●	○	●	○	●	○	●	●
802	$4'/m$	$\bar{4}'$	2	○	○	○	○	○	●	○	●
803	$4'/m$	$4'$	2	○	●	○	●	○	○	○	○
804	$4/m'$	$\bar{4}'$	2	○	○	○	○	○	○	○	●
805	$4/m'$	$2/m_\vert$	2	●	○	○	○	○	○	○	●
806	$4/m'$	m'	4	◐	●	○	○	○	○	●	●
807	$4/m'$	$\bar{1}'$	4	●	○	●	○	○	●	○	●
808	$4'/m'$	$4'$	2	○	●	○	●	○	○	○	○
809	$4'/m'$	$2/m_\vert$	2	●	○	○	○	●	○	●	●
810	$4'/m'$	$m'_\vert$	4	◐	●	○	○	○	◐	●	●
811	$4'/m'$	$\bar{1}'$	4	●	○	●	○	○	●	○	●
812	$42'2'$	$2_\vert 2'2'$	2	●	○	○	●	○	○	○	○
813	$42'2'$	$2'$	4	●	●	●	●	○	○	●	●
814	$4'22'$	$4'$	2	○	●	●	○	○	○	○	○
815	$4'22'$	$2_\vert 2'2'$	2	●	○	○	●	○	○	●	●
816	$4'22'$	$2'$	4	●	●	●	●	○	●	●	○
817	$4'mm'$	$4'$	2	○	○	●	○	○	○	○	○
818	$4'mm'$	$2_\vert m'm'$	2	●	○	○	○	○	○	●	●
819	$4'mm'$	m'_-	4	●	●	●	○	○	●	●	●
820	$4m'm'$	$2_\vert m'm'$	2	●	○	○	○	○	○	○	●
821	$4m'm'$	m'_-	4	●	●	●	○	○	●	●	●
822	$\bar{4}'2m'$	$\bar{4}'$	2	○	○	●	○	○	●	○	●
823	$\bar{4}'2m'$	$2_\vert m'm'$	2	●	●	○	○	○	○	○	●
824	$\bar{4}'2m'$	m'_-	4	●	●	●	○	●	●	●	●
825	$\bar{4}'2m$	$\bar{4}'$	2	○	○	○	○	○	●	○	●
826	$\bar{4}'2m$	$2_\vert 2'2'$	2	●	○	○	●	○	○	○	●
827	$\bar{4}'2m$	$2'_-$	4	●	●	●	●	○	○	●	○
828	$\bar{4}2'm'$	$2_\vert m'm'$	2	●	●	○	○	○	○	○	●
829	$\bar{4}2'm'$	$2_\vert 2'2'$	2	●	○	○	●	○	●	○	○
830	$\bar{4}2'm'$	m'_-	4	●	●	●	○	○	●	●	●
831	$\bar{4}2'm'$	$2'_-$	4	●	●	●	●	○	●	●	○
832	$4'/mmm'$	$\bar{4}'2m$	2	○	○	○	○	●	○	○	○
833	$4'/mmm'$	$\bar{4}'2m'$	2	○	○	○	○	○	○	○	●
834	$4'/mmm'$	$4'mm'$	2	○	●	○	○	○	●	○	○
835	$4'/mmm'$	$4'22'$	2	○	○	○	●	○	○	●	○
836	$4'/mmm'$	$4'/m$	2	○	○	●	○	○	○	○	●
837	$4'/mmm'$	$\bar{4}'$	4	○	○	◐	○	○	◐	○	◐
838	$4'/mmm'$	$4'$	4	○	◐	◐	◐	○	○	○	○
839	$4'/mmm'$	$m_\vert m'm'$	2	●	○	○	○	○	○	●	○
840	$4'/mmm'$	$2'_-m_\vert m'$	4	◐	●	○	○	○	●	●	○
841	$4'/mmm'$	$2_\vert m'm'$	4	◐	◐	○	○	○	◐	◐	●
842	$4'/mmm'$	$2_\vert 2'2'$	4	◐	○	○	●	○	◐	○	○
843	$4'/mmm'$	$2'/m'_-$	4	●	○	●	○	○	●	○	●
844	$4'/mmm'$	m'_-	8	◐	●	◐	○	○	◐	◐	●
845	$4'/mmm'$	$2'_-$	8	◐	◐	◐	●	○	●	◐	○
846	$4/mm'm'$	$\bar{4}2'm'$	2	○	○	○	○	○	○	○	○
847	$4/mm'm'$	$4m'm'$	2	○	●	○	○	○	○	○	●
848	$4/mm'm'$	$42'2'$	2	○	○	○	●	○	●	○	○
849	$4/mm'm'$	$m_\vert m'm'$	2	●	○	○	○	○	○	○	○
850	$4/mm'm'$	$2'_-m_\vert m'$	4	◐	●	○	○	○	●	○	○
851	$4/mm'm'$	$2_\vert m'm'$	4	◐	○	○	○	○	○	●	●
852	$4/mm'm'$	$2_\vert 2'2'$	4	◐	○	○	●	○	◐	○	○
853	$4/mm'm'$	$2'/m'_-$	4	●	○	●	○	○	○	●	○
854	$4/mm'm'$	m'_-	8	◐	●	◐	○	○	◐	◐	●
855	$4/mm'm'$	$2'_-$	8	◐	◐	◐	●	○	●	◐	○
856	$4/m'm'm'$	$\bar{4}'2m'$	2	○	○	○	○	○	●	○	○
857	$4/m'm'm'$	$4m'm'$	2	○	●	○	○	○	●	○	○
858	$4/m'm'm'$	$4/m'$	2	○	○	●	○	○	●	○	○
859	$4/m'm'm'$	$\bar{4}'$	4	○	○	◐	○	○	◐	○	○
860	$4/m'm'm'$	$m'm'm'$	2	●	○	○	○	○	○	○	○
861	$4/m'm'm'$	$2_\vert m'm'$	4	◐	◐	○	○	○	○	◐	◐
862	$4/m'm'm'$	$2_-m'm'$	4	◐	●	○	○	○	○	●	●
863	$4/m'm'm'$	$2/m_\vert$	4	●	○	◐	○	○	◐	○	●
864	$4/m'm'm'$	$2/m'_-$	4	●	○	●	○	○	●	○	●
865	$4/m'm'm'$	$m'_\vert$	8	◐	●	●	○	○	◐	●	◐
866	$4/m'm'm'$	m'_-	8	◐	●	○	○	○	◐	●	●
867	$4/m'm'm'$	$\bar{1}'$	8	●	○	●	○	○	●	○	●
868	$4/m'mm$	$\bar{4}'2m$	2	○	○	○	○	○	●	○	○
869	$4/m'mm$	$42'2'$	2	○	○	○	●	○	○	●	○
870	$4/m'mm$	$4/m'$	2	○	○	●	○	○	○	○	●
871	$4/m'mm$	$\bar{4}'$	4	○	○	◐	○	○	○	○	◐
872	$4/m'mm$	$m_\vert mm$	2	●	○	○	○	○	○	●	○
873	$4/m'mm$	$2'_-m_-m'$	4	◐	●	○	○	○	○	●	○
874	$4/m'mm$	$2_\vert 2'2'$	4	◐	○	○	●	○	◐	○	●
875	$4/m'mm$	$2'/m_-$	4	●	○	●	○	○	●	○	○
876	$4/m'mm$	$2/m_\vert$	4	●	○	◐	○	○	◐	○	●
877	$4/m'mm$	$m'_\vert$	8	◐	●	◐	○	○	○	●	◐
878	$4/m'mm$	$2'_-$	8	◐	◐	◐	●	○	◐	●	○
879	$4/m'mm$	$\bar{1}'$	8	●	○	●	○	○	●	○	●
880	$4'/m'm'm$	$\bar{4}2'm'$	2	○	○	○	○	○	●	○	○
881	$4'/m'm'm$	$4'mm'$	2	○	●	○	○	○	○	●	○
882	$4'/m'm'm$	$4'22'$	2	○	●	○	●	○	○	○	○
883	$4'/m'm'm$	$4'/m'$	2	○	●	○	●	○	○	○	●
884	$4'/m'm'm$	$4'$	4	○	◐	◐	◐	○	○	○	○
885	$4'/m'm'm$	$m_\vert mm$	2	●	○	○	○	○	●	○	○
886	$4'/m'm'm$	$m'm'm'$	2	●	○	○	○	○	○	○	○
887	$4'/m'm'm$	$2'_-m_-m'$	4	◐	●	○	○	○	◐	●	○
888	$4'/m'm'm$	$2_\vert m'm'$	4	◐	◐	○	○	○	○	◐	◐
889	$4'/m'm'm$	$2_-m'm'$	4	●	◐	○	○	○	○	●	●
890	$4'/m'm'm$	$2_\vert 2'2'$	4	◐	○	○	●	○	◐	○	◐
891	$4'/m'm'm$	$2'/m_-$	4	●	○	●	○	○	●	○	○
892	$4'/m'm'm$	$2/m_\vert$	4	●	○	◐	○	○	◐	○	●
893	$4'/m'm'm$	$2/m'_-$	4	●	○	●	○	○	●	○	●
894	$4'/m'm'm$	$m'_\vert$	8	◐	●	◐	○	○	◐	●	●
895	$4'/m'm'm$	m'_-	8	●	●	◐	○	○	◐	●	●
896	$4'/m'm'm$	$2'_-$	8	◐	◐	◐	●	○	◐	●	○

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
897	$4'/m'm'm$	$\bar{1}'$	8	●	○	●	○	○	●	○	●
898	$\bar{3}'$	$\bar{1}'$	3	●	○	○	●	○	●	○	●
899	$32'$	$2'_-$	3	●	●	●	●	○	●	●	○
900	$3m'$	m'_-	3	●	●	●	○	○	●	●	●
901	$\bar{3}m'$	$3m'$	2	○	●	○	○	○	○	○	●
902	$\bar{3}m'$	$32'$	2	○	○	○	●	○	●	○	○
903	$\bar{3}m'$	$2'/m'_-$	3	●	○	●	○	○	○	●	○
904	$\bar{3}m'$	m'_-	6	◐	●	◐	○	○	●	◐	●
905	$\bar{3}m'$	$2'_-$	6	◐	●	◐	●	○	●	◐	○
906	$\bar{3}'m'$	$3m'$	2	●	●	○	○	○	○	●	○
907	$\bar{3}'m'$	$\bar{3}'$	2	○	●	○	○	○	●	○	●
908	$\bar{3}'m'$	$2/m'_-$	3	●	○	●	○	○	○	●	●
909	$\bar{3}'m'$	m'_-	6	◐	●	◐	○	○	◐	●	◐
910	$\bar{3}'m'$	$\bar{1}'$	6	●	○	●	○	○	●	○	●
911	$\bar{3}'m$	$32'$	2	○	○	○	●	○	○	●	○
912	$\bar{3}'m$	$\bar{3}'$	2	○	○	●	○	○	○	○	●
913	$\bar{3}'m$	$2'/m_-$	3	●	○	●	○	○	●	○	○
914	$\bar{3}'m$	$2'_-$	6	◐	●	◐	●	○	◐	●	●
915	$\bar{3}'m$	$\bar{1}'$	6	●	○	●	○	●	●	○	●
916	$6'$	$2'_|$	3	●	○	○	●	○	●	○	○
917	$\bar{6}'$	$m'_|$	3	●	●	●	○	○	○	●	●
918	$6'/m'$	$\bar{6}'$	2	○	○	○	○	○	●	○	●
919	$6'/m'$	$6'$	2	○	●	○	●	○	○	○	●
920	$6'/m'$	$2'/m'_|$	3	●	○	○	○	○	●	○	●
921	$6'/m'$	$m'_|$	6	◐	●	○	○	○	◐	●	●
922	$6'/m'$	$2'_|$	6	◐	◐	○	●	○	●	◐	○
923	$6/m'$	$\bar{6}'$	2	○	○	○	○	○	○	○	○
924	$6/m'$	$\bar{3}'$	2	○	○	○	○	○	○	○	○
925	$6/m'$	$2/m'_|$	3	●	○	○	○	○	○	○	●
926	$6/m'$	$m'_|$	6	◐	●	○	○	○	●	○	◐
927	$6/m'$	$\bar{1}'$	6	●	○	●	○	○	●	○	●
928	$6'/m$	$6'$	2	○	●	○	●	○	○	○	○
929	$6'/m$	$\bar{3}'$	2	○	○	○	○	●	○	○	●
930	$6'/m$	$2'/m_|$	3	●	○	○	○	○	●	○	○
931	$6'/m$	$2'_|$	6	◐	◐	○	●	○	◐	●	○
932	$6'/m$	$\bar{1}'$	6	●	○	●	○	●	●	○	●
933	$6'22'$	$6'$	2	○	●	●	●	○	○	○	○
934	$6'22'$	$32'$	2	○	○	○	○	●	●	●	○
935	$6'22'$	$2_-2'2'$	3	●	○	●	○	●	●	◐	○
936	$6'22'$	$2'_|$	6	●	◐	◐	●	○	●	●	○
937	$6'22'$	$2'_-$	6	●	●	●	●	○	●	●	○
938	$62'2'$	$32'$	2	○	○	○	○	○	○	○	○
939	$62'2'$	$2_|2'2'$	3	●	○	○	●	○	○	○	○
940	$62'2'$	$2'_-$	6	●	●	●	●	○	●	●	○
941	$6'mm'$	$6'$	2	○	○	●	●	○	○	○	○
942	$6'mm'$	$3m'$	2	○	○	○	○	○	○	●	●
943	$6'mm'$	$2'_|mm'$	3	●	○	○	○	●	○	○	○
944	$6'mm'$	m'_-	6	●	●	●	○	○	◐	●	●
945	$6'mm'$	$2'_|$	6	●	○	◐	●	○	●	●	○
946	$6m'm'$	$3m'$	2	○	○	○	○	○	○	○	○
947	$6m'm'$	$2_|m'm'$	3	●	○	○	○	○	○	○	●
948	$6m'm'$	m'_-	6	●	●	●	●	○	○	●	●
949	$\bar{6}'m'2$	$\bar{6}'$	2	○	○	●	○	○	●	○	○
950	$\bar{6}'m'2$	$3m'$	2	○	●	○	○	○	○	●	○
951	$\bar{6}'m'2$	$2_-m'm'$	3	●	●	○	○	○	○	●	●
952	$\bar{6}'m'2$	$m'_|$	6	●	●	◐	○	○	◐	●	●
953	$\bar{6}'m'2$	m'_-	6	●	●	●	○	○	●	●	●
954	$\bar{6}'m2'$	$\bar{6}'$	2	○	○	●	○	○	○	○	●
955	$\bar{6}'m2'$	$32'$	2	○	○	○	●	○	○	●	○
956	$\bar{6}'m2'$	$2'_-m_-m'$	3	●	●	○	○	○	●	○	○
957	$\bar{6}'m2'$	$m'_|$	6	●	●	◐	○	○	●	●	●
958	$\bar{6}'m2'$	$2'_-$	6	●	◐	●	●	○	●	●	●
959	$\bar{6}m'2'$	$3m'$	2	○	●	○	○	○	○	○	●
960	$\bar{6}m'2'$	$32'$	2	○	○	○	●	○	●	○	○
961	$\bar{6}m'2'$	$2'_-m_|m'$	3	●	●	○	○	○	●	○	○
962	$\bar{6}m'2'$	m'_-	6	●	●	●	○	○	◐	●	●
963	$\bar{6}m'2'$	$2'_-$	6	●	◐	●	●	○	●	●	●
964	$6'/m'mm$	$\bar{6}'m2'$	2	○	○	○	○	○	●	○	○
965	$6'/m'mm$	$\bar{6}'m'2$	2	○	○	○	○	○	○	○	●
966	$6'/m'mm$	$6'mm'$	2	○	●	○	○	○	○	●	○
967	$6'/m'mm$	$6'22'$	2	○	○	○	●	○	○	○	○
968	$6'/m'mm$	$6'/m'$	2	○	○	●	○	○	●	○	○
969	$6'/m'mm$	$\bar{6}'$	4	○	○	◐	○	○	◐	○	◐
970	$6'/m'mm$	$6'$	4	○	◐	◐	◐	○	○	○	○
971	$6'/m'mm$	$\bar{3}m'$	2	○	○	○	○	○	●	○	●
972	$6'/m'mm$	$3m'$	4	○	◐	○	○	○	○	◐	◐
973	$6'/m'mm$	$32'$	4	○	○	○	◐	○	◐	○	◐
974	$6'/m'mm$	$m_-m'm'$	3	●	○	○	○	○	○	●	○
975	$6'/m'mm$	$2'_|mm'$	6	◐	◐	○	○	○	●	◐	○
976	$6'/m'mm$	$2'_-m_-m'$	6	◐	●	○	○	○	◐	◐	●
977	$6'/m'mm$	$2_-m'm'$	6	◐	●	○	○	○	○	◐	●
978	$6'/m'mm$	$2_-2'2'$	6	◐	○	○	●	○	●	◐	○
979	$6'/m'mm$	$2'/m'_|$	6	●	○	◐	○	○	○	●	●
980	$6'/m'mm$	$2'/m'_-$	6	●	○	●	○	○	●	○	●
981	$6'/m'mm$	$m'_|$	12	◐	●	◐	○	○	◐	◐	●
982	$6'/m'mm$	m'_-	12	◐	●	◐	○	○	◐	◐	●
983	$6'/m'mm$	$2'_|$	12	◐	◐	◐	●	○	◐	●	○
984	$6'/m'mm$	$2'_-$	12	◐	◐	◐	●	○	●	◐	○
985	$6/mm'm'$	$\bar{6}m'2'$	2	○	○	○	○	○	○	●	○
986	$6/mm'm'$	$6m'm'$	2	○	●	○	○	○	○	○	●
987	$6/mm'm'$	$62'2'$	2	○	○	○	●	○	●	○	○
988	$6/mm'm'$	$\bar{3}m'$	2	○	○	○	○	○	○	○	●
989	$6/mm'm'$	$3m'$	4	○	◐	○	○	○	○	○	◐
990	$6/mm'm'$	$32'$	4	○	○	○	◐	○	◐	○	◐
991	$6/mm'm'$	$m_|m'm'$	3	●	○	○	○	○	○	●	●
992	$6/mm'm'$	$2'_-m_|m'$	6	◐	●	○	○	○	●	○	●
993	$6/mm'm'$	$2_|m'm'$	6	◐	○	○	○	○	●	○	●
994	$6/mm'm'$	$2_|2'2'$	6	◐	○	○	●	○	◐	○	○
995	$6/mm'm'$	$2'/m'_-$	6	●	○	●	○	○	○	●	○
996	$6/mm'm'$	m'_-	12	◐	●	◐	○	○	◐	●	◐
997	$6/mm'm'$	$2'_-$	12	◐	◐	◐	●	○	●	◐	○
998	$6/m'm'm'$	$\bar{6}'m'2$	2	○	○	○	○	○	○	○	○
999	$6/m'm'm'$	$6m'm'$	2	○	●	○	○	○	○	●	○
1000	$6/m'm'm'$	$6/m'$	2	○	○	●	○	○	●	○	○
1001	$6/m'm'm'$	$\bar{6}'$	4	○	○	◐	○	○	◐	○	◐
1002	$6/m'm'm'$	$\bar{3}'m'$	2	○	○	○	○	○	○	○	○
1003	$6/m'm'm'$	$3m'$	4	○	◐	○	○	○	○	◐	○
1004	$6/m'm'm'$	$\bar{3}'$	4	○	○	◐	○	○	◐	○	◐
1005	$6/m'm'm'$	$m'm'm'$	3	●	○	○	○	○	○	○	●
1006	$6/m'm'm'$	$2_|m'm'$	6	◐	◐	○	○	○	○	◐	◐
1007	$6/m'm'm'$	$2_-m'm'$	6	◐	●	○	○	○	○	●	◐
1008	$6/m'm'm'$	$2/m'_|$	6	●	○	◐	○	○	◐	○	○
1009	$6/m'm'm'$	$2/m'_-$	6	●	○	●	○	○	●	○	●
1010	$6/m'm'm'$	$m'_|$	12	◐	●	◐	○	○	○	●	◐
1011	$6/m'm'm'$	m'_-	12	◐	●	◐	○	○	○	●	◐
1012	$6/m'm'm'$	$\bar{1}'$	12	●	○	●	○	○	●	○	●
1013	$6/m'mm$	$\bar{6}'m2'$	2	○	○	○	○	○	○	○	○
1014	$6/m'mm$	$62'2'$	2	○	○	○	●	○	○	●	○
1015	$6/m'mm$	$6/m'$	2	○	○	●	○	○	●	○	○
1016	$6/m'mm$	$\bar{6}'$	4	○	○	◐	○	○	◐	○	◐
1017	$6/m'mm$	$\bar{3}'m$	2	○	○	○	○	○	○	○	○
1018	$6/m'mm$	$32'$	4	○	○	○	◐	○	◐	○	○
1019	$6/m'mm$	$\bar{3}'$	4	○	○	◐	○	○	○	○	◐
1020	$6/m'mm$	$m'_|mm$	3	●	○	○	○	○	○	●	○
1021	$6/m'mm$	$2'_-m_-m'$	6	◐	●	○	○	○	○	●	○
1022	$6/m'mm$	$2_|2'2'$	6	◐	○	○	●	○	◐	○	○
1023	$6/m'mm$	$2'/m_-$	6	●	○	●	○	○	●	○	○
1024	$6/m'mm$	$2/m'_|$	6	●	○	◐	○	○	○	○	●
1025	$6/m'mm$	$m'_|$	12	◐	●	◐	○	○	○	●	◐
1026	$6/m'mm$	$2'_-$	12	◐	◐	◐	●	○	◐	●	○
1027	$6/m'mm$	$\bar{1}'$	12	●	○	●	○	○	●	○	●
1028	$6'/mmm'$	$\bar{6}m'2'$	2	○	○	○	○	○	○	●	○
1029	$6'/mmm'$	$6'mm'$	2	○	●	○	○	○	○	○	○
1030	$6'/mmm'$	$6'22'$	2	○	○	○	●	○	○	○	○
1031	$6'/mmm'$	$6'/m$	2	○	○	●	○	○	○	○	○
1032	$6'/mmm'$	$6'$	4	○	◐	◐	◐	○	○	○	○
1033	$6'/mmm'$	$\bar{3}'m$	2	○	○	○	○	○	●	○	○
1034	$6'/mmm'$	$\bar{3}'m'$	2	○	○	○	○	○	○	○	●
1035	$6'/mmm'$	$3m'$	4	○	◐	○	○	○	○	◐	◐
1036	$6'/mmm'$	$32'$	4	○	○	○	◐	○	◐	○	◐
1037	$6'/mmm'$	$\bar{3}'$	4	○	○	◐	○	○	◐	○	◐
1038	$6'/mmm'$	m'_-mm	3	●	○	○	○	○	●	○	○
1039	$6'/mmm'$	$2'_|mm'$	6	◐	◐	○	○	○	○	●	○
1040	$6'/mmm'$	$2'_-m_|m'$	6	◐	●	○	○	○	◐	○	○
1041	$6'/mmm'$	$2_-2'2'$	6	◐	○	○	●	○	◐	●	○
1042	$6'/mmm'$	$2'/m_|$	6	●	○	◐	○	○	○	●	○
1043	$6'/mmm'$	$2'/m_-$	6	●	○	●	○	○	●	○	○
1044	$6'/mmm'$	$2/m'_-$	6	●	○	●	○	○	◐	○	●
1045	$6'/mmm'$	m'_-	12	◐	●	◐	○	○	●	●	◐
1046	$6'/mmm'$	$2'_|$	12	◐	◐	◐	●	○	◐	●	○

Left table:

No.	G	F	n
1047	$6'/mmm'$	$2'_-$	12
1048	$6'/mmm'$	$\bar{1}'$	12
1049	$m'\bar{3}'$	$\bar{3}'$	4
1050	$m'\bar{3}'$	$m'm'_+m'$	3
1051	$m'\bar{3}'$	$2_+m'_+m'$	6
1052	$m'\bar{3}'$	$2/m'_+$	6
1053	$m'\bar{3}'$	m'_+	12
1054	$m'\bar{3}'$	$\bar{1}'$	12
1055	$4'32'$	$32'$	4
1056	$4'32'$	$4'2_+2'$	3
1057	$4'32'$	$4'$	6
1058	$4'32'$	$2_+2'2'$	6
1059	$4'32'$	$2'$	12
1060	$\bar{4}'3m'$	$3m'$	4
1061	$\bar{4}'3m'$	$\bar{4}'2_+m'$	3
1062	$\bar{4}'3m'$	$\bar{4}'$	6
1063	$\bar{4}'3m'$	$2_+m'm'$	6
1064	$\bar{4}'3m'$	m'	12
1065	$m\bar{3}m'$	$\bar{4}'3m'$	2
1066	$m\bar{3}m'$	$4'32'$	2
1067	$m\bar{3}m'$	$\bar{3}m'$	4
1068	$m\bar{3}m'$	$3m'$	8
1069	$m\bar{3}m'$	$32'$	8
1070	$m\bar{3}m'$	$4'/mm'm'$	3
1071	$m\bar{3}m'$	$\bar{4}'2_+m$	6
1072	$m\bar{3}m'$	$\bar{4}'2_+m'$	6
1073	$m\bar{3}m'$	$4'm'm'$	6
1074	$m\bar{3}m'$	$4'2_+2'$	6
1075	$m\bar{3}m'$	$4'/m$	6
1076	$m\bar{3}m'$	$\bar{4}'$	12
1077	$m\bar{3}m'$	$4'$	12
1078	$m\bar{3}m'$	$m_+m'm'$	6
1079	$m\bar{3}m'$	$2'm_+m'$	12
1080	$m\bar{3}m'$	$2_+m'm'$	12
1081	$m\bar{3}m'$	$2_+2'2'$	12
1082	$m\bar{3}m'$	$2'/m'$	12
1083	$m\bar{3}m'$	m'	24
1084	$m\bar{3}m'$	$2'$	24
1085	$m'\bar{3}'m'$	$\bar{4}'3m'$	2
1086	$m'\bar{3}'m'$	$m'\bar{3}'$	2
1087	$m'\bar{3}'m'$	$\bar{3}'m'$	4
1088	$m'\bar{3}'m'$	$3m'$	8
1089	$m'\bar{3}'m'$	$\bar{3}'$	8
1090	$m'\bar{3}'m'$	$4/m'm'm'$	3
1091	$m'\bar{3}'m'$	$\bar{4}'2_+m'$	6
1092	$m'\bar{3}'m'$	$\bar{4}'2'm'$	6
1093	$m'\bar{3}'m'$	$4m'm'$	6
1094	$m'\bar{3}'m'$	$4/m'$	6
1095	$m'\bar{3}'m'$	$\bar{4}'$	12
1096	$m'\bar{3}'m'$	$m'm'_+m'$	6

Middle table:

No.	G	F	n
1097	$m'\bar{3}'m'$	$m'm'm'$	6
1098	$m'\bar{3}'m'$	$2_+m'_+m'$	12
1099	$m'\bar{3}'m'$	$2_+m'm'$	12
1100	$m'\bar{3}'m'$	$2m'm'$	12
1101	$m'\bar{3}'m'$	$2/m'_+$	12
1102	$m'\bar{3}'m'$	$2/m'$	12
1103	$m'\bar{3}'m'$	m'_+	24
1104	$m'\bar{3}'m'$	m'	24
1105	$m'\bar{3}'m'$	$\bar{1}'$	24
1106	$m'\bar{3}m$	$4'32'$	2
1107	$m'\bar{3}m$	$m'\bar{3}'$	2
1108	$m'\bar{3}m$	$\bar{3}'m$	4
1109	$m'\bar{3}m$	$32'$	8
1110	$m'\bar{3}m$	$\bar{3}'$	8
1111	$m'\bar{3}m$	$4'/m'm'_+m$	3
1112	$m'\bar{3}m$	$\bar{4}'2'm'$	6
1113	$m'\bar{3}m$	$4'm_+m'$	6
1114	$m'\bar{3}m$	$4'2_+2'$	6
1115	$m'\bar{3}m$	$4'/m$	6
1116	$m'\bar{3}m$	$4'$	12
1117	$m'\bar{3}m$	$m'_+m'm$	6
1118	$m'\bar{3}m$	$m'm'_+m'$	6
1119	$m'\bar{3}m$	$2'm'm'$	12
1120	$m'\bar{3}m$	$2_+m'_+m'$	12
1121	$m'\bar{3}m$	$2_+2'2'$	12
1122	$m'\bar{3}m$	$2'/m'$	12
1123	$m'\bar{3}m$	$2/m'_+$	12
1124	$m'\bar{3}m$	m'_+	24
1125	$m'\bar{3}m$	$2'$	24
1126	$m'\bar{3}m$	$\bar{1}'$	24

black-white > black

No.	G	F	n
1127	$\bar{1}'$	1	2
1128	$2'$	1	2
1129	m'	1	2
1130	$2'/m'$	$\bar{1}$	2
1131	$2'/m'$	1	4
1132	$2/m'$	2	2
1133	$2/m'$	1	4
1134	$2'/m$	m	2
1135	$2'/m$	1	4
1136	$22'2'$	2	2
1137	$22'2'$	1	4
1138	$2m'm'$	2	2
1139	$2m'm'$	1	4
1140	$2'mm'$	m	2
1141	$2'mm'$	1	4
1142	$mm'm'$	$2/m$	2
1143	$mm'm'$	m	4
1144	$mm'm'$	2	4
1145	$mm'm'$	$\bar{1}$	4

Right table:

No.	G	F	n
1146	$mm'm'$	1	8
1147	$m'm'm'$	222	2
1148	$m'm'm'$	2	4
1149	$m'm'm'$	1	8
1150	$m'mm$	$2mm$	2
1151	$m'mm$	m	4
1152	$m'mm$	2	4
1153	$m'mm$	1	8
1154	$4'$	$2_|$	2
1155	$4'$	1	4
1156	$\bar{4}'$	$2_|$	2
1157	$\bar{4}'$	1	4
1158	$4'/m$	$2/m_|$	2
1159	$4'/m$	$m_|$	4
1160	$4'/m$	$2_|$	4
1161	$4'/m$	$\bar{1}$	4
1162	$4'/m$	1	8
1163	$4/m'$	4	2
1164	$4/m'$	$2_|$	4
1165	$4/m'$	1	8
1166	$4'/m'$	$\bar{4}$	2
1167	$4'/m'$	$2_|$	4
1168	$4'/m'$	1	8
1169	$42'2'$	4	2
1170	$42'2'$	$2_|$	4
1171	$42'2'$	1	8
1172	$4'22'$	222	2
1173	$4'22'$	$2_|$	4
1174	$4'22'$	2_-	4
1175	$4'22'$	1	8
1176	$4'mm'$	$2_|mm$	2
1177	$4'mm'$	m_-	4
1178	$4'mm'$	$2_|$	4
1179	$4'mm'$	1	8
1180	$4m'm'$	4	2
1181	$4m'm'$	$2_|$	4
1182	$4m'm'$	1	8
1183	$\bar{4}'2m'$	222	2
1184	$\bar{4}'2m'$	$2_|$	4
1185	$\bar{4}'2m'$	2_-	4
1186	$\bar{4}'2m'$	1	8
1187	$\bar{4}'2m$	$2_|mm$	2
1188	$\bar{4}'2m$	m_-	4
1189	$\bar{4}'2m$	$2_|$	4
1190	$\bar{4}'2m$	1	8
1191	$\bar{4}2'm'$	$\bar{4}$	2
1192	$\bar{4}2'm'$	$2_|$	4
1193	$\bar{4}2'm'$	1	8
1194	$4'/mmm'$	mmm	2
1195	$4'/mmm'$	$2_|mm$	4

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
1196	$4'/mmm'$	2_mm	4	◐	●	○	○	◐	●	○	○
1197	$4'/mmm'$	222	4	◐	○	○	●	◐	○	○	●
1198	$4'/mmm'$	$2/m_|$	4	●	○	◐	○	●	○	◐	○
1199	$4'/mmm'$	$2/m_-$	4	●	○	●	○	●	●	○	
1200	$4'/mmm'$	$m_|$	8	◐	●	○	○	◐	●	○	○
1201	$4'/mmm'$	m_-	8	◐	●	○	○	◐	●	○	○
1202	$4'/mmm'$	$2_|$	8	○	◐	○	●	○	◐	○	●
1203	$4'/mmm'$	2_-	8	○	◐	○	●	○	◐	○	●
1204	$4'/mmm'$	$\bar{1}$	8	●	○	●	○	●	○	●	○
1205	$4'/mmm'$	1	16	◐	●	◐	●	◐	●	◐	●
1206	$4/mm'm'$	$4/m$	2	○	○	●	○	●	○	○	○
1207	$4/mm'm'$	$\bar{4}$	4	○	○	◐	○	◐	○	○	○
1208	$4/mm'm'$	4	4	○	○	◐	◐	◐	◐	○	○
1209	$4/mm'm'$	$2/m_|$	4	●	○	◐	○	●	○	○	○
1210	$4/mm'm'$	$m_|$	8	◐	●	○	○	◐	●	○	○
1211	$4/mm'm'$	$2_|$	8	○	◐	○	●	○	◐	○	●
1212	$4/mm'm'$	$\bar{1}$	8	●	○	●	○	●	○	●	○
1213	$4/mm'm'$	1	16	◐	●	◐	●	◐	●	◐	●
1214	$4/m'm'm'$	422	2	○	○	○	●	●	○	○	○
1215	$4/m'm'm'$	4	4	○	◐	◐	◐	◐	◐	◐	○
1216	$4/m'm'm'$	222	4	◐	○	○	●	●	○	○	◐
1217	$4/m'm'm'$	$2_|$	8	◐	○	◐	●	◐	○	◐	●
1218	$4/m'm'm'$	2_-	8	◐	○	◐	●	◐	○	◐	●
1219	$4/m'm'm'$	1	16	◐	●	●	●	◐	●	●	◐
1220	$4/m'mm$	$4mm$	2	○	●	○	○	●	○	○	○
1221	$4/m'mm$	4	4	○	◐	◐	◐	◐	○	◐	◐
1222	$4/m'mm$	$2_|mm$	4	◐	◐	○	○	●	○	○	○
1223	$4/m'mm$	m_-	8	◐	●	○	◐	●	○	○	
1224	$4/m'mm$	$2_|$	8	○	◐	○	●	◐	○	○	●
1225	$4/m'mm$	1	16	◐	●	◐	●	◐	●	◐	●
1226	$4'/m'm'm$	$\bar{4}2m$	2	○	○	○	●	○	○	○	○
1227	$4'/m'm'm$	$\bar{4}$	4	○	○	◐	○	◐	○	◐	○
1228	$4'/m'm'm$	$2_|mm$	4	◐	◐	○	○	●	○	◐	○
1229	$4'/m'm'm$	222	4	◐	○	○	●	●	○	○	◐
1230	$4'/m'm'm$	m_-	8	◐	●	○	◐	●	◐	○	◐
1231	$4'/m'm'm$	$2_|$	8	○	◐	◐	●	◐	○	◐	◐
1232	$4'/m'm'm$	2_-	8	○	◐	◐	●	◐	○	◐	◐
1233	$4'/m'm'm$	1	16	◐	●	◐	●	◐	●	◐	●
1234	$\bar{3}'$	3	2	○	●	○	●	○	●	●	○
1235	$\bar{3}'$	1	6	●	○	●	○	●	○	●	○
1236	$32'$	3	2	○	●	●	○	●	○	○	●
1237	$32'$	1	6	●	●	●	●	●	●	●	●
1238	$3m'$	3	2	○	○	●	●	●	●	○	○
1239	$3m'$	1	6	●	●	●	●	●	●	●	●
1240	$\bar{3}m'$	$\bar{3}$	2	○	○	●	○	●	○	○	○
1241	$\bar{3}m'$	3	4	○	◐	◐	◐	◐	◐	◐	◐
1242	$\bar{3}m'$	$\bar{1}$	6	●	○	●	○	●	○	●	○
1243	$\bar{3}m'$	1	12	◐	●	●	◐	●	◐	●	◐
1244	$\bar{3}'m'$	32	2	○	○	○	●	●	○	○	○
1245	$\bar{3}'m'$	3	4	○	◐	◐	◐	◐	◐	◐	○

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
1246	$\bar{3}'m'$	2_-	6	◐	●	◐	●	●	◐	●	◐
1247	$\bar{3}'m'$	1	12	◐	●	◐	●	◐	●	◐	●
1248	$\bar{3}'m$	$3m$	2	○	●	○	○	●	○	○	○
1249	$\bar{3}'m$	3	4	○	◐	◐	◐	◐	◐	○	◐
1250	$\bar{3}'m$	m_-	6	◐	●	●	○	●	◐	●	○
1251	$\bar{3}'m$	1	12	◐	●	●	◐	●	◐	●	◐
1252	$6'$	3	2	○	○	○	○	●	●	●	●
1253	$6'$	1	6	●	●	●	●	●	●	●	●
1254	$\bar{6}'$	3	2	○	●	○	●	●	○	●	○
1255	$\bar{6}'$	1	6	●	●	●	●	●	●	●	●
1256	$6'/m'$	$\bar{3}$	2	○	○	○	○	●	○	●	○
1257	$6'/m'$	3	4	◐	◐	○	◐	◐	◐	◐	◐
1258	$6'/m'$	$\bar{1}$	6	●	○	●	○	●	○	●	○
1259	$6'/m'$	1	12	◐	●	●	◐	●	◐	●	◐
1260	$6/m'$	6	2	○	●	○	●	●	○	●	○
1261	$6/m'$	3	4	○	◐	○	◐	◐	○	◐	○
1262	$6/m'$	$2_|$	6	◐	◐	○	●	●	○	○	◐
1263	$6/m'$	1	12	◐	●	●	●	◐	●	●	◐
1264	$6'/m$	$\bar{6}$	2	○	○	○	●	●	○	●	○
1265	$6'/m$	3	4	○	◐	○	◐	◐	◐	○	◐
1266	$6'/m$	$m_|$	6	●	○	●	◐	●	◐	○	◐
1267	$6'/m$	1	12	◐	●	●	◐	●	◐	●	◐
1268	$6'22'$	32	2	○	○	○	●	○	○	○	●
1269	$6'22'$	3	4	○	◐	◐	○	◐	◐	◐	○
1270	$6'22'$	2_-	6	●	●	●	●	●	◐	○	●
1271	$6'22'$	1	12	●	●	●	●	●	●	●	●
1272	$62'2'$	6	2	○	●	○	●	●	○	○	●
1273	$62'2'$	3	4	○	◐	◐	○	◐	○	○	◐
1274	$62'2'$	$2_|$	6	●	◐	○	●	◐	●	○	○
1275	$62'2'$	1	12	●	●	●	●	●	●	●	●
1276	$6'mm'$	$3m$	2	○	○	○	●	●	○	○	○
1277	$6'mm'$	3	4	○	◐	○	◐	◐	◐	○	◐
1278	$6'mm'$	m_-	6	●	●	●	○	●	●	◐	○
1279	$6'mm'$	1	12	●	●	●	●	●	●	●	●
1280	$6m'm'$	6	2	○	○	●	●	●	●	○	○
1281	$6m'm'$	3	4	○	◐	◐	◐	◐	◐	◐	○
1282	$6m'm'$	$2_|$	6	●	◐	○	●	◐	●	○	○
1283	$6m'm'$	1	12	●	●	●	●	●	●	●	●
1284	$\bar{6}'m'2$	32	2	○	○	○	●	○	○	○	●
1285	$\bar{6}'m'2$	3	4	○	◐	◐	○	◐	◐	◐	○
1286	$\bar{6}'m'2$	2_-	6	●	●	●	●	●	◐	○	●
1287	$\bar{6}'m'2$	1	12	●	●	●	●	●	●	●	●
1288	$\bar{6}'m2'$	$3m$	2	○	●	○	○	●	○	○	○
1289	$\bar{6}'m2'$	3	4	○	◐	○	◐	◐	◐	○	◐
1290	$\bar{6}'m2'$	m_-	6	●	●	●	○	●	●	◐	○
1291	$\bar{6}'m2'$	1	12	●	●	●	●	●	●	●	●
1292	$\bar{6}m'2'$	$\bar{6}$	2	○	○	●	○	●	○	○	○
1293	$\bar{6}m'2'$	3	4	○	◐	◐	◐	◐	◐	○	◐
1294	$\bar{6}m'2'$	$m_|$	6	●	●	●	○	●	●	○	◐
1295	$\bar{6}m'2'$	1	12	●	●	●	●	●	●	●	●

No.	G	F	n	↕	↑	⊬	⊉	↕	↑	⊬	⊉
1296	$6'/m'mm'$	$\bar{3}m$	2	○	○	○	○	●	○	○	○
1297	$6'/m'mm'$	$3m$	4	○	◐	○	○	◐	◐	○	○
1298	$6'/m'mm'$	32	4	○	○	○	◐	◐	◐	○	○
1299	$6'/m'mm'$	$\bar{3}$	4	○	○	◐	○	◐	○	◐	○
1300	$6'/m'mm'$	3	8	○	◐	◐	◐	◐	◐	◐	◐
1301	$6'/m'mm'$	$2/m_-$	6	●	○	●	○	●	◐	○	○
1302	$6'/m'mm'$	m_-	12	◐	●	●	○	◐	●	●	○
1303	$6'/m'mm'$	2_-	12	○	◐	○	●	○	◐	○	●
1304	$6'/m'mm'$	$\bar{1}$	12	●	○	●	○	●	○	●	○
1305	$6'/m'mm'$	1	24	◐	●	◐	●	◐	●	◐	●
1306	$6/mm'm'$	$6/m$	2	○	○	●	○	●	○	○	○
1307	$6/mm'm'$	$\bar{6}$	4	○	○	◐	○	◐	○	○	○
1308	$6/mm'm'$	6	4	○	◐	◐	◐	◐	○	◐	○
1309	$6/mm'm'$	$\bar{3}$	4	○	○	◐	○	◐	○	◐	○
1310	$6/mm'm'$	3	8	○	◐	◐	◐	◐	◐	◐	◐
1311	$6/mm'm'$	$2/m_|$	6	●	○	◐	○	●	○	○	○
1312	$6/mm'm'$	$m_|$	12	◐	●	○	○	◐	●	○	○
1313	$6/mm'm'$	$2_|$	12	○	◐	○	●	○	◐	○	●
1314	$6/mm'm'$	$\bar{1}$	12	●	○	●	○	●	○	●	○
1315	$6/mm'm'$	1	24	◐	●	◐	●	◐	●	◐	●
1316	$6/m'm'm'$	622	2	○	○	○	●	●	○	○	○
1317	$6/m'm'm'$	6	4	○	◐	◐	◐	◐	○	◐	○
1318	$6/m'm'm'$	32	4	○	○	○	◐	◐	◐	○	○
1319	$6/m'm'm'$	3	8	○	◐	◐	◐	◐	◐	◐	○
1320	$6/m'm'm'$	222	6	◐	○	○	●	●	○	○	◐
1321	$6/m'm'm'$	$2_|$	12	◐	○	◐	●	◐	○	◐	●
1322	$6/m'm'm'$	2_-	12	◐	○	◐	●	◐	○	◐	●
1323	$6/m'm'm'$	1	24	◐	●	●	●	◐	●	●	◐
1324	$6/m'mm$	$6mm$	2	○	●	○	○	●	○	○	○
1325	$6/m'mm$	6	4	○	◐	◐	○	◐	○	◐	○
1326	$6/m'mm$	$3m$	4	○	◐	○	◐	◐	◐	○	◐
1327	$6/m'mm$	3	8	○	◐	◐	◐	◐	◐	◐	◐
1328	$6/m'mm$	$2_|mm$	6	◐	◐	○	○	●	○	○	○
1329	$6/m'mm$	m_-	12	◐	●	○	◐	●	◐	○	○
1330	$6/m'mm$	$2_|$	12	○	◐	○	●	◐	○	○	●
1331	$6/m'mm$	1	24	◐	●	◐	●	◐	●	◐	●
1332	$6'/mmm'$	$\bar{6}m2$	2	○	○	○	○	●	○	○	○
1333	$6'/mmm'$	$\bar{6}$	4	○	○	◐	○	◐	○	◐	○
1334	$6'/mmm'$	$3m$	4	○	◐	○	◐	◐	◐	○	◐
1335	$6'/mmm'$	32	4	○	○	○	◐	◐	◐	○	○
1336	$6'/mmm'$	3	8	○	◐	◐	◐	◐	◐	◐	○
1337	$6'/mmm'$	2_mm	6	◐	●	○	○	●	◐	○	○
1338	$6'/mmm'$	$m_|$	12	◐	●	●	○	◐	●	○	○
1339	$6'/mmm'$	m_-	12	◐	●	●	○	●	◐	○	○
1340	$6'/mmm'$	2_-	12	○	◐	○	●	○	◐	○	●
1341	$6'/mmm'$	1	24	◐	●	◐	●	◐	●	◐	●
1342	$m'\bar{3}'$	23	2	○	○	○	○	○	○	○	○
1343	$m'\bar{3}'$	3	8	●	○	●	●	●	●	○	●
1344	$m'\bar{3}'$	22_+2	6	◐	○	○	●	●	○	○	◐
1345	$m'\bar{3}'$	2_+	12	◐	○	◐	●	●	◐	○	○

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
1346	$m'\bar{3}'$	1	24	◐	●	◐	●	●	●	●	●
1347	$4'32'$	23	2	○	○	○	○	○	○	○	○
1348	$4'32'$	3	8	◐	●	●	●	◐	◐	●	◐
1349	$4'32'$	22_+2	6	●	○	○	●	●	○	○	●
1350	$4'32'$	2_+	12	●	◐	◐	●	●	●	◐	◐
1351	$4'32'$	1	24	●	●	●	●	●	●	●	●
1352	$\bar{4}'3m'$	23	2	○	○	○	○	○	○	○	○
1353	$\bar{4}'3m'$	3	8	◐	◐	●	●	●	●	◐	◐
1354	$\bar{4}'3m'$	22_+2	6	●	○	○	●	●	○	○	●
1355	$\bar{4}'3m'$	2_+	12	●	◐	◐	●	●	●	◐	◐
1356	$\bar{4}'3m'$	1	24	●	●	●	●	●	●	●	●
1357	$m\bar{3}m'$	$m\bar{3}$	2	○	○	○	○	○	○	○	○
1358	$m\bar{3}m'$	23	4	○	○	○	○	○	○	○	○
1359	$m\bar{3}m'$	$\bar{3}$	8	◐	○	●	○	●	○	◐	○
1360	$m\bar{3}m'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1361	$m\bar{3}m'$	mm_+m	6	●	○	○	○	●	○	○	○
1362	$m\bar{3}m'$	2_+m_+m	12	◐	○	○	○	◐	◐	○	○
1363	$m\bar{3}m'$	22_+2	12	◐	○	○	●	◐	○	○	●
1364	$m\bar{3}m'$	$2/m_+$	12	●	○	◐	○	●	◐	○	◐
1365	$m\bar{3}m'$	m_+	24	◐	●	◐	○	◐	●	◐	○
1366	$m\bar{3}m'$	2_+	24	◐	◐	○	●	◐	◐	◐	◐
1367	$m\bar{3}m'$	$\bar{1}$	24	●	○	●	○	●	○	●	○
1368	$m\bar{3}m'$	1	48	◐	●	◐	●	●	◐	●	◐
1369	$m'\bar{3}'m'$	432	2	○	○	○	○	○	○	○	○
1370	$m'\bar{3}'m'$	23	4	○	○	○	○	○	○	○	○
1371	$m'\bar{3}'m'$	32	8	◐	○	○	●	●	○	○	◐
1372	$m'\bar{3}'m'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1373	$m'\bar{3}'m'$	422	6	○	○	○	●	●	○	○	●
1374	$m'\bar{3}'m'$	4	12	◐	◐	◐	◐	◐	◐	◐	◐
1375	$m'\bar{3}'m'$	22_+2	12	◐	○	○	●	●	○	○	◐
1376	$m'\bar{3}'m'$	$22_\backslash 2$	12	◐	○	○	●	●	○	○	◐
1377	$m'\bar{3}'m'$	2_+	24	◐	◐	◐	●	●	◐	◐	◐
1378	$m'\bar{3}'m'$	$2_\backslash$	24	◐	◐	◐	●	●	◐	◐	◐
1379	$m'\bar{3}'m'$	1	48	◐	●	◐	●	●	◐	●	◐
1380	$m'\bar{3}'m$	$\bar{4}3m$	2	○	○	○	○	○	○	○	○
1381	$m'\bar{3}'m$	23	4	○	○	○	○	○	○	○	○
1382	$m'\bar{3}'m$	$3m$	8	◐	●	○	○	●	◐	○	○
1383	$m'\bar{3}'m$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1384	$m'\bar{3}'m$	$\bar{4}2_+m$	6	◐	○	○	○	●	○	○	◐
1385	$m'\bar{3}'m$	$\bar{4}$	12	◐	○	◐	○	●	○	◐	○
1386	$m'\bar{3}'m$	$2_+m_\backslash m$	12	◐	◐	○	○	●	◐	○	○
1387	$m'\bar{3}'m$	22_+2	12	◐	○	○	●	●	○	○	◐
1388	$m'\bar{3}'m$	$m_\backslash$	24	◐	●	◐	○	●	◐	◐	○
1389	$m'\bar{3}'m$	2_+	24	◐	◐	◐	●	●	◐	◐	◐
1390	$m'\bar{3}'m$	1	48	◐	●	◐	●	●	◐	●	◐

black > black

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
1391	$\bar{1}$	1	2	○	●	○	●	○	●	○	●
1392	2	1	2	●	●	●	●	●	●	●	●
1393	m	1	2	●	●	●	●	●	●	●	●
1394	$2/m$	m	2	○	●	○	○	○	●	○	○

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
1395	$2/m$	2	2	○	●	○	●	○	○	○	●
1396	$2/m$	$\bar{1}$	2	●	○	●	○	●	○	●	○
1397	$2/m$	1	4	◐	●	◐	●	◐	●	◐	●
1398	222	2	2	●	●	●	●	●	●	●	●
1399	222	1	4	●	●	●	●	●	●	●	●
1400	2mm	m	2	●	●	●	○	●	●	●	○
1401	2mm	2	2	●	○	●	●	●	○	●	●
1402	2mm	1	4	●	●	●	●	●	●	●	●
1403	mmm	2mm	2	○	●	○	○	○	●	○	○
1404	mmm	222	2	○	○	○	●	○	○	○	●
1405	mmm	$2/m$	2	●	○	●	○	●	○	●	○
1406	mmm	m	4	◐	●	◐	○	◐	●	◐	○
1407	mmm	2	4	◐	○	◐	●	◐	○	◐	●
1408	mmm	$\bar{1}$	4	●	○	●	○	●	○	●	○
1409	mmm	1	8	◐	●	◐	●	◐	●	◐	●
1410	4	$2_\vert$	2	●	○	○	●	●	○	○	●
1411	4	1	4	●	●	●	●	●	●	●	●
1412	$\bar{4}$	$2_\vert$	2	●	●	○	●	●	●	○	●
1413	$\bar{4}$	1	4	●	●	●	●	●	●	●	●
1414	$4/m$	$\bar{4}$	2	○	○	○	○	○	○	○	○
1415	$4/m$	4	2	○	●	○	●	○	●	○	●
1416	$4/m$	$2/m_\vert$	2	●	○	○	○	●	○	○	○
1417	$4/m$	$m_\vert$	4	◐	●	○	○	◐	●	○	○
1418	$4/m$	$2_\vert$	4	◐	○	○	●	◐	○	○	●
1419	$4/m$	$\bar{1}$	4	●	○	●	○	●	○	●	○
1420	$4/m$	1	8	◐	●	◐	●	●	◐	●	◐
1421	422	4	2	○	●	○	●	○	●	○	●
1422	422	222	2	●	○	○	●	●	○	○	●
1423	422	$2_\vert$	4	●	◐	○	●	●	◐	○	●
1424	422	2_-	4	●	●	●	●	●	●	●	●
1425	422	1	8	●	●	●	●	●	●	●	●
1426	4mm	4	2	○	○	●	●	○	○	●	●
1427	4mm	$2_\vert mm$	2	●	○	○	○	●	○	○	○
1428	4mm	m_-	4	●	●	●	○	●	●	●	○
1429	4mm	$2_\vert$	4	●	◐	○	●	●	●	◐	○
1430	4mm	1	8	●	●	●	●	●	●	●	●
1431	$\bar{4}2m$	$\bar{4}$	2	○	○	●	○	○	●	○	●
1432	$\bar{4}2m$	$2_\vert mm$	2	●	●	○	○	●	●	○	○
1433	$\bar{4}2m$	222	2	●	○	○	●	●	○	○	●
1434	$\bar{4}2m$	m_-	4	●	●	●	○	●	●	●	○
1435	$\bar{4}2m$	$2_\vert$	4	●	◐	○	●	●	◐	○	●
1436	$\bar{4}2m$	2_-	4	●	●	●	●	●	●	●	●
1437	$\bar{4}2m$	1	8	●	●	●	●	●	●	●	●
1438	$4/mmm$	$\bar{4}2m$	2	○	○	○	○	○	○	○	○
1439	$4/mmm$	4mm	2	○	●	○	○	○	●	○	○
1440	$4/mmm$	422	2	○	○	●	○	○	○	●	○
1441	$4/mmm$	$4/m$	2	○	○	●	○	○	○	●	○
1442	$4/mmm$	$\bar{4}$	4	○	○	◐	○	○	○	◐	○
1443	$4/mmm$	4	4	○	◐	◐	◐	○	◐	◐	◐
1444	$4/mmm$	mmm	2	●	○	○	○	●	○	○	○

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
1445	$4/mmm$	$2_\vert mm$	4	◐	○	○	○	◐	◐	○	○
1446	$4/mmm$	2_-mm	4	◐	●	○	○	◐	◐	●	○
1447	$4/mmm$	222	4	○	○	◐	●	○	○	○	●
1448	$4/mmm$	$2/m_\vert$	4	●	○	◐	○	●	○	◐	○
1449	$4/mmm$	$2/m_-$	4	●	○	●	○	●	○	●	○
1450	$4/mmm$	$m_\vert$	8	◐	●	●	○	◐	◐	●	○
1451	$4/mmm$	m_-	8	◐	●	●	○	◐	◐	●	○
1452	$4/mmm$	$2_\vert$	8	◐	○	○	●	◐	○	○	●
1453	$4/mmm$	2_-	8	◐	●	●	●	◐	●	○	●
1454	$4/mmm$	$\bar{1}$	8	●	○	●	○	●	○	●	○
1455	$4/mmm$	1	16	◐	●	◐	●	●	◐	●	◐
1456	3	1	3	●	●	●	●	●	●	●	●
1457	$\bar{3}$	3	2	○	●	◐	○	●	◐	●	○
1458	$\bar{3}$	$\bar{1}$	3	●	○	●	○	●	●	○	●
1459	$\bar{3}$	1	6	◐	●	◐	●	◐	●	◐	●
1460	32	3	2	○	●	●	○	○	●	●	○
1461	32	2_-	3	●	●	●	●	●	●	●	●
1462	32	1	6	●	●	●	●	●	●	●	●
1463	3m	3	2	○	○	●	●	○	●	○	○
1464	3m	m_-	3	●	●	●	●	●	●	●	○
1465	3m	1	6	●	●	●	●	●	●	●	●
1466	$\bar{3}m$	3m	2	○	○	○	○	○	●	○	○
1467	$\bar{3}m$	32	2	○	○	○	●	○	○	●	○
1468	$\bar{3}m$	$\bar{3}$	2	○	○	●	○	○	●	○	○
1469	$\bar{3}m$	3	4	○	◐	◐	○	○	◐	◐	◐
1470	$\bar{3}m$	$2/m_-$	3	●	○	●	○	●	○	●	○
1471	$\bar{3}m$	m_-	6	◐	●	●	○	◐	●	●	○
1472	$\bar{3}m$	2_-	6	◐	●	●	●	◐	●	●	●
1473	$\bar{3}m$	$\bar{1}$	6	●	○	●	○	●	○	●	○
1474	$\bar{3}m$	1	12	◐	●	◐	●	◐	●	◐	●
1475	6	3	2	○	○	○	○	○	○	○	○
1476	6	$2_\vert$	3	●	○	●	○	●	○	○	●
1477	6	1	6	●	●	●	●	●	●	●	●
1478	$\bar{6}$	3	2	○	●	○	●	●	○	●	○
1479	$\bar{6}$	$m_\vert$	3	●	●	●	○	○	●	●	○
1480	$\bar{6}$	1	6	●	●	●	●	●	●	●	●
1481	$6/m$	$\bar{6}$	2	○	○	○	○	○	○	○	○
1482	$6/m$	6	2	○	●	○	●	○	●	○	●
1483	$6/m$	$\bar{3}$	2	○	○	○	○	○	○	○	○
1484	$6/m$	3	4	○	◐	○	◐	○	○	◐	◐
1485	$6/m$	$2/m_\vert$	3	●	○	○	○	●	○	○	○
1486	$6/m$	$m_\vert$	6	◐	●	○	○	◐	●	○	○
1487	$6/m$	$2_\vert$	6	◐	○	○	●	◐	○	○	●
1488	$6/m$	$\bar{1}$	6	●	○	●	○	●	○	●	○
1489	$6/m$	1	12	◐	●	◐	●	●	◐	●	◐
1490	622	6	2	○	●	○	○	●	●	○	○
1491	622	32	2	○	○	○	○	○	○	○	○
1492	622	3	4	○	◐	○	○	○	◐	○	◐
1493	622	222	3	●	○	○	●	●	○	○	●
1494	622	$2_\vert$	6	●	◐	◐	●	●	◐	◐	●

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1495	622	2_-	6	●	●	●	●	●	●	●	●
1496	622	1	12	●	●	●	●	●	●	●	●
1497	6mm	6	2	○	○	●	●	○	○	●	●
1498	6mm	3m	2	○	○	○	○	○	○	○	○
1499	6mm	3	4	○	○	◐	◐	○	○	◐	◐
1500	6mm	$2_|mm$	3	●	○	○	○	●	○	○	○
1501	6mm	m_-	6	●	●	●	○	●	●	●	○
1502	6mm	$2_|$	6	●	○	◐	●	●	○	●	●
1503	6mm	1	12	●	●	●	●	●	●	●	●
1504	$\bar{6}m2$	$\bar{6}$	2	○	○	●	○	○	○	●	○
1505	$\bar{6}m2$	3m	2	○	●	○	○	○	●	○	○
1506	$\bar{6}m2$	32	2	○	○	○	●	○	○	○	●
1507	$\bar{6}m2$	3	4	○	◐	◐	◐	○	◐	◐	◐
1508	$\bar{6}m2$	2_-mm	3	●	●	○	○	●	●	○	○
1509	$\bar{6}m2$	$m_|$	6	●	●	●	○	●	●	●	○
1510	$\bar{6}m2$	m_-	6	●	●	●	○	●	●	●	○
1511	$\bar{6}m2$	2_-	6	●	◐	●	●	●	●	◐	●
1512	$\bar{6}m2$	1	12	●	●	●	●	●	●	●	●
1513	6/mmm	$\bar{6}m2$	2	○	○	○	○	○	○	○	○
1514	6/mmm	6mm	2	○	●	○	○	○	●	○	○
1515	6/mmm	622	2	○	○	○	●	○	○	○	●
1516	6/mmm	6/m	2	○	○	●	○	○	○	●	○
1517	6/mmm	$\bar{6}$	4	○	◐	○	○	○	◐	○	○
1518	6/mmm	6	4	○	◐	◐	◐	○	◐	◐	◐
1519	6/mmm	$\bar{3}m$	2	○	○	○	○	○	○	○	○
1520	6/mmm	3m	4	○	◐	○	○	○	○	◐	○
1521	6/mmm	32	4	○	○	○	◐	○	○	○	◐
1522	6/mmm	$\bar{3}$	4	○	○	◐	○	○	○	◐	○
1523	6/mmm	3	8	○	◐	◐	◐	○	◐	◐	◐
1524	6/mmm	mmm	3	●	○	○	○	●	○	○	○
1525	6/mmm	$2_|mm$	6	◐	◐	○	○	◐	◐	○	○
1526	6/mmm	2_-mm	6	◐	●	○	○	◐	●	○	○
1527	6/mmm	222	6	◐	○	○	●	◐	○	○	●
1528	6/mmm	$2/m_|$	6	●	○	◐	○	●	○	◐	○
1529	6/mmm	$2/m_-$	6	●	○	●	○	●	○	●	○
1530	6/mmm	$m_|$	12	◐	●	◐	○	◐	◐	●	○

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1531	6/mmm	m_-	12	◐	●	◐	○	◐	◐	●	○
1532	6/mmm	$2_|$	12	◐	◐	●	●	◐	◐	◐	●
1533	6/mmm	2_-	12	◐	◐	●	●	◐	◐	◐	●
1534	6/mmm	$\bar{1}$	12	●	○	●	●	●	○	●	○
1535	6/mmm	1	24	◐	●	◐	●	◐	◐	●	●
1536	23	3	4	●	●	●	●	●	●	●	●
1537	23	22_+2	3	●	○	○	●	●	○	○	●
1538	23	2_+	6	●	●	●	●	●	●	●	●
1539	23	1	12	●	●	●	●	●	●	●	●
1540	$m\bar{3}$	23	2	○	○	○	○	○	○	○	○
1541	$m\bar{3}$	$\bar{3}$	4	●	○	●	●	●	○	●	○
1542	$m\bar{3}$	3	8	◐	●	◐	●	◐	●	●	●
1543	$m\bar{3}$	mm_+m	3	●	○	○	○	●	○	○	○
1544	$m\bar{3}$	2_+m_+m	6	◐	●	○	○	◐	●	○	○
1545	$m\bar{3}$	22_+2	6	◐	○	○	●	◐	○	○	●
1546	$m\bar{3}$	$2/m_+$	6	●	○	●	●	●	○	●	○
1547	$m\bar{3}$	m_+	12	◐	●	◐	○	◐	●	●	○
1548	$m\bar{3}$	2_+	12	◐	◐	●	●	◐	◐	◐	●
1549	$m\bar{3}$	$\bar{1}$	12	●	○	●	●	●	○	●	○
1550	$m\bar{3}$	1	24	◐	●	◐	●	◐	●	◐	●
1551	432	23	2	○	○	○	○	○	○	○	○
1552	432	32	4	●	○	○	○	●	○	○	○
1553	432	3	8	◐	●	●	◐	◐	●	●	◐
1554	432	422	3	●	○	○	●	●	○	○	●
1555	432	4	6	◐	●	●	◐	◐	●	●	◐
1556	432	22_+2	6	●	○	●	●	●	○	○	●
1557	432	$22_\backslash 2$	6	●	○	●	●	●	○	○	●
1558	432	2_+	12	●	◐	◐	●	●	◐	◐	●
1559	432	$2_\backslash$	12	●	●	●	●	●	●	●	●
1560	432	1	24	●	●	●	●	●	●	●	●
1561	$\bar{4}3m$	23	2	○	○	○	○	○	○	○	○
1562	$\bar{4}3m$	3m	4	●	●	○	○	●	●	○	○
1563	$\bar{4}3m$	3	8	◐	◐	●	●	◐	◐	●	●
1564	$\bar{4}3m$	$\bar{4}2_+m$	3	●	○	○	●	●	○	○	●
1565	$\bar{4}3m$	$\bar{4}$	6	◐	○	●	●	◐	○	●	○
1566	$\bar{4}3m$	$2_+m_\backslash m$	6	●	●	○	○	●	●	○	○

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1567	$\bar{4}3m$	22_+2	6	●	○	○	●	●	○	○	●
1568	$\bar{4}3m$	$m_\backslash$	12	●	●	●	○	●	●	●	○
1569	$\bar{4}3m$	2_+	12	●	●	◐	●	●	◐	◐	●
1570	$\bar{4}3m$	1	24	●	●	●	●	●	●	●	●
1571	$m\bar{3}m$	$\bar{4}3m$	2	○	○	○	○	○	○	○	○
1572	$m\bar{3}m$	432	2	○	○	○	○	○	○	○	○
1573	$m\bar{3}m$	$m\bar{3}$	2	○	○	○	○	○	○	○	○
1574	$m\bar{3}m$	23	4	○	○	○	○	○	○	○	○
1575	$m\bar{3}m$	$\bar{3}m$	4	●	○	○	○	●	○	○	○
1576	$m\bar{3}m$	3m	8	◐	●	○	○	◐	●	○	○
1577	$m\bar{3}m$	32	8	◐	○	○	●	◐	○	○	●
1578	$m\bar{3}m$	$\bar{3}$	8	◐	○	●	○	◐	○	●	○
1579	$m\bar{3}m$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1580	$m\bar{3}m$	4/mmm	3	●	○	○	○	●	○	○	○
1581	$m\bar{3}m$	$\bar{4}2_+m$	6	◐	○	○	◐	◐	○	○	○
1582	$m\bar{3}m$	$\bar{4}2_\backslash m$	6	◐	○	○	◐	◐	○	○	○
1583	$m\bar{3}m$	4mm	6	◐	●	○	○	◐	●	○	○
1584	$m\bar{3}m$	422	6	◐	○	○	●	◐	○	○	●
1585	$m\bar{3}m$	4/m	6	◐	○	●	○	◐	○	●	○
1586	$m\bar{3}m$	$\bar{4}$	12	◐	○	◐	○	◐	○	◐	○
1587	$m\bar{3}m$	4	12	◐	◐	◐	◐	◐	◐	◐	◐
1588	$m\bar{3}m$	mm_+m	6	●	○	○	○	●	○	○	○
1589	$m\bar{3}m$	$mm_\backslash m$	6	●	○	○	○	●	○	○	○
1590	$m\bar{3}m$	2_+m_+m	12	◐	◐	○	○	◐	◐	○	○
1591	$m\bar{3}m$	$2_+m_\backslash m$	12	◐	◐	○	○	◐	◐	○	○
1592	$m\bar{3}m$	$2_\backslash mm$	12	◐	●	○	○	◐	●	○	○
1593	$m\bar{3}m$	22_+2	12	◐	○	○	●	◐	○	○	●
1594	$m\bar{3}m$	$22_\backslash 2$	12	◐	○	○	●	◐	○	○	●
1595	$m\bar{3}m$	$2/m_+$	12	●	○	◐	●	●	○	◐	○
1596	$m\bar{3}m$	$2/m_\backslash$	12	●	○	●	●	●	○	●	○
1597	$m\bar{3}m$	m_+	24	◐	●	○	○	●	◐	●	○
1598	$m\bar{3}m$	$m_\backslash$	24	◐	●	○	●	◐	●	●	○
1599	$m\bar{3}m$	2_+	24	◐	◐	●	◐	◐	◐	◐	●
1600	$m\bar{3}m$	$2_\backslash$	24	◐	◐	●	●	◐	◐	◐	●
1601	$m\bar{3}m$	$\bar{1}$	24	●	○	●	●	●	○	●	○
1602	$m\bar{3}m$	1	48	◐	●	◐	●	◐	●	◐	●

D. Genera of Vectorlike Transitions

The following pages contain a table of 212 genera of macroscopic symmetry-breaking structural phase transitions between magnetic point groups. Each is listed as in Attachment C; only the order varies (as can be seen in the first column). The ordering here is described by the header bars "Genus #n F > G".

Chapter 8 describes the motivation and methods for creating this table as well as discussing the results and presenting some conclusions about its contents, limitations, and possible applications.

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
Genus #1 1̄ > 1											
1127	1̄'	1	2	○	●	○	●	●	○	●	○
1391	1̄	1	2	○	●	○	●	○	●	○	●
1	1̄1'	11'	2	○	●	○	●	○	○	○	○
533	1̄1'	1	4	○	◐	○	◐	◐	●	○	○
Genus #2 2 > 1											
1128	2'	1	2	●	●	●	●	●	●	●	●
1392	2	1	2	●	●	●	●	●	●	●	●
2	21'	11'	2	●	●	●	●	○	○	○	○
535	21'	1	4	◐	○	◐	○	●	●	●	●
Genus #3 m > 1											
1393	m	1	2	●	●	●	●	●	●	●	●
1129	m'	1	2	●	●	●	●	●	●	●	●
3	m1'	11'	2	●	●	●	●	○	○	○	○
537	m1'	1	4	◐	○	◐	◐	●	●	●	●
Genus #4 2/m > m											
1134	2'/m	m	2	○	●	○	○	●	○	●	○
775	2'/m'	m'	2	○	●	○	○	○	○	●	○
1394	2/m	m	2	○	●	○	○	○	●	○	○
777	2/m'	m'	2	○	●	○	○	○	○	●	○
4	2/m1'	m1'	2	○	●	○	○	○	○	○	○
539	2/m1'	m	4	○	◐	○	○	◐	◐	◐	○
219	2/m1'	m'	4	○	◐	○	○	○	◐	●	◐
Genus #5 2/m > 2											
1132	2/m'	2	2	○	●	○	●	●	○	●	○
1395	2/m	2	2	○	●	○	●	○	●	○	●
776	2'/m'	2'	2	○	●	○	●	●	○	○	○
779	2'/m	2'	2	○	●	○	●	○	●	○	●
5	2/m1'	21'	2	○	●	○	●	○	○	○	○
540	2/m1'	2	4	○	◐	○	◐	◐	◐	◐	◐
220	2/m1'	2'	4	○	◐	○	◐	◐	○	◐	◐
Genus #6 2/m > 1̄											
1396	2/m	1̄	2	●	○	●	○	●	●	●	○
1130	2'/m'	1̄	2	●	○	●	○	●	●	●	○
780	2'/m	1̄'	2	●	○	●	○	○	●	●	●
778	2/m'	1̄'	2	●	○	●	○	○	●	○	●
6	2/m1'	1̄1'	2	●	○	●	○	○	○	○	○
541	2/m1'	1̄	4	◐	○	◐	○	●	●	●	○
221	2/m1'	1̄'	4	◐	○	◐	○	○	●	○	●
Genus #7 2/m > 1											
1133	2/m'	1	4	◐	●	◐	○	●	●	◐	●
1135	2'/m	1	4	◐	●	◐	●	●	●	◐	◐
1131	2'/m'	1	4	◐	●	◐	●	●	◐	●	●
1397	2/m	1	4	◐	●	◐	●	◐	●	◐	●
7	2/m1'	11'	4	◐	●	◐	●	○	○	○	○
542	2/m1'	1	8	◐	◐	◐	◐	◐	◐	◐	◐
Genus #8 222 > 2											
1398	222	2	2	●	●	●	●	●	●	●	●
1136	22'2'	2	2	●	●	●	●	●	○	○	●
781	22'2'	2'	2	●	●	●	●	○	●	●	○

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
8	2221'	21'	2	●	●	●	●	○	○	○	○
544	2221'	2	4	◐	◐	◐	◐	●	●	◐	●
223	2221'	2'	4	◐	◐	◐	◐	○	●	●	○
Genus #9 222 > 1											
1399	222	1	4	●	●	●	●	●	●	●	●
1137	22'2'	1	4	●	●	●	●	●	●	●	●
9	2221'	11'	4	●	●	●	●	○	○	○	○
545	2221'	1	8	◐	◐	◐	◐	●	●	●	●
Genus #10 2mm > m											
1400	2mm	m	2	●	●	●	○	●	●	●	○
1140	2'mm'	m	2	●	●	●	○	●	○	○	●
782	2m'm'	m'	2	●	●	●	○	●	○	○	●
783	2'mm'	m'	2	●	●	●	○	○	●	●	○
10	2mm1'	m1'	2	●	●	●	○	○	○	○	○
547	2mm1'	m	4	◐	◐	◐	○	◐	●	●	◐
226	2mm1'	m'	4	◐	◐	◐	○	○	◐	●	●
Genus #11 2mm > 2											
1138	2m'm'	2	2	●	○	●	●	●	●	●	●
1401	2mm	2	2	●	○	●	●	●	○	●	●
784	2'mm'	2'	2	●	○	●	●	●	●	●	○
11	2mm1'	21'	2	●	○	●	●	○	○	○	○
548	2mm1'	2	4	◐	○	◐	◐	●	●	●	●
227	2mm1'	2'	4	◐	○	◐	◐	○	●	●	○
Genus #12 2mm > 1											
1402	2mm	1	4	●	●	●	●	●	●	●	●
1139	2m'm'	1	4	●	●	●	●	●	●	●	●
1141	2'mm'	1	4	●	●	●	●	●	●	●	●
12	2mm1'	11'	4	●	●	●	●	○	○	○	○
549	2mm1'	1	8	◐	◐	◐	◐	●	●	●	●
Genus #13 mmm > 2mm											
1150	m'mm	2mm	2	○	●	○	○	●	○	○	○
785	mm'm'	2'mm'	2	○	●	○	○	●	○	○	○
1403	mmm	2mm	2	○	●	○	○	○	●	○	○
791	m'm'm'	2m'm'	2	○	●	○	○	○	○	●	○
795	m'mm	2'mm'	2	○	●	○	○	○	○	●	○
786	mm'm'	2m'm'	2	○	●	○	○	○	○	○	●
13	mmm1'	2mm1'	2	○	●	○	○	○	○	○	○
551	mmm1'	2mm	4	○	◐	○	○	◐	◐	○	○
231	mmm1'	2'mm'	4	○	◐	○	○	○	◐	●	○
232	mmm1'	2m'm'	4	○	◐	○	○	○	○	◐	◐
Genus #14 mmm > 222											
1147	m'm'm'	222	2	○	○	○	●	●	○	○	○
787	mm'm'	22'2'	2	○	○	○	●	○	●	○	○
796	m'mm	22'2'	2	○	○	○	●	○	○	●	○
1404	mmm	222	2	○	○	○	●	○	●	○	●
14	mmm1'	2221'	2	○	○	○	●	○	○	○	○
552	mmm1'	222	4	○	○	○	◐	◐	○	○	◐
233	mmm1'	22'2'	4	○	○	○	◐	○	◐	○	◐
Genus #15 mmm > 2/m											
1405	mmm	2/m	2	●	○	●	○	●	○	●	○

No.	G	F	n	↕	↑	⊬	⊭	↕	↑	⊬	⊭
1142	mm'm'	2/m	2	●	○	●	○	●	○	○	○
792	m'm'm'	2/m'	2	●	○	●	○	○	●	○	○
797	m'mm	2'/m	2	●	○	●	○	●	○	○	○
788	mm'm'	2'/m'	2	●	○	●	○	○	○	●	○
798	m'mm	2/m'	2	●	○	●	○	○	○	●	○
15	mmm1'	2/m1'	2	●	○	●	○	○	○	○	○
553	mmm1'	2/m	4	◐	○	◐	○	●	○	◐	○
234	mmm1'	2'/m	4	◐	○	◐	○	●	○	◐	○
235	mmm1'	2/m'	4	◐	○	◐	○	○	◐	●	◐
236	mmm1'	2'/m'	4	◐	○	◐	○	○	○	●	◐
Genus #16 mmm > m											
1151	m'mm	m	4	◐	●	◐	○	●	◐	◐	○
1406	mmm	m	4	◐	●	◐	○	◐	●	◐	○
1143	mm'm'	m	4	◐	●	◐	○	●	◐	○	○
793	m'm'm'	m'	4	◐	●	◐	○	○	◐	●	◐
789	mm'm'	m'	4	◐	●	◐	○	○	◐	●	●
799	m'mm	m'	4	◐	●	◐	○	○	○	●	◐
16	mmm1'	m1'	4	◐	●	◐	○	○	○	○	○
554	mmm1'	m	8	◐	◐	◐	○	◐	◐	◐	◐
237	mmm1'	m'	8	◐	◐	◐	○	○	◐	●	◐
Genus #17 mmm > 2											
1148	m'm'm'	2	4	◐	○	◐	●	●	○	◐	◐
1152	m'mm	2	4	◐	○	◐	●	●	○	◐	○
1407	mmm	2	4	◐	○	◐	●	◐	●	◐	●
1144	mm'm'	2	4	◐	○	◐	●	◐	◐	○	●
790	mm'm'	2'	4	◐	○	◐	●	○	●	○	◐
800	m'mm	2'	4	◐	○	◐	●	○	●	●	○
17	mmm1'	21'	4	◐	○	◐	●	○	○	○	○
555	mmm1'	2	8	◐	○	◐	◐	◐	◐	◐	◐
238	mmm1'	2'	8	◐	○	◐	◐	○	◐	○	◐
Genus #18 mmm > 1̄											
1145	mm'm'	1̄	4	●	○	●	○	●	○	●	○
1408	mmm	1̄	4	●	○	●	○	●	○	●	○
801	m'mm	1̄'	4	●	○	●	○	●	○	●	○
794	m'm'm'	1̄'	4	●	○	●	○	●	○	●	○
18	mmm1'	1̄1'	4	●	○	●	○	○	○	○	○
556	mmm1'	1̄	8	◐	○	◐	○	●	○	●	◐
239	mmm1'	1̄'	8	◐	○	◐	○	●	○	○	●
Genus #19 mmm > 1											
1149	m'm'm'	1	8	◐	●	◐	●	●	◐	◐	◐
1153	m'mm	1	8	◐	●	◐	●	●	◐	◐	◐
1146	mm'm'	1	8	◐	●	◐	●	●	◐	◐	◐
1409	mmm	1	8	◐	●	◐	●	◐	●	◐	●
19	mmm1'	11'	8	◐	●	◐	●	○	○	○	○
557	mmm1'	1	16	◐	◐	◐	◐	◐	◐	◐	◐
Genus #20 4 > 2₍											
1154	4'	2₍	2	●	○	○	●	●	●	●	●
1410	4	2₍	2	●	○	○	●	●	○	○	●
20	41'	2₍1'	2	●	○	○	●	○	○	○	○
559	41'	2₍	4	◐	○	○	◐	●	◐	●	●

94

No.	G	F	n	symbols
241	41'	2'|	4	◐ ○ ○ ○ ◐ ○ ● ●

No.	G	F	n	symbols
1155	4'	1	4	● ● ● ● ● ● ● ●
1411	4	1	4	● ● ● ● ● ● ● ●
21	41'	11'	4	● ● ● ● ○ ○ ○ ○
560	41'	1	8	◐ ◐ ◐ ◐ ● ● ● ●

Genus #22 4̄ > 2|

1412	4̄	2		2	● ● ○ ● ● ● ○ ●
1156	4̄'	2		2	● ● ○ ● ● ● ○ ●
22	4̄1'	2	1'	2	● ● ○ ● ○ ○ ○ ○
562	4̄1'	2		4	◐ ◐ ○ ○ ◐ ● ◐ ●
243	4̄1'	2'		4	◐ ◐ ○ ○ ◐ ○ ● ●

Genus #23 4̄ > 1

1157	4̄'	1	4	● ● ● ● ● ● ● ●
1413	4̄	1	4	● ● ● ● ● ● ● ●
23	4̄1'	11'	4	● ● ● ● ○ ○ ○ ○
563	4̄1'	1	8	◐ ◐ ◐ ◐ ● ● ● ●

Genus #24 4/m > 4̄

1166	4'/m'	4̄	2	○ ○ ○ ○ ○ ● ○ ●
565	4/m1'	4̄	4	○ ○ ○ ○ ○ ◐ ○ ◐
802	4'/m	4̄'	2	○ ○ ○ ○ ○ ● ○ ●
247	4/m1'	4̄'	4	○ ○ ○ ○ ○ ◐ ○ ◐
24	4/m1'	4̄1'	2	○ ○ ○ ○ ○ ○ ○ ○
804	4/m'	4̄'	2	○ ○ ○ ○ ○ ○ ○ ○
1414	4/m	4̄	2	○ ○ ○ ○ ○ ○ ○ ○

Genus #25 4/m > 4

1163	4/m'	4	2	○ ● ○ ● ● ○ ● ○
1415	4/m	4	2	○ ● ○ ● ○ ● ● ○
25	4/m1'	41'	2	○ ● ○ ● ○ ○ ○ ○
808	4'/m'	4'	2	○ ● ○ ● ○ ○ ○ ○
803	4'/m	4'	2	○ ● ○ ● ○ ○ ○ ○
566	4/m1'	4	4	○ ◐ ○ ◐ ◐ ◐ ◐ ◐
248	4/m1'	4'	4	○ ◐ ○ ◐ ○ ○ ○ ○

Genus #26 4/m > 2/m|

1158	4'/m	2/m		2	● ○ ○ ○ ● ○ ● ○
1416	4/m	2/m		2	● ○ ○ ○ ● ○ ● ○
809	4'/m'	2/m'		2	● ○ ○ ○ ○ ● ○ ●
805	4/m'	2/m'		2	● ○ ○ ○ ○ ● ○ ●
26	4/m1'	2/m	1'	2	● ○ ○ ○ ○ ○ ○ ○
567	4/m1'	2/m		4	◐ ○ ○ ○ ● ○ ◐ ○
249	4/m1'	2'/m		4	◐ ○ ○ ○ ○ ● ○ ◐
250	4/m1'	2/m'		4	◐ ○ ○ ○ ◐ ○ ◐ ○
251	4/m1'	2'/m'		4	◐ ○ ○ ○ ○ ○ ● ○

Genus #27 4/m > m|

1159	4'/m	m		4	◐ ● ○ ○ ◐ ● ◐ ○
1417	4/m	m		4	◐ ● ○ ○ ◐ ● ◐ ○
810	4'/m'	m'		4	◐ ● ○ ○ ○ ◐ ● ◐
806	4/m'	m'		4	◐ ● ○ ○ ○ ◐ ● ◐
27	4/m1'	m	1'	4	◐ ● ○ ○ ○ ○ ○ ○
568	4/m1'	m		8	◐ ◐ ○ ○ ◐ ◐ ◐ ◐

No.	G	F	n	symbols
252	4/m1'	m'|	8	◐ ◐ ○ ○ ○ ◐ ◐ ◐

Genus #28 4/m > 2|

1167	4'/m'	2		4	○ ○ ○ ● ● ● ○ ◐
1164	4/m'	2		4	○ ○ ○ ● ● ○ ◐ ◐
1160	4'/m	2		4	○ ○ ○ ◐ ◐ ◐ ○ ●
1418	4/m	2		4	○ ○ ○ ● ● ○ ○ ●
28	4/m1'	2	1'	4	○ ○ ● ○ ○ ○ ○ ○
569	4/m1'	2		8	○ ○ ○ ◐ ○ ◐ ○ ◐
253	4/m1'	2'		8	○ ○ ○ ◐ ○ ◐ ○ ○

Genus #29 4/m > 1̄

1161	4'/m	1̄	4	● ○ ● ○ ● ○ ● ○
1419	4/m	1̄	4	● ○ ● ○ ● ○ ● ○
811	4'/m'	1̄'	4	○ ● ○ ○ ○ ● ● ●
807	4/m'	1̄'	4	○ ● ○ ● ○ ● ○ ●
29	4/m1'	1̄1'	4	● ○ ● ○ ● ○ ○ ○
570	4/m1'	1̄	8	◐ ○ ○ ◐ ● ○ ● ○
254	4/m1'	1̄'	8	◐ ○ ○ ◐ ○ ○ ● ●

Genus #30 4/m > 1

1165	4/m'	1	8	◐ ● ○ ● ● ● ◐ ◐
1168	4'/m'	1	8	◐ ● ● ● ● ● ◐ ◐
1162	4'/m	1	8	◐ ● ● ● ◐ ● ◐ ◐
1420	4/m	1	8	◐ ● ● ● ● ◐ ◐ ◐
30	4/m1'	11'	8	◐ ● ○ ● ○ ○ ○ ○
571	4/m1'	1	16	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐

Genus #31 422 > 4

1169	42'2'	4	2	○ ● ● ○ ● ○ ○ ●
1421	422	4	2	○ ● ● ○ ○ ● ● ○
31	4221'	41'	2	○ ● ● ○ ○ ○ ○ ○
814	4'22'	4'	2	● ○ ● ● ○ ○ ○ ○
573	4221'	4	4	○ ◐ ◐ ○ ○ ◐ ◐ ○
257	4221'	4'	4	○ ◐ ◐ ○ ○ ○ ○ ○

Genus #32 422 > 222

1172	4'22'	222	2	● ○ ○ ○ ● ○ ○ ●
1422	422	222	2	● ○ ○ ○ ● ○ ○ ●
815	4'22'	2|2'2'	2	● ○ ○ ● ○ ● ● ○
32	4221'	2221'	2	● ○ ○ ○ ● ○ ○ ○
812	42'2'	2|2'2'	2	● ○ ○ ● ○ ○ ● ○
574	4221'	222	4	○ ○ ○ ◐ ○ ◐ ○ ●
259	4221'	2_2'2'	4	○ ○ ○ ◐ ○ ● ● ○
258	4221'	2|2'2'	4	○ ○ ○ ◐ ○ ◐ ○ ○

Genus #33 422 > 2|

1423	422	2		4	● ○ ○ ● ● ○ ◐ ●
1173	4'22'	2		4	● ○ ◐ ● ● ◐ ○ ●
1170	42'2'	2		4	● ○ ◐ ● ● ○ ○ ●
33	4221'	2	1'	4	● ○ ◐ ● ○ ○ ○ ○
575	4221'	2		8	○ ○ ◐ ● ○ ◐ ○ ●
260	4221'	2'		8	○ ○ ◐ ● ○ ○ ● ○

Genus #34 422 > 2_

| 1424 | 422 | 2_ | 4 | ● ● ● ● ● ● ● ● |
| 1174 | 4'22' | 2_ | 4 | ● ● ● ● ● ● ● ● |

No.	G	F	n	symbols
813	42'2'	2'_	4	● ● ● ● ○ ● ● ○
816	4'22'	2'_	4	● ● ● ● ○ ● ● ○
34	4221'	2_1'	4	● ● ● ● ○ ○ ○ ○
576	4221'	2_	8	◐ ◐ ◐ ◐ ● ● ◐ ●
261	4221'	2'_	8	◐ ◐ ◐ ◐ ○ ● ● ○

Genus #35 422 > 1

1175	4'22'	1	8	● ● ● ● ● ● ● ●
1425	422	1	8	● ● ● ● ● ● ● ●
1171	42'2'	1	8	● ● ● ● ● ● ● ●
35	4221'	11'	8	● ● ● ● ○ ○ ○ ○
577	4221'	1	16	◐ ◐ ◐ ◐ ● ● ● ●

Genus #36 4mm > 4

1180	4m'm'	4	2	○ ○ ● ● ● ● ○ ○
1426	4mm	4	2	○ ○ ● ● ● ● ○ ○
36	4mm1'	41'	2	○ ○ ● ● ○ ○ ○ ○
817	4'mm'	4'	2	○ ○ ● ● ○ ○ ● ●
579	4mm1'	4	4	○ ○ ◐ ◐ ◐ ◐ ◐ ◐
264	4mm1'	4'	4	○ ○ ◐ ◐ ○ ○ ○ ○

Genus #37 4mm > 2|mm

1176	4'mm'	2	mm	2	● ○ ○ ○ ● ● ○ ○
1427	4mm	2	mm	2	● ○ ○ ○ ● ● ○ ○
818	4'mm'	2	m'm'	2	● ○ ○ ○ ○ ○ ● ●
820	4m'm'	2	m'm'	2	● ○ ○ ○ ○ ○ ● ●
37	4mm1'	2	mm1'	2	● ○ ○ ○ ○ ○ ○ ○
580	4mm1'	2	mm	4	◐ ○ ○ ○ ● ◐ ○ ○
265	4mm1'	2'mm'	4	◐ ○ ○ ○ ○ ○ ● ●
266	4mm1'	2	m'm'	4	◐ ○ ○ ○ ○ ○ ◐ ●

Genus #38 4mm > m_

1428	4mm	m_	4	● ● ● ○ ● ● ● ○
1177	4'mm'	m_	4	● ● ● ○ ● ● ● ○
819	4'mm'	m'_	4	● ● ● ○ ● ● ● ○
821	4m'm'	m'_	4	● ● ● ○ ● ● ● ○
38	4mm1'	m_1'	4	● ● ● ○ ○ ○ ○ ○
581	4mm1'	m_	8	◐ ○ ◐ ○ ● ● ◐ ○
267	4mm1'	m'_	8	◐ ○ ◐ ○ ○ ○ ◐ ●

Genus #39 4mm > 2|

1178	4'mm'	2		4	● ○ ◐ ● ● ◐ ◐ ●
1181	4m'm'	2		4	● ○ ◐ ● ● ◐ ○ ●
1429	4mm	2		4	● ○ ● ● ● ○ ◐ ●
39	4mm1'	2	1'	4	● ○ ● ● ○ ○ ○ ○
582	4mm1'	2		8	◐ ○ ◐ ● ○ ◐ ● ●
268	4mm1'	2'		8	◐ ○ ● ◐ ○ ● ● ○

Genus #40 4mm > 1

1182	4m'm'	1	8	● ● ● ● ● ● ● ●
1179	4'mm'	1	8	● ● ● ● ● ● ● ●
1430	4mm	1	8	● ● ● ● ● ● ● ●
40	4mm1'	11'	8	● ● ● ● ○ ○ ○ ○
583	4mm1'	1	16	◐ ◐ ◐ ◐ ● ● ● ●

Genus #41 4̄2m > 4̄

| 1191 | 4̄2'm' | 4̄ | 2 | ○ ○ ● ○ ● ○ ○ ○ |

95

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
822	$\bar{4}2'm'$	$\bar{4}$	2	○	○	●	●	○	○	●	○
1431	$\bar{4}2m$	$\bar{4}$	2	○	○	●	●	○	○	○	●
825	$\bar{4}'2'm$	$\bar{4}$	2	○	○	●	●	○	○	○	●
41	$\bar{4}2m1'$	$\bar{4}1'$	2	○	○	●	●	○	○	○	○
585	$\bar{4}2m1'$	$\bar{4}$	4	○	○	◐	◐	○	◐	○	◐
272	$\bar{4}2m1'$	$\bar{4}'$	4	○	○	◐	○	○	○	◐	◐

Genus #42 $\bar{4}2m > 2_|mm$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1432	$\bar{4}2m$	$2_|mm$	2	●	●	○	○	●	●	○	○
1187	$\bar{4}'2'm$	$2_|mm$	2	●	●	○	○	●	○	○	○
823	$\bar{4}'2'm$	$2_|m'm'$	2	●	●	○	○	○	○	●	●
828	$\bar{4}2'm'$	$2_|m'm'$	2	●	●	○	○	○	○	●	●
42	$\bar{4}2m1'$	$2_|mm1'$	2	●	●	○	○	○	○	○	○
586	$\bar{4}2m1'$	$2_|mm$	4	◐	◐	○	○	●	◐	○	○
273	$\bar{4}2m1'$	$2'_|mm'$	4	◐	◐	○	○	○	◐	●	○
274	$\bar{4}2m1'$	$2_|m'm'$	4	◐	◐	○	○	○	○	◐	●

Genus #43 $\bar{4}2m > 222$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1433	$\bar{4}2m$	222	2	●	○	○	●	●	○	○	●
1183	$\bar{4}'2'm$	222	2	●	○	○	●	●	○	○	●
829	$\bar{4}2'm'$	$2_|2'2'$	2	●	○	○	●	○	●	○	○
826	$\bar{4}'2'm$	$2_|2'2'$	2	●	○	○	●	○	○	●	○
43	$\bar{4}2m1'$	$2221'$	2	●	○	○	●	○	○	○	○
587	$\bar{4}2m1'$	222	4	◐	○	○	◐	●	○	○	●
276	$\bar{4}2m1'$	$2_|2'2'$	4	◐	○	○	◐	○	●	●	○
275	$\bar{4}2m1'$	$2_|2'2'$	4	◐	○	○	◐	○	○	◐	○

Genus #44 $\bar{4}2m > m_-$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1434	$\bar{4}2m$	m_-	4	●	●	●	○	●	●	●	○
1188	$\bar{4}'2'm$	m_-	4	●	●	●	○	●	●	●	○
830	$\bar{4}2'm'$	m'_-	4	●	●	●	○	●	●	●	○
824	$\bar{4}'2'm$	m'_-	4	●	●	●	○	●	●	●	○
44	$\bar{4}2m1'$	m_-1'	4	●	●	●	○	○	○	○	○
588	$\bar{4}2m1'$	m_-	8	◐	◐	◐	○	●	●	●	○
277	$\bar{4}2m1'$	m'_-	8	◐	◐	◐	○	○	◐	◐	●

Genus #45 $\bar{4}2m > 2_|$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1435	$\bar{4}2m$	$2_|$	4	●	◐	○	●	●	◐	○	●
1184	$\bar{4}'2'm$	$2_|$	4	●	◐	○	●	●	◐	○	●
1192	$\bar{4}2'm'$	$2_|$	4	●	◐	○	●	●	◐	○	●
1189	$\bar{4}'2'm$	$2_|$	4	●	◐	○	●	○	●	◐	●
45	$\bar{4}2m1'$	$2_|1'$	4	●	◐	○	●	○	○	○	○
589	$\bar{4}2m1'$	$2_|$	8	◐	◐	◐	◐	●	◐	○	●
278	$\bar{4}2m1'$	$2'_|$	8	◐	◐	◐	◐	○	●	●	○

Genus #46 $\bar{4}2m > 2_-$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1436	$\bar{4}2m$	2_-	4	●	●	●	●	●	●	●	●
1185	$\bar{4}'2'm$	2_-	4	●	●	●	●	●	●	●	●
831	$\bar{4}2'm'$	$2'_-$	4	●	●	●	○	●	●	●	○
827	$\bar{4}'2'm$	$2'_-$	4	●	●	●	○	●	●	●	○
46	$\bar{4}2m1'$	$2_-1'$	4	●	●	●	●	○	○	○	○
590	$\bar{4}2m1'$	2_-	8	◐	◐	◐	◐	●	●	●	●
279	$\bar{4}2m1'$	$2'_-$	8	◐	◐	◐	◐	○	●	●	○

Genus #47 $\bar{4}2m > 1$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1437	$\bar{4}2m$	1	8	●	●	●	●	●	●	●	●

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1193	$\bar{4}2'm'$	1	8	●	●	●	●	●	●	●	●
1186	$\bar{4}'2'm$	1	8	●	●	●	●	●	●	●	●
1190	$\bar{4}'2'm$	1	8	●	●	●	●	●	●	●	●
47	$\bar{4}2m1'$	$11'$	8	●	●	●	●	○	○	○	○
591	$\bar{4}2m1'$	1	16	◐	◐	◐	◐	●	●	●	●

Genus #48 $4/mmm > \bar{4}2m$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1226	$4'/m'm'm$	$\bar{4}2m$	2	○	○	○	○	●	○	○	○
593	$4/mmm1'$	$\bar{4}2m$	4	○	○	○	○	◐	○	○	○
832	$4'/mmm'$	$\bar{4}'2'm$	2	○	○	○	○	●	○	○	○
286	$4/mmm1'$	$\bar{4}'2'm$	4	○	○	○	○	◐	○	○	○
285	$4/mmm1'$	$\bar{4}'2m'$	4	○	○	○	○	○	◐	○	○
880	$4'/m'm'm$	$\bar{4}2'm'$	2	○	○	○	○	○	●	○	○
833	$4'/mmm'$	$\bar{4}'2m'$	2	○	○	○	○	○	○	●	○
287	$4/mmm1'$	$\bar{4}'2m'$	4	○	○	○	○	○	○	○	◐
48	$4/mmm1'$	$\bar{4}2m1'$	2	○	○	○	○	○	○	○	○
856	$4'/m'm'm$	$\bar{4}'2m'$	2	○	○	○	○	○	○	○	○
868	$4/m'mm$	$\bar{4}'2'm$	2	○	○	○	○	○	○	○	○
846	$4/mm'm'$	$\bar{4}2'm'$	2	○	○	○	○	○	○	○	○
1438	$4/mmm$	$\bar{4}2m$	2	○	○	○	○	○	○	○	○

Genus #49 $4/mmm > 4mm$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1220	$4/m'mm$	$4mm$	2	○	●	○	○	○	●	○	○
1439	$4/mmm$	$4mm$	2	○	●	○	○	○	●	○	○
857	$4/m'm'm'$	$4m'm'$	2	○	●	○	○	○	●	○	○
847	$4/mm'm'$	$4m'm'$	2	○	●	○	○	○	○	○	●
49	$4/mmm1'$	$4mm1'$	2	○	●	○	○	○	○	○	○
834	$4'/mmm'$	$4'mm'$	2	○	●	○	○	○	●	○	○
881	$4'/m'm'm$	$4'mm'$	2	○	●	○	○	○	●	○	○
594	$4/mmm1'$	$4mm$	4	○	◐	○	○	○	◐	○	○
288	$4/mmm1'$	$4m'm'$	4	○	◐	○	○	○	○	◐	◐
289	$4/mmm1'$	$4'mm'$	4	○	◐	○	○	○	○	○	○

Genus #50 $4/mmm > 422$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1214	$4/m'm'm'$	422	2	○	○	○	●	●	○	○	○
848	$4/mm'm'$	$42'2'$	2	○	○	○	●	○	●	○	○
869	$4/m'mm$	$42'2'$	2	○	○	○	●	○	●	○	○
1440	$4/mmm$	422	2	○	○	○	●	○	○	●	○
50	$4/mmm1'$	$4221'$	2	○	○	○	●	○	○	○	○
835	$4'/mmm'$	$4'22'$	2	●	○	○	●	○	○	●	○
882	$4'/m'm'm$	$4'22'$	2	○	○	○	●	○	●	○	○
595	$4/mmm1'$	422	4	○	○	○	◐	◐	○	○	◐
291	$4/mmm1'$	$42'2'$	4	○	○	○	◐	○	◐	◐	○
290	$4/mmm1'$	$4'22'$	4	○	○	○	◐	○	○	○	○

Genus #51 $4/mmm > 4/m$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1206	$4/mm'm'$	$4/m$	2	○	○	●	○	○	●	○	○
858	$4/m'm'm'$	$4/m'$	2	○	○	●	○	○	●	○	○
1441	$4/mmm$	$4/m$	2	○	○	●	○	○	○	●	○
870	$4/m'mm$	$4/m'$	2	○	○	●	○	○	○	○	●
51	$4/mmm1'$	$4/m1'$	2	○	○	●	○	○	○	○	○
836	$4'/mmm'$	$4'/m$	2	○	○	●	○	○	○	○	○
883	$4'/m'm'm$	$4'/m'$	2	○	○	●	○	○	○	○	○
596	$4/mmm1'$	$4/m$	4	○	○	◐	○	◐	○	◐	○

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
293	$4/mmm1'$	$4/m'$	4	○	○	◐	○	○	◐	○	◐
294	$4/mmm1'$	$4'/m$	4	○	○	◐	○	○	○	◐	○
292	$4/mmm1'$	$4'/m'$	4	○	○	◐	○	○	○	○	○

Genus #52 $4/mmm > \bar{4}$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
597	$4/mmm1'$	$\bar{4}$	8	○	○	◐	○	◐	○	◐	○
1227	$4'/m'm'm$	$\bar{4}$	4	○	○	◐	○	●	○	◐	○
1207	$4/mm'm'$	$\bar{4}$	4	○	○	◐	○	○	●	◐	○
837	$4'/mmm'$	$\bar{4}'$	4	○	○	◐	○	○	●	○	◐
295	$4/mmm1'$	$\bar{4}'$	8	○	○	◐	○	○	◐	○	◐
859	$4/m'm'm'$	$\bar{4}'$	4	○	○	◐	○	○	●	○	○
1442	$4/mmm$	$\bar{4}$	4	○	○	◐	○	○	◐	○	○
871	$4/m'mm$	$\bar{4}'$	4	○	○	◐	○	○	○	○	◐
52	$4/mmm1'$	$\bar{4}1'$	4	○	○	◐	○	○	○	○	○

Genus #53 $4/mmm > 4$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
598	$4/mmm1'$	4	8	○	◐	◐	◐	◐	◐	◐	◐
1215	$4/m'm'm'$	4	4	○	◐	◐	◐	●	◐	◐	○
1208	$4/mm'm'$	4	4	○	◐	◐	◐	●	●	◐	○
1221	$4/m'mm$	4	4	○	◐	◐	◐	○	◐	◐	◐
1443	$4/mmm$	4	4	○	◐	◐	◐	○	◐	◐	◐
53	$4/mmm1'$	$41'$	4	○	◐	◐	◐	○	○	○	○
296	$4/mmm1'$	$4'$	8	○	◐	◐	◐	○	◐	◐	◐
838	$4'/mmm'$	$4'$	4	○	◐	◐	◐	○	●	◐	○
884	$4'/m'm'm$	$4'$	4	○	◐	◐	◐	○	○	◐	○

Genus #54 $4/mmm > mmm$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1444	$4/mmm$	mmm	2	●	○	○	○	●	○	○	○
1194	$4'/mmm'$	mmm	2	●	○	○	○	●	○	○	○
885	$4'/m'm'm$	$m'_|mm$	2	●	○	○	○	○	●	○	○
839	$4'/mmm'$	$m_|m'm'$	2	●	○	○	○	○	○	●	○
860	$4/m'm'm'$	$m'm'm'$	2	●	○	○	○	○	○	○	●
886	$4'/m'm'm$	$m'm'm'$	2	●	○	○	○	○	○	○	●
54	$4/mmm1'$	$mmm1'$	2	●	○	○	○	○	○	○	○
849	$4/mm'm'$	$m_|m'm'$	2	●	○	○	○	○	○	○	○
872	$4/m'mm$	$m'_|mm$	2	●	○	○	○	○	○	○	○
599	$4/mmm1'$	mmm	4	◐	○	○	○	●	○	○	○
298	$4/mmm1'$	$m'_|mm$	4	◐	○	○	○	○	●	○	○
297	$4/mmm1'$	$m'_|mm$	4	◐	○	○	○	○	◐	○	○
301	$4/mmm1'$	$m_|m'm'$	4	◐	○	○	○	○	○	●	○
300	$4/mmm1'$	$m_|m'm'$	4	◐	○	○	○	○	○	◐	○
299	$4/mmm1'$	$m'm'm'$	4	◐	○	○	○	○	○	○	●

Genus #55 $4/mmm > 2_|mm$

No.	G	F	n	↕	↑	⊥	⯑	↕	↑	⊥	⯑
1228	$4'/m'm'm$	$2_|mm$	4	◐	◐	○	○	●	◐	○	○
1222	$4/m'mm$	$2_|mm$	4	◐	◐	○	○	●	◐	○	○
1195	$4'/mmm'$	$2_|mm$	4	◐	◐	○	○	●	◐	○	○
1445	$4/mmm$	$2_|mm$	4	◐	◐	○	○	◐	◐	○	○
600	$4/mmm1'$	$2_|mm$	8	◐	◐	○	○	◐	◐	○	○
302	$4/mmm1'$	$2'_|mm'$	8	◐	◐	○	○	○	◐	◐	○
841	$4'/mmm'$	$2_|m'm'$	4	◐	◐	○	○	○	○	◐	●
861	$4/m'm'm'$	$2_|m'm'$	4	◐	◐	○	○	○	○	○	◐
305	$4/mmm1'$	$2_|m'm'$	8	◐	◐	○	○	○	○	◐	◐
888	$4'/m'm'm$	$2_|m'm'$	4	◐	◐	○	○	○	○	◐	◐

Column 1

No.	G	F	n
851	$4/mm'm'$	$2_|m'm'$	4
55	$4/mmm1'$	$2_|mm1'$	4

Genus #56 $4/mmm > 2\_mm$

No.	G	F	n
1446	$4/mmm$	2_mm	4
1196	$4'/mmm'$	2_mm	4
840	$4'/mmm'$	$2'\_m_|m'$	4
850	$4/mm'm'$	$2'\_m_|m'$	4
887	$4'/m'm'm$	$2'\_m\_m'$	4
862	$4/m'm'm'$	$2\_m'm'$	4
889	$4'/m'm'm$	$2\_m'm'$	4
873	$4/m'mm$	$2'\_m\_m'$	4
56	$4/mmm1'$	$2\_mm1'$	4
601	$4/mmm1'$	2_mm	8
304	$4/mmm1'$	$2'\_m_|m'$	8
303	$4/mmm1'$	$2'\_m\_m'$	8
306	$4/mmm1'$	$2\_m'm'$	8

Genus #57 $4/mmm > 222$

No.	G	F	n
1229	$4'/m'm'm$	222	4
1216	$4/m'm'm'$	222	4
1197	$4'/mmm'$	222	4
1447	$4/mmm$	222	4
890	$4'/m'm'm$	$2_|2'2'$	4
842	$4'/mmm'$	$2_|2'2'$	4
852	$4/mm'm'$	$2_|2'2'$	4
874	$4/m'mm$	$2_|2'2'$	4
57	$4/mmm1'$	$2221'$	4
602	$4/mmm1'$	222	8
307	$4/mmm1'$	$2_|2'2'$	8
308	$4/mmm1'$	$2\_2'2'$	8

Genus #58 $4/mmm > 2/m_|$

No.	G	F	n
1198	$4'/mmm'$	$2/m_|$	4
1448	$4/mmm$	$2/m_|$	4
1209	$4/mm'm'$	$2/m_|$	4
863	$4/m'm'm'$	$2/m'_|$	4
892	$4'/m'm'm$	$2/m'_|$	4
876	$4/m'mm$	$2/m'_|$	4
58	$4/mmm1'$	$2/m_|1'$	4
603	$4/mmm1'$	$2/m_|$	8
309	$4/mmm1'$	$2'/m_|$	8
311	$4/mmm1'$	$2/m'_|$	8
313	$4/mmm1'$	$2'/m'_|$	8

Genus #59 $4/mmm > 2/m\_$

No.	G	F	n
1199	$4'/mmm'$	$2/m\_$	4
1449	$4/mmm$	$2/m\_$	4
864	$4/m'm'm'$	$2/m\_$	4
893	$4'/m'm'm$	$2/m\_$	4
891	$4'/m'm'm$	$2'/m\_$	4
875	$4/m'mm$	$2'/m\_$	4
853	$4/mm'm'$	$2'/m'\_$	4
843	$4'/mmm'$	$2'/m'\_$	4

Column 2

No.	G	F	n
59	$4/mmm1'$	$2/m\_1'$	4
604	$4/mmm1'$	$2/m\_$	8
310	$4/mmm1'$	$2'/m\_$	8
312	$4/mmm1'$	$2/m'\_$	8
314	$4/mmm1'$	$2'/m'\_$	8

Genus #60 $4/mmm > m_|$

No.	G	F	n
1200	$4'/mmm'$	$m_|$	8
1450	$4/mmm$	$m_|$	8
1210	$4/mm'm'$	$m_|$	8
865	$4/m'm'm'$	$m'_|$	8
894	$4'/m'm'm$	$m'_|$	8
877	$4/m'mm$	$m'_|$	8
60	$4/mmm1'$	$m_|1'$	8
605	$4/mmm1'$	$m_|$	16
315	$4/mmm1'$	$m'_|$	16

Genus #61 $4/mmm > m\_$

No.	G	F	n
1223	$4/m'mm$	$m\_$	8
1230	$4'/m'm'm$	$m\_$	8
1451	$4/mmm$	$m\_$	8
1201	$4'/mmm'$	$m\_$	8
866	$4/m'm'm'$	$m'\_$	8
895	$4'/m'm'm$	$m'\_$	8
854	$4/mm'm'$	$m'\_$	8
844	$4'/mmm'$	$m'\_$	8
61	$4/mmm1'$	m_1'	8
606	$4/mmm1'$	$m\_$	16
316	$4/mmm1'$	$m'\_$	16

Genus #62 $4/mmm > 2_|$

No.	G	F	n
1231	$4'/m'm'm$	$2_|$	8
1217	$4/m'm'm'$	$2_|$	8
1224	$4/m'mm$	$2_|$	8
1452	$4/mmm$	$2_|$	8
1202	$4'/mmm'$	$2_|$	8
1211	$4/mm'm'$	$2_|$	8
62	$4/mmm1'$	$2_|1'$	8
607	$4/mmm1'$	$2_|$	16
317	$4/mmm1'$	$2'_|$	16

Genus #63 $4/mmm > 2\_$

No.	G	F	n
1232	$4'/m'm'm$	$2\_$	8
1218	$4/m'm'm'$	$2\_$	8
1453	$4/mmm$	$2\_$	8
1203	$4'/mmm'$	$2\_$	8
845	$4'/mmm'$	$2'\_$	8
855	$4/mm'm'$	$2'\_$	8
878	$4/m'mm$	$2'\_$	8
896	$4'/m'm'm$	$2'\_$	8
63	$4/mmm1'$	$2\_1'$	8
608	$4/mmm1'$	$2\_$	16
318	$4/mmm1'$	$2'\_$	16

Genus #64 $4/mmm > \bar{1}$

Column 3

No.	G	F	n
1204	$4'/mmm'$	$\bar{1}$	8
1212	$4/mm'm'$	$\bar{1}$	8
1454	$4/mmm$	$\bar{1}$	8
897	$4'/m'm'm$	$\bar{1}'$	8
879	$4/m'mm$	$\bar{1}'$	8
867	$4/m'm'm'$	$\bar{1}'$	8
64	$4/mmm1'$	$\bar{1}1'$	8
609	$4/mmm1'$	$\bar{1}$	16
319	$4/mmm1'$	$\bar{1}'$	16

Genus #65 $4/mmm > 1$

No.	G	F	n
1225	$4/m'mm$	1	16
1219	$4/m'm'm'$	1	16
1233	$4'/m'm'm$	1	16
1213	$4/mm'm'$	1	16
1205	$4'/mmm'$	1	16
1455	$4/mmm$	1	16
65	$4/mmm1'$	$11'$	16
610	$4/mmm1'$	1	32

Genus #66 $3 > 1$

No.	G	F	n
1456	3	1	3
66	$31'$	$11'$	3
612	$31'$	1	6

Genus #67 $\bar{3} > 3$

No.	G	F	n
1234	$\bar{3}'$	3	2
1457	$\bar{3}$	3	2
67	$\bar{3}1'$	$31'$	2
614	$\bar{3}1'$	3	4

Genus #68 $\bar{3} > \bar{1}$

No.	G	F	n
1458	$\bar{3}$	$\bar{1}$	3
898	$\bar{3}'$	$\bar{1}'$	3
68	$\bar{3}1'$	$\bar{1}1'$	3
615	$\bar{3}1'$	$\bar{1}$	6
321	$\bar{3}1'$	$\bar{1}'$	6

Genus #69 $\bar{3} > 1$

No.	G	F	n
1235	$\bar{3}'$	1	6
1459	$\bar{3}$	1	6
69	$\bar{3}1'$	$11'$	6
616	$\bar{3}1'$	1	12

Genus #70 $32 > 3$

No.	G	F	n
1236	$32'$	3	2
1460	32	3	2
70	$321'$	$31'$	2
618	$321'$	3	4

Genus #71 $32 > 2\_$

No.	G	F	n
1461	32	$2\_$	3
899	$32'$	$2'\_$	3
71	$321'$	$2\_1'$	3
619	$321'$	$2\_$	6
323	$321'$	$2'\_$	6

Genus #72 $32 > 1$

Column 1

No.	G	F	n	↕	↑	⟊	⚔	↕	↑	⟊	⚔
1462	32	1	6	●	●	●	●	●	●	●	●
1237	32′	1	6	●	●	●	●	●	●	●	●
72	321′	11′	6	●	●	●	●	○	○	○	○
620	321′	1	12	◐	◐	◐	◐	●	●	●	●

Genus #73   3m > 3

No.	G	F	n								
1238	3m′	3	2	○	○	●	●	●	●	○	○
1463	3m	3	2	○	○	●	●	○	●	●	●
73	3m1′	31′	2	○	○	●	●	○	○	○	○
622	3m1′	3	4	○	○	◐	◐	◐	◐	◐	◐

Genus #74   3m > m_

No.	G	F	n								
1464	3m	m_	3	●	●	●	○	●	●	●	○
900	3m′	m′	3	●	●	●	○	○	●	●	●
74	3m1′	m_1′	3	●	●	●	○	○	○	○	○
623	3m1′	m_	6	◐	◐	◐	◐	●	●	●	●
325	3m1′	m′_	6	◐	◐	◐	○	○	●	●	●

Genus #75   3m > 1

No.	G	F	n								
1465	3m	1	6	●	●	●	●	●	●	●	●
1239	3m′	1	6	●	●	●	●	●	●	●	●
75	3m1′	11′	6	●	●	●	●	○	○	○	○
624	3m1′	1	12	◐	◐	◐	◐	●	●	●	●

Genus #76   3̄m > 3m

No.	G	F	n								
1248	3̄′m	3m	2	○	●	○	○	●	○	○	○
1466	3̄m	3m	2	○	●	○	○	○	●	○	○
906	3̄′m′	3m′	2	○	●	○	○	○	○	●	○
901	3̄′m′	3m′	2	○	●	○	○	○	○	○	●
76	3̄m1′	3m1′	2	○	●	○	○	○	○	○	○
626	3̄m1′	3m	4	○	◐	○	○	◐	◐	○	○
329	3̄m1′	3m′	4	○	◐	○	○	○	○	◐	◐

Genus #77   3̄m > 32

No.	G	F	n								
1244	3̄′m′	32	2	○	○	○	●	●	○	○	○
902	3̄′m′	32′	2	○	○	○	●	○	●	○	○
911	3̄′m	32′	2	○	○	○	●	○	○	●	○
1467	3̄m	32	2	○	○	○	●	○	○	○	●
77	3̄m1′	321′	2	○	○	○	●	○	○	○	○
627	3̄m1′	32	4	○	○	○	◐	◐	○	○	◐
330	3̄m1′	32′	4	○	○	○	◐	○	◐	◐	○

Genus #78   3̄m > 3̄

No.	G	F	n								
1240	3̄′m′	3̄	2	○	○	●	○	●	○	○	○
907	3̄′m′	3̄′	2	○	○	●	○	○	●	○	○
1468	3̄m	3̄	2	○	○	●	○	○	○	●	○
912	3̄′m	3̄′	2	○	○	●	○	○	○	○	●
78	3̄m1′	3̄1′	2	○	○	●	○	○	○	○	○
628	3̄m1′	3̄	4	○	○	◐	○	◐	○	◐	○
331	3̄m1′	3̄′	4	○	○	◐	○	○	◐	○	◐

Genus #79   3̄m > 3

No.	G	F	n								
629	3̄m1′	3	8	○	◐	◐	◐	◐	◐	◐	◐
1245	3̄′m′	3	4	○	◐	◐	◐	◐	○	◐	○
1241	3̄′m′	3	4	○	◐	◐	◐	○	◐	○	◐
1249	3̄′m	3	4	○	◐	◐	◐	◐	○	◐	○
1469	3̄m	3	4	○	◐	◐	◐	○	◐	○	◐

Column 2

No.	G	F	n								
79	3̄m1′	31′	4	○	◐	●	◐	○	○	○	○

Genus #80   3̄m > 2/m_

No.	G	F	n								
1470	3̄m	2/m_	3	●	○	●	○	●	○	●	○
908	3̄′m′	2/m′_	3	●	○	●	○	○	●	○	●
913	3̄′m	2′/m_	3	●	○	●	○	○	●	○	●
903	3̄′m′	2′/m′_	3	●	○	●	○	○	○	●	○
80	3̄m1′	2/m_1′	3	●	○	●	○	○	○	○	○
630	3̄m1′	2/m_	6	◐	○	◐	○	●	○	●	○
333	3̄m1′	2/m′_	6	◐	○	◐	○	○	●	●	○
332	3̄m1′	2′/m_	6	◐	○	◐	○	○	●	○	○
334	3̄m1′	2′/m′_	6	◐	○	◐	○	○	○	●	○

Genus #81   3̄m > m_

No.	G	F	n								
1250	3̄′m	m_	6	◐	●	●	○	○	●	◐	●
1471	3̄m	m_	6	●	●	○	○	◐	●	◐	○
904	3̄′m′	m′	6	◐	●	◐	○	○	●	◐	●
909	3̄′m′	m′	6	◐	●	◐	○	○	◐	◐	●
81	3̄m1′	m_1′	6	●	●	◐	○	○	○	○	○
631	3̄m1′	m_	12	◐	○	◐	○	◐	○	◐	○
335	3̄m1′	m′_	12	◐	○	◐	○	○	○	◐	○

Genus #82   3̄m > 2_

No.	G	F	n								
1246	3̄′m′	2_	6	◐	●	◐	●	●	●	◐	●
1472	3̄m	2_	6	◐	●	◐	●	◐	●	◐	○
905	3̄′m′	2′_	6	◐	●	◐	●	○	●	◐	○
914	3̄′m	2′_	6	◐	●	◐	●	○	◐	●	○
82	3̄m1′	2_1′	6	◐	●	◐	●	○	○	○	○
632	3̄m1′	2_	12	◐	○	◐	◐	◐	◐	◐	◐
336	3̄m1′	2′_	12	◐	○	◐	◐	◐	○	◐	○

Genus #83   3̄m > 1̄

No.	G	F	n								
1242	3̄m′	1̄	6	●	○	●	○	●	○	●	○
1473	3̄m	1̄	6	●	○	●	○	○	●	○	●
910	3̄′m′	1̄′	6	●	○	●	○	○	●	●	●
915	3̄′m	1̄′	6	●	○	●	○	●	○	●	●
83	3̄m1′	1̄11′	6	●	○	●	○	○	○	○	○
633	3̄m1′	1̄	12	◐	○	◐	○	●	○	●	●
337	3̄m1′	1̄′	12	◐	○	◐	○	○	●	○	●

Genus #84   3̄m > 1

No.	G	F	n								
1247	3̄m′	1	12	◐	●	◐	●	●	◐	●	◐
1251	3̄′m	1	12	◐	●	◐	●	●	●	◐	●
1243	3̄m′	1	12	◐	●	◐	●	●	●	◐	●
1474	3̄m	1	12	◐	●	◐	●	◐	●	◐	●
84	3̄m1′	11′	12	◐	●	◐	●	○	○	○	○
634	3̄m1′	1	24	◐	◐	◐	◐	◐	◐	◐	◐

Genus #85   6 > 3

No.	G	F	n								
1252	6′	3	2	○	○	○	○	●	●	●	●
636	61′	3	4	○	○	○	○	◐	◐	◐	◐
85	61′	31′	2	○	○	○	○	●	●	○	○
1475	6	3	2	○	○	○	○	●	○	●	○

Genus #86   6 > 2_|

No.	G	F	n								
1476	6	2_|	3	●	○	○	●	●	○	○	●
916	6′	2′_|	3	●	○	○	●	○	●	●	○

Column 3

No.	G	F	n								
86	61′	2_|1′	3	●	○	○	●	○	○	○	○
637	61′	2_|	6	◐	○	○	◐	●	◐	◐	●
339	61′	2′_|	6	◐	○	○	◐	○	●	●	○

Genus #87   6 > 1

No.	G	F	n								
1253	6′	1	6	●	●	●	●	●	●	●	●
1477	6	1	6	●	●	●	●	●	●	●	●
87	61′	11′	6	●	●	●	●	○	○	○	○
638	61′	1	12	◐	◐	◐	◐	●	●	●	●

Genus #88   6̄ > 3

No.	G	F	n								
1254	6̄′	3	2	○	●	○	●	●	○	●	○
1478	6̄	3	2	○	●	○	●	○	●	○	●
88	6̄1′	31′	2	○	●	○	●	●	○	○	○
640	6̄1′	3	4	○	◐	○	◐	○	◐	○	◐

Genus #89   6̄ > m_|

No.	G	F	n								
1479	6̄	m_|	3	●	●	○	○	●	●	○	○
917	6̄′	m′_|	3	●	●	○	○	○	○	●	●
89	6̄1′	m_|1′	3	●	●	○	○	○	○	○	○
641	6̄1′	m_|	6	◐	◐	○	○	●	●	◐	◐
341	6̄1′	m′_|	6	◐	◐	○	○	○	○	●	●

Genus #90   6̄ > 1

No.	G	F	n								
1255	6̄′	1	6	●	●	●	●	●	●	●	●
1480	6̄	1	6	●	●	●	●	●	●	●	●
90	6̄1′	11′	6	●	●	●	●	○	○	○	○
642	6̄1′	1	12	◐	◐	◐	◐	●	●	●	●

Genus #91   6/m > 6̄

No.	G	F	n								
1264	6′/m	6̄	2	○	○	○	○	●	○	●	○
644	6/m1′	6̄	4	○	○	○	○	◐	○	◐	◐
918	6′/m′	6̄′	2	○	○	○	○	○	●	●	●
345	6/m1′	6̄′	4	○	○	○	○	○	◐	○	◐
91	6/m1′	6̄1′	2	○	○	○	○	●	○	○	○
923	6′/m′	6̄′	2	○	○	○	○	○	●	●	●
1481	6/m	6̄	2	○	○	○	○	○	●	○	●

Genus #92   6/m > 6

No.	G	F	n								
1260	6/m′	6	2	○	●	○	●	●	○	●	○
1482	6/m	6	2	○	●	○	●	○	●	○	●
92	6/m1′	61′	2	○	●	○	●	○	○	○	○
928	6′/m	6′	2	○	●	○	●	○	○	○	○
919	6′/m′	6′	2	○	●	○	●	○	○	○	○
645	6/m1′	6	4	○	◐	○	◐	◐	○	◐	○
346	6/m1′	6′	4	○	◐	○	◐	○	○	○	○

Genus #93   6/m > 3̄

No.	G	F	n								
1256	6′/m′	3̄	2	○	○	○	○	●	○	●	○
646	6/m1′	3̄	4	○	○	○	○	◐	○	◐	◐
929	6′/m	3̄′	2	○	○	○	○	○	●	○	●
347	6/m1′	3̄′	4	○	○	○	○	◐	○	◐	○
93	6/m1′	3̄1′	2	○	○	○	○	●	○	○	○
924	6′/m′	3̄′	2	○	○	○	○	○	●	○	●
1483	6/m	3̄	2	○	○	○	○	○	○	○	○

Genus #94   6/m > 3

No.	G	F	n								
647	6/m1′	3	8	○	◐	○	◐	◐	◐	◐	◐

Left column

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1257	6′/m′	3	4	○	◑	○	○	◑	◑	◑	◑
1265	6′/m	3	4	○	◑	○	○	◑	◑	◑	◑
1261	6/m′	3	4	○	◑	○	○	◑	◑	○	○
1484	6/m	3	4	○	◑	○	○	◑	○	◑	○
94	6/m1′	31′	4	○	◑	○	○	◑	○	○	○

Genus #95   6/m > 2/m|

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1485	6/m	2/m|	3	●	○	○	○	●	○	○	○
930	6′/m	2′/m|	3	●	○	○	○	○	●	○	○
920	6′/m′	2′/m′|	3	●	○	○	○	○	○	●	○
925	6/m′	2/m′|	3	●	○	○	○	○	○	○	●
95	6/m1′	2/m|1′	3	●	○	○	○	○	○	○	○
648	6/m1′	2′/m|	6	◑	○	○	○	●	○	○	○
348	6/m1′	2′/m|	6	◑	○	○	○	○	●	○	○
349	6/m1′	2/m′|	6	◑	○	○	○	○	○	◑	●
350	6/m1′	2′/m′|	6	◑	○	○	○	○	○	●	○

Genus #96   6/m > m|

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1266	6′/m	m|	6	◑	●	○	○	●	◑	◑	○
1486	6/m	m|	6	◑	●	○	○	◑	●	◑	○
921	6′/m′	m′|	6	◑	●	○	○	○	◑	◑	●
926	6/m′	m′|	6	◑	●	○	○	○	○	●	◑
96	6/m1′	m|1′	6	◑	●	○	○	○	○	○	○
649	6/m1′	m|	12	◑	◑	○	○	◑	◑	◑	◑
351	6/m1′	m′|	12	◑	◑	○	○	○	○	◑	◑

Genus #97   6/m > 2|

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1262	6/m′	2|	6	◑	◑	○	●	●	○	○	○
1487	6/m	2|	6	◑	◑	○	●	◑	◑	○	●
922	6′/m′	2′|	6	◑	◑	○	●	○	●	○	◑
931	6′/m	2′|	6	◑	◑	○	●	○	◑	◑	●
97	6/m1′	2|1′	6	◑	◑	○	●	○	○	○	○
650	6/m1′	2|	12	◑	◑	○	○	◑	◑	◑	◑
352	6/m1′	2′|	12	◑	◑	○	◑	○	◑	◑	○

Genus #98   6/m > 1̄

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1488	6/m	1̄	6	●	○	●	○	○	●	○	●
1258	6′/m′	1̄	6	●	○	●	○	●	○	●	○
927	6/m′	1̄′	6	●	○	●	○	○	●	○	●
932	6′/m	1̄′	6	●	○	●	○	○	●	●	○
98	6/m1′	1̄1′	6	●	○	●	○	○	○	○	○
651	6/m1′	1̄	12	◑	○	◑	●	○	◑	●	○
353	6/m1′	1̄′	12	◑	○	◑	○	○	●	○	◑

Genus #99   6/m > 1

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1267	6′/m	1	12	◑	●	◑	●	◑	●	◑	◑
1263	6/m′	1	12	◑	●	◑	●	◑	●	●	◑
1259	6′/m′	1	12	◑	●	●	◑	◑	●	◑	◑
1489	6/m	1	12	◑	●	◑	●	◑	●	◑	◑
99	6/m1′	11′	12	◑	●	◑	●	○	○	○	○
652	6/m1′	1	24	◑	◑	◑	◑	◑	◑	◑	◑

Genus #100   622 > 6

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1272	62′2′	6	2	○	●	●	○	●	●	○	●
1490	622	6	2	○	●	●	○	○	●	●	○
100	6221′	61′	2	○	●	●	○	○	○	○	○

Middle column

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
933	6′22′	6′	2	○	●	●	○	○	○	○	○
654	6221′	6	4	○	◑	○	◑	◑	◑	◑	◑
356	6221′	6′	4	○	◑	◑	○	○	○	○	○

Genus #101   622 > 32

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1268	6′22′	32	2	○	○	○	○	●	○	○	●
655	6221′	32	4	○	○	○	○	◑	○	○	◑
934	6′22′	32′	2	○	○	○	○	○	●	●	○
357	6221′	32′	4	○	○	○	○	◑	◑	○	○
101	6221′	321′	2	○	○	○	○	○	○	○	○
938	62′2′	32′	2	○	○	○	○	○	●	●	○
1491	622	32	2	○	○	○	○	○	○	○	○

Genus #102   622 > 3

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1269	6′22′	3	4	○	◑	◑	○	◑	○	◑	◑
656	6221′	3	8	○	◑	◑	○	◑	◑	◑	◑
1273	62′2′	3	4	○	◑	◑	○	◑	○	◑	○
1492	622	3	4	○	◑	◑	○	○	◑	◑	○
102	6221′	31′	4	○	◑	◑	○	○	○	○	○

Genus #103   622 > 222

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1493	622	222	3	●	○	○	●	●	○	○	●
935	6′22′	2_2′2′	3	●	○	○	●	●	●	●	○
103	6221′	2221′	3	●	○	○	●	○	○	○	○
939	62′2′	2|2′2′	3	●	○	○	●	○	●	●	○
657	6221′	222	6	◑	○	○	◑	●	○	●	○
359	6221′	2_2′2′	6	◑	○	○	◑	○	●	●	○
358	6221′	2|2′2′	6	◑	○	○	◑	○	◑	◑	○

Genus #104   622 > 2|

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1494	622	2|	6	●	●	○	●	●	●	◑	◑
1274	62′2′	2|	6	●	◑	○	●	●	○	○	◑
936	6′22′	2′|	6	●	◑	○	●	●	○	●	○
104	6221′	2|1′	6	●	◑	○	●	○	○	○	○
658	6221′	2|	12	◑	◑	○	◑	●	●	◑	●
360	6221′	2′|	12	◑	◑	○	◑	◑	●	●	○

Genus #105   622 > 2_

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1495	622	2_	6	●	●	●	●	●	●	●	●
1270	6′22′	2_	6	●	●	●	●	●	●	◑	●
937	6′22′	2′_	6	●	●	●	●	○	●	●	●
940	62′2′	2′_	6	●	●	●	●	○	●	●	●
105	6221′	2_1′	6	●	●	●	●	○	○	○	○
659	6221′	2_	12	◑	◑	◑	◑	●	●	◑	●
361	6221′	2′_	12	◑	◑	◑	◑	○	●	●	◑

Genus #106   622 > 1

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1271	6′22′	1	12	●	●	●	●	●	●	●	●
1275	62′2′	1	12	●	●	●	●	●	●	●	●
1496	622	1	12	●	●	●	●	●	●	●	●
106	6221′	11′	12	●	●	●	●	○	○	○	○
660	6221′	1	24	◑	◑	◑	◑	●	●	●	●

Genus #107   6mm > 6

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1280	6m′m′	6	2	○	○	●	●	●	●	○	○
1497	6mm	6	2	○	○	●	●	○	○	●	●
107	6mm1′	61′	2	○	○	●	●	○	○	○	○

Right column

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
941	6′mm′	6′	2	○	○	●	●	○	○	○	○
662	6mm1′	6	4	○	○	◑	◑	◑	◑	◑	◑
364	6mm1′	6′	4	○	○	◑	◑	○	○	○	○

Genus #108   6mm > 3m

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1276	6′mm′	3m	2	○	○	○	○	●	●	○	○
663	6mm1′	3m	4	○	○	○	○	◑	◑	○	○
942	6′mm′	3m′	2	○	○	○	○	○	○	●	●
365	6mm1′	3m′	4	○	○	○	○	○	○	◑	◑
108	6mm1′	3m1′	2	○	○	○	○	○	○	○	○
946	6m′m′	3m′	2	○	○	○	○	○	○	●	●
1498	6mm	3m	2	○	○	○	○	○	○	○	○

Genus #109   6mm > 3

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
664	6mm1′	3	8	○	○	◑	◑	◑	◑	◑	◑
1277	6′mm′	3	4	○	○	◑	◑	◑	◑	◑	◑
1281	6m′m′	3	4	○	○	◑	◑	◑	◑	○	○
1499	6mm	3	4	○	○	◑	◑	○	◑	◑	○
109	6mm1′	31′	4	○	○	◑	◑	○	○	○	○

Genus #110   6mm > 2|mm

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1500	6mm	2|mm	3	●	○	○	○	●	○	○	○
943	6′mm′	2′|mm	3	●	○	○	○	●	●	○	○
947	6m′m′	2|m′m′	3	●	○	○	○	○	○	○	●
110	6mm1′	2|mm1′	3	●	○	○	○	○	○	○	○
665	6mm1′	2|mm	6	◑	○	○	○	●	○	◑	○
366	6mm1′	2′|mm	6	◑	○	○	○	○	●	●	○
367	6mm1′	2|m′m′	6	◑	○	○	○	○	○	◑	●

Genus #111   6mm > m_

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1501	6mm	m_	6	●	●	●	○	●	●	●	●
1278	6′mm′	m_	6	●	●	●	●	●	●	◑	○
948	6m′m′	m′	6	●	●	●	○	○	●	●	●
944	6′mm′	m′_	6	●	●	●	○	○	●	●	●
111	6mm1′	m_1′	6	●	●	●	○	○	○	○	○
666	6mm1′	m_	12	◑	◑	◑	○	●	●	◑	○
368	6mm1′	m′_	12	◑	◑	◑	○	○	◑	●	●

Genus #112   6mm > 2|

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1282	6m′m′	2|	6	●	○	◑	●	●	○	○	●
1502	6mm	2|	6	●	○	◑	●	●	○	◑	○
945	6′mm′	2′|	6	●	○	◑	●	○	●	●	●
112	6mm1′	2|1′	6	●	○	◑	●	○	○	○	○
667	6mm1′	2|	12	◑	○	◑	◑	●	◑	◑	●
369	6mm1′	2′|	12	◑	○	◑	◑	○	●	●	○

Genus #113   6mm > 1

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1283	6m′m′	1	12	●	●	●	●	●	●	●	●
1279	6′mm′	1	12	●	●	●	●	●	●	●	●
1503	6mm	1	12	●	●	●	●	●	●	●	●
113	6mm1′	11′	12	●	●	●	●	○	○	○	○
668	6mm1′	1	24	◑	◑	◑	◑	●	●	●	●

Genus #114   6̄m2 > 6̄

No.	G	F	n	↕	↑	⊹	⚲	↕	↑	⊹	⚲
1292	6̄m′2′	6̄	2	○	○	●	○	●	○	○	○
949	6̄′m′2	6̄′	2	○	○	●	○	○	●	○	○
1504	6̄m2	6̄	2	○	○	●	○	○	○	●	○

Column 1

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
954	6̄'m2'	6̄'	2	○	○	●	○	○	○	○	●
114	6̄m21'	6̄1'	2	○	○	●	○	○	○	○	○
670	6̄m21'	6̄	4	○	○	◐	○	◐	○	◐	○
373	6̄m21'	6̄'	4	○	○	◐	○	○	◐	○	◐

Genus #115   6̄m2 > 3m

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1288	6̄'m2'	3m	2	○	●	○	○	○	●	○	○
1505	6̄m2	3m	2	○	●	○	○	○	○	●	○
950	6̄'m2	3m'	2	○	●	○	○	○	○	●	○
959	6̄m'2'	3m'	2	○	●	○	○	○	○	○	●
115	6̄m21'	3m1'	2	○	●	○	○	○	○	○	○
671	6̄m21'	3m	4	○	◐	○	○	○	◐	◐	○
374	6̄m21'	3m'	4	○	◐	○	○	○	○	◐	◐

Genus #116   6̄m2 > 32

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1284	6̄'m'2	32	2	○	○	○	●	●	○	○	○
960	6̄m'2'	32'	2	○	○	○	●	○	●	○	○
955	6̄'m2'	32'	2	○	○	○	●	○	○	●	○
1506	6̄m2	32	2	○	○	○	●	○	○	○	●
116	6̄m21'	321'	2	○	○	○	●	○	○	○	○
672	6̄m21'	32	4	○	○	○	◐	◐	○	○	◐
375	6̄m21'	32'	4	○	○	○	◐	○	◐	○	◐

Genus #117   6̄m2 > 3

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
673	6̄m21'	3	8	○	◐	◐	◐	◐	◐	◐	◐
1285	6̄'m'2	3	4	○	◐	◐	◐	◐	◐	○	○
1293	6̄m'2'	3	4	○	◐	◐	◐	◐	○	○	◐
1289	6̄'m2'	3	4	○	◐	◐	◐	○	○	◐	◐
1507	6̄m2	3	4	○	◐	◐	◐	◐	○	◐	○
117	6̄m21'	31'	4	○	◐	◐	◐	○	○	○	○

Genus #118   6̄m2 > 2_mm

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1508	6̄m2	2_mm	3	●	●	○	○	●	●	○	○
961	6̄m'2'	2'm|m'	3	●	●	○	○	○	●	○	○
951	6̄'m2	2_m'm'	3	●	●	○	○	○	○	●	●
956	6̄'m2'	2'm_m'	3	●	●	○	○	○	○	●	○
118	6̄m21'	2_mm1'	3	●	●	○	○	○	○	○	○
674	6̄m21'	2_mm	6	◐	◐	○	○	○	●	●	○
377	6̄m21'	2'm|m'	6	◐	◐	○	○	○	○	●	◐
376	6̄m21'	2'm_m'	6	◐	◐	○	○	○	◐	●	○
378	6̄m21'	2_m'm'	6	◐	◐	○	○	○	○	●	●

Genus #119   6̄m2 > m|

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1509	6̄m2	m|	6	●	●	◐	○	○	●	●	◐
1294	6̄m'2'	m|	6	●	●	◐	○	○	●	●	○
952	6̄'m2	m|'	6	●	●	◐	○	○	○	◐	●
957	6̄'m2'	m|'	6	●	●	◐	○	○	○	●	●
119	6̄m21'	m|1'	6	●	●	◐	○	○	○	○	○
675	6̄m21'	m|	12	◐	◐	◐	○	○	●	●	◐
379	6̄m21'	m|'	12	◐	◐	◐	○	○	○	◐	●

Genus #120   6̄m2 > m_

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1510	6̄m2	m_	6	●	●	●	○	○	●	●	●
1290	6̄'m2'	m_	6	●	●	●	○	○	●	●	◐
953	6̄'m'2	m_'	6	●	●	●	○	○	●	●	●
962	6̄m'2'	m_'	6	●	●	●	○	○	◐	●	●

Column 2

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
120	6̄m21'	m_1'	6	●	●	●	○	○	○	○	○
676	6̄m21'	m_	12	◐	◐	◐	○	●	●	●	○
380	6̄m21'	m_'	12	◐	◐	◐	○	○	○	◐	●

Genus #121   6̄m2 > 2_

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1286	6̄'m'2	2_	6	●	◐	●	●	●	●	●	●
1511	6̄m2	2_	6	●	◐	●	●	●	◐	●	●
963	6̄m'2'	2'_	6	●	◐	●	●	○	●	●	○
958	6̄'m2'	2'_	6	●	◐	●	●	○	●	●	○
121	6̄m21'	2_1'	6	●	◐	●	●	○	○	○	○
677	6̄m21'	2_	12	◐	◐	◐	◐	●	◐	◐	●
381	6̄m21'	2'_	12	◐	◐	◐	◐	○	●	●	○

Genus #122   6̄m2 > 1

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1291	6̄'m2'	1	12	●	●	●	●	●	●	●	●
1287	6̄'m'2	1	12	●	●	●	●	●	●	●	●
1512	6̄m2	1	12	●	●	●	●	●	●	●	●
1295	6̄m'2'	1	12	●	●	●	●	●	●	●	●
122	6̄m21'	11'	12	●	●	●	●	○	○	○	○
678	6̄m21'	1	24	◐	◐	◐	◐	●	●	●	●

Genus #123   6/mmm > 6̄m2

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1332	6'/mmm'	6̄m2	2	○	○	○	○	●	○	○	○
680	6/mmm1'	6̄m2	4	○	○	○	○	◐	○	○	○
964	6'/m'mm	6̄m2'	2	○	○	○	○	○	●	○	○
388	6/mmm1'	6̄m2'	4	○	○	○	○	○	◐	○	○
1028	6'/mmm'	6̄m'2'	2	○	○	○	○	○	●	○	○
387	6/mmm1'	6̄m'2'	4	○	○	○	○	○	◐	○	○
965	6'/m'mm	6̄m'2	2	○	○	○	○	○	○	○	●
389	6/mmm1'	6̄m'2	4	○	○	○	○	○	○	○	◐
123	6/mmm1'	6̄m21'	2	○	○	○	○	○	○	○	○
998	6/m'm'm'	6̄'m2	2	○	○	○	○	○	○	○	○
1013	6/m'mm	6̄'m2	2	○	○	○	○	○	○	○	○
985	6/mm'm'	6̄'m2'	2	○	○	○	○	○	○	○	○
1513	6/mmm	6̄m2	2	○	○	○	○	○	○	○	○

Genus #124   6/mmm > 6mm

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1324	6/m'mm	6mm	2	○	●	○	○	●	○	○	○
1514	6/mmm	6mm	2	○	●	○	○	○	●	○	○
999	6/m'm'm'	6m'm'	2	●	○	○	○	○	●	○	○
986	6/mm'm'	6m'm'	2	○	●	○	○	○	○	○	●
124	6/mmm1'	6mm1'	2	○	●	○	○	○	○	○	○
1029	6'/mmm'	6'mm'	2	○	●	○	○	○	○	○	○
966	6'/m'mm	6'mm'	2	○	●	○	○	○	○	●	○
681	6/mmm1'	6mm	4	○	◐	○	○	○	◐	◐	○
390	6/mmm1'	6m'm'	4	○	◐	○	○	○	○	◐	◐
391	6/mmm1'	6'mm'	4	◐	◐	○	○	○	◐	○	○

Genus #125   6/mmm > 622

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1316	6/m'm'm'	622	2	○	○	○	●	●	○	○	○
987	6/mm'm'	62'2'	2	○	○	○	●	○	●	○	○
1014	6/m'mm	62'2'	2	○	○	○	●	○	○	●	○
1515	6/mmm	622	2	○	○	○	●	○	○	○	●
125	6/mmm1'	6221'	2	○	○	○	●	○	○	○	○
1030	6'/mmm'	6'22'	2	○	○	○	●	○	○	○	○

Column 3

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
967	6'/m'mm'	6'22'	2	○	○	○	●	○	○	○	○
682	6/mmm1'	622	4	○	○	○	◐	◐	○	○	◐
392	6/mmm1'	62'2'	4	○	○	○	◐	○	◐	◐	○
393	6/mmm1'	6'22'	4	○	○	○	◐	○	○	○	○

Genus #126   6/mmm > 6/m

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1306	6/mm'm'	6/m	2	○	○	○	●	○	●	○	○
1000	6/m'm'm'	6/m'	2	○	○	○	●	○	○	●	○
1516	6/mmm	6/m	2	○	○	○	●	○	○	●	○
1015	6/m'mm	6/m'	2	○	○	○	●	○	○	○	●
126	6/mmm1'	6/m1'	2	○	○	●	○	○	○	○	○
968	6'/m'mm'	6'/m'	2	○	○	●	○	○	○	○	○
1031	6'/mmm'	6'/m	2	○	○	●	○	○	○	○	○
683	6/mmm1'	6/m	4	○	○	◐	○	◐	○	◐	○
395	6/mmm1'	6/m'	4	○	○	◐	○	○	◐	○	◐
396	6/mmm1'	6'/m'	4	○	○	◐	○	○	◐	○	◐
394	6/mmm1'	6'/m	4	○	○	◐	○	○	○	○	○

Genus #127   6/mmm > 6̄

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1333	6'/mmm'	6̄	4	○	○	◐	○	◐	○	◐	○
684	6/mmm1'	6̄	8	○	○	◐	○	◐	○	◐	○
1307	6/mm'm'	6̄	4	○	○	◐	○	◐	○	◐	○
397	6/mmm1'	6̄'	8	○	○	◐	○	○	◐	○	◐
969	6'/m'mm'	6̄'	4	○	○	◐	○	○	◐	○	◐
1001	6/m'm'm'	6̄'	4	○	○	◐	○	○	◐	○	◐
1517	6/mmm	6̄	4	○	○	◐	○	○	◐	○	○
1016	6/m'mm	6̄'	4	○	○	◐	○	○	○	○	◐
127	6/mmm1'	6̄1'	4	○	○	◐	○	○	○	○	○

Genus #128   6/mmm > 6

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
685	6/mmm1'	6	8	○	◐	◐	◐	◐	◐	◐	◐
1317	6/m'm'm'	6	4	◐	◐	◐	◐	◐	◐	○	○
1308	6/mm'm'	6	4	○	◐	◐	◐	◐	◐	○	◐
1325	6/m'mm	6	4	○	◐	◐	◐	◐	○	◐	◐
1518	6/mmm	6	4	○	◐	◐	◐	○	◐	◐	◐
128	6/mmm1'	61'	4	○	◐	◐	◐	○	○	○	○
1032	6'/mmm'	6'	4	○	◐	◐	◐	○	○	○	○
970	6'/m'mm'	6'	4	○	◐	◐	◐	○	○	○	○
398	6/mmm1'	6'	8	○	◐	◐	◐	○	○	○	○

Genus #129   6/mmm > 3̄m

No.	G	F	n	↕	↑	⊬	⊰	↕	↑	⊬	⊰
1296	6'/m'mm'	3̄m	2	○	○	○	○	●	○	○	○
686	6/mmm1'	3̄m	4	○	○	○	○	◐	○	○	○
1033	6'/mmm'	3̄'m	2	○	○	○	○	○	●	○	○
399	6/mmm1'	3̄'m	4	○	○	○	○	○	◐	○	○
971	6'/m'mm'	3̄m'	2	○	○	○	○	○	○	●	○
401	6/mmm1'	3̄m'	4	○	○	○	○	○	○	◐	○
1034	6'/mmm'	3̄'m'	2	○	○	○	○	○	○	○	●
400	6/mmm1'	3̄'m'	4	○	○	○	○	○	○	○	◐
129	6/mmm1'	3̄m1'	2	○	○	○	○	○	○	○	○
988	6/mm'm'	3̄m'	2	○	○	○	○	○	○	○	○
1002	6/m'm'm'	3̄'m'	2	○	○	○	○	○	○	○	○
1017	6/m'mm	3̄'m	2	○	○	○	○	○	○	○	○
1519	6/mmm	3̄m	2	○	○	○	○	○	○	○	○

Genus #130   6/mmm > 3m

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1334	6'/mmm'	3m	4	○	◐	○	○	○	◐	◐	○
687	6/mmm1'	3m	8	○	◐	○	○	○	◐	◐	○
1297	6'/m'mm	3m	4	○	◐	○	○	○	◐	◐	○
1326	6/m'mm	3m	4	○	◐	○	○	○	◐	○	○
1520	6/mmm	3m	4	○	◐	○	○	○	○	◐	○
972	6'/m'mm'	3m'	4	○	◐	○	○	○	○	◐	◐
1035	6'/mmm'	3m'	4	○	◐	○	○	○	○	◐	◐
402	6/mmm1'	3m'	8	○	◐	○	○	○	○	◐	◐
1003	6/m'm'm'	3m'	4	○	◐	○	○	○	○	◐	○
989	6/mm'm'	3m'	4	○	◐	○	○	○	○	○	◐
130	6/mmm1'	3m1'	4	○	◐	○	○	○	○	○	○

Genus #131   6/mmm > 32

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1298	6'/m'mm'	32	4	○	○	○	◐	◐	○	○	◐
688	6/mmm1'	32	8	○	○	○	◐	◐	○	○	◐
1335	6'/mmm'	32	4	○	○	○	◐	◐	○	○	◐
1318	6/m'm'm'	32	4	○	○	○	◐	◐	○	○	◐
403	6/mmm1'	32'	8	○	○	○	◐	○	◐	◐	○
973	6'/m'mm'	32'	4	○	○	○	◐	○	◐	◐	○
1036	6'/mmm'	32'	4	○	○	○	◐	○	◐	◐	○
990	6/mm'm'	32'	4	○	○	○	◐	○	◐	○	○
1018	6/m'mm	32'	4	○	○	○	◐	○	◐	○	○
1521	6/mmm	32	4	○	○	○	◐	○	○	○	◐
131	6/mmm1'	321'	4	○	○	○	◐	○	○	○	○

Genus #132   6/mmm > 3̄

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1299	6'/m'mm'	3̄	4	○	○	◐	○	◐	○	◐	○
689	6/mmm1'	3̄	8	○	○	◐	○	◐	○	◐	○
1309	6/mm'm'	3̄	4	○	○	◐	○	◐	○	○	○
404	6/mmm1'	3̄'	8	○	○	◐	○	○	◐	◐	○
1037	6'/mmm'	3̄'	4	○	◐	○	○	○	◐	◐	○
1004	6/m'm'm'	3̄'	4	○	○	◐	○	○	◐	○	○
1522	6/mmm	3̄	4	○	○	◐	○	○	◐	○	○
1019	6/m'mm	3̄'	4	○	○	◐	○	○	○	○	◐
132	6/mmm1'	3̄1'	4	○	○	◐	○	○	○	○	○

Genus #133   6/mmm > 3

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
690	6/mmm1'	3	16	○	◐	◐	◐	◐	◐	◐	◐
1300	6'/m'mm'	3	8	○	◐	◐	◐	◐	◐	◐	◐
1336	6'/mmm'	3	8	○	◐	◐	◐	◐	◐	◐	◐
1319	6/m'm'm'	3	8	○	◐	◐	◐	◐	◐	◐	○
1310	6/mm'm'	3	8	○	◐	◐	◐	◐	◐	○	◐
1327	6/m'mm	3	8	○	◐	◐	◐	◐	○	◐	◐
1523	6/mmm	3	8	○	◐	◐	◐	○	◐	◐	◐
133	6/mmm1'	31'	8	○	◐	◐	◐	○	○	○	○

Genus #134   6/mmm > mmm

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1524	6/mmm	mmm	3	●	○	○	○	●	○	○	○
1038	6'/mmm'	m'mm	3	●	○	○	○	○	●	○	○
974	6'/m'mm'	m_m'm'	3	●	○	○	○	○	○	●	○
1005	6/m'm'm'	m'm'm'	3	●	○	○	○	○	○	○	●
134	6/mmm1'	mmm1'	3	●	○	○	○	○	○	○	○
991	6/mm'm'	m|m'm'	3	●	○	○	○	○	○	○	○

(Genus #134 continued)

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1020	6/m'mm	m|'mm	3	●	○	○	○	○	○	○	○
691	6/mmm1'	mmm	6	●	○	○	○	●	○	○	○
406	6/mmm1'	m'_mm	6	●	○	○	○	○	●	○	○
405	6/mmm1'	m|'mm	6	●	○	○	○	○	◐	○	○
409	6/mmm1'	m_m'm'	6	●	○	○	○	○	○	●	○
408	6/mmm1'	m|m'm'	6	●	○	○	○	○	○	◐	○
407	6/mmm1'	m'm'm'	6	●	○	○	○	○	○	○	●

Genus #135   6/mmm > 2|mm

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1328	6/m'mm	2|mm	6	○	◐	○	○	○	●	○	○
692	6/mmm1'	2|mm	12	◐	○	○	○	○	◐	◐	○
1525	6/mmm	2|mm	6	◐	○	○	○	○	◐	◐	○
975	6'/m'mm'	2|'mm	6	◐	○	○	○	○	●	◐	○
1039	6'/mmm'	2|'mm'	6	◐	○	○	○	○	◐	●	○
410	6/mmm1'	2|'mm'	12	◐	○	○	○	○	◐	◐	○
413	6/mmm1'	2|m'm'	12	◐	○	○	○	○	○	◐	◐
1006	6/m'm'm'	2|m'm'	6	◐	○	○	○	○	○	◐	◐
993	6/mm'm'	2|m'm'	6	◐	○	○	○	○	○	○	●
135	6/mmm1'	2|mm1'	6	◐	○	○	○	○	○	○	○

Genus #136   6/mmm > 2_mm

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1337	6'/mmm'	2_mm	6	○	◐	●	○	○	●	◐	○
1526	6/mmm	2_mm	6	○	◐	●	○	○	◐	●	○
992	6/mm'm'	2'_m|m'	6	○	◐	●	○	○	○	●	○
976	6'/m'mm'	2_m_m'	6	○	◐	●	○	○	○	◐	○
1007	6/m'm'm'	2_m'm'	6	○	◐	○	○	○	○	●	●
1021	6/m'mm	2'_m_m'	6	○	◐	○	○	○	○	●	○
977	6'/mmm'	2_m'm'	6	○	◐	●	○	○	○	◐	●
136	6/mmm1'	2_mm1'	6	○	◐	●	○	○	○	○	○
693	6/mmm1'	2_mm	12	◐	○	○	○	○	◐	◐	○
412	6/mmm1'	2'_m|m'	12	◐	○	○	○	○	○	◐	○
411	6/mmm1'	2'_m_m'	12	◐	○	○	○	○	○	◐	○
414	6/mmm1'	2_m'm'	12	◐	○	○	○	○	○	◐	◐

Genus #137   6/mmm > 222

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1320	6/m'm'm'	222	6	○	◐	○	●	●	○	○	◐
1527	6/mmm	222	6	○	◐	○	●	◐	○	○	●
978	6'/m'mm'	2_2'2'	6	○	◐	○	●	○	●	○	○
1041	6'/mmm'	2_2'2'	6	○	◐	○	●	○	○	●	○
994	6/mm'm'	2|2'2'	6	○	◐	○	●	○	○	◐	○
1022	6/m'mm	2|2'2'	6	○	◐	○	●	○	○	◐	○
137	6/mmm1'	2221'	6	○	◐	○	●	○	○	○	○
694	6/mmm1'	222	12	◐	○	○	◐	◐	○	○	◐
415	6/mmm1'	2|2'2'	12	◐	○	○	◐	○	◐	◐	○
416	6/mmm1'	2_2'2'	12	◐	○	○	◐	○	○	◐	○

Genus #138   6/mmm > 2/m|

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1528	6/mmm	2/m|	6	●	○	◐	○	●	○	◐	○
1311	6/mm'm'	2/m|	6	●	○	◐	○	●	○	○	○
1042	6'/mmm'	2'/m|	6	●	○	◐	○	○	●	○	○
1008	6/m'm'm'	2/m|'	6	●	○	◐	○	○	○	◐	●
979	6'/m'mm'	2'/m|'	6	●	○	◐	○	○	○	●	○
1024	6/m'mm	2/m|'	6	●	○	◐	○	○	○	○	●

(Genus #138 continued)

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
138	6/mmm1'	2/m|1'	6	●	○	◐	○	○	○	○	○
695	6/mmm1'	2/m|	12	◐	○	◐	○	●	○	◐	○
417	6/mmm1'	2'/m|	12	◐	○	◐	○	○	●	○	○
419	6/mmm1'	2/m|'	12	◐	○	◐	○	○	○	◐	●
421	6/mmm1'	2'/m|'	12	◐	○	◐	○	○	○	●	○

Genus #139   6/mmm > 2/m_

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1529	6/mmm	2/m_	6	●	○	●	○	●	○	●	○
1301	6'/m'mm'	2/m_	6	●	○	●	○	●	○	◐	○
1009	6/m'm'm'	2/m_'	6	●	○	●	○	○	○	●	●
1043	6'/mmm'	2'/m_	6	●	○	●	○	○	●	○	○
1023	6/m'mm	2'/m_	6	●	○	●	○	○	●	○	○
1044	6'/mmm'	2/m_'	6	●	○	●	○	○	◐	○	●
980	6'/m'mm'	2'/m_'	6	●	○	●	○	○	○	●	○
995	6/mm'm'	2'/m_'	6	●	○	●	○	○	○	●	○
139	6/mmm1'	2/m_1'	6	●	○	●	○	○	○	○	○
696	6/mmm1'	2/m_	12	◐	○	◐	○	●	○	◐	○
418	6/mmm1'	2'/m_	12	◐	○	◐	○	○	●	○	○
420	6/mmm1'	2/m_'	12	◐	○	◐	○	○	○	◐	●
422	6/mmm1'	2'/m_'	12	◐	○	◐	○	○	○	●	○

Genus #140   6/mmm > m|

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1338	6'/mmm'	m|	12	◐	●	○	○	●	◐	◐	○
1530	6/mmm	m|	12	◐	●	○	○	◐	●	◐	○
1312	6/mm'm'	m|	12	◐	●	○	○	◐	●	○	○
1010	6/m'm'm'	m|'	12	◐	●	○	○	○	○	●	◐
981	6'/m'mm'	m|'	12	◐	●	○	○	○	○	◐	●
1025	6/m'mm	m|'	12	◐	●	○	○	○	○	●	◐
140	6/mmm1'	m|1'	12	◐	●	○	○	○	○	○	○
697	6/mmm1'	m|	24	◐	○	○	○	◐	○	◐	○
423	6/mmm1'	m|'	24	◐	○	○	○	○	◐	◐	◐

Genus #141   6/mmm > m_

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1329	6/m'mm	m_	12	◐	●	○	○	●	◐	◐	○
1339	6'/mmm'	m_	12	◐	●	○	○	●	◐	◐	○
1531	6/mmm	m_	12	◐	●	○	○	◐	●	◐	○
1302	6'/m'mm'	m_	12	◐	●	○	○	◐	●	◐	○
1045	6'/mmm'	m_'	12	◐	●	○	○	○	○	●	◐
1011	6/m'm'm'	m_'	12	◐	●	○	○	○	○	●	◐
996	6/mm'm'	m_'	12	◐	●	○	○	○	○	◐	●
982	6'/m'mm'	m_'	12	◐	●	○	○	○	○	◐	●
141	6/mmm1'	m_1'	12	◐	●	○	○	○	○	○	○
698	6/mmm1'	m_	24	◐	○	○	○	◐	○	◐	○
424	6/mmm1'	m_'	24	◐	○	○	○	○	◐	◐	◐

Genus #142   6/mmm > 2|

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1321	6/m'm'm'	2|	12	◐	○	○	●	●	◐	◐	○
1330	6/m'mm	2|	12	◐	○	◐	○	●	◐	○	◐
1532	6/mmm	2|	12	◐	○	◐	○	◐	◐	◐	○
1313	6/mm'm'	2|	12	◐	○	◐	○	◐	◐	○	●
983	6'/m'mm'	2|'	12	◐	○	◐	○	○	●	◐	○
1046	6'/mmm'	2|'	12	◐	○	◐	●	○	◐	◐	○
142	6/mmm1'	2|1'	12	◐	○	◐	●	○	○	○	○
699	6/mmm1'	2|	24	◐	○	◐	○	◐	○	◐	◐

Genus #143 $6/mmm > 2_-$

No.	G	F	n	pattern
425	$6/mmm1'$	$2'_-$	24	◐ ◐ ◐ ◐ ○ ◐ ◐ ○
1322	$6/m'm'm'$	2_-	12	◐ ◐ ◐ ◐ ● ◐ ◐ ◐
1340	$6'/mmm'$	2_-	12	◐ ◐ ◐ ● ● ◐ ◐ ◐
1303	$6'/m'mm'$	2_-	12	◐ ◐ ◐ ● ◐ ◐ ◐ ●
1533	$6/mmm$	2_-	12	◐ ◐ ◐ ◐ ● ◐ ◐ ●
984	$6'/m'm'm$	$2'_-$	12	◐ ◐ ◐ ● ○ ◐ ● ○
997	$6/m'm'm'$	$2'_-$	12	◐ ◐ ◐ ● ○ ◐ ● ○
1047	$6'/mmm'$	$2'_-$	12	◐ ◐ ◐ ● ○ ◐ ● ○
1026	$6/m'mm$	$2'_-$	12	◐ ◐ ◐ ● ○ ◐ ● ○
143	$6/mmm1'$	$2_-1'$	12	◐ ◐ ◐ ● ○ ○ ○ ○
700	$6/mmm1'$	2_-	24	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐
426	$6/mmm1'$	$2'_-$	24	◐ ◐ ◐ ◐ ○ ◐ ◐ ○

Genus #144 $6/mmm > \bar{1}$

No.	G	F	n	pattern
1314	$6/mm'm'$	$\bar{1}$	12	● ○ ● ● ○ ● ● ○
1304	$6'/m'mm'$	$\bar{1}$	12	● ○ ● ● ○ ● ● ○
1534	$6/mmm$	$\bar{1}$	12	● ○ ● ● ○ ● ● ○
1012	$6'/m'm'm$	$\bar{1}'$	12	● ○ ● ○ ○ ● ○ ●
1027	$6/m'mm$	$\bar{1}'$	12	● ○ ● ○ ○ ● ○ ●
1048	$6'/mmm'$	$\bar{1}'$	12	● ○ ● ○ ○ ● ○ ●
144	$6/mmm1'$	$\bar{1}1'$	12	● ○ ● ○ ○ ○ ○ ○
701	$6/mmm1'$	$\bar{1}$	24	◐ ○ ◐ ○ ● ○ ● ○
427	$6/mmm1'$	$\bar{1}'$	24	◐ ○ ◐ ○ ○ ● ○ ●

Genus #145 $6/mmm > 1$

No.	G	F	n	pattern
1341	$6'/mmm'$	1	24	◐ ● ◐ ● ◐ ● ● ◐
1331	$6/m'mm$	1	24	◐ ● ◐ ● ● ◐ ● ◐
1323	$6/m'm'm'$	1	24	◐ ● ◐ ● ● ● ◐ ●
1315	$6/mm'm'$	1	24	◐ ● ◐ ◐ ● ◐ ● ●
1305	$6'/m'mm'$	1	24	◐ ● ◐ ◐ ● ● ◐ ●
1535	$6/mmm$	1	24	◐ ● ◐ ● ◐ ● ● ◐
145	$6/mmm1'$	$11'$	24	◐ ● ◐ ● ○ ○ ○ ○
702	$6/mmm1'$	1	48	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐

Genus #146 $23 > 3$

No.	G	F	n	pattern
1536	23	3	4	● ● ● ● ● ● ● ●
146	$231'$	$31'$	4	● ● ● ● ○ ○ ○ ○
704	$231'$	3	8	◐ ◐ ◐ ◐ ● ● ● ●

Genus #147 $23 > 22_+2$

No.	G	F	n	pattern
1537	23	22_+2	3	● ○ ○ ● ● ○ ○ ●
147	$231'$	$22_+21'$	3	● ○ ○ ● ○ ○ ○ ○
705	$231'$	22_+2	6	◐ ○ ○ ◐ ● ○ ○ ●
428	$231'$	$2_+2'_+2'$	6	○ ○ ○ ◐ ◐ ● ● ○

Genus #148 $23 > 2_+$

No.	G	F	n	pattern
1538	23	2_+	6	● ● ● ● ● ● ● ●
148	$231'$	$2_+1'$	6	● ● ● ● ○ ○ ○ ○
706	$231'$	2_+	12	◐ ◐ ◐ ◐ ● ● ◐ ●
429	$231'$	$2'_+$	12	◐ ◐ ◐ ◐ ○ ● ● ○

Genus #149 $23 > 1$

No.	G	F	n	pattern
1539	23	1	12	● ● ● ● ● ● ● ●
149	$231'$	$11'$	12	● ● ● ● ○ ○ ○ ○
707	$231'$	1	24	◐ ◐ ◐ ◐ ● ● ● ●

Genus #150 $m\bar{3} > 23$

No.	G	F	n	pattern
150	$m\bar{3}1'$	$231'$	2	○ ○ ○ ○ ○ ○ ○ ○
1540	$m\bar{3}$	23	2	○ ○ ○ ○ ○ ○ ○ ○
709	$m\bar{3}1'$	23	4	○ ○ ○ ○ ○ ○ ○ ○
1342	$m'\bar{3}'$	23	2	○ ○ ○ ○ ○ ○ ○ ○

Genus #151 $m\bar{3} > \bar{3}$

No.	G	F	n	pattern
1541	$m\bar{3}$	$\bar{3}$	4	● ○ ● ○ ● ○ ● ○
1049	$m'\bar{3}'$	$\bar{3}'$	4	● ○ ● ○ ● ○ ● ●
151	$m\bar{3}1'$	$\bar{3}1'$	4	● ○ ● ○ ● ○ ● ○
710	$m\bar{3}1'$	$\bar{3}$	8	◐ ○ ◐ ○ ◐ ○ ◐ ○
431	$m\bar{3}1'$	$\bar{3}'$	8	◐ ○ ◐ ○ ○ ◐ ○ ◐

Genus #152 $m\bar{3} > 3$

No.	G	F	n	pattern
1343	$m'\bar{3}'$	3	8	◐ ● ● ● ● ◐ ◐ ●
1542	$m\bar{3}$	3	8	◐ ● ● ◐ ● ● ◐ ●
152	$m\bar{3}1'$	$31'$	8	◐ ● ● ◐ ○ ○ ◐ ●
711	$m\bar{3}1'$	3	16	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐

Genus #153 $m\bar{3} > mm_+m$

No.	G	F	n	pattern
1543	$m\bar{3}$	mm_+m	3	● ○ ○ ○ ● ○ ○ ○
1050	$m'\bar{3}'$	$m'm'_+m'$	3	● ○ ○ ○ ○ ○ ○ ●
153	$m\bar{3}1'$	mm_+m1'	3	● ○ ○ ○ ○ ○ ○ ○
712	$m\bar{3}1'$	mm_+m	6	◐ ○ ○ ○ ● ○ ○ ○
432	$m\bar{3}1'$	m'_+m_+m	6	◐ ○ ○ ○ ○ ● ○ ○
434	$m\bar{3}1'$	$m_+m'_+m'$	6	◐ ○ ○ ○ ○ ○ ● ○
433	$m\bar{3}1'$	$m'm'_+m'$	6	◐ ○ ○ ○ ○ ○ ○ ●

Genus #154 $m\bar{3} > 2_+m_+m$

No.	G	F	n	pattern
1544	$m\bar{3}$	2_+m_+m	6	◐ ● ○ ○ ◐ ● ○ ○
1051	$m'\bar{3}'$	$2_+m'_+m'$	6	◐ ● ○ ○ ○ ○ ● ●
154	$m\bar{3}1'$	$2_+m_+m1'$	6	◐ ● ○ ○ ○ ○ ○ ○
713	$m\bar{3}1'$	2_+m_+m	12	◐ ◐ ○ ○ ◐ ◐ ○ ○
435	$m\bar{3}1'$	$2'_+m_+m'$	12	◐ ◐ ○ ○ ○ ◐ ◐ ○
436	$m\bar{3}1'$	$2_+m'_+m'$	12	◐ ◐ ○ ○ ○ ○ ◐ ◐

Genus #155 $m\bar{3} > 22_+2$

No.	G	F	n	pattern
1344	$m'\bar{3}'$	22_+2	6	◐ ○ ○ ● ● ○ ○ ◐
1545	$m\bar{3}$	22_+2	6	◐ ○ ○ ● ◐ ○ ○ ●
155	$m\bar{3}1'$	$22_+21'$	6	◐ ○ ○ ● ○ ○ ○ ○
714	$m\bar{3}1'$	22_+2	12	◐ ○ ○ ◐ ◐ ○ ○ ◐
437	$m\bar{3}1'$	$2_+2'_+2'$	12	◐ ○ ○ ◐ ○ ◐ ◐ ○

Genus #156 $m\bar{3} > 2/m_+$

No.	G	F	n	pattern
1546	$m\bar{3}$	$2/m_+$	6	● ○ ● ○ ● ○ ● ○
1052	$m'\bar{3}'$	$2/m'_+$	6	● ○ ● ○ ○ ● ○ ●
156	$m\bar{3}1'$	$2/m_+1'$	6	● ○ ● ○ ○ ○ ○ ○
715	$m\bar{3}1'$	$2/m_+$	12	◐ ○ ◐ ○ ◐ ○ ◐ ○
438	$m\bar{3}1'$	$2'/m_+$	12	◐ ○ ◐ ○ ○ ● ○ ○
439	$m\bar{3}1'$	$2/m'_+$	12	◐ ○ ◐ ○ ○ ◐ ○ ●
440	$m\bar{3}1'$	$2'/m'_+$	12	◐ ○ ◐ ○ ○ ○ ● ○

Genus #157 $m\bar{3} > m_+$

No.	G	F	n	pattern
1547	$m\bar{3}$	m_+	12	◐ ● ◐ ○ ◐ ◐ ● ○
1053	$m'\bar{3}'$	m'_+	12	◐ ● ◐ ○ ○ ◐ ● ●
157	$m\bar{3}1'$	m_+1'	12	◐ ● ◐ ○ ○ ○ ○ ○
716	$m\bar{3}1'$	m_+	24	◐ ◐ ◐ ○ ◐ ◐ ◐ ○

Genus #158 $m\bar{3} > 2_+$

No.	G	F	n	pattern
441	$m\bar{3}1'$	m'_+	24	◐ ◐ ◐ ○ ○ ◐ ◐ ◐
1345	$m'\bar{3}'$	2_+	12	◐ ◐ ◐ ● ● ◐ ◐ ◐
1548	$m\bar{3}$	2_+	12	◐ ◐ ◐ ● ◐ ◐ ◐ ●
158	$m\bar{3}1'$	$2_+1'$	12	◐ ◐ ◐ ● ○ ○ ○ ○
717	$m\bar{3}1'$	2_+	24	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐
442	$m\bar{3}1'$	$2'_+$	24	◐ ◐ ◐ ◐ ○ ◐ ◐ ○

Genus #159 $m\bar{3} > \bar{1}$

No.	G	F	n	pattern
1549	$m\bar{3}$	$\bar{1}$	12	● ○ ● ○ ● ○ ● ○
1054	$m'\bar{3}'$	$\bar{1}'$	12	● ○ ● ○ ● ○ ● ●
159	$m\bar{3}1'$	$\bar{1}1'$	12	● ○ ● ○ ○ ○ ○ ○
718	$m\bar{3}1'$	$\bar{1}$	24	◐ ○ ◐ ○ ● ○ ● ○
443	$m\bar{3}1'$	$\bar{1}'$	24	◐ ○ ◐ ○ ○ ○ ● ●

Genus #160 $m\bar{3} > 1$

No.	G	F	n	pattern
1346	$m'\bar{3}'$	1	24	◐ ● ◐ ● ● ◐ ● ◐
1550	$m\bar{3}$	1	24	◐ ● ◐ ● ◐ ● ◐ ●
160	$m\bar{3}1'$	$11'$	24	◐ ● ◐ ● ○ ○ ○ ○
719	$m\bar{3}1'$	1	48	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐

Genus #161 $432 > 23$

No.	G	F	n	pattern
161	$4321'$	$231'$	2	○ ○ ○ ○ ○ ○ ○ ○
1551	432	23	2	○ ○ ○ ○ ○ ○ ○ ○
721	$4321'$	23	4	○ ○ ○ ○ ○ ○ ○ ○
1347	$4'32'$	23	2	○ ○ ○ ○ ○ ○ ○ ○

Genus #162 $432 > 32$

No.	G	F	n	pattern
1552	432	32	4	● ○ ○ ● ● ○ ○ ●
1055	$4'32'$	$32'$	4	● ○ ○ ● ● ○ ○ ●
162	$4321'$	$321'$	4	● ○ ○ ● ○ ○ ○ ○
722	$4321'$	32	8	◐ ○ ○ ◐ ◐ ○ ○ ◐
445	$4321'$	$32'$	8	◐ ○ ○ ◐ ○ ● ● ○

Genus #163 $432 > 3$

No.	G	F	n	pattern
1348	$4'32'$	3	8	◐ ● ● ◐ ● ◐ ◐ ●
1553	432	3	8	◐ ● ● ◐ ◐ ● ● ◐
163	$4321'$	$31'$	8	◐ ● ● ◐ ○ ○ ○ ○
723	$4321'$	3	16	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐

Genus #164 $432 > 422$

No.	G	F	n	pattern
1554	432	422	3	● ○ ○ ● ● ○ ○ ●
164	$4321'$	$4221'$	3	● ○ ○ ● ○ ○ ○ ○
1056	$4'32'$	$4'2_+2'$	3	● ○ ○ ● ○ ○ ○ ○
724	$4321'$	422	6	◐ ○ ○ ◐ ◐ ○ ○ ◐
448	$4321'$	$42'2'$	6	◐ ○ ○ ◐ ○ ● ● ○
446	$4321'$	$4'2_+2'$	6	◐ ○ ○ ◐ ○ ○ ○ ○
447	$4321'$	$4'2\backslash 2'$	6	◐ ○ ○ ◐ ○ ○ ○ ○

Genus #165 $432 > 4$

No.	G	F	n	pattern
1555	432	4	6	◐ ● ● ◐ ◐ ● ● ◐
165	$4321'$	$41'$	6	◐ ● ● ◐ ○ ○ ○ ○
1057	$4'32'$	$4'$	6	◐ ● ● ◐ ○ ○ ○ ○
725	$4321'$	4	12	◐ ◐ ◐ ◐ ◐ ◐ ◐ ◐
449	$4321'$	$4'$	12	◐ ◐ ◐ ◐ ○ ○ ○ ○

Genus #166 $432 > 22_+2$

No.	G	F	n	pattern
1349	$4'32'$	22_+2	6	● ○ ○ ● ● ○ ○ ●

Column 1

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1556	432	22₊2	6	●	○	○	●	●	●	○	○
166	4321'	22₊21'	6	●	○	○	●	○	○	○	○
726	4321'	22₊2	12	◐	○	○	○	◐	●	○	○
450	4321'	2₊2'₊2'	12	◐	○	○	○	◐	○	◐	◐

Genus #167   432 > 22\2

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1557	432	22\2	6	●	○	○	●	●	●	○	○
1058	4'32'	2₊2'2'	6	●	○	○	●	○	●	●	○
167	4321'	22\21'	6	●	○	○	●	○	○	○	○
727	4321'	22\2	12	◐	○	○	○	◐	●	○	○
452	4321'	2\2'2'	12	◐	○	○	○	◐	○	●	●
451	4321'	2₊2'2'	12	◐	○	○	○	◐	○	◐	◐

Genus #168   432 > 2₊

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1350	4'32'	2₊	12	●	●	◐	●	●	●	◐	◐
1558	432	2₊	12	●	●	◐	●	●	●	◐	◐
168	4321'	2₊1'	12	●	●	◐	●	○	○	○	○
728	4321'	2₊	24	◐	◐	◐	◐	●	◐	◐	◐
453	4321'	2'₊	24	◐	◐	◐	◐	◐	○	●	●

Genus #169   432 > 2

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1559	432	2\	12	●	●	●	●	●	●	●	●
1059	4'32'	2\	12	●	●	●	●	○	●	●	○
169	4321'	2\1'	12	●	●	●	●	○	○	○	○
729	4321'	2\	24	◐	◐	◐	◐	●	◐	◐	◐
454	4321'	2'	24	◐	◐	◐	◐	○	●	●	○

Genus #170   432 > 1

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1560	432	1	24	●	●	●	●	●	●	●	●
1351	4'32'	1	24	●	●	●	●	●	●	●	●
170	4321'	11'	24	●	●	●	●	○	○	○	○
730	4321'	1	48	◐	◐	◐	◐	◐	●	●	●

Genus #171   ̄43m > 23

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
171	̄43m1'	231'	2	○	○	○	○	○	○	○	○
1352	̄4'3m'	23	2	○	○	○	○	○	○	○	○
732	̄43m1'	23	4	○	○	○	○	○	○	○	○
1561	̄43m	23	2	○	○	○	○	○	○	○	○

Genus #172   ̄43m > 3m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1562	̄43m	3m	4	●	●	○	○	○	●	●	○
1060	̄4'3m'	3m'	4	●	●	○	○	○	○	●	●
172	̄43m1'	3m1'	4	●	●	○	○	○	○	○	○
733	̄43m1'	3m	8	◐	○	◐	○	○	●	●	○
456	̄43m1'	3m'	8	◐	○	◐	○	○	○	●	●

Genus #173   ̄43m > 3

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1353	̄4'3m'	3	8	◐	○	◐	●	●	●	●	◐
1563	̄43m	3	8	◐	○	◐	●	●	◐	●	●
173	̄43m1'	31'	8	◐	○	●	●	○	○	○	○
734	̄43m1'	3	16	◐	○	◐	◐	◐	◐	◐	◐

Genus #174   ̄43m > ̄42₊m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1564	̄43m	̄42₊m	3	●	○	○	○	●	○	○	○
1061	̄4'3m'	̄4'2₊m'	3	●	○	○	○	○	○	○	●
174	̄43m1'	̄42₊m1'	3	●	○	○	○	○	○	○	○
735	̄43m1'	̄42₊m	6	◐	○	○	○	●	○	○	○
458	̄43m1'	̄4'2'₊m	6	◐	○	○	○	○	●	○	○

Column 2

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
457	̄43m1'	̄42'₊m'	6	◐	○	○	○	○	○	○	●
459	̄43m1'	̄4'2₊m'	6	◐	○	○	○	○	○	○	●

Genus #175   ̄43m > ̄4

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1565	̄43m	̄4	6	◐	○	○	●	○	◐	○	●
1062	̄4'3m'	̄4'	6	◐	○	●	○	○	●	○	◐
175	̄43m1'	̄41'	6	◐	○	○	●	○	○	○	○
736	̄43m1'	̄4	12	◐	○	○	◐	○	◐	○	◐
460	̄43m1'	̄4'	12	◐	○	○	◐	○	○	◐	◐

Genus #176   ̄43m > 2₊m\m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1566	̄43m	2₊m\m	6	●	●	○	○	●	●	○	○
1063	̄4'3m'	2₊m'm'	6	●	●	○	○	○	○	●	●
176	̄43m1'	2₊m\m1'	6	●	●	○	○	○	○	○	○
737	̄43m1'	2₊m\m	12	◐	◐	○	○	○	●	◐	○
461	̄43m1'	2'₊m\m'	12	◐	◐	○	○	○	○	●	●
462	̄43m1'	2₊m'm'	12	◐	◐	○	○	○	○	◐	●

Genus #177   ̄43m > 22₊2

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1567	̄43m	22₊2	6	●	○	○	●	●	○	○	○
1354	̄4'3m'	22₊2	6	●	○	○	●	●	○	○	○
177	̄43m1'	22₊21'	6	●	○	○	●	○	○	○	○
738	̄43m1'	22₊2	12	◐	○	○	○	◐	●	○	○
463	̄43m1'	2₊2'₊2'	12	◐	○	○	○	◐	○	○	◐

Genus #178   ̄43m > m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1568	̄43m	m\	12	●	●	●	○	●	●	●	○
1064	̄4'3m'	m'	12	●	●	●	○	○	●	●	◐
178	̄43m1'	m\1'	12	●	●	●	○	○	○	○	○
739	̄43m1'	m\	24	◐	◐	◐	○	●	●	◐	○
464	̄43m1'	m'	24	◐	◐	◐	○	○	◐	●	●

Genus #179   ̄43m > 2₊

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1569	̄43m	2₊	12	●	◐	◐	●	●	◐	◐	●
1355	̄4'3m'	2₊	12	●	◐	◐	●	●	◐	◐	●
179	̄43m1'	2₊1'	12	●	◐	◐	●	○	○	○	○
740	̄43m1'	2₊	24	◐	◐	◐	◐	●	◐	◐	◐
465	̄43m1'	2'₊	24	◐	◐	◐	◐	◐	○	●	●

Genus #180   ̄43m > 1

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1356	̄4'3m'	1	24	●	●	●	●	●	●	●	●
1570	̄43m	1	24	●	●	●	●	●	●	●	●
180	̄43m1'	11'	24	●	●	●	●	○	○	○	○
741	̄43m1'	1	48	◐	◐	◐	◐	●	●	●	●

Genus #181   m̄3m > ̄43m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
181	m̄3m1'	̄43m1'	2	○	○	○	○	○	○	○	○
469	m̄3m1'	̄4'3m'	4	○	○	○	○	○	○	○	○
1085	m'̄3'm'	̄4'3m'	2	○	○	○	○	○	○	○	○
1065	m̄3m'	̄4'3m'	2	○	○	○	○	○	○	○	○
1571	m̄3m	̄43m	2	○	○	○	○	○	○	○	○
1380	m'̄3'm	̄43m	2	○	○	○	○	○	○	○	○
743	m̄3m1'	̄43m	4	○	○	○	○	○	○	○	○

Genus #182   m̄3m > 432

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
182	m̄3m1'	4321'	2	○	○	○	○	○	○	○	○
1106	m'̄3'm	4'32'	2	○	○	○	○	○	○	○	○
1066	m̄3m'	4'32'	2	○	○	○	○	○	○	○	○

Column 3

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
470	m̄3m1'	4'32'	4	○	○	○	○	○	○	○	○
1572	m̄3m	432	2	○	○	○	○	○	○	○	○
1369	m'̄3'm'	432	2	○	○	○	○	○	○	○	○
744	m̄3m1'	432	4	○	○	○	○	○	○	○	○

Genus #183   m̄3m > m̄3

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
183	m̄3m1'	m̄31'	2	○	○	○	○	○	○	○	○
1086	m'̄3'm'	m'̄3'	2	○	○	○	○	○	○	○	○
1107	m'̄3'm	m'̄3'	2	○	○	○	○	○	○	○	○
471	m̄3m1'	m'̄3'	4	○	○	○	○	○	○	○	○
1357	m̄3m'	m̄3	2	○	○	○	○	○	○	○	○
1573	m̄3m	m̄3	2	○	○	○	○	○	○	○	○
745	m̄3m1'	m̄3	4	○	○	○	○	○	○	○	○

Genus #184   m̄3m > 23

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
184	m̄3m1'	231'	4	○	○	○	○	○	○	○	○
1381	m'̄3'm	23	4	○	○	○	○	○	○	○	○
1370	m'̄3'm'	23	4	○	○	○	○	○	○	○	○
746	m̄3m1'	23	8	○	○	○	○	○	○	○	○
1574	m̄3m	23	4	○	○	○	○	○	○	○	○
1358	m̄3m'	23	4	○	○	○	○	○	○	○	○

Genus #185   m̄3m > ̄3m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1575	m̄3m	̄3m	4	●	○	○	○	●	○	○	○
1108	m'̄3'm	̄3'm	4	●	○	○	○	○	●	○	○
1067	m̄3m'	̄3m'	4	●	○	○	○	○	○	●	○
1087	m'̄3'm'	̄3'm'	4	●	○	○	○	○	○	○	●
185	m̄3m1'	̄3m1'	4	●	○	○	○	○	○	○	○
747	m̄3m1'	̄3m	8	◐	○	○	○	●	○	○	○
472	m̄3m1'	̄3'm	8	◐	○	○	○	○	●	○	○
474	m̄3m1'	̄3m'	8	◐	○	○	○	○	○	●	○
473	m̄3m1'	̄3'm'	8	◐	○	○	○	○	○	○	●

Genus #186   m̄3m > 3m

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1382	m'̄3'm	3m	8	◐	●	○	○	●	◐	○	○
1576	m̄3m	3m	8	◐	●	○	○	◐	●	○	○
1088	m'̄3'm'	3m'	8	◐	●	○	○	○	○	●	◐
1068	m̄3m'	3m'	8	◐	○	●	○	○	◐	●	●
186	m̄3m1'	3m1'	8	◐	●	○	○	○	○	○	○
748	m̄3m1'	3m	16	◐	◐	○	○	○	◐	◐	○
475	m̄3m1'	3m'	16	◐	◐	○	○	○	○	○	◐

Genus #187   m̄3m > 32

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1371	m'̄3'm'	32	8	◐	○	○	●	●	○	○	◐
1577	m̄3m	32	8	◐	○	○	●	◐	○	○	●
1069	m̄3m'	32'	8	◐	○	○	●	○	●	◐	○
1109	m'̄3'm	32'	8	◐	○	○	●	○	◐	●	○
187	m̄3m1'	321'	8	◐	○	○	●	○	○	○	○
749	m̄3m1'	32	16	◐	○	○	◐	◐	○	○	◐
476	m̄3m1'	32'	16	◐	○	○	◐	○	◐	◐	○

Genus #188   m̄3m > ̄3

No.	G	F	n	↕	↑	⊬	⚓	↕	↑	⊬	⚓
1359	m̄3m'	̄3	8	◐	○	●	○	●	○	○	◐
1578	m̄3m	̄3	8	◐	○	●	○	◐	○	●	○
1089	m'̄3'm'	̄3'	8	◐	○	●	○	○	●	○	◐
1110	m'̄3'm	̄3'	8	◐	○	●	○	○	◐	○	●

Column 1

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
188	$m\bar{3}m1'$	$\bar{3}1'$	8	◐	○	●	○	○	○	○	○
750	$m\bar{3}m1'$	$\bar{3}$	16	◐	○	◐	○	◐	○	○	◐
477	$m\bar{3}m1'$	$\bar{3}'$	16	◐	○	◐	○	○	○	◐	◐

Genus #189 $m\bar{3}m > 3$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1579	$m\bar{3}m$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1372	$m'\bar{3}'m'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
1383	$m'\bar{3}'m$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
751	$m\bar{3}m1'$	3	32	◐	◐	◐	◐	◐	◐	◐	◐
1360	$m\bar{3}m'$	3	16	◐	◐	◐	◐	◐	◐	◐	◐
189	$m\bar{3}m1'$	$\bar{3}1'$	16	◐	◐	◐	◐	◐	○	○	○

Genus #190 $m\bar{3}m > 4/mmm$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1580	$m\bar{3}m$	$4/mmm$	3	●	○	○	○	●	○	○	○
1090	$m'\bar{3}'m'$	$4/m'm'm'$	3	●	○	○	○	○	○	○	●
190	$m\bar{3}m1'$	$4/mmm1'$	3	●	○	○	○	○	○	○	○
1070	$m\bar{3}m'$	$4'/mm\backslash m'$	3	●	○	○	○	○	○	○	○
1111	$m'\bar{3}'m$	$4'/m'm'_+m$	3	●	○	○	○	○	○	○	○
752	$m\bar{3}m1'$	$4/mmm$	6	◐	○	○	○	●	○	○	○
480	$m\bar{3}m1'$	$4/m'mm$	6	◐	○	○	○	●	○	○	○
482	$m\bar{3}m1'$	$4/mm'm'$	6	◐	○	○	○	●	○	○	○
481	$m\bar{3}m1'$	$4/m'm'm'$	6	◐	○	○	○	○	○	○	●
483	$m\bar{3}m1'$	$4'/mm_+m'$	6	◐	○	○	○	○	○	○	○
484	$m\bar{3}m1'$	$4'/mm\backslash m'$	6	◐	○	○	○	○	○	○	○
478	$m\bar{3}m1'$	$4'/m'm'_+m$	6	◐	○	○	○	○	○	○	○
479	$m\bar{3}m1'$	$4'/m'm\backslash m$	6	◐	○	○	○	○	○	○	○

Genus #191 $m\bar{3}m > \bar{4}2_+m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1384	$m'\bar{3}'m$	$\bar{4}2_+m$	6	◐	○	○	○	●	○	○	○
1581	$m\bar{3}m$	$\bar{4}2_+m$	6	◐	○	○	○	◐	○	○	○
753	$m\bar{3}m1'$	$\bar{4}2_+m$	12	◐	○	○	○	◐	○	○	○
487	$m\bar{3}m1'$	$\bar{4}'2'_+m$	12	◐	○	○	○	○	◐	○	○
485	$m\bar{3}m1'$	$\bar{4}2'_+m'$	12	◐	○	○	○	○	◐	○	○
1072	$m\bar{3}m'$	$\bar{4}'2_+m'$	6	◐	○	○	○	○	○	○	●
1091	$m'\bar{3}'m'$	$\bar{4}'2_+m'$	6	◐	○	○	○	○	○	○	◐
489	$m\bar{3}m1'$	$\bar{4}'2_+m'$	12	◐	○	○	○	○	○	○	◐
191	$m\bar{3}m1'$	$\bar{4}2_+m1'$	6	◐	○	○	○	○	○	○	○

Genus #192 $m\bar{3}m > \bar{4}2\backslash m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
754	$m\bar{3}m1'$	$\bar{4}2\backslash m$	12	◐	○	○	○	◐	○	○	○
1582	$m\bar{3}m$	$\bar{4}2\backslash m$	6	◐	○	○	○	◐	○	○	○
1071	$m\bar{3}m'$	$\bar{4}'2\backslash m$	6	◐	○	○	○	●	○	○	○
488	$m\bar{3}m1'$	$\bar{4}'2\backslash m$	12	◐	○	○	○	○	◐	○	○
1112	$m'\bar{3}'m$	$\bar{4}2\backslash m'$	6	◐	○	○	○	○	●	○	○
486	$m\bar{3}m1'$	$\bar{4}2\backslash m'$	12	◐	○	○	○	○	◐	○	○
490	$m\bar{3}m1'$	$\bar{4}'2\backslash m'$	12	◐	○	○	○	○	○	○	◐
1092	$m'\bar{3}'m'$	$\bar{4}'2\backslash m'$	6	◐	○	○	○	○	○	○	◐
192	$m\bar{3}m1'$	$\bar{4}2\backslash m1'$	6	◐	○	○	○	○	○	○	○

Genus #193 $m\bar{3}m > 4mm$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1583	$m\bar{3}m$	$4mm$	6	◐	●	○	○	◐	●	○	○
1093	$m'\bar{3}'m'$	$4m'm'$	6	◐	●	○	○	○	○	●	◐
193	$m\bar{3}m1'$	$4mm1'$	6	◐	●	○	○	○	○	○	○
1113	$m'\bar{3}'m$	$4'm_+m'$	6	◐	●	○	○	○	○	○	○
1073	$m\bar{3}m'$	$4'm\backslash m'$	6	◐	●	○	○	○	○	○	○
755	$m\bar{3}m1'$	$4mm$	12	◐	●	○	○	◐	●	○	○
491	$m\bar{3}m1'$	$4m'm'$	12	◐	●	○	○	○	○	◐	◐
492	$m\bar{3}m1'$	$4'm_+m'$	12	◐	●	○	○	○	○	○	○
493	$m\bar{3}m1'$	$4'm\backslash m'$	12	◐	●	○	○	○	○	○	○

Genus #194 $m\bar{3}m > 422$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1373	$m'\bar{3}'m'$	422	6	○	○	○	●	●	○	○	◐
1584	$m\bar{3}m$	422	6	○	○	○	●	◐	○	○	●
194	$m\bar{3}m1'$	$4221'$	6	○	○	○	●	○	○	○	○
1074	$m\bar{3}m'$	$4'2_+2'$	6	○	○	○	●	○	○	○	○
1114	$m'\bar{3}'m$	$4'2_+2'$	6	○	○	○	●	○	○	○	○
756	$m\bar{3}m1'$	422	12	◐	○	○	○	◐	◐	○	◐
496	$m\bar{3}m1'$	$42'2'$	12	◐	○	○	○	◐	○	◐	◐
494	$m\bar{3}m1'$	$4'2'_+2'$	12	◐	○	○	◐	○	◐	○	○
495	$m\bar{3}m1'$	$4'2\backslash 2'$	12	◐	○	○	○	◐	○	○	○

Genus #195 $m\bar{3}m > 4/m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1585	$m\bar{3}m$	$4/m$	6	◐	○	○	●	○	◐	○	●
1094	$m'\bar{3}'m'$	$4/m'$	6	◐	○	○	●	○	◐	○	●
195	$m\bar{3}m1'$	$4/m1'$	6	◐	○	●	○	○	○	○	○
1075	$m\bar{3}m'$	$4'/m$	6	◐	○	●	○	○	○	○	○
1115	$m'\bar{3}'m$	$4'/m'$	6	◐	●	○	○	○	○	○	○
757	$m\bar{3}m1'$	$4/m$	12	◐	○	◐	○	◐	○	◐	○
498	$m\bar{3}m1'$	$4/m'$	12	◐	○	◐	○	◐	○	◐	◐
499	$m\bar{3}m1'$	$4'/m$	12	◐	○	◐	○	◐	○	○	○
497	$m\bar{3}m1'$	$4'/m'$	12	◐	○	◐	○	○	○	○	○

Genus #196 $m\bar{3}m > \bar{4}$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1385	$m'\bar{3}'m$	$\bar{4}$	12	◐	○	◐	○	○	◐	○	○
1586	$m\bar{3}m$	$\bar{4}$	12	◐	○	◐	○	○	◐	○	○
758	$m\bar{3}m1'$	$\bar{4}$	24	◐	○	◐	○	◐	○	◐	○
1095	$m'\bar{3}'m'$	$\bar{4}'$	12	◐	○	○	○	◐	◐	○	◐
500	$m\bar{3}m1'$	$\bar{4}'$	24	◐	○	◐	○	◐	○	◐	◐
1076	$m\bar{3}m'$	$\bar{4}'$	12	◐	○	○	○	○	◐	○	◐
196	$m\bar{3}m1'$	$\bar{4}1'$	12	◐	○	◐	○	○	○	○	○

Genus #197 $m\bar{3}m > 4$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
759	$m\bar{3}m1'$	4	24	◐	◐	◐	◐	◐	◐	◐	◐
1374	$m'\bar{3}'m'$	4	12	◐	◐	◐	◐	◐	◐	◐	◐
1587	$m\bar{3}m$	4	12	◐	◐	◐	◐	◐	◐	◐	◐
197	$m\bar{3}m1'$	$41'$	12	◐	◐	◐	◐	◐	○	○	○
1116	$m'\bar{3}'m$	$4'$	12	◐	◐	◐	◐	○	○	○	○
501	$m\bar{3}m1'$	$4'$	24	◐	◐	◐	◐	○	○	○	○
1077	$m\bar{3}m'$	$4'$	12	◐	◐	◐	◐	○	○	○	○

Genus #198 $m\bar{3}m > mm_+m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1588	$m\bar{3}m$	mm_+m	6	●	○	○	○	●	○	○	○
1361	$m\bar{3}m'$	mm_+m	6	●	○	○	○	●	○	○	○
1118	$m'\bar{3}'m$	$m'm'_+m'$	6	●	○	○	○	○	○	○	●
1096	$m'\bar{3}'m'$	$m'm'_+m'$	6	●	○	○	○	○	○	○	●
198	$m\bar{3}m1'$	mm_+m1'	6	●	○	○	○	○	○	○	○
760	$m\bar{3}m1'$	mm_+m	12	◐	○	○	○	●	○	○	○
502	$m\bar{3}m1'$	m'_+m_+m	12	◐	○	○	○	◐	○	○	○
507	$m\bar{3}m1'$	$m_+m'_+m'$	12	◐	○	○	○	○	○	◐	○
505	$m\bar{3}m1'$	$m'm'_+m'$	12	◐	○	○	○	○	○	○	●

Genus #199 $m\bar{3}m > mm\backslash m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1589	$m\bar{3}m$	$mm\backslash m$	6	●	○	○	○	●	○	○	○
1117	$m'\bar{3}'m$	$m'_+m\backslash m$	6	●	○	○	○	●	○	○	○
1078	$m\bar{3}m'$	$m_+m'_+m'$	6	●	○	○	○	○	○	●	○
1097	$m'\bar{3}'m'$	$m'm'_+m'$	6	●	○	○	○	○	○	○	●
199	$m\bar{3}m1'$	$mm\backslash m1'$	6	●	○	○	○	○	○	○	○
761	$m\bar{3}m1'$	$mm\backslash m$	12	◐	○	○	○	●	○	○	○
504	$m\bar{3}m1'$	m'_+mm	12	◐	○	○	○	●	○	○	○
503	$m\bar{3}m1'$	$m'_+m\backslash m$	12	◐	○	○	○	◐	○	○	○
509	$m\bar{3}m1'$	$m\backslash m'm'$	12	◐	○	○	○	○	○	●	○
508	$m\bar{3}m1'$	$m_+m'_+m'$	12	◐	○	○	○	○	○	◐	○
506	$m\bar{3}m1'$	$m'm'\backslash m'$	12	◐	○	○	○	○	○	○	●

Genus #200 $m\bar{3}m > 2_+m_+m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
762	$m\bar{3}m1'$	2_+m_+m	24	◐	◐	○	○	◐	◐	○	○
1362	$m\bar{3}m'$	2_+m_+m	12	◐	◐	○	○	◐	◐	○	○
1590	$m\bar{3}m$	2_+m_+m	12	◐	◐	○	○	◐	◐	○	○
510	$m\bar{3}m1'$	$2'_+m_+m'$	24	◐	◐	○	○	◐	◐	○	○
1098	$m'\bar{3}'m'$	$2_+m'_+m'$	12	◐	◐	○	○	○	○	◐	◐
1120	$m'\bar{3}'m$	$2_+m'_+m'$	12	◐	◐	○	○	○	○	◐	◐
514	$m\bar{3}m1'$	$2_+m'_+m'$	24	◐	◐	○	○	◐	◐	○	○
200	$m\bar{3}m1'$	$2_+m_+m1'$	12	◐	◐	○	○	○	○	○	○

Genus #201 $m\bar{3}m > 2_+m\backslash m$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1386	$m'\bar{3}'m$	$2_+m\backslash m$	12	◐	◐	○	○	●	◐	○	○
1591	$m\bar{3}m$	$2_+m\backslash m$	12	◐	◐	○	○	○	◐	◐	○
763	$m\bar{3}m1'$	$2_+m\backslash m$	24	◐	◐	○	○	◐	◐	○	○
511	$m\bar{3}m1'$	$2'_+m\backslash m'$	24	◐	◐	○	○	◐	◐	○	○
1080	$m\bar{3}m'$	$2_+m'_+m'$	12	◐	◐	○	○	○	○	○	●
1099	$m'\bar{3}'m'$	$2_+m'_+m'$	12	◐	◐	○	○	○	○	◐	◐
515	$m\bar{3}m1'$	$2_+m'_+m'$	24	◐	◐	○	○	◐	◐	○	○
201	$m\bar{3}m1'$	$2_+m\backslash m1'$	12	◐	◐	○	○	○	○	○	○

Genus #202 $m\bar{3}m > 2\backslash mm$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1592	$m\bar{3}m$	$2\backslash mm$	12	◐	●	○	○	◐	●	○	○
1079	$m\bar{3}m'$	$2'\backslash m_+m'$	12	◐	●	○	○	◐	●	○	○
1119	$m'\bar{3}'m$	$2'\backslash m_+m'$	12	◐	●	○	○	○	●	◐	○
1100	$m'\bar{3}'m'$	$2\backslash m'm'$	12	◐	●	○	○	○	○	●	◐
202	$m\bar{3}m1'$	$2\backslash mm1'$	12	◐	●	○	○	○	○	○	○
764	$m\bar{3}m1'$	$2\backslash mm$	24	◐	◐	○	○	◐	◐	○	○
513	$m\bar{3}m1'$	$2'\backslash m_+m'$	24	◐	◐	○	○	◐	◐	○	○
512	$m\bar{3}m1'$	$2\backslash m'm'$	24	◐	◐	○	○	○	◐	◐	○
516	$m\bar{3}m1'$	$2\backslash m'm'$	24	◐	◐	○	○	○	○	◐	◐

Genus #203 $m\bar{3}m > 22_+2$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1375	$m'\bar{3}'m'$	22_+2	12	◐	○	○	●	●	○	○	◐
1387	$m'\bar{3}'m$	22_+2	12	◐	○	○	●	●	○	○	◐
1593	$m\bar{3}m$	22_+2	12	◐	○	○	●	◐	○	○	●
1363	$m\bar{3}m'$	22_+2	12	◐	○	○	●	◐	○	○	●
203	$m\bar{3}m1'$	$22_+21'$	12	◐	○	○	●	○	○	○	○
765	$m\bar{3}m1'$	22_+2	24	◐	○	○	◐	◐	○	○	◐
517	$m\bar{3}m1'$	$2_+2'_+2'$	24	◐	○	○	◐	○	◐	○	◐

Genus #204 $m\bar{3}m > 22\backslash 2$

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1376	$m'\bar{3}'m'$	$22\backslash 2$	12	◐	○	○	●	●	○	○	◐

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
1594	$m\bar{3}m$	$22\backslash2$	12	◐	○	○	○	●	◐	○	○	●
1081	$m\bar{3}m'$	$2_+2'2'$	12	◐	○	○	○	●	○	◐	◐	○
1121	$m'\bar{3}'m$	$2_+2'2'$	12	◐	○	○	○	●	○	◐	◐	○
204	$m\bar{3}m1'$	$22\backslash21'$	12	◐	○	○	○	●	○	○	○	○
766	$m\bar{3}m1'$	$22\backslash2$	24	◐	○	○	○	◐	◐	○	○	◐
519	$m\bar{3}m1'$	$2\backslash2'2'$	24	◐	○	○	○	◐	○	◐	◐	○
518	$m\bar{3}m1'$	$2_+2'2'$	24	◐	○	○	○	◐	○	◐	◐	○
Genus #205		$m\bar{3}m > 2/m_+$										
1364	$m\bar{3}m'$	$2/m_+$	12	●	○	○	◐	●	○	○	◐	○
1595	$m\bar{3}m$	$2/m_+$	12	●	○	◐	○	●	○	◐	○	○
1101	$m'\bar{3}'m$	$2/m'_+$	12	●	○	○	◐	○	○	◐	○	●
1123	$m'\bar{3}'m$	$2/m'_+$	12	●	○	○	◐	○	○	◐	○	●
205	$m\bar{3}m1'$	$2/m_+1'$	12	●	○	◐	○	○	○	○	○	○
767	$m\bar{3}m1'$	$2/m_+$	24	◐	○	○	◐	○	●	○	◐	○
520	$m\bar{3}m1'$	$2'/m_+$	24	◐	○	○	◐	○	○	●	○	○
522	$m\bar{3}m1'$	$2/m'_+$	24	◐	○	○	◐	○	○	◐	○	●
524	$m\bar{3}m1'$	$2'/m'_+$	24	◐	○	○	◐	○	○	○	●	○
Genus #206		$m\bar{3}m > 2/m_\backslash$										
1596	$m\bar{3}m$	$2/m_\backslash$	12	●	○	●	○	●	○	●	○	
1102	$m'\bar{3}'m$	$2/m'_\backslash$	12	●	○	●	○	●	○	●	○	
1122	$m'\bar{3}'m$	$2'/m_\backslash$	12	●	○	●	○	●	○	○	○	
1082	$m\bar{3}m'$	$2'/m'_\backslash$	12	●	○	●	○	○	●	○	○	
206	$m\bar{3}m1'$	$2/m_\backslash1'$	12	●	○	◐	○	○	○	○	○	
768	$m\bar{3}m1'$	$2/m_\backslash$	24	◐	○	○	◐	○	●	○	◐	○
521	$m\bar{3}m1'$	$2'/m_\backslash$	24	◐	○	○	◐	○	○	●	○	○

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
523	$m\bar{3}m1'$	$2/m'_\backslash$	24	◐	○	◐	○	○	○	◐	○	●
525	$m\bar{3}m1'$	$2'/m'_\backslash$	24	◐	○	◐	○	○	○	○	●	○
Genus #207		$m\bar{3}m > m_+$										
1365	$m\bar{3}m'$	m_+	24	◐	●	○	◐	○	◐	●	◐	○
1597	$m\bar{3}m$	m_+	24	◐	●	○	◐	○	◐	●	◐	○
1103	$m'\bar{3}'m'$	m'_+	24	◐	●	○	◐	○	○	◐	●	◐
1124	$m'\bar{3}'m$	m'_+	24	◐	●	○	◐	○	◐	◐	●	◐
207	$m\bar{3}m1'$	m_+1'	24	◐	●	○	◐	○	○	○	○	○
769	$m\bar{3}m1'$	m_+	48	◐	◐	○	◐	○	◐	◐	○	◐
526	$m\bar{3}m1'$	m'_+	48	◐	◐	○	◐	○	○	◐	◐	◐
Genus #208		$m\bar{3}m > m_\backslash$										
1388	$m'\bar{3}'m$	$m_\backslash$	24	◐	●	◐	○	○	●	◐	◐	○
1598	$m\bar{3}m$	$m_\backslash$	24	◐	●	◐	○	◐	●	◐	○	
1104	$m'\bar{3}'m'$	$m'_\backslash$	24	◐	●	◐	○	○	◐	●	◐	
1083	$m\bar{3}m'$	$m'_\backslash$	24	◐	●	◐	○	○	◐	●	◐	
208	$m\bar{3}m1'$	$m_\backslash1'$	24	◐	●	◐	○	○	○	○	○	
770	$m\bar{3}m1'$	$m_\backslash$	48	◐	◐	◐	○	○	◐	◐	◐	
527	$m\bar{3}m1'$	$m'_\backslash$	48	◐	◐	◐	○	○	◐	◐	◐	
Genus #209		$m\bar{3}m > 2_+$										
1389	$m'\bar{3}'m$	2_+	24	◐	◐	◐	●	●	◐	◐	◐	
1377	$m'\bar{3}'m'$	2_+	24	◐	◐	◐	●	●	◐	◐	◐	
1366	$m\bar{3}m'$	2_+	24	◐	◐	◐	●	◐	◐	●		
1599	$m\bar{3}m$	2_+	24	◐	◐	◐	●	◐	◐	●		
209	$m\bar{3}m1'$	$2_+1'$	24	◐	◐	◐	●	○	○	○	○	
771	$m\bar{3}m1'$	2_+	48	◐	◐	◐	◐	◐	◐	◐	◐	

No.	G	F	n	↕	↑	⊹	⚓	↕	↑	⊹	⚓
528	$m\bar{3}m1'$	$2'_+$	48	◐	◐	◐	◐	◐	○	◐	◐	○
Genus #210		$m\bar{3}m > 2_\backslash$										
1378	$m'\bar{3}'m'$	$2_\backslash$	24	◐	◐	◐	●	●	◐	◐	◐	
1600	$m\bar{3}m$	$2_\backslash$	24	◐	◐	◐	●	◐	◐	◐	●	
1084	$m\bar{3}m'$	$2_\backslash$	24	◐	◐	◐	●	◐	●	◐	○	
1125	$m'\bar{3}'m$	$2'_\backslash$	24	◐	◐	◐	●	○	◐	●	○	
210	$m\bar{3}m1'$	$2_\backslash1'$	24	◐	◐	◐	●	○	○	○	○	
772	$m\bar{3}m1'$	$2_\backslash$	48	◐	◐	◐	◐	◐	◐	◐	◐	
529	$m\bar{3}m1'$	$2'_\backslash$	48	◐	◐	◐	◐	○	◐	◐	◐	
Genus #211		$m\bar{3}m > \bar{1}$										
1367	$m\bar{3}m'$	$\bar{1}$	24	●	○	●	○	●	○	●	○	
1601	$m\bar{3}m$	$\bar{1}$	24	●	○	●	○	●	○	●	○	
1105	$m'\bar{3}'m'$	$\bar{1}'$	24	●	●	○	○	●	○	●	●	
1126	$m'\bar{3}'m$	$\bar{1}'$	24	●	●	○	○	●	○	○	○	
211	$m\bar{3}m1'$	$\bar{1}1'$	24	●	●	○	○	○	○	○	○	
773	$m\bar{3}m1'$	$\bar{1}$	48	◐	○	◐	○	●	○	●	○	
530	$m\bar{3}m1'$	$\bar{1}'$	48	◐	○	◐	○	○	○	●	●	
Genus #212		$m\bar{3}m > 1$										
1390	$m'\bar{3}'m$	1	48	◐	●	◐	●	●	◐	●	◐	
1379	$m'\bar{3}'m'$	1	48	◐	●	◐	●	◐	●	◐		
1602	$m\bar{3}m$	1	48	◐	●	◐	●	◐	●	●		
1368	$m\bar{3}m'$	1	48	◐	●	◐	●	◐	●	●		
212	$m\bar{3}m1'$	$11'$	48	◐	●	◐	●	○	○	○	○	
774	$m\bar{3}m1'$	1	96	◐	◐	◐	◐	◐	◐	◐	◐	

E. Magnetic Point Group Character Tables

The following pages contain character tables of the 122 magnetic point groups. Each irreducible representation is presented with both its character and the vector or bidirector quantities which transform as that irrep.

Which component a vector symbol corresponds to or which axis the bidirector is aligned with, is given as a subscript. In the case of vectors, it is customary to include x, y, and z as different possible basis functions. This is true even in monoclinic, triclinic, and orthorhombic classes where there is no such distinction. This is because an arbitrary three-dimensional vector may have such components; but, since a bidirector cannot be worked with in terms of components, we include only the symbol with no subscript in those classes. In the tetragonal and hexagonal classes we use the subscript z to indicate an axis parallel to the unique axis of the group and p to indicate an axis perpendicular to z.

The Mulliken symbols used here are intended to correspond to other tables already in print except that we chose to use the highest order rotation *or improper rotation* on the principal rotation axis when determining whether to use A or B. An example of this choice is found in comparing groups $\bar{4}$ and $\bar{6}$. In both groups, our algorithm for generating Mulliken symbols uses the sign of the character of the highest-order improper rotation and thus gives A and B for the one-dimensional irreps.

The symbols for isometries used here always include a subscript to make clear the spatial orientation of the axes. For mirror planes, the axis normal to the plane is used.

Chapter 9 describes the motivation and methods for creating this table as well as discussing the results and presenting some conclusions about its contents, limitations, and possible applications.

11′	1	1′	Vector	Bidirector
A	1	1	$\uparrow_x\uparrow_y\uparrow_z\downarrow_x\downarrow_y\downarrow_z$	⚓↕
mA	1	-1	$\uparrow_x\uparrow_y\uparrow_z\downarrow_x\downarrow_y\downarrow_z$	⚓↕

$\bar{1}1′$	1	$\bar{1}$	1′	$\bar{1}′$	Vector	Bidirector
A_g	1	1	1	1	$\downarrow_x\downarrow_y\downarrow_z$	↕
A_u	1	-1	1	-1	$\uparrow_x\uparrow_y\uparrow_z$	⚓
mA_g	1	1	-1	-1	$\downarrow_x\downarrow_y\downarrow_z$	↕
mA_u	1	-1	-1	1	$\uparrow_x\uparrow_y\uparrow_z$	⚓

21′	1	2_z	1′	$2_z′$	Vector	Bidirector
A	1	1	1	1	$\uparrow_z\downarrow_z$	⚓↕
B	1	-1	1	-1	$\uparrow_x\uparrow_y\downarrow_x\downarrow_y$	
mA	1	1	-1	-1	$\uparrow_z\downarrow_z$	⚓↕
mB	1	-1	-1	1	$\uparrow_x\uparrow_y\downarrow_x\downarrow_y$	

$m1′$	1	m_z	1′	$m_z′$	Vector	Bidirector
A'	1	1	1	1	$\uparrow_x\uparrow_y\downarrow_z$	↕
A''	1	-1	1	-1	$\uparrow_z\downarrow_x\downarrow_y$	⚓
mA'	1	1	-1	-1	$\uparrow_x\uparrow_y\downarrow_z$	↕
mA''	1	-1	-1	1	$\uparrow_z\downarrow_x\downarrow_y$	⚓

$2/m1′$	1	2_z	$\bar{1}$	m_z	1′	$2_z′$	$\bar{1}′$	$m_z′$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	$\downarrow_z$	↕
B_g	1	-1	1	-1	1	-1	1	-1	$\downarrow_x\downarrow_y$	
A_u	1	1	-1	-1	1	1	-1	-1	$\uparrow_z$	⚓
B_u	1	-1	-1	1	1	-1	-1	1	$\uparrow_x\uparrow_y$	
mA_g	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	↕
mB_g	1	-1	1	-1	-1	1	-1	1	$\downarrow_x\downarrow_y$	
mA_u	1	1	-1	-1	-1	-1	1	1	$\uparrow_z$	⚓
mB_u	1	-1	-1	1	-1	1	1	-1	$\uparrow_x\uparrow_y$	

2221′	1	2_z	2_x	2_y	1′	$2_z′$	$2_x′$	$2_y′$	Vector	Bidirector
A	1	1	1	1	1	1	1	1		⚓↕
B_1	1	1	-1	-1	1	1	-1	-1	$\uparrow_z\downarrow_z$	
B_2	1	-1	1	-1	1	-1	1	-1	$\uparrow_x\downarrow_x$	
B_3	1	-1	-1	1	1	-1	-1	1	$\uparrow_y\downarrow_y$	
mA	1	1	1	1	-1	-1	-1	-1		⚓↕
mB_1	1	1	-1	-1	-1	-1	1	1	$\uparrow_z\downarrow_z$	
mB_2	1	-1	1	-1	-1	1	-1	1	$\uparrow_x\downarrow_x$	
mB_3	1	-1	-1	1	-1	1	1	-1	$\uparrow_y\downarrow_y$	

2mm1′	1	2_z	m_x	m_y	1′	$2_z′$	$m_x′$	$m_y′$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	$\uparrow_z$	↕
A_2	1	1	-1	-1	1	1	-1	-1	$\downarrow_z$	⚓
B_1	1	-1	1	-1	1	-1	1	-1	$\uparrow_y\downarrow_x$	
B_2	1	-1	-1	1	1	-1	-1	1	$\uparrow_x\downarrow_y$	
mA_1	1	1	1	1	-1	-1	-1	-1	$\uparrow_z$	↕
mA_2	1	1	-1	-1	-1	-1	1	1	$\downarrow_z$	⚓
mB_1	1	-1	1	-1	-1	1	-1	1	$\uparrow_y\downarrow_x$	
mB_2	1	-1	-1	1	-1	1	1	-1	$\uparrow_x\downarrow_y$	

$mmm1'$	1	2_z	2_x	2_y	$\bar{1}$	m_z	m_x	m_y	$1'$	$2_z{}'$	$2_x{}'$	$2_y{}'$	$\bar{1}'$	$m_z{}'$	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow$
B_{1g}	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	$\dashv_z$	
B_{2g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	$\dashv_x$	
B_{3g}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	$\dashv_y$	
A_u	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1		$\updownarrow$
B_{1u}	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{2u}	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	$\uparrow_x$	
B_{3u}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	$\uparrow_y$	
mA_g	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1		$\updownarrow$
mB_{1g}	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	$\dashv_z$	
mB_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	$\dashv_x$	
mB_{3g}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	$\dashv_y$	
mA_u	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1		$\updownarrow$
mB_{1u}	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	$\uparrow_z$	
mB_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	$\uparrow_x$	
mB_{3u}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	$\uparrow_y$	

$41'$	1	4_z	2_z	4_z^3	$1'$	$4_z{}'$	$2_z{}'$	$4_z^{3}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	$\uparrow_z\dashv_z$	$\updownarrow_z\updownarrow_z$
B	1	-1	1	-1	1	-1	1	-1		
E	2	0	-2	0	2	0	-2	0	$\{\uparrow_x,\uparrow_y\}\{\dashv_x,\dashv_y\}$	
mA	1	1	1	1	-1	-1	-1	-1	$\uparrow_z\dashv_z$	$\updownarrow_z\updownarrow_z$
mB	1	-1	1	-1	-1	1	-1	1		
mE	2	0	-2	0	-2	0	2	0	$\{\uparrow_x,\uparrow_y\}\{\dashv_x,\dashv_y\}$	

$\bar{4}1'$	1	$\bar{4}_z$	2_z	$\bar{4}_z^3$	$1'$	$\bar{4}_z{}'$	$2_z{}'$	$\bar{4}_z^{3}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	$\dashv_z$	$\updownarrow_z$
B	1	-1	1	-1	1	-1	1	-1	$\uparrow_z$	$\updownarrow_z$
E	2	0	-2	0	2	0	-2	0	$\{\uparrow_x,\uparrow_y\}\{\dashv_x,\dashv_y\}$	
mA	1	1	1	1	-1	-1	-1	-1	$\dashv_z$	$\updownarrow_z$
mB	1	-1	1	-1	-1	1	-1	1	$\uparrow_z$	$\updownarrow_z$
mE	2	0	-2	0	-2	0	2	0	$\{\uparrow_x,\uparrow_y\}\{\dashv_x,\dashv_y\}$	

$4/m1'$	1	4_z	2_z	4_z^3	$\bar{1}$	$\bar{4}_z$	m_z	$\bar{4}_z^3$	$1'$	$4_z{}'$	$2_z{}'$	$4_z^{3}{}'$	$\bar{1}'$	$\bar{4}_z{}'$	$m_z{}'$	$\bar{4}_z^{3}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\dashv_z$	$\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_g	2	0	-2	0	2	0	-2	0	2	0	-2	0	2	0	-2	0	$\{\dashv_x,\dashv_y\}$	
A_u	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
B_u	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1		
E_u	2	0	-2	0	-2	0	2	0	2	0	-2	0	-2	0	2	0	$\{\uparrow_x,\uparrow_y\}$	
mA_g	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	$\dashv_z$	$\updownarrow_z$
mB_g	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1		
mE_g	2	0	-2	0	2	0	-2	0	-2	0	2	0	-2	0	2	0	$\{\dashv_x,\dashv_y\}$	
mA_u	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
mB_u	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1		
mE_u	2	0	-2	0	-2	0	2	0	-2	0	2	0	2	0	-2	0	$\{\uparrow_x,\uparrow_y\}$	

$4221'$	1	2_z	4_z	2_x	2_{xy}	$1'$	$2_z{}'$	$4_z{}'$	$2_x{}'$	$2_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	1	1		⇅z↕z
A_2	1	1	1	-1	-1	1	1	1	-1	-1	↑z↓z	
B_1	1	1	-1	1	-1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1	1	1	-1	-1	1		
E	2	-2	0	0	0	2	-2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	
mA_1	1	1	1	1	1	-1	-1	-1	-1	-1		⇅z↕z
mA_2	1	1	1	-1	-1	-1	-1	-1	1	1	↑z↓z	
mB_1	1	1	-1	1	-1	-1	-1	1	-1	1		
mB_2	1	1	-1	-1	1	-1	-1	1	1	-1		
mE	2	-2	0	0	0	-2	2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	

$4mm1'$	1	2_z	4_z	m_x	m_{xy}	$1'$	$2_z{}'$	$4_z{}'$	$m_x{}'$	$m_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	1	1	↑z	↕z
A_2	1	1	1	-1	-1	1	1	1	-1	-1	↓z	⇅z
B_1	1	1	-1	1	-1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1	1	1	-1	-1	1		
E	2	-2	0	0	0	2	-2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	
mA_1	1	1	1	1	1	-1	-1	-1	-1	-1	↑z	↕z
mA_2	1	1	1	-1	-1	-1	-1	-1	1	1	↓z	⇅z
mB_1	1	1	-1	1	-1	-1	-1	1	-1	1		
mB_2	1	1	-1	-1	1	-1	-1	1	1	-1		
mE	2	-2	0	0	0	-2	2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	

$\bar{4}2m1'$	1	2_z	$\bar{4}_z$	2_x	m_{xy}	$1'$	$2_z{}'$	$\bar{4}_z{}'$	$2_x{}'$	$m_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	1	1		↕z
A_2	1	1	1	-1	-1	1	1	1	-1	-1	↓z	
B_1	1	1	-1	1	-1	1	1	-1	1	-1		⇅z
B_2	1	1	-1	-1	1	1	1	-1	-1	1	↑z	
E	2	-2	0	0	0	2	-2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	
mA_1	1	1	1	1	1	-1	-1	-1	-1	-1		↕z
mA_2	1	1	1	-1	-1	-1	-1	-1	1	1	↓z	
mB_1	1	1	-1	1	-1	-1	-1	1	-1	1		⇅z
mB_2	1	1	-1	-1	1	-1	-1	1	1	-1	↑z	
mE	2	-2	0	0	0	-2	2	0	0	0	{ ↑x,↑y } { ↓x,↓y }	

$4/mmm1'$	1	2_z	4_z	2_x	2_{xy}	$\bar{1}$	m_z	$\bar{4}_z$	m_x	m_{xy}	$1'$	$2_z{}'$	$4_z{}'$	$2_x{}'$	$2_{xy}{}'$	$\bar{1}'$	$m_z{}'$	$\bar{4}_z{}'$	$m_x{}'$	$m_{xy}{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	$\curvearrowright_z$	
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1	1	1	-1	1	-1		
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1	1	1	-1	-1	1		
E_g	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	2	-2	0	0	0	$\{\curvearrowright_x,\curvearrowright_y\}$	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1		$\Updownarrow_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1	1	1	-1	1	-1	-1	-1	1	-1	1		
B_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	1	-1		
E_u	2	-2	0	0	0	-2	2	0	0	0	2	-2	0	0	0	-2	2	0	0	0	$\{\uparrow_x,\uparrow_y\}$	
mA_{1g}	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1		$\updownarrow_z$
mA_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	$\curvearrowright_z$	
mB_{1g}	1	1	-1	1	-1	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1	1		
mB_{2g}	1	1	-1	-1	1	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	-1		
mE_g	2	-2	0	0	0	2	-2	0	0	0	-2	2	0	0	0	-2	2	0	0	0	$\{\curvearrowright_x,\curvearrowright_y\}$	
mA_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1		$\Updownarrow_z$
mA_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	1	1	1	1	1	-1	-1	$\uparrow_z$	
mB_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1	-1	-1	1	-1	1	1	1	-1	1	-1		
mB_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1	-1	-1	1	1	-1	1	1	-1	-1	1		
mE_u	2	-2	0	0	0	-2	2	0	0	0	-2	2	0	0	0	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}$	

$31'$	1	3_z	3_z^2	$1'$	$3_z'$	$3_z^{2}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	$\uparrow_z\curvearrowright_z$	$\Updownarrow_z\updownarrow_z$
E	2	-1	-1	2	-1	-1	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	
mA	1	1	1	-1	-1	-1	$\uparrow_z\curvearrowright_z$	$\Updownarrow_z\updownarrow_z$
mE	2	-1	-1	-2	1	1	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	

$\bar{3}1'$	1	3_z	3_z^2	$\bar{1}$	$\bar{3}_z$	$\bar{3}_z^2$	$1'$	$3_z'$	$3_z^{2}{}'$	$\bar{1}'$	$\bar{3}_z{}'$	$\bar{3}_z^{2}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$\curvearrowright_z$	$\updownarrow_z$
E_g	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	$\{\curvearrowright_x,\curvearrowright_y\}$	
A_u	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	$\uparrow_z$	$\Updownarrow_z$
E_u	2	-1	-1	-2	1	1	2	-1	-1	-2	1	1	$\{\uparrow_x,\uparrow_y\}$	
mA_g	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$\curvearrowright_z$	$\updownarrow_z$
mE_g	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	$\{\curvearrowright_x,\curvearrowright_y\}$	
mA_u	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	$\uparrow_z$	$\Updownarrow_z$
mE_u	2	-1	-1	-2	1	1	-2	1	1	2	-1	-1	$\{\uparrow_x,\uparrow_y\}$	

$321'$	1	3_z	2_x	$1'$	$3_z'$	$2_x{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1		$\Updownarrow_z\updownarrow_z$
A_2	1	1	-1	1	1	-1	$\uparrow_z\curvearrowright_z$	
E	2	-1	0	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	
mA_1	1	1	1	-1	-1	-1		$\Updownarrow_z\updownarrow_z$
mA_2	1	1	-1	-1	-1	1	$\uparrow_z\curvearrowright_z$	
mE	2	-1	0	-2	1	0	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	

$3m1'$	1	3_z	m_x	$1'$	$3_z'$	$m_x{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_2	1	1	-1	1	1	-1	$\curvearrowright_z$	$\Updownarrow_z$
E	2	-1	0	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	
mA_1	1	1	1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
mA_2	1	1	-1	-1	-1	1	$\curvearrowright_z$	$\Updownarrow_z$
mE	2	-1	0	-2	1	0	$\{\uparrow_x,\uparrow_y\}\{\curvearrowright_x,\curvearrowright_y\}$	

$\bar{3}m1'$

$\bar{3}m1'$	1	3_z	2_x	$\bar{1}$	$\bar{3}_z$	m_x	$1'$	$3_z{}'$	$2_x{}'$	$\bar{1}'$	$\bar{3}_z{}'$	$m_x{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	-1	1	1	-1	1	1	-1	1	1	-1	$\downarrow\!\!\!\uparrow_z$	
E_g	2	-1	0	2	-1	0	2	-1	0	2	-1	0	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
A_{1u}	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1		$\updownarrow\!\!\!\updownarrow_z$
A_{2u}	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	$\uparrow_z$	
E_u	2	-1	0	-2	1	0	2	-1	0	-2	1	0	$\{\,\uparrow_x,\uparrow_y\,\}$	
mA_{1g}	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		$\updownarrow_z$
mA_{2g}	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	$\downarrow\!\!\!\uparrow_z$	
mE_g	2	-1	0	2	-1	0	-2	1	0	-2	1	0	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
mA_{1u}	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1		$\updownarrow\!\!\!\updownarrow_z$
mA_{2u}	1	1	-1	-1	-1	1	-1	-1	1	1	1	-1	$\uparrow_z$	
mE_u	2	-1	0	-2	1	0	-2	1	0	2	-1	0	$\{\,\uparrow_x,\uparrow_y\,\}$	

$61'$

$61'$	1	6_z	3_z	2_z	3_z^2	6_z^5	$1'$	$6_z{}'$	$3_z{}'$	$2_z{}'$	$3_z^{2'}$	$6_z^{5'}$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	1	1	1	1	$\uparrow_z\,\downarrow\!\!\!\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z\,\updownarrow_z$
B	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_1	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
E_2	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1		
mA	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$\uparrow_z\,\downarrow\!\!\!\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z\,\updownarrow_z$
mB	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
mE_1	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
mE_2	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1		

$\bar{6}1'$

$\bar{6}1'$	1	3_z	3_z^2	$\bar{6}_z$	m_z	$\bar{6}_z^5$	$1'$	$3_z{}'$	$3_z^{2'}$	$\bar{6}_z{}'$	$m_z{}'$	$\bar{6}_z^{5'}$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	1	1	1	1	$\downarrow\!\!\!\uparrow_z$	$\updownarrow_z$
B	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	$\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z$
E'	2	-1	-1	-1	2	-1	2	-1	-1	-1	2	-1	$\{\,\uparrow_x,\uparrow_y\,\}$	
E''	2	-1	-1	1	-2	1	2	-1	-1	1	-2	1	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
mA	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$\downarrow\!\!\!\uparrow_z$	$\updownarrow_z$
mB	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	$\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z$
mE'	2	-1	-1	-1	2	-1	-2	1	1	1	-2	1	$\{\,\uparrow_x,\uparrow_y\,\}$	
mE''	2	-1	-1	1	-2	1	-2	1	1	-1	2	-1	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	

$6/m1'$

$6/m1'$	1	6_z	3_z	2_z	3_z^2	6_z^5	$\bar{1}$	$\bar{6}_z$	$\bar{3}_z$	m_z	$\bar{3}_z^2$	$\bar{6}_z^5$	$1'$	$6_z{}'$	$3_z{}'$	$2_z{}'$	$3_z^{2'}$	$6_z^{5'}$	$\bar{1}'$	$\bar{6}_z{}'$	$\bar{3}_z{}'$	$m_z{}'$	$\bar{3}_z^{2'}$	$\bar{6}_z^{5'}$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	$\downarrow\!\!\!\uparrow_z$	$\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
E_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1		
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z$
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
E_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{\,\uparrow_x,\uparrow_y\,\}$	
E_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1		
mA_g	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	$\downarrow\!\!\!\uparrow_z$	$\updownarrow_z$
mB_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1		
mE_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	-2	-1	1	2	1	-1	$\{\,\downarrow\!\!\!\uparrow_x,\downarrow\!\!\!\uparrow_y\,\}$	
mE_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	-2	1	1	-2	1	1		
mA_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow\!\!\!\updownarrow_z$
mB_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1		
mE_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	-2	-1	1	2	1	-1	2	1	-1	-2	-1	1	$\{\,\uparrow_x,\uparrow_y\,\}$	
mE_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	-2	1	1	-2	1	1	2	-1	-1	2	-1	-1		

$6221'$	1	2_z	3_z	6_z	2_x	2_y	$1'$	$2_z{}'$	$3_z{}'$	$6_z{}'$	$2_x{}'$	$2_y{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	1	1	1	1		⇕$_z$↕$_z$
A_2	1	1	1	1	-1	-1	1	1	1	1	-1	-1	↑$_z$⥾$_z$	
B_1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	2	-2	-1	1	0	0	{ ↑$_x$,↑$_y$ } { ⥾$_x$,⥾$_y$ }	
E_2	2	2	-1	-1	0	0	2	2	-1	-1	0	0		
mA_1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		⇕$_z$↕$_z$
mA_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	↑$_z$⥾$_z$	
mB_1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
mB_2	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
mE_1	2	-2	-1	1	0	0	-2	2	1	-1	0	0	{ ↑$_x$,↑$_y$ } { ⥾$_x$,⥾$_y$ }	
mE_2	2	2	-1	-1	0	0	-2	-2	1	1	0	0		

$6mm1'$	1	2_z	3_z	6_z	m_x	m_y	$1'$	$2_z{}'$	$3_z{}'$	$6_z{}'$	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	1	1	1	1	1	1	↑$_z$	↕$_z$
A_2	1	1	1	1	-1	-1	1	1	1	1	-1	-1	⥾$_z$	⇕$_z$
B_1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	2	-2	-1	1	0	0	{ ↑$_x$,↑$_y$ } { ⥾$_x$,⥾$_y$ }	
E_2	2	2	-1	-1	0	0	2	2	-1	-1	0	0		
mA_1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	↑$_z$	↕$_z$
mA_2	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	⥾$_z$	⇕$_z$
mB_1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
mB_2	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
mE_1	2	-2	-1	1	0	0	-2	2	1	-1	0	0	{ ↑$_x$,↑$_y$ } { ⥾$_x$,⥾$_y$ }	
mE_2	2	2	-1	-1	0	0	-2	-2	1	1	0	0		

$\bar{6}m_x2_y1'$	1	3_z	2_y	m_z	$\bar{6}_z$	m_x	$1'$	$3_z{}'$	$2_y{}'$	$m_z{}'$	$\bar{6}_z{}'$	$m_x{}'$	Vector	Bidirector
$A_1^{`}$	1	1	1	1	1	1	1	1	1	1	1	1		↕$_z$
$A_2^{`}$	1	1	-1	1	1	-1	1	1	-1	1	1	-1	⥾$_z$	
$B_1^{"}$	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1		⇕$_z$
$B_2^{"}$	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	↑$_z$	
$E^{`}$	2	-1	0	2	-1	0	2	-1	0	2	-1	0	{ ↑$_x$,↑$_y$ }	
$E^{"}$	2	-1	0	-2	1	0	2	-1	0	-2	1	0	{ ⥾$_x$,⥾$_y$ }	
$mA_1^{`}$	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1		↕$_z$
$mA_2^{`}$	1	1	-1	1	1	-1	-1	-1	1	-1	-1	1	⥾$_z$	
$mB_1^{"}$	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1		⇕$_z$
$mB_2^{"}$	1	1	-1	-1	-1	1	-1	-1	1	1	1	-1	↑$_z$	
$mE^{`}$	2	-1	0	2	-1	0	-2	1	0	-2	1	0	{ ↑$_x$,↑$_y$ }	
$mE^{"}$	2	-1	0	-2	1	0	-2	1	0	2	-1	0	{ ⥾$_x$,⥾$_y$ }	

$6/mmm1'$	1	2_z	3_z	6_z	$\bar{1}$	m_z	$\bar{3}_z$	$\bar{6}_z$	2_x	2_y	m_x	m_y	$1'$	$2_z{}'$	$3_z{}'$	$6_z{}'$	$\bar{1}'$	$m_z{}'$	$\bar{3}_z{}'$	$\bar{6}_z{}'$	$2_x{}'$	$2_y{}'$	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\curvearrowright_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\curvearrowright_x,\curvearrowright_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0	2	2	-1	-1	2	2	-1	-1	0	0	0	0		
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1		$\updownarrow\!\!\!\!\text{-}_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x,\uparrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0	2	2	-1	-1	-2	-2	1	1	0	0	0	0		
mA_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1		$\updownarrow_z$
mA_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1	$\curvearrowright_z$	
mB_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1		
mB_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1		
mE_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	-2	2	1	-1	-2	2	1	-1	0	0	0	0	$\{\curvearrowright_x,\curvearrowright_y\}$	
mE_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0	-2	-2	1	1	-2	-2	1	1	0	0	0	0		
mA_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	1	1		$\updownarrow\!\!\!\!\text{-}_z$
mA_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	$\uparrow_z$	
mB_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	1	-1	1	-1	-1	1	1	-1		
mB_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	1	-1	1	-1	1	-1	-1	1		
mE_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	-2	2	1	-1	2	-2	-1	1	0	0	0	0	$\{\uparrow_x,\uparrow_y\}$	
mE_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0	-2	-2	1	1	2	2	-1	-1	0	0	0	0		

$231'$	1	2_z	3_{xyz}	3^2_{xyz}	$1'$	$2_z{}'$	$3_{xyz}{}'$	$3^2_{xyz}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	1	1		
E	2	2	-1	-1	2	2	-1	-1		
T	3	-1	0	0	3	-1	0	0	$\{\uparrow_x,\uparrow_y,\uparrow_z\}\ \{\curvearrowright_x,\curvearrowright_y,\curvearrowright_z\}$	
mA	1	1	1	1	-1	-1	-1	-1		
mE	2	2	-1	-1	-2	-2	1	1		
mT	3	-1	0	0	-3	1	0	0	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\ \{\curvearrowright_x,\curvearrowright_y,\curvearrowright_z\}$	

$m\bar{3}1'$	1	2_z	$\bar{1}$	m_z	3_{xyz}	3^2_{xyz}	$\bar{3}_{xyz}$	$\bar{3}^2_{xyz}$	$1'$	$2_z{}'$	$\bar{1}'$	$m_z{}'$	$3_{xyz}{}'$	$3^2_{xyz}{}'$	$\bar{3}^-_{xyz}{}'$	$\bar{3}^{2-}_{xyz}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
E_g	2	2	2	2	-1	-1	-1	-1	2	2	2	2	-1	-1	-1	-1		
T_g	3	-1	3	-1	0	0	0	0	3	-1	3	-1	0	0	0	0	$\{\curvearrowright_x,\curvearrowright_y,\curvearrowright_z\}$	
A_u	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1		
E_u	2	2	-2	-2	-1	-1	1	1	2	2	-2	-2	-1	-1	1	1		
T_u	3	-1	-3	1	0	0	0	0	3	-1	-3	1	0	0	0	0	$\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
mA_g	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1		
mE_g	2	2	2	2	-1	-1	-1	-1	-2	-2	-2	-2	1	1	1	1		
mT_g	3	-1	3	-1	0	0	0	0	-3	1	-3	1	0	0	0	0	$\{\curvearrowright_x,\curvearrowright_y,\curvearrowright_z\}$	
mA_u	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1		
mE_u	2	2	-2	-2	-1	-1	1	1	-2	-2	2	2	1	1	-1	-1		
mT_u	3	-1	-3	1	0	0	0	0	-3	1	3	-1	0	0	0	0	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}$	

4321'	1	2_z	4_z	2_{xy}	3_{xyz}	1'	$2_z{}'$	$4_z{}'$	$2_{xy}{}'$	$3_{xyz}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	1	1		
B	1	1	-1	-1	1	1	1	-1	-1	1		
E	2	2	0	0	-1	2	2	0	0	-1		
T_1	3	-1	1	-1	0	3	-1	1	-1	0	{ ↑$_x$,↑$_y$,↑$_z$ } { +$_x$,+$_y$,+$_z$ }	
T_2	3	-1	-1	1	0	3	-1	-1	1	0		
mA	1	1	1	1	1	-1	-1	-1	-1	-1		
mB	1	1	-1	-1	1	-1	-1	1	1	-1		
mE	2	2	0	0	-1	-2	-2	0	0	1		
mT_1	3	-1	1	-1	0	-3	1	-1	1	0	{ ↑$_x$,↑$_y$,↑$_z$ } { +$_x$,+$_y$,+$_z$ }	
mT_2	3	-1	-1	1	0	-3	1	1	-1	0		

$\bar{4}3m1'$	1	2_z	$\bar{4}_z$	3_{xyz}	m_{xy}	1'	$2_z{}'$	$\bar{4}_z{}'$	$3_{xyz}{}'$	$m_{xy}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	1	1	1	1		
B	1	1	-1	1	-1	1	1	-1	1	-1		
E	2	2	0	-1	0	2	2	0	-1	0		
T_1	3	-1	1	0	-1	3	-1	1	0	-1	{ +$_x$,+$_y$,+$_z$ }	
T_2	3	-1	-1	0	1	3	-1	-1	0	1	{ ↑$_x$,↑$_y$,↑$_z$ }	
mA	1	1	1	1	1	-1	-1	-1	-1	-1		
mB	1	1	-1	1	-1	-1	-1	1	-1	1		
mE	2	2	0	-1	0	-2	-2	0	1	0		
mT_1	3	-1	1	0	-1	-3	1	-1	0	1	{ +$_x$,+$_y$,+$_z$ }	
mT_2	3	-1	-1	0	1	-3	1	1	0	-1	{ ↑$_x$,↑$_y$,↑$_z$ }	

$m\bar{3}m1'$	1	2_z	$\bar{1}$	m_z	3_{xyz}	$\bar{3}_{xyz}$	4_z	$\bar{4}_z$	2_{xy}	m_{xy}	1'	$2_z{}'$	$\bar{1}'$	$m_z{}'$	$3_{xyz}{}'$	$\bar{3}_{xyz}^{-}{}'$	$4_z{}'$	$\bar{4}_z{}'$	$2_{xy}{}'$	$m_{xy}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1		
B_g	1	1	1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1		
E_g	2	2	2	2	-1	-1	0	0	0	0	2	2	2	2	-1	-1	0	0	0	0		
T_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	3	-1	3	-1	0	0	1	1	-1	-1	{ +$_x$,+$_y$,+$_z$ }	
T_{2g}	3	-1	3	-1	0	0	-1	-1	1	1	3	-1	3	-1	0	0	-1	-1	1	1		
A_u	1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	-1	1	-1	1	-1	1	-1		
B_u	1	1	-1	-1	1	-1	-1	1	-1	1	1	1	-1	-1	1	-1	-1	1	-1	1		
E_u	2	2	-2	-2	-1	1	0	0	0	0	2	2	-2	-2	-1	1	0	0	0	0		
T_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	3	-1	-3	1	0	0	1	-1	-1	1	{ ↑$_x$,↑$_y$,↑$_z$ }	
T_{2u}	3	-1	-3	1	0	0	-1	1	1	-1	3	-1	-3	1	0	0	-1	1	1	-1		
mA_g	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1		
mB_g	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1		
mE_g	2	2	2	2	-1	-1	0	0	0	0	-2	-2	-2	-2	1	1	0	0	0	0		
mT_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	-3	1	-3	1	0	0	-1	-1	1	1	{ +$_x$,+$_y$,+$_z$ }	
mT_{2g}	3	-1	3	-1	0	0	-1	-1	1	1	-3	1	-3	1	0	0	1	1	-1	-1		
mA_u	1	1	-1	-1	1	-1	1	-1	1	-1	-1	-1	1	1	-1	1	-1	1	-1	1		
mB_u	1	1	-1	-1	1	-1	-1	1	-1	1	-1	-1	1	1	-1	1	1	-1	1	-1		
mE_u	2	2	-2	-2	-1	1	0	0	0	0	-2	-2	2	2	1	-1	0	0	0	0		
mT_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	-3	1	3	-1	0	0	-1	1	1	-1	{ ↑$_x$,↑$_y$,↑$_z$ }	
mT_{2u}	3	-1	-3	1	0	0	-1	1	1	-1	-3	1	3	-1	0	0	1	-1	-1	1		

$\bar{1}'$	1	$\bar{1}'$	Vector	Bidirector
A_g	1	1	+$_x$+$_y$+$_z$↑$_x$↑$_y$↑$_z$	↕⤫
A_u	1	-1	↑$_x$↑$_y$↑$_z$+$_x$+$_y$+$_z$	⤫↕

2'	1	$2_z{}'$	Vector	Bidirector
A	1	1	↑$_z$+$_z$↑$_x$↑$_y$+$_x$+$_y$	⤫↕
B	1	-1	↑$_x$↑$_y$+$_x$+$_y$↑$_z$+$_z$	⤫↕

m'	1	$m_z{'}$	Vector	Bidirector
A'	1	1	$\uparrow_x\uparrow_y\downarrow_z\uparrow_z\downarrow_x\downarrow_y$	↕↧
A''	1	-1	$\uparrow_z\downarrow_x\downarrow_y\uparrow_x\uparrow_y\downarrow_z$	↧↕

$2'/m'$	1	$\bar{1}$	$2_z{'}$	$m_z{'}$	Vector	Bidirector
A_g	1	1	1	1	$\downarrow_z\downarrow_x\downarrow_y$	↕
B_g	1	1	-1	-1	$\downarrow_x\uparrow_y\downarrow_z$	↕
A_u	1	-1	1	-1	$\uparrow_z\uparrow_x\uparrow_y$	↧
B_u	1	-1	-1	1	$\uparrow_x\uparrow_y\uparrow_z$	↧

$2/m'$	1	2_z	$\bar{1}'$	$m_z{'}$	Vector	Bidirector
A_g	1	1	1	1	$\downarrow_z\uparrow_z$	↕↧
B_g	1	-1	1	-1	$\downarrow_x\downarrow_y\uparrow_x\uparrow_y$	
A_u	1	1	-1	-1	$\uparrow_z\downarrow_z$	↧↕
B_u	1	-1	-1	1	$\uparrow_x\uparrow_y\downarrow_x\downarrow_y$	

$2'/m$	1	m_z	$2_z{'}$	$\bar{1}'$	Vector	Bidirector
A_g	1	1	1	1	$\downarrow_z\uparrow_x\uparrow_y$	↕
B_g	1	-1	-1	1	$\downarrow_x\downarrow_y\uparrow_z$	↧
A_u	1	-1	1	-1	$\uparrow_z\downarrow_x\downarrow_y$	↧
B_u	1	1	-1	-1	$\uparrow_x\uparrow_y\downarrow_z$	↕

$22'2'$	1	2_z	$2_x{'}$	$2_y{'}$	Vector	Bidirector
A_1	1	1	1	1	$\uparrow_z\downarrow_z$	↧↕
A_2	1	1	-1	-1	$\uparrow_z\downarrow_z$	↧↕
B_1	1	-1	1	-1	$\uparrow_x\downarrow_x\uparrow_y\downarrow_y$	
B_2	1	-1	-1	1	$\uparrow_y\downarrow_y\uparrow_x\downarrow_x$	

$2m'm'$	1	$m_x{'}$	$m_y{'}$	2_z	Vector	Bidirector
A_1	1	1	1	1	$\uparrow_z\downarrow_z$	↕↧
A_2	1	-1	-1	1	$\downarrow_z\uparrow_z$	↧↕
B_1	1	1	-1	-1	$\uparrow_y\downarrow_x\uparrow_x\downarrow_y$	
B_2	1	-1	1	-1	$\uparrow_x\downarrow_y\uparrow_y\downarrow_x$	

$2'mm'$	1	m_x	$m_y{'}$	$2_z{'}$	Vector	Bidirector
A_1	1	1	1	1	$\uparrow_z\uparrow_y\downarrow_x$	↕
A_2	1	-1	-1	1	$\downarrow_z\uparrow_x\downarrow_y$	↧
B_1	1	1	-1	-1	$\uparrow_y\downarrow_x\uparrow_z$	↕
B_2	1	-1	1	-1	$\uparrow_x\downarrow_y\downarrow_z$	↧

$mm'm'$	1	$\bar{1}$	2_z	m_z	$2_x{'}$	$m_x{'}$	$2_y{'}$	$m_y{'}$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	$\downarrow_z$	↕
A_{2g}	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	↕
B_{1g}	1	1	-1	-1	1	1	-1	-1	$\downarrow_x\downarrow_y$	
B_{2g}	1	1	-1	-1	-1	-1	1	1	$\downarrow_y\downarrow_x$	
A_{1u}	1	-1	1	-1	1	-1	1	-1	$\uparrow_z$	↧
A_{2u}	1	-1	1	-1	-1	1	-1	1	$\uparrow_z$	↧
B_{1u}	1	-1	-1	1	1	-1	-1	1	$\uparrow_x\uparrow_y$	
B_{2u}	1	-1	-1	1	-1	1	1	-1	$\uparrow_y\uparrow_x$	

$m'm'm'$	1	2_z	2_x	2_y	$\bar{1}'$	$m_z{}'$	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1		$\updownarrow$
A_{2g}	1	1	-1	-1	1	1	-1	-1	$\downarrow_z\uparrow_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	$\downarrow_x\uparrow_x$	
B_{2g}	1	-1	-1	1	1	-1	-1	1	$\downarrow_y\uparrow_y$	
A_{1u}	1	1	1	1	-1	-1	-1	-1		$\updownarrow$
A_{2u}	1	1	-1	-1	-1	-1	1	1	$\uparrow_z\downarrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	$\uparrow_x\downarrow_x$	
B_{2u}	1	-1	-1	1	-1	1	1	-1	$\uparrow_y\downarrow_y$	

$m'mm$	1	$\bar{1}'$	2_z	$m_z{}'$	$2_x{}'$	m_x	$2_y{}'$	m_y	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow$
A_{2g}	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	$\updownarrow$
B_{1g}	1	1	-1	-1	1	1	-1	-1	$\downarrow_x\uparrow_y$	
B_{2g}	1	1	-1	-1	-1	-1	1	1	$\downarrow_y\uparrow_x$	
A_{1u}	1	-1	1	-1	-1	1	-1	1	$\uparrow_z$	$\updownarrow$
A_{2u}	1	-1	1	-1	1	-1	1	-1	$\downarrow_z$	$\updownarrow$
B_{1u}	1	-1	-1	1	-1	1	1	-1	$\uparrow_y\downarrow_x$	
B_{2u}	1	-1	-1	1	1	-1	-1	1	$\uparrow_x\downarrow_y$	

$4'$	1	$4_z{}'$	2_z	$4_z^{3'}$	Vector	Bidirector
A	1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B	1	-1	1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E	2	0	-2	0	$\{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}\ \{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}$	

$\bar{4}'$	1	$\bar{4}_z{}'$	2_z	$\bar{4}_z^{3'}$	Vector	Bidirector
A	1	1	1	1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
B	1	-1	1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E	2	0	-2	0	$\{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}\ \{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}$	

$4'/m$	1	$4_z{}'$	2_z	$4_z^{3'}$	$\bar{1}$	$\bar{4}_z{}'$	m_z	$\bar{4}_z^{3'}$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1	$\downarrow_z$	$\updownarrow_z$
E_g	2	0	-2	0	2	0	-2	0	$\{\,\downarrow_x,\downarrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}$	
A_u	1	1	1	1	-1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
B_u	1	-1	1	-1	-1	1	-1	1	$\uparrow_z$	$\updownarrow_z$
E_u	2	0	-2	0	-2	0	2	0	$\{\,\uparrow_x,\uparrow_y\,\}\ \{\,\uparrow_x,\uparrow_y\,\}$	

$4/m'$	1	4_z	2_z	4_z^3	$\bar{1}'$	$\bar{4}_z{}'$	$m_z{}'$	$\bar{4}_z^{3'}$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1		
E_g	2	0	-2	0	2	0	-2	0	$\{\,\downarrow_x,\downarrow_y\,\}\ \{\,\uparrow_x,\uparrow_y\,\}$	
A_u	1	1	1	1	-1	-1	-1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B_u	1	-1	1	-1	-1	1	-1	1		
E_u	2	0	-2	0	-2	0	2	0	$\{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}$	

$4'/m'$	1	$4_z{}'$	2_z	$4_z^{3'}$	$\bar{1}'$	$\bar{4}_z$	$m_z{}'$	$\bar{4}_z^3$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1	$\uparrow_z$	$\updownarrow_z$
E_g	2	0	-2	0	2	0	-2	0	$\{\,\downarrow_x,\downarrow_y\,\}\ \{\,\uparrow_x,\uparrow_y\,\}$	
A_u	1	1	1	1	-1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
B_u	1	-1	1	-1	-1	1	-1	1	$\downarrow_z$	$\updownarrow_z$
E_u	2	0	-2	0	-2	0	2	0	$\{\,\uparrow_x,\uparrow_y\,\}\ \{\,\downarrow_x,\downarrow_y\,\}$	

$42'2'$	1	2_z	4_z	$2_x{}'$	$2_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	1	-1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B_1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1		
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$4'22'$	1	2_z	$4_z{}'$	2_x	$2_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_2	1	1	1	-1	-1	$\uparrow_z\downarrow_z$	
B_1	1	1	-1	1	-1		$\updownarrow_z\updownarrow_z$
B_2	1	1	-1	-1	1	$\uparrow_z\downarrow_z$	
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$4'mm'$	1	2_z	$4_z{}'$	$m_x{}'$	m_{xy}	Vector	Bidirector
A_1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_2	1	1	1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_1	1	1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
B_2	1	1	-1	1	-1	$\downarrow_z$	$\updownarrow_z$
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$4m'm'$	1	2_z	4_z	$m_x{}'$	$m_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	1	-1	-1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
B_1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1		
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$\bar{4}'2m'$	1	2_z	$\bar{4}_z{}'$	2_x	$m_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_2	1	1	1	-1	-1	$\downarrow_z\uparrow_z$	
B_1	1	1	-1	1	-1		$\updownarrow_z\updownarrow_z$
B_2	1	1	-1	-1	1	$\uparrow_z\downarrow_z$	
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$\bar{4}'2'm$	1	2_z	$\bar{4}_z{}'$	$2_x{}'$	m_{xy}	Vector	Bidirector
A_1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_2	1	1	1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_1	1	1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
B_2	1	1	-1	1	-1	$\downarrow_z$	$\updownarrow_z$
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$\bar{4}2'm'$	1	2_z	$\bar{4}_z$	$2_x{}'$	$m_{xy}{}'$	Vector	Bidirector
A_1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
A_2	1	1	1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_1	1	1	-1	1	-1	$\uparrow_z$	$\updownarrow_z$
B_2	1	1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
E	2	-2	0	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$4'/mmm'$	1	2_z	$4_z{'}$	$2_x{'}$	2_{xy}	$\bar{1}$	m_z	$\bar{4}_z{'}$	$m_x{'}$	m_{xy}	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1		↕$_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	⥮$_z$	
B_{1g}	1	1	-1	1	-1	1	1	-1	-1	1		↕$_z$
B_{2g}	1	1	-1	-1	1	1	1	-1	1	-1	⥮$_z$	
E_g	2	-2	0	0	0	2	-2	0	0	0	{ ⥮$_x$,⥮$_y$ } { ⥮$_x$,⥮$_y$ }	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		⇅$_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	↑$_z$	
B_{1u}	1	1	-1	-1	1	-1	-1	1	1	-1		⇅$_z$
B_{2u}	1	1	-1	1	-1	-1	-1	1	-1	1	↑$_z$	
E_u	2	-2	0	0	0	-2	2	0	0	0	{ ↑$_x$,↑$_y$ } { ↑$_x$,↑$_y$ }	

$4/mm'm'$	1	2_z	4_z	$2_x{'}$	$2_{xy}{'}$	$\bar{1}$	m_z	$\bar{4}_z$	$m_x{'}$	$m_{xy}{'}$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	⥮$_z$	↕$_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	⥮$_z$	↕$_z$
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1		
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1		
E_g	2	-2	0	0	0	2	-2	0	0	0	{ ⥮$_x$,⥮$_y$ } { ⥮$_x$,⥮$_y$ }	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1	↑$_z$	⇅$_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	↑$_z$	⇅$_z$
B_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1		
B_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1		
E_u	2	-2	0	0	0	-2	2	0	0	0	{ ↑$_x$,↑$_y$ } { ↑$_x$,↑$_y$ }	

$4/m'm'm'$	1	2_z	4_z	2_x	2_{xy}	$\bar{1}{'}$	$m_z{'}$	$\bar{4}_z{'}$	$m_x{'}$	$m_{xy}{'}$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1		↕$_z$⇅$_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	⥮$_z$↑$_z$	
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1		
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1		
E_g	2	-2	0	0	0	2	-2	0	0	0	{ ⥮$_x$,⥮$_y$ } { ↑$_x$,↑$_y$ }	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		⇅$_z$↕$_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	↑$_z$⥮$_z$	
B_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1		
B_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1		
E_u	2	-2	0	0	0	-2	2	0	0	0	{ ↑$_x$,↑$_y$ } { ⥮$_x$,⥮$_y$ }	

$4/m'mm$	1	2_z	4_z	$2_x{'}$	$2_{xy}{'}$	$\bar{1}{'}$	$m_z{'}$	$\bar{4}_z{'}$	m_x	m_{xy}	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	↑$_z$	↕$_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	⥮$_z$	⇅$_z$
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1		
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1		
E_g	2	-2	0	0	0	2	-2	0	0	0	{ ⥮$_x$,⥮$_y$ } { ↑$_x$,↑$_y$ }	
A_{1u}	1	1	1	-1	-1	-1	-1	-1	1	1	↑$_z$	↕$_z$
A_{2u}	1	1	1	1	1	-1	-1	-1	-1	-1	⥮$_z$	⇅$_z$
B_{1u}	1	1	-1	-1	1	-1	-1	1	1	-1		
B_{2u}	1	1	-1	1	-1	-1	-1	1	-1	1		
E_u	2	-2	0	0	0	-2	2	0	0	0	{ ↑$_x$,↑$_y$ } { ⥮$_x$,⥮$_y$ }	

119

$4'/m'm'm$	1	2_z	$4_z{'}$	2_x	$2_{xy}{'}$	$\bar{1}'$	m_z'	$\bar{4}_z$	m_x'	m_{xy}	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	$\downarrow_z$	
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1		$\updownarrow_z$
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1	$\uparrow_z$	
E_g	2	-2	0	0	0	2	-2	0	0	0	$\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}$	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		$\updownarrow_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1		$\updownarrow_z$
B_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1	$\downarrow_z$	
E_u	2	-2	0	0	0	-2	2	0	0	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$\bar{3}'$	1	3_z	3_z^2	$\bar{1}'$	$\bar{3}_z{'}$	$\bar{3}_z^{2}{'}$	Vector	Bidirector
A_g	1	1	1	1	1	1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
E_g	2	-1	-1	2	-1	-1	$\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}$	
A_u	1	1	1	-1	-1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E_u	2	-1	-1	-2	1	1	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$32'$	1	3_z	$2_x{'}$	Vector	Bidirector
A_1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E	2	-1	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$3m'$	1	3_z	$m_x{'}$	Vector	Bidirector
A_1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	-1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
E	2	-1	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$\bar{3}m'$	1	3_z	$2_x{'}$	$\bar{1}$	$\bar{3}_z$	$m_x{'}$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
A_{2g}	1	1	-1	1	1	-1	$\downarrow_z$	$\updownarrow_z$
E_g	2	-1	0	2	-1	0	$\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	
A_{1u}	1	1	1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
A_{2u}	1	1	-1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
E_u	2	-1	0	-2	1	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}$	

$\bar{3}'m'$	1	3_z	2_x	$\bar{1}'$	$\bar{3}_z{'}$	$m_x{'}$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_{2g}	1	1	-1	1	1	-1	$\downarrow_z\uparrow_z$	
E_g	2	-1	0	2	-1	0	$\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}$	
A_{1u}	1	1	1	-1	-1	-1		$\updownarrow_z\updownarrow_z$
A_{2u}	1	1	-1	-1	-1	1	$\uparrow_z\downarrow_z$	
E_u	2	-1	0	-2	1	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$\bar{3}'m$	1	3_z	$2_x{'}$	$\bar{1}'$	$\bar{3}_z{'}$	m_x	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_{2g}	1	1	-1	1	1	-1	$\downarrow_z$	$\updownarrow_z$
E_g	2	-1	0	2	-1	0	$\{\,\downarrow_x,\downarrow_y\,\}\,\{\,\uparrow_x,\uparrow_y\,\}$	
A_{1u}	1	1	-1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
A_{2u}	1	1	1	-1	-1	-1	$\downarrow_z$	$\updownarrow_z$
E_u	2	-1	0	-2	1	0	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$6'$	1	$6_z{'}$	3_z	$2_z{'}$	3_z^2	$6_z^{5}{'}$	Vector	Bidirector
A	1	1	1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B	1	-1	1	-1	1	-1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E_1	2	1	-1	-2	-1	1	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	
E_2	2	-1	-1	2	-1	-1	$\{\,\uparrow_x,\uparrow_y\,\}\,\{\,\downarrow_x,\downarrow_y\,\}$	

$\bar{6}'$	1	3_z	3_z^2	$\bar{6}_z{}'$	$m_z{}'$	$\bar{6}_z^{5}{}'$	Vector	Bidirector
A	1	1	1	1	1	1	$↓_z↑_z$	$↕_z↯_z$
B	1	1	1	-1	-1	-1	$↑_z↓_z$	$↯_z↕_z$
E'	2	-1	-1	-1	2	-1	$\{↑_x,↑_y\}\ \{↓_x,↓_y\}$	
E''	2	-1	-1	1	-2	1	$\{↓_x,↓_y\}\ \{↑_x,↑_y\}$	

$6'/m'$	1	$6_z{}'$	3_z	$2_z{}'$	3_z^2	$6_z^{5}{}'$	$\bar{1}$	$\bar{6}_z{}'$	$\bar{3}_z$	$m_z{}'$	$\bar{3}_z^2$	$\bar{6}_z^{5}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$↓_z$	$↕_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	$↓_z$	$↕_z$
E_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{↓_x,↓_y\}$	
E_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	$\{↓_x,↓_y\}$	
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$↑_z$	$↯_z$
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	$↑_z$	$↯_z$
E_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{↑_x,↑_y\}$	
E_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	$\{↑_x,↑_y\}$	

$6/m'$	1	6_z	3_z	2_z	3_z^2	6_z^5	$\bar{1}'$	$\bar{6}_z{}'$	$\bar{3}_z{}'$	$m_z{}'$	$\bar{3}_z^2{}'$	$\bar{6}_z^{5}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$↓_z↑_z$	$↕_z↯_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{↓_x,↓_y\}\ \{↑_x,↑_y\}$	
E_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1		
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$↑_z↓_z$	$↯_z↕_z$
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
E_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{↑_x,↑_y\}\ \{↓_x,↓_y\}$	
E_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1		

$6'/m$	1	$6_z{}'$	3_z	$2_z{}'$	3_z^2	$6_z^{5}{}'$	$\bar{1}'$	$\bar{6}_z$	$\bar{3}_z{}'$	m_z	$\bar{3}_z^2{}'$	$\bar{6}_z^{5}$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$↓_z$	$↕_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	$↑_z$	$↯_z$
E_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{↓_x,↓_y\}$	
E_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1	$\{↑_x,↑_y\}$	
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$↑_z$	$↯_z$
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	$↓_z$	$↕_z$
E_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{↑_x,↑_y\}$	
E_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1	$\{↓_x,↓_y\}$	

$6'22'$	1	$2_z{}'$	3_z	$6_z{}'$	2_x	$2_y{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1		$↯_z↕_z$
A_2	1	1	1	1	-1	-1	$↑_z↓_z$	
B_1	1	-1	1	-1	1	-1		$↯_z↕_z$
B_2	1	-1	1	-1	-1	1	$↓_z↑_z$	
E_1	2	-2	-1	1	0	0	$\{↑_x,↑_y\}\ \{↓_x,↓_y\}$	
E_2	2	2	-1	-1	0	0	$\{↓_x,↓_y\}\ \{↑_x,↑_y\}$	

$62'2'$	1	2_z	3_z	6_z	$2_x{}'$	$2_y{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	$↑_z↓_z$	$↯_z↕_z$
A_2	1	1	1	1	-1	-1	$↑_z↓_z$	$↯_z↕_z$
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	$\{↑_x,↑_y\}\ \{↓_x,↓_y\}\ \{↑_x,↑_y\}\ \{↓_x,↓_y\}$	
E_2	2	2	-1	-1	0	0		

121

$6'mm'$	1	$2_z{}'$	3_z	$6_z{}'$	$m_x{}'$	m_y	Vector	Bidirector
A_1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_2	1	1	1	1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_1	1	-1	1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
B_2	1	-1	1	-1	1	-1	$\downarrow_z$	$\updownarrow_z$
E_1	2	-2	-1	1	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_2	2	2	-1	-1	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$6m'm'$	1	2_z	3_z	6_z	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_1	1	1	1	1	1	1	$\uparrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	1	1	-1	-1	$\downarrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_2	2	2	-1	-1	0	0		

$\bar{6}'m_x'2_y$	1	3_z	2_y	$m_z{}'$	$\bar{6_z}'$	$m_x{}'$	Vector	Bidirector
$A_1^{`}$	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
$A_2^{`}$	1	1	-1	1	1	-1	$\downarrow_z\uparrow_z$	
$B_1^{"}$	1	1	1	-1	-1	-1		$\updownarrow_z\updownarrow_z$
$B_2^{"}$	1	1	-1	-1	-1	1	$\uparrow_z\downarrow_z$	
$E^{`}$	2	-1	0	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	
$E^{"}$	2	-1	0	-2	1	0	$\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}$	

$\bar{6}'m_x2_y'$	1	3_z	$2_y{}'$	$m_z{}'$	$\bar{6_z}'$	m_x	Vector	Bidirector
A_1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_2	1	1	-1	1	1	-1	$\downarrow_z$	$\updownarrow_z$
B_1	1	1	-1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
B_2	1	1	1	-1	-1	-1	$\downarrow_z$	$\updownarrow_z$
E_1	2	-1	0	-2	1	0	$\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}$	
E_2	2	-1	0	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$\bar{6}m_x'2_y'$	1	3_z	$2_y{}'$	m_z	$\bar{6_z}$	$m_x{}'$	Vector	Bidirector
$A_1^{`}$	1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
$A_2^{`}$	1	1	-1	1	1	-1	$\downarrow_z$	$\updownarrow_z$
$B_1^{"}$	1	1	1	-1	-1	-1	$\uparrow_z$	$\updownarrow_z$
$B_2^{"}$	1	1	-1	-1	-1	1	$\uparrow_z$	$\updownarrow_z$
$E^{`}$	2	-1	0	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\uparrow_x,\uparrow_y\}$	
$E^{"}$	2	-1	0	-2	1	0	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	

$6'/m'mm'$	1	$2_z{}'$	3_z	$6_z{}'$	$\bar{1}$	$m_z{}'$	$\bar{3}_z$	$\bar{6_z}'$	$2_x{}'$	2_y	$m_x{}'$	m_y	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		$\updownarrow_z$
B_{2g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	$\downarrow_z$	
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x,\downarrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0	$\{\downarrow_x,\downarrow_y\}$	
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1		$\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		$\updownarrow_z$
B_{2u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1	$\uparrow_z$	
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x,\uparrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0	$\{\uparrow_x,\uparrow_y\}$	

$6/mm'm'$	1	2_z	3_z	6_z	$\bar{1}$	m_z	$\bar{3}_z$	$\bar{6}_z$	$2_x{}'$	$2_y{}'$	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	$\downarrow_z$	$\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x, \downarrow_y\}\{\downarrow_x, \downarrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0		
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	$\uparrow_z$	$\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	$\updownarrow_z$
B_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x, \uparrow_y\}\{\uparrow_x, \uparrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0		

$6/m'm'm'$	1	2_z	3_z	6_z	$\bar{1}'$	$m_z{}'$	$\bar{3}_z{}'$	$\bar{6}_z{}'$	2_x	2_y	$m_x{}'$	$m_y{}'$	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z\uparrow_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x, \downarrow_y\}\{\uparrow_x, \uparrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0		
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1		$\updownarrow_z\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z\downarrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x, \uparrow_y\}\{\downarrow_x, \downarrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0		

$6/m'mm$	1	2_z	3_z	6_z	$\bar{1}'$	$m_z{}'$	$\bar{3}_z{}'$	$\bar{6}_z{}'$	$2_x{}'$	$2_y{}'$	m_x	m_y	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1	$\uparrow_z$	$\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x, \downarrow_y\}\{\uparrow_x, \uparrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0		
A_{1u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	$\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1	$\downarrow_z$	$\updownarrow_z$
B_{1u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x, \uparrow_y\}\{\downarrow_x, \downarrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0		

$6'/mmm'$	1	$2_z{}'$	3_z	$6_z{}'$	$\bar{1}'$	m_z	$\bar{3}_z{}'$	$\bar{6}_z$	2_x	$2_y{}'$	$m_x{}'$	m_y	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		$\updownarrow_z$
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	$\uparrow_z$	
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x, \downarrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0	$\{\uparrow_x, \uparrow_y\}$	
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1		$\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\uparrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		$\updownarrow_z$
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1	$\downarrow_z$	
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\uparrow_x, \uparrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0	$\{\downarrow_x, \downarrow_y\}$	

$m'\bar{3}'$	1	2_z	$\bar{1}'$	m_z'	3_{xyz}	3_{xyz}^2	$3_{xyz}^{-}{}'$	$3_{xyz}^{2-}{}'$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1		
E_g	2	2	2	2	-1	-1	-1	-1		
T_g	3	-1	3	-1	0	0	0	0	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\;\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
A_u	1	1	-1	-1	1	1	-1	-1		
E_u	2	2	-2	-2	-1	-1	1	1		
T_u	3	-1	-3	1	0	0	0	0	$\{\uparrow_x,\uparrow_y,\uparrow_z\}\;\{\downarrow_x,\downarrow_y,\downarrow_z\}$	

$4'32'$	1	2_z	$4_z'$	$2_{xy}'$	3_{xyz}	Vector	Bidirector
A	1	1	1	1	1		
B	1	1	-1	-1	1		
E	2	2	0	0	-1		
T_1	3	-1	1	-1	0	$\{\uparrow_x,\uparrow_y,\uparrow_z\}\;\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_2	3	-1	-1	1	0	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\;\{\uparrow_x,\uparrow_y,\uparrow_z\}$	

$\bar{4}'3m'$	1	2_z	$\bar{4}_z'$	3_{xyz}	m_{xy}'	Vector	Bidirector
A	1	1	1	1	1		
B	1	1	-1	1	-1		
E	2	2	0	-1	0		
T_1	3	-1	1	0	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\;\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
T_2	3	-1	-1	0	1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}\;\{\downarrow_x,\downarrow_y,\downarrow_z\}$	

$m\bar{3}m'$	1	2_z	$\bar{1}$	m_z	3_{xyz}	$\bar{3}_{xyz}$	$4_z'$	$\bar{4}_z'$	$2_{xy}'$	m_{xy}'	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1		
B_g	1	1	1	1	1	1	-1	-1	-1	-1		
E_g	2	2	2	2	-1	-1	0	0	0	0		
T_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_{2g}	3	-1	3	-1	0	0	-1	-1	1	1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
A_u	1	1	-1	-1	1	-1	1	-1	1	-1		
B_u	1	1	-1	-1	1	-1	-1	1	-1	1		
E_u	2	2	-2	-2	-1	1	0	0	0	0		
T_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
T_{2u}	3	-1	-3	1	0	0	-1	1	1	-1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}$	

$m'\bar{3}'m'$	1	2_z	$\bar{1}'$	m_z'	3_{xyz}	$\bar{3}_{xyz}'$	4_z	$\bar{4}_z'$	2_{xy}	m_{xy}'	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1		
B_g	1	1	1	1	1	1	-1	-1	-1	-1		
E_g	2	2	2	2	-1	-1	0	0	0	0		
T_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\;\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
T_{2g}	3	-1	3	-1	0	0	-1	-1	1	1		
A_u	1	1	-1	-1	1	-1	1	-1	1	-1		
B_u	1	1	-1	-1	1	-1	-1	1	-1	1		
E_u	2	2	-2	-2	-1	1	0	0	0	0		
T_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}\;\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_{2u}	3	-1	-3	1	0	0	-1	1	1	-1		

$m'\bar{3}'m$	1	2_z	$\bar{1}'$	m_z'	3_{xyz}	$\bar{3}_{xyz}'$	$4_z'$	$\bar{4}_z$	$2_{xy}'$	m_{xy}	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1		
B_g	1	1	1	1	1	1	-1	-1	-1	-1		
E_g	2	2	2	2	-1	-1	0	0	0	0		
T_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_{2g}	3	-1	3	-1	0	0	-1	-1	1	1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
A_u	1	1	-1	-1	1	-1	1	-1	1	-1		
B_u	1	1	-1	-1	1	-1	-1	1	-1	1		
E_u	2	2	-2	-2	-1	1	0	0	0	0		
T_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	$\{\uparrow_x,\uparrow_y,\uparrow_z\}$	
T_{2u}	3	-1	-3	1	0	0	-1	1	1	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}$	

1	1	Vector	Bidirector
A	1	$\uparrow_x\uparrow_y\uparrow_z\downarrow_x\downarrow_y\downarrow_z\uparrow_x\uparrow_y\uparrow_z\downarrow_x\downarrow_y\downarrow_z$	⚓⚓⚓⚓

$\bar{1}$	1	$\bar{1}$	Vector	Bidirector
A_g	1	1	$\downarrow_x\downarrow_y\downarrow_z\downarrow_x\downarrow_y\downarrow_z$	⚓⚓
A_u	1	-1	$\uparrow_x\uparrow_y\uparrow_z\uparrow_x\uparrow_y\uparrow_z$	⚓⚓

2	1	2_z	Vector	Bidirector
A	1	1	$\uparrow_z\downarrow_z\uparrow_z\downarrow_z$	⚓⚓⚓⚓
B	1	-1	$\uparrow_x\uparrow_y\downarrow_x\downarrow_y\uparrow_x\uparrow_y\downarrow_x\downarrow_y$	

m	1	m_z	Vector	Bidirector
A'	1	1	$\uparrow_x\uparrow_y\downarrow_z\uparrow_x\uparrow_y\downarrow_z$	⚓⚓
A''	1	-1	$\uparrow_z\downarrow_x\downarrow_y\uparrow_z\downarrow_x\downarrow_y$	⚓⚓

$2/m$	1	2_z	$\bar{1}$	m_z	Vector	Bidirector
A_g	1	1	1	1	$\downarrow_z\downarrow_z$	⚓⚓
B_g	1	-1	1	-1	$\downarrow_x\downarrow_y\downarrow_x\downarrow_y$	
A_u	1	1	-1	-1	$\uparrow_z\uparrow_z$	⚓⚓
B_u	1	-1	-1	1	$\uparrow_x\uparrow_y\uparrow_x\uparrow_y$	

222	1	2_z	2_x	2_y	Vector	Bidirector
A	1	1	1	1		⚓⚓⚓⚓
B_1	1	1	-1	-1	$\uparrow_z\downarrow_z\uparrow_z\downarrow_z$	
B_2	1	-1	1	-1	$\uparrow_x\downarrow_x\uparrow_x\downarrow_x$	
B_3	1	-1	-1	1	$\uparrow_y\downarrow_y\uparrow_y\downarrow_y$	

$2mm$	1	2_z	m_x	m_y	Vector	Bidirector
A_1	1	1	1	1	$\uparrow_z\uparrow_z$	⚓⚓
A_2	1	1	-1	-1	$\downarrow_z\downarrow_z$	⚓⚓
B_1	1	-1	1	-1	$\uparrow_y\downarrow_x\uparrow_y\downarrow_x$	
B_2	1	-1	-1	1	$\uparrow_x\downarrow_y\uparrow_x\downarrow_y$	

mmm	1	2_z	2_x	2_y	$\bar{1}$	m_z	m_x	m_y	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1		⚓⚓
B_{1g}	1	1	-1	-1	1	1	-1	-1	$\downarrow_z\downarrow_z$	
B_{2g}	1	-1	1	-1	1	-1	1	-1	$\downarrow_x\downarrow_x$	
B_{3g}	1	-1	-1	1	1	-1	-1	1	$\downarrow_y\downarrow_y$	
A_u	1	1	1	1	-1	-1	-1	-1		⚓⚓
B_{1u}	1	1	-1	-1	-1	-1	1	1	$\uparrow_z\uparrow_z$	
B_{2u}	1	-1	1	-1	-1	1	-1	1	$\uparrow_x\uparrow_x$	
B_{3u}	1	-1	-1	1	-1	1	1	-1	$\uparrow_y\uparrow_y$	

4	1	4_z	2_z	4_z^3	Vector	Bidirector
A	1	1	1	1	↑$_z$↓$_z$↕$_z$↕$_z$	↯$_z$↕$_z$↯$_z$↕$_z$
B	1	-1	1	-1		
E	2	0	-2	0	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$\bar{4}$	1	$\bar{4}_z$	2_z	$\bar{4}_z^3$	Vector	Bidirector
A	1	1	1	1	↓$_z$↕$_z$	↕$_z$↕$_z$
B	1	-1	1	-1	↑$_z$↕$_z$	↯$_z$↯$_z$
E	2	0	-2	0	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$4/m$	1	4_z	2_z	4_z^3	$\bar{1}$	$\bar{4}_z$	m_z	$\bar{4}_z^3$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	↓$_z$↕$_z$	↕$_z$↕$_z$
B_g	1	-1	1	-1	1	-1	1	-1		
E_g	2	0	-2	0	2	0	-2	0	{ ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ }	
A_u	1	1	1	1	-1	-1	-1	-1	↑$_z$↕$_z$	↯$_z$↯$_z$
B_u	1	-1	1	-1	-1	1	-1	1		
E_u	2	0	-2	0	-2	0	2	0	{ ↑$_x$,↑$_y$ } { ↕$_x$,↕$_y$ }	

422	1	2_z	4_z	2_x	2_{xy}	Vector	Bidirector
A_1	1	1	1	1	1		↯$_z$↕$_z$↯$_z$↕$_z$
A_2	1	1	1	-1	-1	↑$_z$↓$_z$↕$_z$↕$_z$	
B_1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1		
E	2	-2	0	0	0	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$4mm$	1	2_z	4_z	m_x	m_{xy}	Vector	Bidirector
A_1	1	1	1	1	1	↑$_z$↕$_z$	↕$_z$↕$_z$
A_2	1	1	1	-1	-1	↓$_z$↕$_z$	↯$_z$↯$_z$
B_1	1	1	-1	1	-1		
B_2	1	1	-1	-1	1		
E	2	-2	0	0	0	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$\bar{4}2m$	1	2_z	$\bar{4}_z$	2_x	m_{xy}	Vector	Bidirector
A_1	1	1	1	1	1		↕$_z$↕$_z$
A_2	1	1	1	-1	-1	↓$_z$↕$_z$	
B_1	1	1	-1	1	-1		↯$_z$↯$_z$
B_2	1	1	-1	-1	1	↑$_z$↕$_z$	
E	2	-2	0	0	0	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$4/mmm$	1	2_z	4_z	2_x	2_{xy}	$\bar{1}$	m_z	$\bar{4}_z$	m_x	m_{xy}	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1		↕$_z$↕$_z$
A_{2g}	1	1	1	-1	-1	1	1	1	-1	-1	↓$_z$↕$_z$	
B_{1g}	1	1	-1	1	-1	1	1	-1	1	-1		
B_{2g}	1	1	-1	-1	1	1	1	-1	-1	1		
E_g	2	-2	0	0	0	2	-2	0	0	0	{ ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ }	
A_{1u}	1	1	1	1	1	-1	-1	-1	-1	-1		↯$_z$↯$_z$
A_{2u}	1	1	1	-1	-1	-1	-1	-1	1	1	↑$_z$↕$_z$	
B_{1u}	1	1	-1	1	-1	-1	-1	1	-1	1		
B_{2u}	1	1	-1	-1	1	-1	-1	1	1	-1		
E_u	2	-2	0	0	0	-2	2	0	0	0	{ ↑$_x$,↑$_y$ } { ↕$_x$,↕$_y$ }	

3	1	3_z	3_z^2	Vector	Bidirector
A	1	1	1	↑$_z$↓$_z$↕$_z$↕$_z$	↯$_z$↕$_z$↯$_z$↕$_z$
E	2	-1	-1	{ ↑$_x$,↑$_y$ } { ↓$_x$,↓$_y$ } { ↕$_x$,↕$_y$ } { ↕$_x$,↕$_y$ }	

$\bar{3}$	1	3_z	3_z^2	$\bar{1}$	$\bar{3}_z$	$\bar{3}_z^2$	Vector	Bidirector
A_g	1	1	1	1	1	1	$\downarrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
E_g	2	-1	-1	2	-1	-1	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	
A_u	1	1	1	-1	-1	-1	$\uparrow_z\uparrow_z$	$\downdownarrows_z\downdownarrows_z$
E_u	2	-1	-1	-2	1	1	$\{\uparrow_x,\uparrow_y\}\{\uparrow_x,\uparrow_y\}$	

32	1	3_z	2_x	Vector	Bidirector
A_1	1	1	1		$\downdownarrows_z\updownarrow_z\downdownarrows_z\updownarrow_z$
A_2	1	1	-1	$\uparrow_z\downarrow_z\uparrow_z\downarrow_z$	
E	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$3m$	1	3_z	m_x	Vector	Bidirector
A_1	1	1	1	$\uparrow_z\uparrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	-1	$\downarrow_z\downarrow_z$	$\downdownarrows_z\downdownarrows_z$
E	2	-1	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	

$\bar{3}m$	1	3_z	2_x	$\bar{1}$	$\bar{3}_z$	m_x	Vector	Bidirector
A_{1g}	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_{2g}	1	1	-1	1	1	-1	$\downarrow_z\downarrow_z$	
E_g	2	-1	0	2	-1	0	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	
A_{1u}	1	1	1	-1	-1	-1		$\downdownarrows_z\downdownarrows_z$
A_{2u}	1	1	-1	-1	-1	1	$\uparrow_z\uparrow_z$	
E_u	2	-1	0	-2	1	0	$\{\uparrow_x,\uparrow_y\}\{\uparrow_x,\uparrow_y\}$	

6	1	6_z	3_z	2_z	3_z^2	6_z^5	Vector	Bidirector
A	1	1	1	1	1	1	$\uparrow_z\downarrow_z\uparrow_z\downarrow_z$	$\downdownarrows_z\updownarrow_z\downdownarrows_z\updownarrow_z$
B	1	-1	1	-1	1	-1		
E_1	2	1	-1	-2	-1	1	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_2	2	-1	-1	2	-1	-1		

$\bar{6}$	1	3_z	3_z^2	$\bar{6}_z$	m_z	$\bar{6}_z^5$	Vector	Bidirector
A	1	1	1	1	1	1	$\downarrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B	1	1	1	-1	-1	-1	$\uparrow_z\uparrow_z$	$\downdownarrows_z\downdownarrows_z$
E'	2	-1	-1	-1	2	-1	$\{\uparrow_x,\uparrow_y\}\{\uparrow_x,\uparrow_y\}$	
E''	2	-1	-1	1	-2	1	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	

$6/m$	1	6_z	3_z	2_z	3_z^2	6_z^5	$\bar{1}$	$\bar{6}_z$	$\bar{3}_z$	m_z	$\bar{3}_z^2$	$\bar{6}_z^5$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1	1	1	$\downarrow_z\downarrow_z$	$\updownarrow_z\updownarrow_z$
B_g	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
E_{1g}	2	1	-1	-2	-1	1	2	1	-1	-2	-1	1	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_{2g}	2	-1	-1	2	-1	-1	2	-1	-1	2	-1	-1		
A_u	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	$\uparrow_z\uparrow_z$	$\downdownarrows_z\downdownarrows_z$
B_u	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1		
E_{1u}	2	1	-1	-2	-1	1	-2	-1	1	2	1	-1	$\{\uparrow_x,\uparrow_y\}\{\uparrow_x,\uparrow_y\}$	
E_{2u}	2	-1	-1	2	-1	-1	-2	1	1	-2	1	1		

622	1	2_z	3_z	6_z	2_x	2_y	Vector	Bidirector
A_1	1	1	1	1	1	1		$\downdownarrows_z\updownarrow_z\downdownarrows_z\updownarrow_z$
A_2	1	1	1	1	-1	-1	$\uparrow_z\downarrow_z\uparrow_z\downarrow_z$	
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	$\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}\{\uparrow_x,\uparrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_2	2	2	-1	-1	0	0		

$6mm$	1	2_z	3_z	6_z	m_x	m_y	Vector	Bidirector
A_1	1	1	1	1	1	1	$\updownarrow_z\updownarrow_z$	$\updownarrow_z\updownarrow_z$
A_2	1	1	1	1	-1	-1	$\downarrow_z\downarrow_z$	$\not\updownarrow_z\not\updownarrow_z$
B_1	1	-1	1	-1	1	-1		
B_2	1	-1	1	-1	-1	1		
E_1	2	-2	-1	1	0	0	$\{\updownarrow_x,\updownarrow_y\}\{\downarrow_x,\downarrow_y\}\{\updownarrow_x,\updownarrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_2	2	2	-1	-1	0	0		

$\bar{6}m_x2_y$	1	3_z	2_y	m_z	$\bar{6}_z$	m_x	Vector	Bidirector
$A_1^{'}$	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
$A_2^{'}$	1	1	-1	1	1	-1	$\downarrow_z\downarrow_z$	
$B_1^{''}$	1	1	1	-1	-1	-1		$\not\updownarrow_z\not\updownarrow_z$
$B_2^{''}$	1	1	-1	-1	-1	1	$\updownarrow_z\updownarrow_z$	
$E^{'}$	2	-1	0	2	-1	0	$\{\updownarrow_x,\updownarrow_y\}\{\updownarrow_x,\updownarrow_y\}$	
$E^{''}$	2	-1	0	-2	1	0	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	

$6/mmm$	1	2_z	3_z	6_z	$\bar{1}$	m_z	$\bar{3}_z$	$\bar{6}_z$	2_x	2_y	m_x	m_y	Vector	Bidirector
A_{1g}	1	1	1	1	1	1	1	1	1	1	1	1		$\updownarrow_z\updownarrow_z$
A_{2g}	1	1	1	1	1	1	1	1	-1	-1	-1	-1	$\downarrow_z\downarrow_z$	
B_{1g}	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1		
B_{2g}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1		
E_{1g}	2	-2	-1	1	2	-2	-1	1	0	0	0	0	$\{\downarrow_x,\downarrow_y\}\{\downarrow_x,\downarrow_y\}$	
E_{2g}	2	2	-1	-1	2	2	-1	-1	0	0	0	0		
A_{1u}	1	1	1	1	-1	-1	-1	-1	1	1	-1	-1		$\not\updownarrow_z\not\updownarrow_z$
A_{2u}	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	$\updownarrow_z\updownarrow_z$	
B_{1u}	1	-1	1	-1	-1	1	-1	1	1	-1	-1	1		
B_{2u}	1	-1	1	-1	-1	1	-1	1	-1	1	1	-1		
E_{1u}	2	-2	-1	1	-2	2	1	-1	0	0	0	0	$\{\updownarrow_x,\updownarrow_y\}\{\updownarrow_x,\updownarrow_y\}$	
E_{2u}	2	2	-1	-1	-2	-2	1	1	0	0	0	0		

23	1	2_z	3_{xyz}	3^2_{xyz}	Vector	Bidirector
A	1	1	1	1		
E	2	2	-1	-1		
T	3	-1	0	0	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}$	

$m\bar{3}$	1	2_z	$\bar{1}$	m_z	3_{xyz}	3^2_{xyz}	$\bar{3}_{xyz}$	$\bar{3}^2_{xyz}$	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1		
E_g	2	2	2	2	-1	-1	-1	-1		
T_g	3	-1	3	-1	0	0	0	0	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
A_u	1	1	-1	-1	1	1	-1	-1		
E_u	2	2	-2	-2	-1	-1	1	1		
T_u	3	-1	-3	1	0	0	0	0	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}$	

432	1	2_z	4_z	2_{xy}	3_{xyz}	Vector	Bidirector
A	1	1	1	1	1		
B	1	1	-1	-1	1		
E	2	2	0	0	-1		
T_1	3	-1	1	-1	0	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_2	3	-1	-1	1	0		

$\bar{4}3m$	1	2_z	$\bar{4}_z$	3_{xyz}	m_{xy}	Vector	Bidirector
A	1	1	1	1	1		
B	1	1	-1	1	-1		
E	2	2	0	-1	0		
T_1	3	-1	1	0	-1	$\{\downarrow_x,\downarrow_y,\downarrow_z\}\{\downarrow_x,\downarrow_y,\downarrow_z\}$	
T_2	3	-1	-1	0	1	$\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}\{\updownarrow_x,\updownarrow_y,\updownarrow_z\}$	

$m\bar{3}m$	1	2_z	$\bar{1}$	m_z	3_{xyz}	$\bar{3}_{xyz}$	4_z	$\bar{4}_z$	2_{xy}	m_{xy}	Vector	Bidirector
A_g	1	1	1	1	1	1	1	1	1	1		
B_g	1	1	1	1	1	1	-1	-1	-1	-1		
E_g	2	2	2	2	-1	-1	0	0	0	0		
T_{1g}	3	-1	3	-1	0	0	1	1	-1	-1	$\{\, {\looparrowright}_x, {\looparrowright}_y, {\looparrowright}_z \,\}\ \{\, {\looparrowright}_x, {\looparrowright}_y, {\looparrowright}_z \,\}$	
T_{2g}	3	-1	3	-1	0	0	-1	-1	1	1		
A_u	1	1	-1	-1	1	-1	1	-1	1	-1		
B_u	1	1	-1	-1	1	-1	-1	1	-1	1		
E_u	2	2	-2	-2	-1	1	0	0	0	0		
T_{1u}	3	-1	-3	1	0	0	1	-1	-1	1	$\{\, {\uparrow}_x, {\uparrow}_y, {\uparrow}_z \,\}\ \{\, {\uparrow}_x, {\uparrow}_y, {\uparrow}_z \,\}$	
T_{2u}	3	-1	-3	1	0	0	-1	1	1	-1		