Posudek

❑vedoucího ☑oponenta
☑diplomové ❑bakalářské práce
Autor/Autorka: Antonín Češík
Název práce: Convex hull properties for parabolic systems of partial differential equations
Jméno oponenta: RNDr. Miroslav Bulíček, Ph. D.

Matematická úroveň:
❑vynikající ❑velmi dobrá ☑průměrná ❑podprůměrná ❑nevyhovující

Grafická, jazyková a formální úroveň:
❑vynikající ❑velmi dobrá ❑průměrná ☑podprůměrná ❑nevyhovující

Výsledky:
❑originální ☑původní i převzaté ❑netriviální kompilace ❑citované z literatury ❑opsané

Použité metody:
❑nestandardní ☑standardní ❑obojí

Aplikovatelnost:
☑přínos pro teorii ❑přínos pro praxi ❑přínos pro praxi i teorii ❑bez přínosu ❑nedovedu posoudit

Věcné chyby:
❑téměř žádné ☑vzhledem k rozsahu a pojednávanému tématu přiměřený počet ❑méně podstatné četné ❑závažné

Tiskové chyby:
☑téměř žádné ❑vzhledem k rozsahu a pojednávanému tématu přiměřený počet ❑četné

Celková úroveň práce:
❑vynikající ❑velmi dobrá ❑průměrná ☑podprůměrná ❑nevyhovující

Práci
☑doporučuji ❑nedoporučuji uznat jako diplomovou.

Detailed report:

The thesis deals with the convex hull properties for elliptic and parabolic systems of partial differential equations. Under some reasonable assumptions on the nonlinear elliptic operator, the author shows that the weak solution of the elliptic/parabolic system satisfies the convex hull property. In addition, he provides two counterexamples for systems that violate the structural assumption. According to my knowledge, these results are new and correct, and I did not find any essential mistake in the proofs.

On the other hand, the quality of the thesis is very unsatisfactory. It is written more in the style of the scientific paper than in the style of the thesis. Very frequently, some details are not proven and it is really not clear whether the author really sees how delicate the problem can be. Also, from the point of view of scientific writing, the thesis is sometimes very vague. The author sometimes writes "Sketch of the proof" but it is rather a "Sketch of the sketch of the proof" – e.g. proof of Theorem 1.1. There are missing precise references to results used in the paper, some results are cited incorrectly, some definitions and notations are nonstandard and misleading, etc.

More detailed list of most irritating inconsistencies that should be answered/discussed during the defense:

1) Already Notation is confusing, e.g., missing subscript "j" in the definition of Δu in the term $\frac{\partial^2 u_j}{\partial x_i^2}$. The correct notation for the dual space should be $W^{-1,p'}$ and not $W^{-1,p}$. In the definition of div F, there are $F_{\{i\}}$ and $F_{ij}$and one may just guess what the meaning is.
2) In the proof of Theorem 1.2: What happens if $sup_{x\in\Omega} u(x) = \infty$. On page 7, it is considered that u is constant on the set Ω_{sup}. But what if u is infinity? One should say that it is not possible.
3) Theorem 1.3 is not true! There are missing structural assumptions. Also, one should quote it more precisely – not only Bildahuer and Fuchs [2002] but rather Lemma 1.3 in …
4) The condition (1.9) is also called splitting condition and is used in the proof of Hoelder continuity of solution.
5) Frequently, the role of N and n is switched, see for example first sentence of Section 1.4., Theorem 1.7, ..
6) The convex hull is defined in Definition 3.1. On the other hand, it is already used in Theorem 1.6. Furthermore, there is a sentence "…we know that conv $u(\partial\Omega)$…" But I would at least expect some reason why know it…
7) Wrong signs in the inequality on page 16, line 13.
8) In Definition 2.9, one should assume that A is symmetric, otherwise it does not generate the scalar product.
9) In theorem 2.13, one also needs to assume that $u \in L^p(0,T;W^{1,p})$, otherwise the duality is not well defined. Also, I would really expect more details in the proof.
10) Should not be the function a in Definition 3.3 at least Carathéodory?
11) In Definition 3.4. In case that p<2, some integrals are not well defined. Moreover, if $c \neq 0$ and satisfies just the assumptions of Definition 3.3 then the last integral in (3.5) can explode easily. So, under which assumptions Theorem 3.6 really holds?
12) In the proof of Theorem 3.6. there is said: "…we use the test function …. In the weak formulation (3.5) and obtain…". But it really does not correspond to (3.5) and must be much more carefully justified.

To summarize, I think that the author obtained a new and interesting result, which is correct and can be even published. However, I also think that the thesis would deserve much more work and attention. Clearly, it could be twice longer with explained details and then I would consider it as an excellent thesis. Unfortunately, in the current situation I can just say that it can be considered as a very average or even not very good diploma thesis.

Prague, September 2, 2019 Miroslav Bulíček